Review of existing design methods for geosynthetic-reinforced pile-supported embankments

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Abstract

Embankment construction over soft foundation soils is a challenging task for geotechnical engineers due to the undesirable characteristics of soft soils, such as excessive settlements and low bearing capacity. Among the various ground-improvement methods available for overcoming these undesirable characteristics, geosynthetic-reinforced pile-supported (GRPS) embankments are considered to be a reliable solution suitable for time-bound construction projects and difficult ground conditions. Various researchers have introduced methods to design GRPS embankments based on different load transfer mechanisms. However, among design engineers, there is uncertainty regarding the applicability of these design methods. This paper investigates the load transfer mechanism of GRPS embankments using two-dimensional and three-dimensional finite element analyses, and currently available design methods are compared with the results of the finite element modelling. A comparison of the design methods was carried out using the stress reduction ratio, the geosynthetic tension and pile efficacy, considering different pile diameters and spacing, and embankment heights, which govern the currently available design methods. Based on these model results, the inconsistencies in the currently available design methods are identified and discussed in detail.

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1. Introduction

Embankments are widely used in infrastructure development projects to elevate the platform of roads, railways and runways. With the rapid world population growth over the past few decades, infrastructure development activities have increased considerably over marginal lands, which were previously considered unsuitable, such as around river estuaries, low-lying marshy areas and harbour foreshore areas characterising deep soft clay deposits. However, the construction of embankments under these ground conditions is a real challenge for geotechnical engineers due to the undesirable characteristics of soft soils, such as low bearing capacity, insufficient shear strength and high compressibility. Therefore, many complications, like local or global instability and excessive post-construction settlements due to the consolidation of the soft soil, arise when embankments are constructed on soft foundation soils.

A variety of techniques is available for overcoming these issues: (i) preloading or staged construction, (ii) the addition of vertical drains, (iii) the use of lightweight fill materials for the embankment fill, over the excavation of soft soil and replacing it with a suitable fill material, (iv) the reduction of the slope of the fill.
embankment and (v) the addition of column supports (Mitchell, 1981; Magnan, 1994; Shen et al., 2005). Column supports can be hard columns, such as piles (Jenck et al., 2009; Han et al., 2012), semi-hard columns, such as deep cement mixed columns (Huang and Han, 2009) or stone columns (Deb et al., 2007; Deb and Mohapatra, 2013). The first four methods listed above are not suitable for fast-track construction projects as they are all consolidation-based methods and consume time. The use of pile supports is considered as a reliable solution for embankment construction on soft foundation soils as the structure can be built in a single stage without prolonged waiting periods and with a significant reduction in total and differential settlements. Moreover, pile supports are effective in difficult or extremely poor ground conditions, such as landfills, brownfield sites and dumps where the engineering behaviour of the soils is not well known and the extracting of the soil properties by routine laboratory tests is difficult. Since the majority of the embankment load is transferred to the piles, detailed knowledge of the mechanical properties of the ground is not required. Also, in a contaminated ground, it is possible to maintain minimal contact with contaminated water squeezing out of the ground due to consolidation, if pile supports are used instead of consolidation-based methods.

Generally, single or multiple layers of geosynthetic reinforcement are installed in pile-supported embankment systems to increase the load transfer to the piles and to reduce the required area replacement ratio (Lawson, 1992; Kempton et al., 1998). Geosynthetic reinforcement, combined with pile supports, is commonly used for bridge approaches, storage tank supports, the widening of existing roads, retaining walls and embankments to create an efficient load transfer platform, as discussed by many researchers (Han and Gabr, 2002; Collin, 2003; Pham et al., 2004; Qian and Ling, 2009).

A large number of numerical and experimental studies have been conducted over the last few decades on pile-supported embankments with and without geosynthetic, to investigate their behaviour and the load transfer mechanism (Low et al., 1994; Han and Wayne, 2000; Li et al., 2003; Collin, 2004; Han et al., 2004; Chen et al., 2010; Hong et al., 2011; Eskisar et al., 2012). Although various studies have been done and many successful case histories have been presented in the literature over the years, the precise mechanism by which the embankment load is transferred to the piles and the foundation soil is still not clearly understood.

Several methods can be found in the literature for calculating the vertical load distribution in pile-supported embankments. A majority of the currently available design methods assumes that the embankment load is transferred to the piles by the soil arching mechanism introduced by Terzaghi (1943). Guido et al. (1987) proposed a design approach based on model tests performed on sand in a rigid box with multiple layers of geogrid reinforcement. Hewlett and Randolph (1988) presented a semi-spherical arching model to describe the load transfer mechanism based on their three-dimensional model tests. However, the effect of geosynthetic reinforcement on the load transfer mechanism was not considered in this method. Low et al. (1994) investigated a piled embankment system which uses cap beams and geosynthetic reinforcement. They improved the method adopted by Hewlett and Randolph (1988) by incorporating the body force into the plane-strain differential equation of equilibrium. Carlsson (1987)
introduced another method to design geosynthetic-reinforced pile-supported (GRPS) embankments assuming plane-strain conditions. The original paper was presented in Swedish, but it was later discussed by Rogbeck et al. (1998) and Horgan and Sarsby (2002). Carlsson’s method assumes a triangular soil wedge with an internal angle of 30° at the apex. It is assumed that the additional overburden above the wedge is directly transferred to the piles. Jenner et al. (1998) also introduced a method to estimate the magnitude of soil arching, but it was later found that this method underpredicts the magnitude of soil arching and tension in the geosynthetic reinforcement when compared with other analytical methods as well as with numerical results (Naughton and Kempton, 2005). Russell et al. (2003) developed a new design method based on the three-dimensional numerical analysis of GRPS embankments presented by Russell and Pierpoint (1997). This method assumes that a cruciform-shaped soil block between the pile caps moves vertically downwards and is supported by the geosynthetic reinforcement while the remaining load from the embankment fill arches onto the piles. Collin (2004) presented a design method by improving Guido et al.’s (1987) method. In this method, multiple layers of reinforcement are used to create a stiff platform of reinforced soil. This reinforced soil mass acts as a beam and transfers the embankment load to the piles. Kempfert et al. (2004) introduced a new design method based on their laboratory model tests of a piled embankment problem. The magnitude of the load on soft foundation soil without geosynthetic reinforcement is first calculated by this method and then the tension in the geosynthetic reinforcement required to carry that load is estimated. A new method was developed to calculate the load on the geosynthetic reinforcement for geosynthetic-reinforced column-supported embankments by Filz and Smith (2006). The design procedure was incorporated into an Excel workbook for convenience. The German standard EBGEO (2010) is based on the work carried out by Zaeske (2001) and Kempfert et al. (2004). Abusharar et al. (2009) presented a new simple method based on the arching effect. This method enables the estimation of the magnitude of arching in the embankment fill and the calculation of the tension in the geosynthetic reinforcement layer. The BS 8006 (2010) design method is an updated version of the original BS 8006 (1995) design code. In the new design code, two methods are provided. The first method is based on the simplified analysis method developed by Jones et al. (1990), using Martinson’s formula, and the second method is the one proposed by Hewlett and Randolph (1988) based on arching theory.

