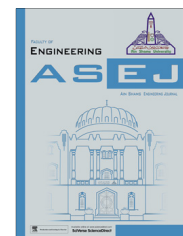




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MECHANICAL ENGINEERING

A hybrid multi-criteria decision modeling approach for the best biodiesel blend selection based on ANP-TOPSIS analysis



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Received 10 January 2014; revised 16 June 2014; accepted 6 August 2014

Available online 13 September 2014

KEYWORDS

IC engine;
MCDM;
ANP;
TOPSIS;
VIKOR;
Best blend

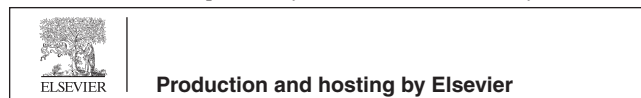
Abstract The ever increasing demand and depletion of fossil fuels had an adverse impact on environmental pollution. The selection of appropriate source of biodiesel and proper blending of biodiesel plays a major role in alternate energy production. This paper describes an application of hybrid Multi Criteria Decision Making (MCDM) technique for the selection of optimum fuel blend in fish oil biodiesel for the IC engine. The proposed model, Analytical Network Process (ANP) is integrated with Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) and VlseKriterijumska Optimizacija I Kompromisno Resenje (in Serbian) (VIKOR) to evaluate the optimum blend. Evaluation of suitable blend is based on the exploratory analysis of the performance, emission and combustion parameters of the single cylinder, constant speed direct injection diesel engine at different load conditions. Here the ANP is used to determine the relative weights of the criteria, whereas TOPSIS and VIKOR are used for obtaining the final ranking of alternative blends. An efficient pair-wise comparison process and ranking of alternatives can be achieved for optimum blend selection through the integration of ANP with TOPSIS and VIKOR. The obtained preference order of the blends for ANP-VIKOR and ANP-TOPSIS are B20 > Diesel > B40 > B60 > B80 > B100 and B20 > B40 > Diesel > B60 > B80 > B100 respectively. Hence by comparing both these methods, B20 is selected as the best blend to operate the internal combustion engines. This paper highlights a new insight into MCDM techniques to evaluate the best fuel blend for the decision makers such as engine manufactures and R& D engineers to meet the fuel economy and emission norms to empower the green revolution.

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Peer review under responsibility of Ain Shams University.



1. Introduction

With energy shortage and growing environmental pollution, all countries are look for a new energy that has long period exploits value and no environmental pollution. There is a need

Nomenclature

| | | | |
|------------------|------------------------------|----------------|--|
| λ_{\max} | Eigen value | D_j^*, D_j^- | the distance between the performance scores of alternates with respect to all criteria |
| W | Eigen vector | CC_j | relative closeness coefficient |
| CR | Consistency Ratio | S_j | utility measure |
| RCI | Random Consistency Index | f_i^* | best value |
| r_{ij} | normalized evaluation matrix | f_i^- | worst value |
| v_{ij} | weighed normalized value | R_i | regret measure |
| W_j | weights | Q_i | VIKOR index |
| V_i^* | positive ideal solution | | |
| V_i^- | negative ideal solution | | |

to search and find ways of using alternative fuels, which are preferably renewable and also emit low levels of gaseous and particulate pollutants in internal combustion engines. Biodiesel is clean chemistry energy of future. Biodiesel has significant superiority over diesel in a way to meet the fuel economy and stringent emission norms. It also reduces global warming and environmental meltdown occurring due to massive carbon footprints left by the fossil fuels. Biodiesel can be produced from renewable resources such as vegetable oil, animal fat and waste cooking oil [1]. The cultivation of crops for biodiesel production poses a threat to food security and contributes to decline in soil fertility [2]. Moreover the oil percentage and the yield per hectare of vegetable oil are very low. On the other hand, the animal fat present in the waste parts of fish serves to be a good source of crude oil for biodiesel. Fish oil can be derived from waste parts of fish like viscera, eyes, fins, head, tails, liver and maw [3,4]. The calorific value of fish oil is similar to that of petroleum distillates [5]. Jayasinghe and Hawboldt reviewed the production process of physical, chemical and thermal properties of biofuel from fish waste and suggested that waste fish oil exhibits better engine performance [6]. A few researchers have used methyl ester of fish oil as a biodiesel to study the performance and emission characteristics of Internal Combustion (IC) engine [7–12]. Godiganur et al. prepared several blends of varying concentration ranging from 0% to 80% in the interval of 20% [7]. Steigers and Behcet attempted with blends from 0% to 100% in the interval of 25% and 50% [10,12]. In this paper, an attempt has been made to use ethyl ester of fish oil as a biodiesel to study the performance, exhaust emissions and combustion parameters. The engine was operated with the biodiesel–diesel blends at a constant speed of 1500 rpm at different load conditions. The performance, emission and combustion characteristic of the engine varies with respect to different load conditions for different biodiesel–diesel blends. It is quite difficult to choose the optimum blend to run the Internal Combustion (IC) engine.

In the existing work, most of the researchers have discussed their operating fuel with reference of NO_x , smoke and BTE performance. Based on the reduction of NO_x and smoke and increase of BTE, the operating fuel is recommended as the best blend without considering other influencing parameters such as HC, CO and EGT [7,12–16]. In this paper, to overcome the shortcomings of the existing research, all the performance, emission and combustion characteristics are considered. In this paper ANP integrated with TOPSIS is proposed as the hybrid

MCDM technique for evaluating and selecting the suitable blend for IC engines.

MCDM is a branch of a general class of operation research models dealing with decision problems under the presence of a plethora of factors and criteria. It provides sophisticated methodological tools that are oriented towards the support of the decision makers in facing complex real-world decisions. The application of MCDM in automobile engineering has been gradually increasing in the past few decades. Milani et al. proposed a methodology for material selection illustrating non-metallic gear nylon material used to drive secondary power system in aircrafts using ANP approach [17]. Atmaca and Basar used ANP technique to select best power plant in turkey based on Technology, sustainability, economy, life, quality, socio-economic criteria [18]. Vinodh et al. did a case study for Indian electronics switches manufacturing company based on FANP approach to select the best supplier [19]. Boran and Goztepe evaluated the best vendor based on FANP approach for commodity purchase [20]. Wu et al. made an attempt to monitor and determine the appropriate marketing strategy using integration of ANP and TOPSIS [21]. Lee et al. compared the five force model which emphasis on factors threatening industrial growth time to time using ANP technique [22]. Tuzkaya et al. addresses the selection of facility location selection using ANP [23]. Hong-cai et al. made an attempt to evaluate better bridge alternative using ANP approach [24]. Gencer and Gurpinar proposed ANP methodology for supplier selection, being the most important factor affecting the performance of the company [25]. From the literature, there is no trace of research that deals with selection of suitable fuel blend based on the performance, combustion and emission characteristics using MCDM technique. Hence in this study, an effort is taken to evaluate the best blend using hybrid MCDM technique, ANP with TOPSIS and VIKOR to achieve the maximum engine performance and environmental benefits by reducing noxious emissions.

The rest of this paper is organized as follows: in the second section, preparation of test fuel and its properties are summarized. In the third section experimental procedure and its specifications are explained. In the fourth section ANP, TOPSIS and VIKOR are summarized. In this section, literature review and methodology for each technique are also given. In the fifth section, the proposed model is explained. In the sixth section, the obtained results are discussed. Finally, in section seven, the paper is concluded with suggestions for future research.

2. Experimental investigations

2.1. Test fuel

The waste parts of sardine fish were cooked, squeezed and centrifuged to extract the fish oil. The water content and impurities in raw fish oil were refined by a series of pre-treatment process. A sample of 1000 ml of fish oil was taken in a container and heated up to 75 °C using heating coil at a minimum stirring speed. A fixed amount of ethanol and 6 g of Potassium Hydroxide (KOH) was vigorously shaken in a conical flask and poured into the container. The container was closed with an air tight lid. The mixture was then stirred for an hour and the solution was transferred to a separator funnel and allowed to settle overnight. The biodiesel process turned the oil into esters, separating out the glycerol. The glycerol sank to bottom and the bio-diesel which can be siphoned off floated on top. The crude ester phase was separated and the glycerol phase in it was washed with warm de-ionized water several times until the washed water became clear. The excess water in the ester phase was removed by evaporation under atmospheric condition. Diesel was added in the ratio of 0%, 20%, 40%, 60%, 80% and 100% with ester to transform it as biodiesel–diesel blends. The prepared biodiesel was examined for physical, chemical and fuel related properties as per ASTM test standards before the engine testing was carried out. The fuel properties of fish oil biodiesel, diesel and biodiesel–diesel blends are given in Table 1.