Van Ekelen et al. (2011) proposed modifications to the British standard considering the three-dimensional configuration of piles. A Dutch Design Guideline (CUR 226, 2010) was published in 2010; it closely follows the German Standard (EBGEO, 2010), although some constraints have been adapted to suit Dutch circumstances.

Even though several design approaches are available for designing GRPS embankments, a universally accepted method has yet to be introduced. A number of researchers have compared some of these methods in the past; they have shown that the methods give inconsistent results (Russell and Pierpoint, 1997; Kempton et al., 1998; Horgan and Sarsby, 2002; Naughton and Kempton, 2005; Ariyarathne et al., 2012; Yapage et al., 2013). This paper presents a numerical study on a GRPS embankment problem in both three-dimensional and two-dimensional plane-strain conditions. The research presented here is different from the previous research published by Ariyarathne et al. (2012) or Yapage et al. (2013). Ariyarathne et al. (2012) used a different embankment problem for the analysis and only five design methods are considered. Furthermore, they did not investigate the development of soil arching during embankment construction. Yapage et al. (2013) considered three design methods and their results are based on a two-dimensional numerical modelling of a Deep Cement Mixed (DCM) column-supported embankment constructed in Finland. They modelled the DCM columns by incorporating the strain-softening behaviour of cement-stabilised soils.

Fully coupled mechanical and hydraulic modelling presented in this paper is carried out using the finite element modelling program ABAQUS/Standard. The results from the analysis are used to discuss the load transfer mechanism from the embankment fill to piles. A number of currently available design methods are selected for the comparison using both three-dimensional and two-dimensional finite element model results. The comparison presented in this paper includes seven design methods, which were not previously compared (new and recently revised design methods) for GRPS embankments. The comparison is further extended considering different values of pile spacings and diameters, and embankment heights. The inconsistencies in the current practice are identified and discussed in detail.

2. Site conditions and geometry of the embankment for the base case

A geosynthetic-reinforced pile-supported embankment problem reported by Liu et al. (2007) was selected as the base case for the numerical modelling in this study. The embankment is located in a northern suburb of Shanghai, China and details of the site conditions, instrumentation, construction process and monitoring were well documented by Liu et al. (2007). Fig. 1 shows the cross section of the embankment. The soil profile near the ground surface consists of a 1.5-m-thick coarse grained fill layer with a 2.3-m-thick silty clay layer below that. Underlying the silty clay layer, there is a soft silty clay layer, 10.2 m in thickness, followed by a medium silty clay layer, 2 m in thickness, and a sandy silt layer, 9 m in thickness. The ground water table is located 1.5 m below the ground surface. The height of the embankment is 5.6 m and it spans 120 m in the direction perpendicular to its cross section. The crest width of the embankment is 35.2 m and the side slopes are 1:1.5 (vertical:horizontal).

The embankment is supported by cast in-situ concrete annulus piles with a wall thickness of 120 mm, an external diameter of 1 m and an embedded length of 16 m. The top 0.5 m of the piles were cast as solid cylindrical piles as a measure to recover any damage to the top part of the annulus caused by the withdrawal of the double wall casing used
during the construction of the piles. Piles support a sandy silt layer and have a centre to centre spacing of 3 m.

The geosynthetic reinforcement layer is sandwiched between two gravel layers. A 0.25-m-thick layer of gravel is placed on top of the pile head level so that the geosynthetic layer can be placed without any damage above the pile heads. After laying the geosynthetic layer, another 0.25-m-thick gravel layer was placed on top of that in order to create a working platform for the embankment construction, making the total thickness of the geosynthetic bearing layer 0.5 m. The embankment was constructed on top of the gravel bed over a period of 55 days.

3. Numerical modelling

The analysis of a piled embankment is truly a three-dimensional problem. Two-dimensional finite element models do not appropriately represent the realistic conditions because they assume that the piles are continuous in the out-of-plane direction and behave as walls. However, two-dimensional finite element modelling requires significantly less computer memory and analysis time than three-dimensional modelling; thus, no high-performance computers are needed to analyse problems within a reasonable time frame. In this study, the numerical modelling is performed with the finite element program ABAQUS/Standard under both three-dimensional and two-dimensional plane-strain conditions.

3.1. Material model and parameters

The material parameters used in this analysis are summarised in Table 1. According to Liu et al. (2007), these parameters were extracted from field and laboratory tests.

The constitutive behaviour of the three silty clay layers, below the coarse grained fill layer and the bottom sandy silt layer shown in Fig. 1, was modelled using the Modified Cam Clay (MCC) model. The parameters required for the MCC model are the slope of the virgin consolidation line, $\lambda$; the slope of the unloading or the reloading line, $\kappa$; the void ratio at unit pressure, $e_1$; the slope of the critical state line, $M$ and Poisson’s ratio, $\nu$. These four layers are considered to be

![Fig. 1. Cross section of the embankment.](image-url)
normally consolidated. A linear elastic-perfectly plastic model with Mohr-Coulomb’s failure criterion was used to model the embankment fill, the coarse grained fill and the gravel bed. The parameters used for this model are effective cohesion \( c' \), effective friction angle \( \phi' \), dilation angle \( \psi \), Young’s modulus \( E \) and Poisson’s ratio \( \nu \). The geosynthetic layer and the piles were modelled as linear elastic materials. Interface friction is considered between the gravel bed and the geosynthetic layer during the analysis, and the interface friction angle is assumed to be the same as the friction angle of gravel. The interaction between the piles and the soil was not considered during the analysis in order to avoid convergence problems.

3.2. Three-dimensional finite element modelling

Due to the symmetry of the embankment along the centreline, only one half of the problem was selected for numerical modelling. The element type used to model the soil layers below the ground water table are 20-node brick elements with reduced integration and pore pressure degrees of freedom at the corner nodes. Due to the high permeability of the coarse grained fill layer, the embankment fill and the gravel bed, compared to the silty clay layers, they were modelled using 20-node brick elements with reduced integration, but without pore pressure degrees of freedom. The piles were modelled using the same element type. The geosynthetic layer was modelled using 8-node quadratic membrane elements with reduced integration. This element type has no bending stiffness and is unable to transfer bending moments; it only transfers in plane stresses. Therefore, this is a realistic representation of the geosynthetic reinforcement layer.

Fig. 2 shows the three-dimensional model for the pile configuration and the finite element mesh used for the analysis of the embankment. The total depth of the foundation soil is taken to be 25 m and the layer below that is assumed to be a rigid impermeable stratum. The horizontal length of the model in the \( x \) direction was extended up to 78 m in order to minimise the boundary effect. The span length of the embankment in the \( y \) direction (longitudinal direction) is 120 m. However, for the numerical modelling, a 6-m-wide section with two rows of piles was selected. Hence, the behaviour of the embankment in both lateral and transverse directions can be evaluated.

The displacements in all three directions were restricted at the bottom boundary at the \( z=0 \) plane. Symmetrical boundary conditions were assigned along the centreline of the embankment (\( x=0 \) plane) as well as two vertical planes, \( y=0 \) and \( y=6 \) m. At the far end of the model (\( x=78 \) m plane), displacements in the \( x \) direction were restricted. All the above boundaries are impermeable and the pore water was allowed to dissipate only through the bottom surface of the coarse grained fill layer by assigning a zero pore pressure boundary condition along that surface (\( z=23.5 \) m plane).

The numerical analysis was started by removing all the elements corresponding to the embankment fill and bringing the foundation soil and piles into a geostatic equilibrium. Then, the elements were added layer by layer in nine lifts until the total height of the embankment was reached over a period of 55 days. Finally, the embankment was left for 125 days to consolidate.

3.3. Two-dimensional finite element modelling

A two-dimensional plane-strain analysis was also carried out in this study for the same embankment problem. Performing a two-dimensional modelling is less time-consuming compared to a three-dimensional modelling. The results can also be achieved with reasonable accuracy. When three-dimensional piles are modelled in a two-dimensional condition, they are idealised into two-dimensional pile walls. There are various two-dimensional idealization methods available, and the best idealization method is the equivalent area method (Ariyarathne et al., 2013). Therefore, the equivalent area method is adopted for the two-dimensional modelling in this study.

The same material properties used for the three-dimensional modelling were used in this analysis as well. The saturated soil layers below the groundwater table were modelled using 8-node plane strain elements with reduced integration and pore pressure degrees of freedom at the corner nodes, and the other soil layers and piles were modelled using 8-node plane strain elements with reduced integration, but without pore pressure degrees of freedom. The geosynthetic layer was modelled using 3-node truss elements which can only transfer tensile axial stresses. Fig. 3 shows the two-dimensional model and the finite element mesh.

3.4. Parametric study

A parametric study was incorporated to compare the existing design methods. Once the basic three-dimensional and two-dimensional models were verified, the analysis was further extended for different pile spacings, pile diameters and embankment heights. This will help in the understanding of why the results from different design techniques vary with the above-mentioned parameters. Only one parameter was changed at a time, while the others were kept at the baseline case values. The details are summarised in Table 2.

4. Comparison of design techniques

There are various design methods available for the design of GRPS embankments. Not all these methods were initially developed for designing GRPS embankments, but they were later adopted for this process. This section presents a description of seven currently available design methods.

4.1. Comparison using the stress reduction ratio

A comparison of the results from these methods is carried out using the stress reduction ratio \( S_{342} \); a parameter introduced by Low et al. (1994) which is defined as the ratio of the average vertical stress, \( P_v \), carried by the reinforcement
4.1.1. Terzaghi (1943)

The arching theory developed by Terzaghi (1943), based on his classic trap door experiment, is used by many authors to describe the load transfer mechanism in pile-supported embankments. Russell and Pierpoint (1997) extended Terzaghi’s analysis and derived an expression for the stress reduction ratio in GRPS embankments by considering the three-dimensional nature. The derived expression is

\[ S_{3D} = \frac{P_r}{\gamma H} \]  

(1)

where \( s \) is the pile spacing, \( a \) is the pile cap width, \( H \) is the embankment height and \( K \) is the coefficient of earth pressure at rest. It is related to the friction angle of the embankment fill material, \( \phi' \), by

\[ K = \left(1 - \sin \phi'\right) \]  

(3)

4.1.2. Guido et al. (1987)

This method is derived from plate loading tests carried out by Guido et al. (1987). For the three-dimensional condition, this method assumes that the load spreads through the fill layer at an angle of 45° and geosynthetic reinforcement is required to support the weight of a soil pyramid which is not supported by piles. Russell and Pierpoint (1997) derived an expression for the stress reduction ratio for this method, as shown in Eq. (4). According to them, this method was used to design the

\[ S_{3D} = \frac{(s^2 - a^2)}{4HaK \tan \phi'} \left(1 - e^{-4HaK \tan \phi'/(s^2 - a^2)}\right) \]  

(2)
Second Severn Crossing embankment in UK.

\[ S_{3D} = \left(1 - \frac{a}{s}\right)^{2(K_p-1)} \left(1 - \frac{s}{2(K_p-1)} \sqrt{2H(2K_p-3)}\right) + \frac{(s-a)2(K_p-1)}{\sqrt{2H(2K_p-3)}} \]  \tag{5}

Conditions at the crown

\[ S_{3D} = \left(\frac{2K_p}{(K_p+1)}\right) \left(1 - \frac{\phi}{\phi'}\right) \left(1 - \frac{\phi}{\phi'}\right) - \left(1 + \frac{\phi}{\phi'} K_p\right) + \left(1 - \frac{\phi}{\phi'}\right) \]  \tag{6}

where, \(K_p\) is the passive earth pressure coefficient and \(\phi'\) is the friction angle of the embankment fill material.