2.2. Experimental setup and testing procedure

Tests were conducted on a single cylinder, air cooled, four stroke, vertical, DI diesel engine with the displacement volume of 661 cm³, compression ratio of 17.5:1, developing 4.4 kW of power at 1500 rpm. The injector opening pressure recommended by the manufacturer was 210 bar and the engine was operated at standard injection timing 21obTDC. Specification details of the engine are given in Table 2. The engine was loaded by electric dynamometer to provide the brake load. The schematic diagram of the engine setup is shown in Fig. 1. The engine was started initially with diesel and allowed to have a warm-up for about 10 min. The AVL 437 smoke meter and AVL 444 Di Gas analyser were used to measure the smoke and exhaust emission of the engine. The exhaust gas was made to pass through the probe of exhaust gas analyzer to measure the levels of carbon monoxide (CO), carbon dioxide (CO₂), oxides of nitrogen (NO_x) and hydrocarbon (HC) and then passed through the probe of smoke meter to measure smoke density. The AVL 615 Indi meter software, along with the necessary instruments and sensors was used to measure the

in-cylinder pressure and crank angle for the combustion. The pressure data were collected over 100 consecutive cycles at each crank angle and averaged. A series of tests were carried out at a constant speed of 1500 rpm and variable loads. Several blends of varying concentrations such as B0, B20, B40, B60, B80 and B100 were used as engine fuel. Each test was repeated three times to ensure the reproducibility of data and shown in Table 3.

3. Experimental results and discussions

3.1. The proposed methodology

The proposed methodology consists of three basic stages: (1) identification of the performance and emission criteria, (2) exploratory observations of the criteria and (3) ANP and TOPSIS to rank the alternatives. The schematic diagram of the proposed methodology for the selection of the best blend is shown in Fig. 2. In the first stage, the alternative blends and their evaluation criteria are identified, and then a decision hierarchy is framed. A single cylinder four stroke naturally aspirated compression ignition engine is operated with a constant speed of 1500 rpm for the different alternatives at variable load for observing the performance, emission and combustion characteristics in the second stage. In the last stage of the proposed methodology, pair-wise comparison matrices are framed using Saaty's scale to compute the relative weights of the performance and emission criteria. TOPSIS is used to rank the alternatives with the use of observed readings and relative weights of the evaluation criteria.

3.2. Criteria for selecting a best blend

In this study, the authors have used the literature survey to identify the evaluation criteria for selection of best blend [8,12,13]. The criteria are arranged in a hierarchical structure as shown in Fig. 3. The group decision making technique gives an opportunity to include the opinions of different IC engine experts and the engine manufacturers in the decision making process. The identified criteria are described below:

3.2.1. Oxides of nitrogen (NO_x)

The formation of nitrogen oxides (NO_x) depends on the peak flame temperature, ignition delay, and the content of nitrogen and oxygen available in the reacting mixture.

3.2.2. Smoke

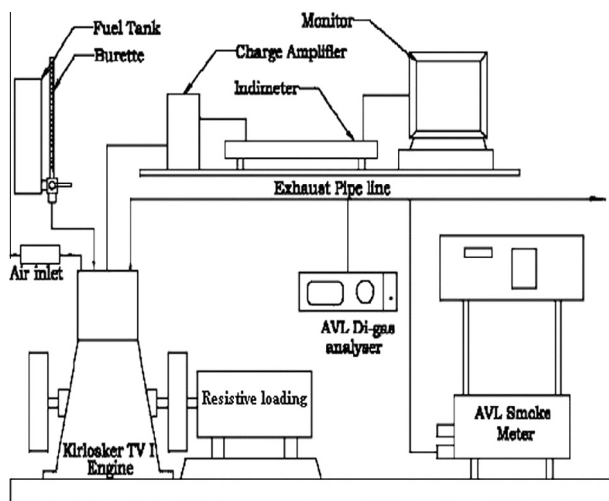
The oxygen content in the molecular structure of the biodiesel is the main factor that has effects on the smoke emission.

Table 1 Properties of blended fuels.

| Properties | Diesel fuel | B100 | B20 | B40 | B60 | B80 |
|------------------------------------|-------------|--------|--------|--------|--------|--------|
| Density (kg/m ³) | 850 | 885 | 852 | 860 | 869 | 879 |
| Specific gravity | 0.85 | 0.885 | 0.852 | 0.860 | 0.869 | 0.879 |
| Kinematic viscosity at 40 °C (Cst) | 3.05 | 4.74 | 4.1 | 4.22 | 4.36 | 4.52 |
| Calorific value (K J/kg) | 42,800 | 40,057 | 41,844 | 41,388 | 40,973 | 40,517 |
| Flash point (°C) | 56 | 114 | 44 | 63 | 79 | 98 |
| Fire point (°C) | 63 | 125 | 52 | 72 | 90 | 107 |

Table 2 Engine specifications.

| Items | Specification |
|--------------------|--|
| Make | Kirloskar |
| Cylinder number | 1 |
| Type | Four-stroke, stationary, constant speed, direct injection, air cooled, diesel engine |
| Bore × stroke | 80 mm × 110 mm |
| Displacement | 661 cc |
| Compression ratio | 17.5:1 |
| Max. power/speed | 4.4 kW/1500 rpm |
| Injection timing | 24° bTDC |
| Injection pressure | 210bar |

**Fig. 1** Schematic diagram of the engine setup.

Smoke emission is made to occur by the thermal cracking with the long chain HC molecules in an oxygen deficient combustion environment.

3.2.3. Brake Thermal Efficiency (BTE)

The brake power of a heat engine as a function of the thermal input from the fuel. It indicates how efficiently the fuel energy is converted into mechanical output.

3.2.4. Carbon dioxide (CO₂)

The CO₂ emissions of diesel engine indicate how efficiently the fuel is burnt inside the combustion chamber. If the combustion is good then most of the carbon will be converted into carbon dioxide during combustion.

3.2.5. Carbon monoxide (CO)

The carbon monoxide emission depends upon oxygen content, carbon content and combustion efficiency of the fuel. The carbon present in the fuel is oxidized with oxygen present in air to form CO.

3.2.6. Hydrocarbon (HC)

Hydrocarbons present in the fuel take part in the combustion reaction in the presence of oxygen and the remaining hydrocarbons come out as unburned hydrocarbons.

3.2.7. Exhaust Gas Temperature (EGT)

EGT represents the exact temperature of the fuel mixture after it is combusted in the cylinder. It indicates the efficiency of the combustion and is an important parameter in analyzing the emission values.

3.2.8. Ignition Delay (ID)

Ignition delay refers to the time difference between the start of injection and start of combustion in which each droplet gets ready for combustion by being atomized, vaporized, mixed with air, ignited through auto ignition and burned.

3.2.9. Combustion Duration (CD)

Combustion duration is the period from the start of the combustion to the end of combustion and can be determined from the heat release data.

3.2.10. Maximum Rate of Pressure Rise (MRPR)

In a CI engine, at the initial stage, pressure rise depends on the combustion rate that is influenced by the amount of fuel taking part in the premixed combustion phase.

4. Methods

4.1. Analytic network process

ANP is a method proposed by Saaty [26]. In ANP the decision problem is structured network to deal with decision without making assumptions about the independence of higher level elements from lower level elements [26]. In the literature, ANP has been applied in many complicated decision making problems. The ANP has its own advantages and has produced ideal results in various fields. Chung et al. proposed ANP for the selection of product mix for efficient manufacturing in a semiconductor fabricator [27]. Coulter et al. developed and tested a comprehensive model for media selection and budget allocation using the ANP technique [28]. Gencer

Table 3 Scale of preference between two elements [68].

| Numerical value | Definition | Explanation |
|-----------------|-------------------------|---|
| 1 | Equally preferred | Two activities contribute equally to the objective |
| 3 | Moderately preferred | Experience and judgement slightly favor one activity over another |
| 5 | Strongly preferred | Experience and judgment strongly or essentially favor one activity over another |
| 7 | Very strongly preferred | An activity is strongly favored over another and its dominance demonstrated in practice |
| 9 | Extremely preferred | The evidence favoring one activity over another is of the highest possible order of affirmation |
| 2, 4, 6, 8 | Intermediate values | When compromise is needed |