4.1.4. Low et al. (1994)

The calculation method introduced by Hewlett and Randolph (1988) was improved by Low et al. (1994); it includes a geosynthetic reinforcement layer and contribution from the foundation soil. The proposed design method was based on laboratory model tests done with a cap beam arrangement. This arrangement represents a two-dimensional numerical model; it will form semi-cylindrical arches between the pile walls. The thickness of each arch is equal to half the width of the pile wall.

Low et al. (1994) developed some equations and charts to evaluate the tension and mobilized strain in the geosynthetic reinforcement layer and the stress reduction over the foundation soil. The deflection of the geosynthetic layer was assumed to be a circular arc with a radius \(R\) and a subtended angle of \(2\theta\) at the centre of the arc. The maximum vertical displacement
of the foundation soil midway between the pile caps is \( t \). The following equations were presented considering the geometry of the problem.

The tension in the reinforcement is given by

\[
T = J\varepsilon
\]  
(8)

where \( J \) is the tensile stiffness of the geosynthetic and \( \varepsilon \) is the axial strain. Based on the shape of deformation assumed, the axial strain is given by

\[
\varepsilon = \frac{\theta - \sin \theta}{\sin \theta}
\]  
(9)

\[
R = \frac{S - a}{2 \sin \theta}
\]  
(10)

Considering the equilibrium of vertical forces, the tension in the geosynthetic is also given by

\[
\frac{T}{R} = \frac{p_0}{\left( \varepsilon_s - \frac{t E_s}{D} \right)}
\]  
(11)

Both the stresses acting on top of the geosynthetic layer and the ground reaction below it vary laterally with a distribution that reaches a maximum midway between the pile caps. However, Low et al. (1994) assumed that their difference is nearly uniform. Hence, \( p_0 \) is the assumed uniform pressure applied on the geosynthetic layer, \( E_s \) is the elastic modulus of the foundation soil and \( D \) is the depth of the foundation soil. The vertical stress acting on the foundation soil midway between the piles, \( \sigma_s \), is given by

\[
\sigma_s = \frac{\gamma(s-a)(K_p-1)}{2(K_p-2)} + \frac{(s-a)K_p-1}{2} \left( \frac{\gamma H}{2} - \frac{\sigma_s}{K_p-2} \right)
\]  
(12)

The following equation has been derived to calculate the subtended angle 2\( \theta \):

\[
\sin \theta = \frac{4\left( t/(s-a) \right)}{1 + 4\left( t/(s-a) \right)^2}
\]  
(13)

In order to solve these equations, a trial and error procedure has to be performed using trial values of \( t \) to calculate \( \theta \) and \( R \) and the geosynthetic tension using Eqs. (8) and (11) separately until the resulting tension from both equations is the same. Then, that \( t \) value can be used to calculate the geosynthetic tension and the stress reduction ratio.

\[
S_{3D} = \left( \varepsilon_s - \frac{t E_s}{D} \right) \frac{\gamma H}{C_0}
\]  
(14)

4.1.5. Kempfert et al. (2004)

The Kempfert et al. (2004) method is developed using three-dimensional instrumented model tests in a scale of 1:3 carried out to investigate the bearing and deformation behavior of piled embankments. In this method, the magnitude of the load on the soft soil, without the inclusion of the reinforcement, is calculated before the tension in the reinforcement is estimated to carry that load. This method allows the support from the foundation soil to be included in the design. The tension in the reinforcement is estimated based on the theory of elastically embedded membranes.

The stress reduction ratio for this method is shown in Eq. (15).

\[
S_{3D} = \frac{1}{\gamma H} \left[ \lambda_1 \left( \frac{\sigma_s D}{\sigma_s} \right) ^{-x} + h_\gamma \left( \lambda_1 + \frac{h_\gamma^2 d_2}{4} \right)^{-x} \right]
\]  
(15)

where,

\[
\lambda_1 = \frac{1}{8}(s_d - d)^2; \quad \lambda_2 = \frac{s_d^2 + 2ds_d - d^2}{s_d^2}; \quad \gamma = \frac{d(K_p - 1)}{h_\gamma s_d}
\]  
(16)

and

\[
h_\gamma = \frac{s_d}{2} \quad \text{for} \quad H \geq \frac{s_d}{2}; \quad h_\gamma = H \quad \text{for} \quad H < \frac{s_d}{2}
\]  
(17)

where \( d \) is the pile diameter, \( K_p \) is the passive lateral earth pressure, \( h_\gamma \) is the arching height, \( q \) is the surcharge, \( H \) is the embankment height, \( \gamma \) is the unit weight of embankment fill and \( s_d \) is the diagonal pile spacing.

4.1.6. Abusharar et al. (2009)

Abusharar et al. (2009) presented a new theoretical analysis for embankments on soft ground supported by a rectangular grid of piles and geosynthetic, similar to the one proposed by Low et al. (1994). The main modifications were the inclusion of a uniform surcharge load on the embankment, the use of individual square pile caps and taking into account the skin friction mechanism at the soil geosynthetic interface. However, the term for surcharge load was neglected in this study because no surcharge loads were applied to the selected embankment.

The vertical stress acting midway between the pile caps, \( \sigma_s \), is given by Eq. (12). Half the subtended angle is given by modifying Eq. (13) as follows:

\[
\theta = \sin^{-1} \left( \frac{4\beta}{1 + 4\beta^2} \right)
\]  
(18)

where \( \beta \) is defined as \( t/(s-a) \).

It can be proven that Eq. (9) can be re written as

\[
\varepsilon = \frac{\theta - \sin \theta}{\sin \theta} = 4\beta^2
\]  
(19)

Combining Eqs. (10), (11) and (18) gives the following expression to calculate the tension in the geosynthetic layer:

\[
T = \left( \frac{1 + 4\beta^2}{8\beta} \right) (s-a) \left( \varepsilon_s - \frac{t E_s}{D} \right)
\]  
(20)

This method takes the skin friction mechanism into account when calculating the tension in the geosynthetic layer. In order to find the total shear stress at the soil–geosynthetic interface, Abusharar et al. (2009) took two different angles of shearing resistance for the fill material and the foundation soil at the top and bottom of the geosynthetic layer, respectively. However, in this study the geosynthetic layer is surrounded by two gravel beds and the total shear stress can be calculated using a common friction angle. The friction angle at the gravel–geosynthetic interface is assumed to be as same as the friction
angle of the gravel. Considering the equilibrium of horizontal forces, the following equation was derived to calculate the tension in the reinforcement.

\[ T = 4\beta^2 J + \frac{1}{4} (s-a) \lambda_3 \tan \phi' \left( \sigma_c + \frac{tE_s}{D} \right) \]  \( \text{(21)} \)

where \( \lambda_3 \) is a dimensionless parameter varying between 0.7 and 0.9 and depending on the type of geosynthetic. In this study it is taken as 0.8.

Combining Eqs. (20) and (21), the following equation can be obtained:

\[ a\beta^2 + b\beta + c = 0 \]  \( \text{(22)} \)

where \( a = 32DJ + 4(s-a)^2E_s; \ b = 2(s-a)^2\lambda_3\tan \phi' - 4(s-a)D\sigma_c; \ c = 2(s-a)\lambda_3D\sigma_c \tan \phi' + (s-a)^2E_s. \)

Eq. (22) can be solved to find \( \beta \) and then the geosynthetic tension is calculated using Eq. (21). Finally, the stress reduction ratio can be obtained using Eq. (14), namely,

\[ S_{3D} = \frac{1}{(s-a)} \left[ s^2 - a^2 \left( \frac{P_C}{\gamma H} \right) \right] \]  \( \text{(27)} \)

For full arching,

\[ S_{3D} = \frac{1.4}{H(s+a)} \left[ s^2 - a^2 \left( \frac{P_C}{\gamma H} \right) \right] \]  \( \text{(28)} \)

### 4.1.7. BS 8006 (2010)

The method adopted in the British Standard for strengthened/reinforced soils and other fills (BS 8006, 2010) was originally developed using the simplified analysis methods developed by Jones et al. (1990) based on the two-dimensional pipeline theory. The amount of load carried by the piles is calculated according to Marston’s formula for positively projecting subsurface conduits. In this design code, two different arching conditions are defined: (i) the partial arching condition, where \( 0.7(s-a) \leq H \leq 1.4(s-a) \) and (ii) the full arching condition, where \( H > 1.4(s-a) \). Equations for the stress reduction ratio can be derived for both conditions using the method adopted by Russell and Pierpoint (1997).

For partial arching,

\[ S_{3D} = \frac{2s}{(s+a)(s^2-a^2)} \left[ s^2 - a^2 \left( \frac{P_C}{\gamma H} \right) \right] \]  \( \text{(23)} \)

For full arching,

\[ S_{3D} = \frac{2.8s}{(s+a)^2H} \left[ s^2 - a^2 \left( \frac{P_C}{\gamma H} \right) \right] \]  \( \text{(24)} \)

\[ \left( \frac{P_C}{\gamma H} \right) = \left[ \frac{C_C\epsilon}{H} \right]^2 \]  \( \text{(25)} \)

where, \( C_C \) is the arching coefficient and \( P_C \) is the vertical stress on the pile. For the friction piles used in this embankment,

\[ C_C = \frac{1.5}{a} - 0.07 \]  \( \text{(26)} \)

These equations were used to calculate the stress reduction ratio for the embankment problems analysed. However, for some embankment problems, these equations yielded stress reduction ratios greater than one, which is impossible. The main reason for this is that BS 8006 does not satisfy the vertical equilibrium when calculating the line load on the geosynthetic layer. Jones et al. made this choice to guarantee sufficient safety so that the stress on the geosynthetic is overpredicted and the outcome is a stronger design. However, this leads to unrealistic results for some embankment problems as observed in this study.

Recently Van Eekelen et al. (2011) proposed some modifications to BS 8006 in order to eliminate the shortcomings when calculating the line load on the geosynthetic layer. The new equations satisfy the vertical equilibrium for the partial arching condition, but not for the full arching condition. However, they give more realistic values. Therefore, the stress reduction ratio is calculated using the newly proposed equations which are given below.

For partial arching,

\[ S_{3D} = \frac{1}{(s^2-a^2)} \left[ s^2 - a^2 \left( \frac{P_C}{\gamma H} \right) \right] \]  \( \text{(27)} \)

For full arching,

\[ S_{3D} = \frac{1.4}{H(s+a)} \left[ s^2 - a^2 \left( \frac{P_C}{\gamma H} \right) \right] \]  \( \text{(28)} \)

### 4.2. Comparison using the geosynthetic tension

In order to find the tension developed in the geosynthetic layer, the following equation given in the British standard was used:

\[ T = \frac{W_T(s-a)}{2a} \sqrt{1 + \frac{1}{6\epsilon}} \]  \( \text{(29)} \)

\( T \) is the tension force per meter run between the pile caps, \( W_T \) is the uniformly distributed load between the pile caps and \( \epsilon \) is the strain in the reinforcement. \( W_T \) can be obtained using the following equations which were modified from BS 8006 by Van Eekelen et al. (2011) after eliminating the double load calculation error in the original British standard.