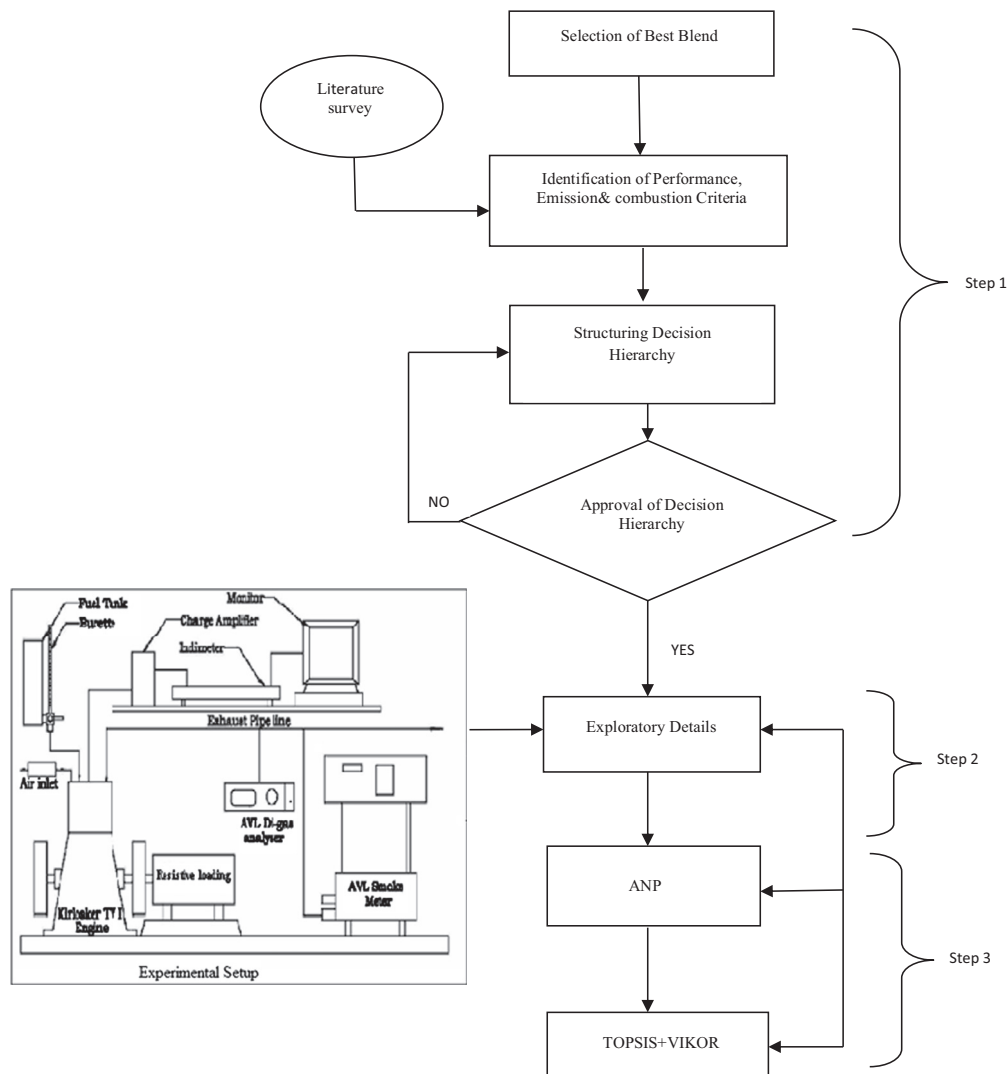


Fig. 2 Proposed methodology.

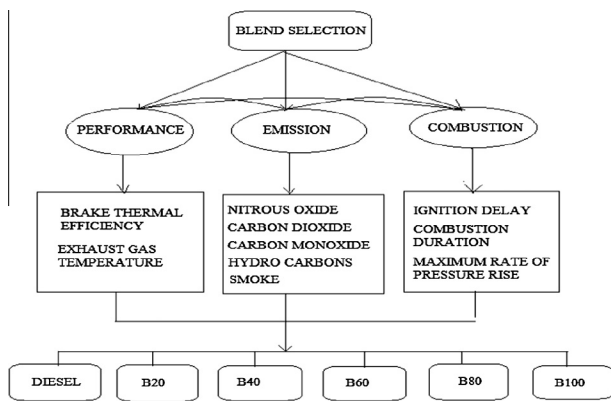


Fig. 3 Decision hierarchy.

and Gurpinar proposed the decision making model based on ANP and implemented in a company of electronic firm [25]. Wu et al. made an attempt to evaluate the favorable management strategies using ANP approach [29]. Chen et al.

incorporated the ANP approach to partner selection for strategic alliance [30]. Yang et al. proposed an evaluation and measurement of manufacturing performance for wafer fabricating industry by implementing ANP technique [31]. Banar et al. applied the multi criteria decision approach for choosing a recycling system using ANP and ELECTRE III [32]. Nikolai Bobylev implemented ANP approach to select optimum underground sewer construction technologies [33]. Pramod et al. applied ANP Analysis in Indian Telecommunication Service Supply Chain [35]. Yazgan et al. presented an ANP method to determine the weights of selection criteria for civil pilots [36]. Das et al. implemented ANP technique for selection of optimum non-traditional machining processes [37]. Toosi et al. used ANP as an effective tool for ranking water transfer projects [38]. Shahroudi et al. used a Multi-Criteria Decision Making approach of ANP-TOPSIS to evaluate suppliers in Iran's auto industry [39]. Chena et al. implemented an ANP technique to select green supply chain management strategies in an electronic industry [40]. Shiue et al. performed an evaluation of optimal recycling strategy for waste in solar energy industry based on BOCR model using ANP technique [41]. Promentilla et al. has proposed an ANP technique to rank

the potential CO₂ sources and sinks to identify sites for CO₂ capture and storage demonstration projects [42].

4.1.1. Modelling with ANP

4.1.1.1. *ANP model construction and problem structuring.* The ANP model consists of three elements, (1) goal of selecting the best alternative, (2) criteria and sub criteria for the model, and (3) alternatives. The elements in the hierarchy of determinants are divided into dimensions and attribute enablers. Identification of dimensions and attribute enablers at each level and definition of inter relationships is necessary for the development of decision model. The objective of hierarchy is ultimately to find out the alternatives.

4.1.1.2. *Establishment of pair wise comparison matrices.* For proper pair wise comparison of matrices, the opinion of experts has been collected using a nine-point scale as listed in Table 3. The number of matrices depends on the number of elements at each level and the order of the matrix at each level depends on the number of elements at the lower level to which it is linked. The pair-wise comparisons are made in terms of, how element A is more important than element B. It uses a ratio scale for each level of criteria, sub criteria, and alternatives, which allows the construction of relative weight matrices. For example, for a given criterion in the first row, if alternative A is “Very Strongly Preferred” over alternative B, then a weight of 7 is entered. If the alternative A is “Strongly Preferred” over alternative C, then a weight of 5 is entered. The judgment matrix A in which every element a_{ij} ($i, j = 1, 2, \dots, n$) is the quotient of weights of the criteria, as shown:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}, \quad (1)$$

$$a_{ii} = 1, \quad a_{ji} = 1/a_{ij}, \quad a_{ij} \neq 0.$$

The decision matrix for the optimum blend selection is formulated by identified criteria and alternatives using Eq. (1) and shown in Table 6 respectively.

At the last step, the mathematical process is commenced to normalize and find the relative weights of each matrix. The relative weights are given by the right Eigen vector (w) corresponding to the largest Eigen value (λ_{\max}), as

$$A_w = \lambda_{\max} w. \quad (2)$$

It should be noted that the quality of output of ANP is strictly related to the consistency of the pair-wise comparison judgments. The consistency is defined by relation between the entries of A : $a_{ij} \times a_{jk} = a_{ik}$. The Consistency Index (CI) is $CI = (\lambda_{\max} - n)/(n - 1)$. (3)

The consistency of the subjective input in the pair-wise comparison matrix can be determined by calculating a Consistency Ratio (CR). In general, the CR having value less than 0.1

is good. The CR for each square matrix is obtained from dividing CI values by Random Consistency Index (RCI) values. RCI is obtained from a large number of simulation runs and varies depending upon the order of matrix. Table 4 lists the value of the Random Consistency Index for the matrices of order 1–15 that is obtained by approximating random indices.

$$CR = CI/RI, \quad (4)$$

where RI is the average index for randomly generated weights.

4.1.1.3. *Pair wise comparison matrices of interdependencies.* In order to compare the all interdependencies in a network, a pair wise comparison has made among all attributes–enablers and corresponding values have been calculated.