For partial arching,

\[ W_T = \frac{\gamma H}{2(s-a)} \left[ s^2 - a^2 \left( \frac{P_C}{\gamma H} \right) \right] \]  \( \text{(30)} \)

For full arching,

\[ W_T = 0.7\gamma \left[ s^2 - a^2 \left( \frac{P_C}{\gamma H} \right) \right] \]  \( \text{(31)} \)

By assuming a constant stiffness \( J \) for the geosynthetic reinforcement, the tension that develops can be calculated using Eq. (8) as well, and a value for the geosynthetic tension can be obtained by solving Eqs. (8) and (29). This procedure was used to calculate the geosynthetic tension for the BS 8006 design method.

Using this method, Russell and Pierpoint (1997) derived an equation with respect to the stress reduction ratio to calculate the tension as follows:

\[ T = \frac{S_{3D}\gamma H(s^2-a^2)}{4a} \sqrt{1 + \frac{1}{6\epsilon}} \]  \( \text{(32)} \)

This equation was used to calculate the reinforcement tension for the Hewlett and Randolph, Guido and Terzaghi methods. A design strain of 5% was used for the calculation, as recommended by BS 8006.
The reinforcement strain for the Kempfert method was calculated by the graphs provided by Kempfert et al. (2004) assuming no support from the foundation soil. Then, Eq. (8) was used to calculate the reinforcement tension. In Low et al.’s method, Eq. (11) is used to calculate the reinforcement tension after obtaining a value for $t$. The reinforcement tension for Abusharar et al.’s method is calculated using Eq. (21) after solving Eq. (22) for $\beta$.

4.3. Comparison using the pile efficacy

The efficacy $E_f$ of the pile support is defined as the proportion of embankment load carried by the pile.

$$E_f = \frac{P}{s^2\gamma H}$$

where $P$ is the total load on the pile.

Assuming no support from the foundation soil, this equation can be adopted to include the stress reduction ratio as follows:

$$E_f = 1 - \frac{S_{1D}(s^2-a^2)}{s^2}$$

The pile efficacy for the Hewlett and Randolph, Guido, Kempfert and Terzaghi methods is calculated using the above equation.

The pile efficacy for the BS 8006 method is calculated using the equations provided by Van Eekelen et al. (2011).

For partial arching,

$$E_f = 1 - \frac{1}{s^3} \left[ s^2-a^2 \left( \frac{P_c}{\gamma H} \right) \right]$$

For full arching,

$$E_f = 1 - \frac{1.4(s-a)}{s^2H} \left[ s^2-a^2 \left( \frac{P_c}{\gamma H} \right) \right]$$

Eq. (37) is developed to calculate the pile efficacy for the methods proposed by Low et al. (1994) and Abusharar et al. (2009), assuming a three-dimensional configuration of the piles.

$$E_f = 1 - \frac{(s^2-a^2)p_0}{s^2\gamma H}$$

5. Analysis of results

The results obtained from this study are presented in the following sections. A verification of the three-dimensional and two-dimensional models is presented by Aryanathne et al. (2013).

5.1. Load transfer from the embankment to piles and foundation soil

The design methods discussed in Section 4 assume that the embankment load is transferred to piles based on the arching mechanism. However, the shape of the arch used in these design methods is not consistent. Hewlett and Randolph (1988), Low et al. (1994), Abusharar et al. (2009) assumed that arches are semicircular in the plane-strain condition and have a uniform thickness equivalent to half of the column width without any overlapping. BS 8006 (2010) also assumed semi-circular arches but introduced a partial and full arching concept, which depends on the embankment height and clear spacing between columns, as discussed in Section 4. Since $H$ changes with the placement of each fill layer, it is possible for a partial arch to convert into a full arch during the embankment construction. Guido et al. (1987) assumed the formation of triangular arches with 45° internal angles with the horizontal direction in the two-dimensional plane-strain condition and a similar pyramid for the three-dimensional situation. Rogbeck et al. (1998) described arches with a triangular shape under the plane-strain condition, but with a 30° angle at the apex of the soil wedge. In the method, proposed by Kempfert et al. (2004), the average vertical pressures acting on the DCM columns and the soft foundation soil are calculated by considering domed arches spanning between pile caps consisting of multi-shell domes. The topmost shell takes the shape of a hemisphere, but towards the pile cap level the radius of the shell domes increase, reducing the curvature. In addition to the inconsistencies related to the shape of the arch, different assumptions and approximations were used in the derivation of equations for the embankment load transferred to the piles and the geosynthetic, and tension developed in the geosynthetic in the previously-discussed design methods. As a result, some parameters (e.g., the elastic modulus of the piles, the friction angle of the fill material and the support provided by the foundation soil) are not included in those design methods, as outlined in Table 3. If a finite element model is developed, all these missing parameters listed in Table 3 can be included in the analysis. It is reasonable to assume that the results given by finite element simulations are close to the field performance of pile supported embankments, if appropriate material models and element sizes are used for the analysis. Therefore, in this section, the load transfer mechanism from the embankment to the piles is investigated in detail using the results of three-dimensional finite element simulations.

The main objective of adding pile supports to the embankment is to transfer a major portion of the embankment load to piles. Due to the presence of soft foundation soil between the piles with high stiffness, the embankment fill in between the piles will tend to settle more than the fill just above the piles. This downward movement will be restrained by the development of shear resistance above the piles. This will reduce the pressure acting on the foundation soil and increase the stress applied on the piles, as shown in Fig. 4. If the load transfer is investigated quantitatively, the load applied over a pile closer to the centre of the embankment is more than five times greater than the total embankment load at the end of embankment construction, which is about 104 kPa. The load transferred to the foundation soil is about 65% of the total embankment load at the end of the embankment construction. These values confirm that there is soil arching within the embankment fill and, as a result, the majority of the embankment load is supported by piles.
Fig. 5 shows the development of shear stresses in between two piles forming shear planes as a result of the differential settlement of the fill material above the pile heads and soft soil. These shear planes will contribute to the formation of arches in the embankment fill above the pile heads. The shape of the arches can be identified by investigating the horizontal stress distributions in both x and y directions. Since the horizontal stress acting over the cross section of a vertically loaded arch is a constant, arch-shaped stress contours are visible, as shown in Fig. 6(a) and (b). As a result of soil arching, in both x and y directions, the combined arches in both directions are assumed to form a dome in a three-dimensional space, as shown in Fig. 6(c). In two-dimensional models, the soil arch will take a cylindrical shape, as shown in Fig. 6(d).