4.1.1.4. *Formation and analyzing of super matrix.* The super matrix has been used to represent the various interdependence effects that exist between different process elements. It represents the relationship between criteria/sub criteria, and also shows the interdependencies between the criteria and sub criteria. Super matrix will be a partitioned matrix, where each sub-matrix is composed with a set of relationships between two graphical models. Initially, the super matrix will be considered as un-weighted matrix because its Eigen vectors may not be equal to 1. The super matrix needs to convert into reasonable priorities. In order to achieve this state un-weighted matrix has been multiplied by the priority weights generated from the clusters; hence the weighted matrix has been derived. The weighted super matrix is then raised to a significantly large power in order to have converged or stable values. This can be done by raising the super-matrix to the power $2k + 1$, where k is an arbitrary large number or it can be found out by using ANP solver.

4.2. TOPSIS

Hwang et al. were the first one to develop the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) [43]. The TOPSIS is relatively simple and fast, with a systematic procedure [44]. It has been proved as one of the best methods in addressing the rank reversal issue. The basic idea of TOPSIS is that the best decision should be made to be closest to the ideal and farthest from the non-ideal. Such ideal and negative-ideal solutions are computed by considering the other over all alternatives [45]. The positive-ideal solution is a solution that maximizes the benefit criteria and minimizes the cost criteria, whereas the negative ideal solution maximizes the cost criteria and minimizes the benefit criteria [45,47]. Many researchers have proposed the TOPSIS to solve the Multi Criteria Decision Making problem. Ho et al. proposed TOPSIS approaches for supplier evaluation and selection [48]. Alemi et al. approached TOPSIS to present the best artificial lift method selection for different circumstances of oil fields [49]. Fausto Cavallaro implemented Fuzzy TOPSIS to evaluate the molten salt as efficient source for storing heat energy in concentrated solar power (CSP) systems [50]. Yan et al.

Table 4 Average Random Consistency Index (RCI) based on matrix size.

| No | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|-----|---|---|------|------|------|------|------|------|------|------|------|------|------|------|------|
| RCI | 0 | 0 | 0.58 | 0.90 | 1.12 | 1.24 | 1.32 | 1.41 | 1.45 | 1.49 | 1.51 | 1.48 | 1.56 | 1.57 | 1.59 |

evaluated coal enterprises implementing hybrid GRD-TOPSIS approach [51]. Peiyue et al. applied TOPSIS based on entropy weight to assess the performance of groundwater quality [52]. Rouhani et al. presented fuzzy TOPSIS for the evaluation of enterprise systems [53]. Etghani et al. (2013) integrated NSGA-II and TOPSIS to optimize performance and emissions features of a diesel engine using biodiesel [54]. Tavana et al. used hybrid approach of ANP and TOPSIS for prioritization of advanced-technology projects at NASA [55]. Wang and Wang modified TOPSIS to evaluate high-tech industrial competitiveness within Chinese province [56]. Soufi et al. implemented TOPSIS to check feasibility of Bio lubricants over available lubricants [57].

4.2.1. Modelling with TOPSIS

(1) *Normalization of the evaluation matrix*: the process is to transform different scales and units among various criteria into common measurable units to allow comparisons across the criteria. Assume f_{ij} to be of the evaluation matrix R of alternative j under evaluation criterion i then an element r_{ij} of the normalized evaluation matrix R can be calculated by many normalization methods to achieve this objective.

$$r_{ij} = \frac{f_{ij}}{\sqrt{\sum_{j=1}^J f_{ij}^2}}, \quad j = 1, 2, 3, \dots, J, \quad i = 1, 2, 3, \dots, n. \quad (5)$$

(2) *Construction of the weighted normalized decision matrix*: the weighted normalized decision matrix can be calculated by multiplying the normalized evaluation matrix r_{ij} with its associated weight w_i to obtain the result

$$v_{ij} = w_i * r_{ij}, \quad j = 1, 2, 3, \dots, J, \quad i = 1, 2, 3, \dots, n, \quad (6)$$

where w_i is given by $\sum_{i=1}^n w_i = 1$

(3) *Determination of the positive and negative ideal solutions*: the positive ideal solution A^* indicates the most preferable alternative and the negative ideal solution A^- indicates the least preferable alternative.

$$A^* = \{v_1^*, \dots, v_n^*\} = \left\{ \left(\max_j v_{ij} | i \in I' \right), \left(\min_j v_{ij} | i \in I'' \right) \right\}, \quad (7)$$

$$A^- = \{v_1^-, \dots, v_n^-\} = \left\{ \left(\min_j v_{ij} | i \in I' \right), \left(\max_j v_{ij} | i \in I'' \right) \right\}. \quad (8)$$

(4) *Calculation of the separation measure*: the separation from the positive and negative ideal for each alternative can be measured by the n-criteria Euclidean distance.

$$D_j^* = \sqrt{\sum_{i=1}^n (v_{ij} - v_i^*)^2}, \quad j = 1, 2, 3, \dots, J, \quad (9)$$

$$D_j^- = \sqrt{\sum_{i=1}^n (v_{ij} - v_i^-)^2}, \quad j = 1, 2, 3, \dots, J. \quad (10)$$

(5) *Calculation of the relative closeness to the ideal solution*: the relative closeness of the i th alternative with respect to ideal solution A^+ is defined as

$$CC_j^* = \frac{D_j^-}{D_j^* + D_j^-}, \quad j = 1, 2, 3, \dots, J. \quad (11)$$

(6) *Ranking the priority*: a set of alternatives then can be preference ranked according to the descending order of CC_j^* .

4.3. VIKOR

VIKOR method was developed by Opricovic [57] to solve MCDM problems with conflicting and non-commensurable criteria. The method is focused on selecting and ranking from a set of alternatives and a compromise solution is obtained with the initial weights of a problem with conflicting criteria. Assuming that each alternative is computed according to each criterion function, the compromise ranking is performed by comparing the measure of closeness to the ideal alternative. Mahmoodzadeh et al. [58] proposed the integration of FAHP and TOPSIS in a project selection problem. But the TOPSIS methodology is not considering the relative distances from the ideal and negative ideal solution. The limitations can be overcome through VIKOR methodology. Only a few research papers have been found in the literature in connection with VIKOR application for various fields. Opricovic and Tzeng [59] have given a detailed comparison of TOPSIS and VIKOR and said that the compromise solution gives a maximum group utility of the group majority and a minimum individual regret of the opponent. Wu et al. [21] developed a hybrid fuzzy model application for the innovation capital indicator assessment of Taiwanese Universities using FAHP and VIKOR. Sanayei [60] proposed a hierarchical MCDM model based on fuzzy set theory and VIKOR method was proposed to deal with the supplier selection problems in the supply chain system. Kaya and Kahraman [61] proposed an integrated VIKOR-AHP methodology to determine the best renewable energy alternative for Istanbul. San Cristobal [62] applied the VIKOR method in the selection of a Renewable Energy project in the Spain. Ilangkumaran and Kumanan [63] applied VIKOR to select a suitable maintenance strategy for the textile spinning mill. Hsu et al. [64] implemented a hybrid DANP VIKOR methodology to select the best vendor for conducting the recycled material in Taiwan. Wang and Tzeng [65] have determined best brand marketing strategies combining DEMATEL with ANP and VIKOR methods. Raei and Jahromi [66] proposed hybrid model of ANP, VIKOR AND TOPSIS to identify an appropriate portfolio. Mehbodniya et al. [67] are used VIKOR method for the network selection.