The differential settlement, within the embankment fill, decreases towards the top of the embankment. The reason for this reduction is the increase in the distance between the fill material and the pile head level at which the stiffness contrast between the piles and the foundation soil is the highest. Therefore, arches are not formed up to the top of the embankment. In addition, the height above which there is no soil arching depends on the embankment geometry as well as the material properties. Fig. 7 shows the vertical stress distribution along the embankment height midway between

<table>
<thead>
<tr>
<th>Design method</th>
<th>Missing factors when calculating $P_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terzaghi (1943)</td>
<td>Subsoil: subsoil support is not considered Piles: elastic modulus Embankment fill: construction rate Geosynthetic: stiffness</td>
</tr>
<tr>
<td>Guido et al. (1987)</td>
<td>Subsoil: subsoil support is not considered Piles: elastic modulus Embankment fill: friction angle, Construction rate Geosynthetic: stiffness</td>
</tr>
<tr>
<td>Low et al. (1994)</td>
<td>Subsoil: subsoil support is considered only using elastic modulus and depth Piles: elastic modulus Embankment fill: construction rate Geosynthetic: stiffness</td>
</tr>
<tr>
<td>Kempfert et al. (2004)</td>
<td>Subsoil: subsoil support is considered only when calculating the geosynthetic tension Piles: elastic modulus Embankment fill: construction rate Geosynthetic: stiffness</td>
</tr>
<tr>
<td>Abusharar et al. (2009)</td>
<td>Subsoil: subsoil support is considered only using elastic modulus and depth Piles: elastic modulus Embankment fill: construction rate Geosynthetic: stiffness</td>
</tr>
<tr>
<td>BS 8006 (2010)</td>
<td>Subsoil: subsoil support is not considered Piles: elastic modulus Embankment fill: friction angle, construction rate Geosynthetic: stiffness</td>
</tr>
</tbody>
</table>
Fig. 5. Shear stress distribution in XZ plane.

Fig. 6. (a) Horizontal stress distribution in x direction; (b) horizontal stress distribution in y direction; (c) formation of domes in three-dimensional space; (d) formation of domes in two-dimensional space.
the pile closest to the centre and pile A marked in Fig. 1. This figure clearly shows the stress redistribution within the embankment fill. The vertical stress increases from the top of the embankment to a certain depth and then starts to decrease due to the stress redistribution occurring as a result of soil arching. Then, near the base of the embankment, the vertical stress increases again by a smaller amount due to the load from the fill below the arch, which is just above the foundation soil.

According to Fig. 7, the vertical stress starts to decrease due to soil arching at a height about 2 m above the pile heads and then starts to increase slightly at a height of 0.5 m above the pile heads. This shows that the outer line of the arch is at a height of 2 m and the inner line of the arch is at a height of 0.5 m above the pile heads midway between the first two piles. Therefore, the thickness of the arch midway between the piles is about 1.5 m. However, several design methods, such as BS 8006 and Hewlett and Randolph (1988), assume that the shape of the arch is a semi-circle in between the piles. If semi-circular arches are to be formed in this embankment problem, their thickness should be about 0.5 m, which is half of the pile diameter. According to Fig. 7, it is clear that the shape of the arch formed in this instance is not a semi-circular one. The shape of the arch will be discussed later in this section.

The development of the soil arch inside the embankment fill during embankment construction can be observed by plotting the vertical stress distribution along the embankment fill in between piles with increasing embankment height, as shown in Fig. 8. The gradient of the vertical stress distribution increases from the top of the embankment with a gradient equal to the unit weight of the fill material up to the crown of the arch. Hence, in Fig. 8, signs of soil arching become visible after placing the third fill layer in the embankment. Then, as the embankment height increases, soil arching is also increased due to the increased differential settlements under increasing embankment loading. The development of the arch during the construction period of the embankment is clearly visible in this figure where the height of the outer line of the arch or the crown at the mid-section between adjacent piles is gradually increased during the construction period, while the inner line remains at a constant height closer to the embankment base. During the post-construction period, soil arching is further increased as a result of the increasing differential settlements within the embankment fill due to the consolidation of the soft foundation soil. This results in a reduction in vertical stress at the base of the arch, as shown in Fig. 8. There is a slight increase in the height of the outer line of the arch during consolidation. However, the inner line of the arch does not show any noticeable change in height during consolidation.

In order to investigate the shape of the arch developing inside the embankment fill, the vertical stress distribution along different vertical panes in between the first two piles is used. The height to the point of the maximum vertical stress
for any such plane considered represents the height of the outer line of the arch and the height to the point where the stress is a minimum represents the inner line of the arch. Fig. 9 shows the shape of the arch developing within the embankment in between the first two piles. The outer line of the arch can be easily located using the vertical stress distribution, but the inner line is very close to the base of the embankment. The increase in thickness of the arch during consolidation, which was explained earlier in this section, is clearly visible in this figure. Furthermore, the shape of the arch does not reflect a semi-circular shape. This arch shows a parabolic shape and it gradually changes the curvature from the outer line to the inner line. The shape of the arch observed in this study is very similar to the multi-arching theory proposed by Kempfert et al. (2004). However, even in the multi-arching theory, the outer arch is taken as a semi-circular arch, while the inner arches have gradually decreasing curvatures. These results clearly confirm that the shape of the arch forming within the embankment fill assumed in the current design methods do not represent the actual shape. Also, the shape of the arch evolves during the embankment construction and subsequent consolidation.

The load transfer mechanism in pile-supported embankments, with and without geosynthetic reinforcement, is similar and it is based on the arching theory. However, the degrees of soil arching within geosynthetic reinforced and unreinforced pile-supported embankments are not the same. When compared with an unreinforced embankment, the geosynthetic reinforcement is expected to reduce the vertical settlement of the embankment fill in between the piles. This reduction will result in a decrease in the differential settlements within the fill, and consequently, a reduction in soil arching. Therefore, the load transferred by soil arching is assumed to be reduced when geosynthetic reinforcement is present. Alternatively, the vertical load transferred to the piles will be increased by the vertical component of the tension developed in the geosynthetic layer. In addition, the geosynthetic layer reduces the loads transferred to the foundation soil in between the piles.