4.3.1. Modelling with TOPSIS

The development of VIKOR is started with the following form of L_p -metric:

$$L_{pj} = \left\{ \sum_{i=1}^n [w_i (f_i^* - f_{ij}) / (f_i^* - f_i^-)]^p \right\}^{1/p}, \quad 1 \leq p \leq \infty, \quad j = 1, 2, \dots, J.$$

In the VIKOR method $L_{1,j}$ (as S_j) and $L_{\infty,j}$ (as R_j) are used to formulate ranking measure. The answers are obtained by $\min_j S_j$ is with the maximum group utility ("majority" rule), and the answer obtained by $\min R_j$ is with a minimum individual regret of the "opponent". The compromise ranking algorithm of VIKOR encompasses the following steps:

Table 5 Experimental performance, emission and combustion readings observed from engine for various alternative blends.

| Load (%) | Criteria | | | | | | | | | | | |
|----------|----------|-----------------------|-----------|---------|------------------------|-----------|----------|----------|----------|----------|----------------|--|
| | Blends | NO _x (ppm) | Smoke (%) | BTE (%) | CO ₂ (%vol) | CO (%vol) | HC (ppm) | EGT (°C) | ID (°CA) | CD (°CA) | MRPR (bar/°CA) | |
| 0 | Diesel | 233 | 9 | 0 | 1.8 | 0.07 | 27 | 142 | 18.58 | 44.68 | 5.88 | |
| | B20 | 227 | 15 | 0 | 1.5 | 0.06 | 26 | 145 | 18.4 | 43.6 | 5.08 | |
| | B40 | 216 | 14.9 | 0 | 1.6 | 0.06 | 25 | 148 | 18.12 | 42.42 | 4.81 | |
| | B60 | 209 | 17.6 | 0 | 1.7 | 0.05 | 24 | 150 | 18.08 | 42.98 | 4.55 | |
| | B80 | 183 | 20.5 | 0 | 1.8 | 0.05 | 23 | 151 | 17.58 | 40.8 | 4.55 | |
| | B100 | 174 | 24 | 0 | 1.8 | 0.04 | 21 | 154 | 17.52 | 39.06 | 4.01 | |
| 25 | Diesel | 501 | 16.1 | 17.92 | 3 | 0.07 | 32 | 196 | 17.68 | 47.18 | 7.95 | |
| | B20 | 505 | 18.7 | 18.71 | 2.6 | 0.06 | 29 | 199 | 17.25 | 45.52 | 7.48 | |
| | B40 | 495 | 21.9 | 17.58 | 2.7 | 0.06 | 26 | 202 | 17.02 | 45.04 | 6.88 | |
| | B60 | 483 | 19.7 | 16.61 | 2.9 | 0.05 | 23 | 205 | 16.92 | 44.6 | 6.35 | |
| | B80 | 476 | 22.7 | 16.29 | 3 | 0.05 | 23 | 208 | 16.48 | 42.76 | 6.08 | |
| | B100 | 475 | 27.9 | 15.43 | 3.1 | 0.04 | 22 | 209 | 15.64 | 40.52 | 5.61 | |
| 50 | Diesel | 989 | 20.4 | 26.22 | 4.4 | 0.06 | 32 | 241 | 16.82 | 49.82 | 9.09 | |
| | B20 | 976 | 23.5 | 28.14 | 3.9 | 0.06 | 30 | 244 | 16.37 | 48.12 | 8.15 | |
| | B40 | 957 | 25.8 | 26.37 | 3.9 | 0.05 | 30 | 247 | 16.02 | 47.1 | 7.28 | |
| | B60 | 939 | 24.2 | 24.77 | 4.2 | 0.05 | 28 | 249 | 15.92 | 46.06 | 6.52 | |
| | B80 | 926 | 29.3 | 24.14 | 4.3 | 0.04 | 25 | 252 | 15.76 | 44.24 | 6.15 | |
| | B100 | 909 | 32 | 23.15 | 4.4 | 0.04 | 24 | 255 | 15.02 | 43.18 | 5.48 | |
| 75 | Diesel | 1376 | 28.5 | 31.45 | 5.9 | 0.07 | 37 | 296 | 15.78 | 51.14 | 9.09 | |
| | B20 | 1331 | 28.7 | 31.94 | 5.5 | 0.06 | 33 | 301 | 15.28 | 49.8 | 8.29 | |
| | B40 | 1323 | 29.6 | 30.14 | 5.6 | 0.06 | 32 | 307 | 15.01 | 49.48 | 8.02 | |
| | B60 | 1317 | 34.1 | 29 | 5.8 | 0.05 | 31 | 311 | 14.76 | 47.82 | 7.22 | |
| | B80 | 1302 | 39.4 | 28.06 | 6 | 0.04 | 29 | 315 | 14.24 | 45.46 | 6.95 | |
| | B100 | 1289 | 42.5 | 27.6 | 6.1 | 0.03 | 26 | 318 | 13.16 | 45.8 | 6.28 | |
| 100 | Diesel | 1686 | 39.2 | 33.38 | 7.9 | 0.08 | 38 | 357 | 14.86 | 52.82 | 9.29 | |
| | B20 | 1651 | 42 | 35 | 7.3 | 0.07 | 34 | 365 | 14.68 | 52.68 | 8.76 | |
| | B40 | 1633 | 43 | 32.85 | 7.4 | 0.06 | 33 | 371 | 14.46 | 51 | 8.29 | |
| | B60 | 1618 | 45 | 30.76 | 7.9 | 0.05 | 33 | 362 | 14.14 | 49.18 | 8.29 | |
| | B80 | 1596 | 51.5 | 30.21 | 8 | 0.05 | 32 | 360 | 14.06 | 47.08 | 7.49 | |
| | B100 | 1584 | 52.8 | 29.68 | 8.1 | 0.04 | 30 | 374 | 13.42 | 46.42 | 7.22 | |

(1) The purpose of *normalizing the performance* matrix is to unify the unit of matrix entries. The determination of normalized values of alternatives x_{ij} is the numerical score of alternative j on criterion i . The corresponding normalized value f_{ij} is defined as follows.

$$f_{ij} = \left\{ \sum_{i=1}^m [x_{ij}^2] \right\}, \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n. \quad (12)$$

(2) Determine the best f_i^* and the worst f_i^- values for each criterion functions, $i = 1, 2, \dots, n$.

$$f_i^* = \max_j f_{ij}, \dots, f_i^- = \min_j f_{ij}. \quad (13)$$

(3) The *utility measure and the regret measure* for each maintenance alternative is given as

$$S_i = \sum_{i=1}^n w_i (f_i^* - f_{ij}) / (f_i^* - f_i^-), \quad (14)$$

$$R_j = \max_i [w_i (f_i^* - f_{ij}) / (f_i^* - f_i^-)], \quad (15)$$

where S_i and R_j represent the utility measure and the regret measure, respectively and w_j is the weight of the j th criterion.

(4) Calculate the *VIKOR index*

$$Q_j = v(S_i - S^*) / (S^- - S^*) + (1 - v)(R_j - R^*) / (R^- - R^*), \quad (16)$$

where $S^* = \min_j S_j$, $S^- = \max_j S_j$, $R^* = \min_j R_j$, $R^- = \max_j R_j$ and v is introduced as weight of the strategy of “the majority of criteria” (or “the maximum group utility”), here $v = 0.5$.

(5) Rank the order of preference The alternative with the smallest VIKOR value is determined to be the best value. Propose as a compromise solution the alternative A' which is ranked the best by the measure Q (minimum) if the following two conditions are satisfied:

C1. Acceptable advantage:

$$Q(A'') - Q(A') \geq DQ,$$

where A'' is the alternative with second position in the ranking list by Q ; $DQ = 1/(m - 1)$; m is the number of alternatives.

C2. Acceptable stability in decision making:

Table 9 Weighted super matrix for criteria.

| Criteria | NO _x | Smoke | CO ₂ | CO | HC | ID | CD | MRPR | BTE | EGT |
|-----------------|-----------------|-------|-----------------|-------|-------|-------|-------|-------|-------|-------|
| NO _x | 0 | 0.392 | 0.332 | 0.325 | 0.329 | 0.294 | 0.294 | 0.294 | 0.317 | 0.317 |
| Smoke | 0.322 | 0 | 0.205 | 0.191 | 0.171 | 0.173 | 0.173 | 0.173 | 0.186 | 0.186 |
| CO ₂ | 0.173 | 0.13 | 0 | 0.095 | 0.093 | 0.092 | 0.092 | 0.092 | 0.099 | 0.099 |
| CO | 0.1 | 0.078 | 0.071 | 0 | 0.056 | 0.056 | 0.056 | 0.056 | 0.061 | 0.061 |
| HC | 0.054 | 0.048 | 0.041 | 0.038 | 0 | 0.034 | 0.034 | 0.034 | 0.036 | 0.036 |
| ID | 0.144 | 0.144 | 0.144 | 0.142 | 0.144 | 0 | 0.209 | 0.186 | 0.155 | 0.155 |
| CD | 0.1 | 0.1 | 0.1 | 0.106 | 0.1 | 0.223 | 0 | 0.093 | 0.108 | 0.108 |
| MRPR | 0.035 | 0.035 | 0.035 | 0.032 | 0.035 | 0.056 | 0.07 | 0 | 0.037 | 0.037 |
| BTE | 0.062 | 0.062 | 0.062 | 0.062 | 0.062 | 0.062 | 0.062 | 0.062 | 0 | 0 |
| EGT | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0 | 0 |

Table 10 Limit matrix for criteria.

| Criteria | NO _x | Smoke | CO ₂ | CO | HC | ID | CD | MRPR | BTE | EGT |
|-----------------|-----------------|-------|-----------------|-------|-------|-------|-------|-------|-------|-------|
| NO _x | 0.249 | 0.248 | 0.248 | 0.248 | 0.248 | 0.248 | 0.248 | 0.248 | 0.248 | 0.248 |
| Smoke | 0.184 | 0.184 | 0.184 | 0.184 | 0.184 | 0.184 | 0.184 | 0.184 | 0.184 | 0.184 |
| CO ₂ | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 | 0.11 |
| CO | 0.069 | 0.069 | 0.069 | 0.069 | 0.069 | 0.069 | 0.069 | 0.069 | 0.069 | 0.069 |
| HC | 0.041 | 0.041 | 0.041 | 0.041 | 0.041 | 0.041 | 0.041 | 0.041 | 0.041 | 0.041 |
| ID | 0.134 | 0.134 | 0.134 | 0.134 | 0.134 | 0.134 | 0.134 | 0.134 | 0.134 | 0.134 |
| CD | 0.107 | 0.107 | 0.107 | 0.107 | 0.107 | 0.107 | 0.107 | 0.107 | 0.107 | 0.107 |
| MRPR | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 |
| BTE | 0.058 | 0.058 | 0.058 | 0.058 | 0.058 | 0.058 | 0.058 | 0.058 | 0.058 | 0.058 |
| EGT | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |

Table 11 Weights of criteria.

| Criteria | NO _x | Smoke | BTE | CO ₂ | CO | HC | EGT | ID | CD | MRPR |
|----------|-----------------|-------|-------|-----------------|-------|-------|-------|-------|-------|-------|
| Weights | 0.266 | 0.182 | 0.175 | 0.107 | 0.086 | 0.066 | 0.038 | 0.038 | 0.027 | 0.014 |

$$A^* = \{0.111, 0.112, 0.110, 0.107, 0.106, 0.105\} = \{0.105\},$$

$$A^- = \{0.111, 0.112, 0.110, 0.107, 0.106, 0.105\} = \{0.112\}.$$

Then the distance of each alternative from the positive ideal solution and the negative ideal solution with respect to each criterion are computed by using Eqs. (9) and (10) and tabulated in Table 15.

$$CC_1^* = \frac{0.042}{0.024 + 0.042} = 0.642; \quad CC_2^* = \frac{0.036}{0.018 + 0.036} = 0.662,$$

$$CC_3^* = \frac{0.025}{0.026 + 0.025} = 0.501; \quad CC_4^* = \frac{0.033}{0.017 + 0.033} = 0.657,$$

$$CC_5^* = \frac{0.025}{0.027 + 0.025} = 0.486; \quad CC_6^* = \frac{0.023}{0.044 + 0.023} = 0.342.$$

$$D_1^* = \sqrt{\frac{(0.12 - 0.09)^2 + (0.04 - 0.04)^2 + (0 - 0)^2 + (0.05 - 0.04)^2 + (0.04 - 0.02)^2}{+(0.03 - 0.02)^2 + (0.02 - 0.02)^2 + (0.02 - 0.02)^2 + (0.01 - 0.01)^2 + (0.01 - 0.01)^2}} = 0.038,$$

$$D_1^- = \sqrt{\frac{(0.12 - 0.12)^2 + (0.038 - 0.10)^2 + (0 - 0)^2 + (0.05 - 0.05)^2 + (0.04 - 0.04)^2}{+(0.03 - 0.03)^2 + (0.02 - 0.02)^2 + (0.02 - 0.02)^2 + (0.01 - 0.01)^2 + (0.01 - 0.01)^2}} = 0.064.$$

The closeness coefficients of the alternatives with respect to the ideal solution are calculated using Eq. (11). Finally, the alternatives are ranked using the relative closeness value.

The developed TOPSIS model for selection of best blend for 25% load is incorporated in Appendix A. A similar computational process is followed for 0%, 50%, 75% and 100% load

Table 12 Weighted normalized decision matrix (v_{ij}).

| Load (%) | Criteria | | | | | | | | | | |
|----------|----------|-----------------|-------|-------|-----------------|-------|-------|-------|-------|-------|-------|
| | Blends | NO _x | Smoke | BTE | CO ₂ | CO | HC | EGT | ID | CD | MRPR |
| 0 | Diesel | 0.457 | 0.210 | 0.000 | 0.431 | 0.512 | 0.452 | 0.391 | 0.420 | 0.431 | 0.495 |
| | B 20 | 0.445 | 0.350 | 0.000 | 0.359 | 0.439 | 0.435 | 0.399 | 0.416 | 0.421 | 0.428 |
| | B 40 | 0.424 | 0.348 | 0.000 | 0.383 | 0.439 | 0.418 | 0.407 | 0.410 | 0.409 | 0.405 |
| | B 60 | 0.410 | 0.411 | 0.000 | 0.407 | 0.366 | 0.401 | 0.413 | 0.409 | 0.415 | 0.383 |
| | B 80 | 0.359 | 0.479 | 0.000 | 0.431 | 0.366 | 0.385 | 0.415 | 0.398 | 0.394 | 0.383 |
| | B 100 | 0.341 | 0.560 | 0.000 | 0.431 | 0.293 | 0.351 | 0.424 | 0.396 | 0.377 | 0.338 |
| 25 | Diesel | 0.418 | 0.306 | 0.427 | 0.424 | 0.512 | 0.501 | 0.394 | 0.429 | 0.435 | 0.479 |
| | B 20 | 0.421 | 0.355 | 0.446 | 0.367 | 0.439 | 0.454 | 0.400 | 0.418 | 0.419 | 0.451 |
| | B 40 | 0.413 | 0.416 | 0.419 | 0.382 | 0.439 | 0.407 | 0.406 | 0.413 | 0.415 | 0.415 |
| | B 60 | 0.403 | 0.374 | 0.396 | 0.410 | 0.366 | 0.360 | 0.412 | 0.410 | 0.411 | 0.383 |
| | B 80 | 0.397 | 0.431 | 0.388 | 0.424 | 0.366 | 0.360 | 0.418 | 0.399 | 0.394 | 0.366 |
| | B 100 | 0.396 | 0.530 | 0.368 | 0.438 | 0.293 | 0.344 | 0.420 | 0.379 | 0.373 | 0.338 |
| 50 | Diesel | 0.425 | 0.319 | 0.419 | 0.429 | 0.483 | 0.461 | 0.397 | 0.429 | 0.438 | 0.514 |
| | B 20 | 0.420 | 0.367 | 0.450 | 0.380 | 0.483 | 0.433 | 0.402 | 0.418 | 0.423 | 0.461 |
| | B 40 | 0.411 | 0.403 | 0.422 | 0.380 | 0.403 | 0.433 | 0.407 | 0.409 | 0.414 | 0.412 |
| | B 60 | 0.404 | 0.378 | 0.396 | 0.409 | 0.403 | 0.404 | 0.410 | 0.406 | 0.405 | 0.369 |
| | B 80 | 0.398 | 0.457 | 0.386 | 0.419 | 0.322 | 0.361 | 0.415 | 0.402 | 0.389 | 0.348 |
| | B 100 | 0.391 | 0.500 | 0.370 | 0.429 | 0.322 | 0.346 | 0.420 | 0.383 | 0.379 | 0.310 |
| 75 | Diesel | 0.425 | 0.340 | 0.432 | 0.414 | 0.535 | 0.479 | 0.392 | 0.437 | 0.432 | 0.482 |
| | B 20 | 0.411 | 0.342 | 0.438 | 0.386 | 0.459 | 0.427 | 0.399 | 0.424 | 0.421 | 0.440 |
| | B 40 | 0.408 | 0.353 | 0.414 | 0.393 | 0.459 | 0.415 | 0.407 | 0.416 | 0.418 | 0.425 |
| | B 60 | 0.406 | 0.407 | 0.398 | 0.407 | 0.382 | 0.402 | 0.412 | 0.409 | 0.404 | 0.383 |
| | B 80 | 0.402 | 0.470 | 0.385 | 0.421 | 0.306 | 0.376 | 0.417 | 0.395 | 0.384 | 0.369 |
| | B 100 | 0.398 | 0.507 | 0.379 | 0.428 | 0.229 | 0.337 | 0.421 | 0.365 | 0.387 | 0.333 |
| 100 | Diesel | 0.423 | 0.349 | 0.425 | 0.415 | 0.546 | 0.464 | 0.399 | 0.425 | 0.432 | 0.460 |
| | B 20 | 0.414 | 0.374 | 0.446 | 0.383 | 0.477 | 0.415 | 0.408 | 0.420 | 0.431 | 0.433 |
| | B 40 | 0.409 | 0.383 | 0.419 | 0.389 | 0.409 | 0.403 | 0.415 | 0.413 | 0.417 | 0.410 |
| | B 60 | 0.406 | 0.401 | 0.392 | 0.415 | 0.341 | 0.403 | 0.405 | 0.404 | 0.402 | 0.410 |
| | B 80 | 0.400 | 0.459 | 0.385 | 0.420 | 0.341 | 0.391 | 0.403 | 0.402 | 0.385 | 0.370 |
| | B 100 | 0.397 | 0.470 | 0.378 | 0.425 | 0.273 | 0.366 | 0.418 | 0.384 | 0.380 | 0.357 |