5.2. Comparison of results using the stress reduction ratio

The stress reduction ratios are calculated for each method separately by varying the pile diameter, pile spacing, and embankment height. The results from each design method are compared against the stress reduction ratios obtained by three-dimensional and two-dimensional numerical models. For the Low et al. and Abusharar et al. methods, the elastic modulus of the foundation soil is required for the $S_{3D}$ calculation. This value was taken to be 5.5 MPa, which is the weighted average of the elastic modulus values of the corresponding soil layers. The variation in $S_{3D}$ with pile spacing is shown in Fig. 10. Out of the seven design methods, the method proposed by Guido et al. significantly underpredicts the stress reduction ratio. Terzaghi’s method gives a close result for the 2-m pile spacing, but as the pile spacing is increased, the results are overpredicted. The design methods proposed by BS 8006, Kempfert and Hewlett and Randolph produce inconsistent results over the range of pile spacings selected. The Low et al. and Abusharar et al. methods highly underpredict the stress reduction ratios. The variation in $S_{3D}$, obtained from these two methods, shows an inverse variation compared to the other design methods and numerical results. This is because the $tE_s/D$ term in Eq. (14) becomes large when $E_s$ and $t$ are high. Therefore, with an increased pile spacing, the equation yields lower stress reduction ratios. The method developed by Abusharar et al. gives close results to the method developed by Low et al., because it was developed with a slight modification to Low et al.’s method by taking into account the skin frictional mechanism at the soil-geosynthetic interface.
A comparison of the design methods for different pile diameters is shown in Fig. 11, when the pile spacing is 3 m. According to the results, the BS 8006 method provides a good agreement with the numerical results. However, BS 8006 produces inconsistent results when the pile spacing is changed from 2 to 4 m, as shown in Fig. 10. The Low et al. and Abusharar et al. methods significantly underpredict the stress reduction ratio.

The analysis was carried out for three different embankment heights, and a comparison of the stress reduction ratios is shown in Fig. 12. The Guido, Low and Abusharar methods highly underpredict the numerical model results here as well.

5.3. Comparison of results using geosynthetic tension

The geosynthetic tension results, obtained using the selected design techniques, are compared with the results from the two-dimensional and three-dimensional finite element model results in this section. Fig. 13 shows the geosynthetic tension for different pile spacings.

According to the results, the Terzaghi, BS 8006, Hewlett and Randolph and Kempfert methods significantly overpredict the geosynthetic tension when the pile spacing is increased. The Guido, Low and Abusharar methods are in better agreement with the numerical results compared to the other methods. However, the small geosynthetic tension given by these three methods cannot be accepted because the calculated tension is based on the highly underpredicted stress reduction ratios

Comparison of the geosynthetic tension for different pile diameters is shown in Fig. 14. The variations in the results are similar to Fig. 13, where the Guido, Low and Abusharar methods give results in agreement with the numerical results. All the other design methods give overly conservative results for the tension developing in the geosynthetic reinforcement, yielding uneconomical designs.

A similar pattern can be observed in Fig. 15 which shows the variation in geosynthetic tension with different embankment heights for the selected design techniques.

5.4. Comparison of results using pile efficacy

Pile efficacies, obtained from the design techniques, are compared with the numerical model results in this section. The variation in pile efficacy with pile spacing, pile diameter and embankment height are shown in Figs. 16–18, respectively.

According to Figs. 16–18, the trends in the results are similar. The Terzaghi, Hewlett and Randolph, BS 8006 and Kempfert methods give inconsistent results over the range of parameters selected, and the Guido, Low and Abusharar methods highly overpredict the efficacy. None of the selected design methods yields results that are in good agreement with the two-dimensional and three-dimensional model results.

6. Summary and conclusions

A numerical study on GRPS embankments under both two-dimensional plane-strain conditions and three-dimensional conditions has been presented in this paper. The load transfer mechanism was studied using both two-dimensional and three-dimensional results. Then, the study was further expanded to compare seven currently available design methods for
designing GRPS embankments for embankments with varying pile spacing, pile diameter and embankment height.

The load transfer from the embankment fill to the piles takes place mainly due to the soil arching mechanism and the stress concentration due to the difference in stiffness between the piles and the foundation soil. The formation of soil arches is visible in the three-dimensional model as a dome which is supported by piles, and in the two-dimensional model as arches which span along the pile walls. The soil arch formation can also be justified using the vertical stress distribution in the embankment fill.

The design techniques used for comparison in this paper are the seven most common methods used in practice. According to the results, these methods differ significantly when predicting the stress reduction ratio, geosynthetic tension and pile efficacy. All the selected design methods provide overpredictions or underpredictions depending on the geometric properties of the embankment. The methods proposed by Guido, Low and Abusharar consistently underpredict the stress reduction ratio for the selected case study. The results obtained from Guido et al.’s method cannot be relied upon because they only consider the pile spacing diameter and the embankment height and no other material parameters. Some important factors missing from the current design methods are tabulated in Table 3.

Therefore, it can be concluded that the design methods proposed by Low and Abusharar are applicable for predicting the tension developing in the geosynthetic layer, but they
significantly underpredict the stress reduction ratio and overpredict the pile efficacy. In order to achieve reliable designs, a numerical modelling should be incorporated into the design process. Further research needs to be carried out in order to improve the available analytical methods by incorporating various aspects of the embankment geometry, soil properties and soil arching mechanism within the embankment fill. In addition, the lateral deformation of the embankment is not considered in the current design practice, but for some embankments this can be significant, and thus, needs to be included. Furthermore, having a gravel layer below the geosynthetic layer can restrain the stretching of the geosynthetic layer, thus reducing the developed strain and tension. However, it is common practice to use a working platform above the pile heads and the current design methods should be improved in the future to incorporate its effect.

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