Table 13 Positive ideal solution (A^*).

| Load (%) | Criteria | | | | | | | | | | |
|----------|-------------------------------|-------|-------|-----------------|-------|-------|-------|-------|-------|-------|--|
| | Positive ideal solution A^* | | | | | | | | | | |
| | NO _x | Smoke | BTE | CO ₂ | CO | HC | EGT | ID | CD | MRPR | |
| 0 | 0.000 | 0.015 | 0.091 | 0.038 | 0.023 | 0.025 | 0.038 | 0.015 | 0.010 | 0.007 | |
| 25 | 0.078 | 0.015 | 0.105 | 0.056 | 0.023 | 0.025 | 0.039 | 0.015 | 0.010 | 0.007 | |
| 50 | 0.079 | 0.015 | 0.104 | 0.058 | 0.023 | 0.028 | 0.040 | 0.015 | 0.010 | 0.007 | |
| 75 | 0.077 | 0.015 | 0.106 | 0.062 | 0.022 | 0.020 | 0.041 | 0.014 | 0.010 | 0.007 | |
| 100 | 0.078 | 0.015 | 0.106 | 0.064 | 0.024 | 0.024 | 0.041 | 0.015 | 0.010 | 0.007 | |

Table 14 Negative ideal solution (A^-).

| Load (%) | Criteria | | | | | | | | | | |
|----------|-------------------------------|-------|-------|-----------------|-------|-------|-------|-------|-------|-------|--|
| | Negative ideal solution A^- | | | | | | | | | | |
| | NO _x | Smoke | BTE | CO ₂ | CO | HC | EGT | ID | CD | MRPR | |
| 0 | 0.000 | 0.016 | 0.121 | 0.102 | 0.030 | 0.044 | 0.046 | 0.016 | 0.012 | 0.005 | |
| 25 | 0.064 | 0.016 | 0.112 | 0.097 | 0.033 | 0.044 | 0.047 | 0.017 | 0.012 | 0.005 | |
| 50 | 0.065 | 0.016 | 0.113 | 0.091 | 0.030 | 0.042 | 0.046 | 0.017 | 0.012 | 0.004 | |
| 75 | 0.066 | 0.016 | 0.113 | 0.092 | 0.032 | 0.046 | 0.046 | 0.017 | 0.012 | 0.005 | |
| 100 | 0.066 | 0.016 | 0.112 | 0.086 | 0.031 | 0.047 | 0.045 | 0.016 | 0.012 | 0.005 | |

Table 15 Distance of alternatives from PIS and NIS (D_j^*, D_j^-).

| Blends | D_j^* | | | | | D_j^- | | | | |
|--------|---------|----------|----------|----------|-----------|---------|----------|----------|----------|-----------|
| | 0% Load | 25% Load | 50% Load | 75% Load | 100% Load | 0% Load | 25% Load | 50% Load | 75% Load | 100% Load |
| Diesel | 0.038 | 0.024 | 0.020 | 0.029 | 0.026 | 0.064 | 0.042 | 0.034 | 0.032 | 0.024 |
| B 20 | 0.040 | 0.018 | 0.019 | 0.021 | 0.019 | 0.040 | 0.036 | 0.029 | 0.033 | 0.023 |
| B 40 | 0.036 | 0.025 | 0.019 | 0.021 | 0.015 | 0.041 | 0.025 | 0.022 | 0.030 | 0.022 |
| B 60 | 0.042 | 0.017 | 0.017 | 0.020 | 0.015 | 0.033 | 0.033 | 0.025 | 0.024 | 0.023 |
| B 80 | 0.050 | 0.027 | 0.028 | 0.027 | 0.024 | 0.033 | 0.025 | 0.019 | 0.023 | 0.020 |
| B 100 | 0.064 | 0.044 | 0.036 | 0.033 | 0.025 | 0.037 | 0.023 | 0.018 | 0.029 | 0.025 |

conditions for the other alternatives for evaluating the optimum blend. The obtained results are tabulated in Table 16. The B20 alternative is positioned at first ranking in 75% load condition. It also obtained the optimum ranking in all other load condition when compare to other blends. So the decision maker can select B20 as the best blend for operating the internal combustion engines. The preference order of alternative blends are B20 > B40 > Diesel > B60 > B80 > B100.

5.3. VIKOR computations

The VIKOR method is also proposed for selecting the optimum fuel blend among the alternative blends. The performance, emission and combustion characteristics of the engine at 0% load are considered to illustrate the computational procedure of the proposed VIKOR.

- Step1: The normalized decision matrix is calculated just as in TOPSIS methodology, the resulting normalized decision matrix is shown in Table 12.
- Step 2: Determination of Ideal Solution f_i^* and Negative Solution f_i^- by using Eq. (13) and the obtained results are given below

$$f_i^* = \{0.0029978, 0.0001159, 0.0000000, 0.0000198, 0.0000007, 0.0003434, 0.0018289, 0.0002393, 0.0005755, 0.0000768\},$$

$$f_i^- = \{0.0030893, 0.0004221, 0.0000000, 0.0000317, 0.0000009, 0.0003883, 0.0027084, 0.0003081, 0.0006888, 0.0000657\}.$$

- Step 3: The values of S_i and R_i is calculated using Eqs. (14) and (15) the obtained results are given in Table 17.
- Step 4: The VIKOR Index is calculated using Eq. (16). The obtained results are shown in Table 18.
- Step 5: As indicated in step five the smallest VIKOR index is determined as the best value and shown in Table 19.

The B20 is positioned at first ranking in 0%, 25% and 100% load conditions whereas diesel is also positioned at first in 75% load conditions respectively. So the B20 blend is the optimum blend for operating the internal combustion engine.

6. Results and discussions

The results obtained through the proposed methodology ANP-VIKOR and ANP-TOPSIS are tabulated in Table 17.

Table 16 Closeness coefficient of alternatives (CC_j^*) and ranking of alternatives.

| Blends | Relative closeness to ideal solution and its ranking | | | | | | | | | |
|--------|--|------|----------|------|----------|------|----------|------|-----------|------|
| | 0% Load | Rank | 25% Load | Rank | 50% Load | Rank | 75% Load | Rank | 100% Load | Rank |
| Diesel | 0.630 | 1 | 0.642 | 3 | 0.632 | 1 | 0.521 | 4 | 0.477 | 5 |
| B 20 | 0.498 | 3 | 0.662 | 1 | 0.599 | 2 | 0.610 | 1 | 0.543 | 3 |
| B 40 | 0.530 | 2 | 0.501 | 4 | 0.533 | 4 | 0.586 | 2 | 0.598 | 1 |
| B 60 | 0.439 | 4 | 0.657 | 2 | 0.590 | 3 | 0.543 | 3 | 0.597 | 2 |
| B 80 | 0.396 | 5 | 0.486 | 5 | 0.403 | 5 | 0.462 | 6 | 0.450 | 6 |
| B 100 | 0.364 | 6 | 0.342 | 6 | 0.337 | 6 | 0.472 | 5 | 0.500 | 4 |

Table 17 S_i and R_i values of alternatives.

| Blends | S_i | | | | | R_i | | | | |
|--------|---------|----------|----------|----------|-----------|---------|----------|----------|----------|-----------|
| | 0% Load | 25% Load | 50% Load | 75% Load | 100% Load | 0% Load | 25% Load | 50% Load | 75% Load | 100% Load |
| Diesel | 0.1338 | 0.3524 | 0.1974 | 0.1924 | 0.1828 | 0.0865 | 0.0865 | 0.0790 | 0.0865 | 0.0865 |
| B 20 | 0.1061 | 0.1374 | 0.2213 | 0.3684 | 0.2834 | 0.0489 | 0.0512 | 0.0865 | 0.1463 | 0.0858 |
| B 40 | 0.2402 | 0.3263 | 0.3416 | 0.4165 | 0.3424 | 0.0599 | 0.0928 | 0.1020 | 0.1646 | 0.1287 |
| B 60 | 0.2464 | 0.4793 | 0.4771 | 0.4677 | 0.4484 | 0.0822 | 0.2158 | 0.1652 | 0.1787 | 0.1839 |
| B 80 | 0.7156 | 0.6060 | 0.5179 | 0.5640 | 0.6382 | 0.2657 | 0.2657 | 0.2076 | 0.2247 | 0.2503 |
| B 100 | 0.6193 | 0.6065 | 0.6619 | 0.6186 | 0.6166 | 0.1821 | 0.2652 | 0.2657 | 0.2657 | 0.2657 |

Table 18 Q_i values for $v = 0.5$.

| Blends | Q_i values for $v = 0.5$ | | | | | | | | | |
|--------|----------------------------|------|----------|------|----------|------|----------|------|-----------|------|
| | 0% Load | Rank | 25% Load | Rank | 50% Load | Rank | 75% Load | Rank | 100% Load | Rank |
| Diesel | 0.1094 | 2 | 0.3115 | 3 | 0.001 | 1 | 0.0012 | 1 | 0.1104 | 2 |
| B 20 | 0.0001 | 1 | 0.0012 | 1 | 0.0459 | 2 | 0.3733 | 2 | 0.0020 | 1 |
| B 40 | 0.1355 | 3 | 0.2984 | 2 | 0.2170 | 3 | 0.4807 | 3 | 0.2946 | 3 |
| B 60 | 0.1919 | 4 | 0.7482 | 4 | 0.5321 | 4 | 0.5803 | 4 | 0.5644 | 4 |
| B 80 | 1.0000 | 6 | 0.9995 | 6 | 0.6894 | 5 | 0.8217 | 5 | 0.9571 | 5 |
| B 100 | 0.7283 | 5 | 0.9990 | 5 | 1.000 | 6 | 1.000 | 6 | 0.9762 | 6 |

Table 19 Ranking of alternatives with the use of ANP-TOPSIS and ANP-VIKOR methodologies.

| Blends | Ranking of ANP-TOPSIS | | | | | Ranking of ANP-VIKOR | | | | |
|--------|-----------------------|----------|----------|----------|-----------|----------------------|----------|----------|----------|-----------|
| | 0% Load | 25% Load | 50% Load | 75% Load | 100% Load | 0% Load | 25% Load | 50% Load | 75% Load | 100% Load |
| Diesel | 1 | 3 | 1 | 4 | 5 | 2 | 3 | 1 | 1 | 2 |
| B 20 | 3 | 1 | 2 | 1 | 3 | 1 | 1 | 2 | 2 | 1 |
| B 40 | 2 | 4 | 4 | 2 | 1 | 3 | 2 | 3 | 3 | 3 |
| B 60 | 4 | 2 | 3 | 3 | 2 | 4 | 4 | 4 | 4 | 4 |
| B 80 | 5 | 5 | 5 | 6 | 6 | 6 | 6 | 5 | 5 | 5 |
| B 100 | 6 | 6 | 6 | 5 | 4 | 5 | 5 | 6 | 6 | 6 |

The ranking order at 50% load is considered to illustrate the result of VIKOR and TOPSIS analysis. The ranking order are positioned in descending order based on VIKOR index (Diesel = 0.001 > B20 = 0.045 > B40 = 0.217 > B60 = 0.532 > B80 = 0.689 < B100 = 1.00). The ranking order by TOPSIS is based on closeness coefficient (Diesel = 0.632 < B20 = 0.599 < B60 = 0.590 < B40 = 0.533 < B80 = 0.403 < B100 = 0.337). B20 is obtained as the best blend for both the methods where as B100 stands last ranking due to the fuel characteristics. The relative density of diesel is 850 kg/m³ and for B100, it is 885 kg/m³, where as B80–B20 increases with increasing concentration of biodiesel in blends. Similarly the flash point and fire point for diesel is 56 °C and 63 °C whereas for B100, it is 114 °C and 125 °C respectively that increases with blend ratio. The calorific value of biodiesel is lower than diesel which is 40,060 kJ/kg for diesel and 42,800 kJ/kg for biodiesel and found to decrease as blend proportion is increased. The effect of BTE, EGT, NO_x, CO, CO₂, HC, smoke, ID, CDU and MRPR in the engine with respect to the load varies with respect to the fuel characteristics. Researchers cannot be able to propose the best blend among B20, B40 and B60 blends, since the fuel characteristics are closer which creates contradiction to meet the emission norms and fuel economy. To overcome the above issue, the mathematical model of ANP-TOPSIS & ANP VIKOR were proposed to assess and prioritise the fuel blends.

Even though the first ranking (B20) is similar in both the methods, the ANP-TOPSIS has the following limitations over the ANP-VIKOR. It requires lengthy calculations to obtain the ideal and negative ideal solution of the criteria and separation measures and the closeness coefficients of the alternatives. The computational timing is more, if the number of alternatives and the criteria increase which was minimized by VIKOR. In TOPSIS, the closeness coefficients of alternatives are not always closest to ideal solution. For example, in VIKOR diesel is obtained at the first position with aggregate function of 1(1 – 0.001), which is very closest to ideal value 1. But in TOPSIS, the same diesel is positioned at first rank

with the closeness coefficient value of 0.632, which is not closest to ideal value 1 as compared to VIKOR aggregate function. Similar evaluation is carried out with 0%, 25%, 75% and 100% load. Thus, the final ranking based on both techniques is B20 > Diesel > B40 > B60 > B80 > B100. The application of hybrid VIKOR is providing valuable assistance for best blend selection decision-making problems.

7. Conclusion

The selection of best blend plays an imperative role for biodiesel usage in internal combustion engines. There are a number of performance, emission and combustion parameters that are to be considered before choosing the best blend which involves a multidimensional perspective. Therefore effective decision-making approach is essential to resolve the problem. ANP integrated TOPSIS and ANP integrated VIKOR decision making methods have been used to evaluate the best blend. ANP is used to compute the evaluation criteria weights and TOPSIS is employed to determine the priorities of the alternatives. Similarly ANP weights are taken as the input for VIKOR and the priorities of alternatives are determined. The proposed decision methods can help the decision makers such as engine manufacturers and R&D engineers to analyze and choose the best blend for the IC engines. The outranking methods are used for precise ranking results of alternatives. It is evident that the proposed approaches are different from the existing literature for the selection of best blend.

For further research, the analysis can be carried out by implementing fuzzy set theory. The biodiesel can be used in other type of engines such as multi cylinder engine and variable compression ratio engine (VCR) with same performance. In addition, group decision making approaches can be developed using various MCDM techniques such as ELECTRE, PROMETHEE & GRA for choosing the best blend. Further experiments can be conducted by varying blending concentrations in the range of 5% and 10% to get more accurate results.

Appendix A. Development of TOPSIS model for selection of best blend for 25% load

Step 1: Normalization of the evaluation matrix:

$$r_{ij} = \frac{f_{ij}}{\sqrt{\sum_{j=1}^J f_{ij}^2}} \quad j = 1, 2, 3, \dots, J; \quad i = 1, 2, 3, \dots, n. \tag{1}$$

| Load (%) | Criteria | | | | | | | | | | |
|--|-----------------------------------|-----------------------|-----------|----------|------------------------|-----------|----------|----------|-----------|----------|-----------|
| | Blends | NO _x (ppm) | Smoke (%) | BTE (%) | CO ₂ (%vol) | CO (%vol) | HC (ppm) | EGT (°C) | ID (%vol) | CD (ppm) | MRPR (°C) |
| <i>f_{ij}</i> | Diesel | 501 | 16.1 | 17.92 | 3 | 0.07 | 32 | 196 | 17.68 | 47.18 | 7.95 |
| | B 20 | 505 | 18.7 | 18.71 | 2.6 | 0.06 | 29 | 199 | 17.25 | 45.52 | 7.48 |
| | B 40 | 495 | 21.9 | 17.58 | 2.7 | 0.06 | 26 | 202 | 17.02 | 45.04 | 6.88 |
| | B 60 | 483 | 19.7 | 16.61 | 2.9 | 0.05 | 23 | 205 | 16.92 | 44.6 | 6.35 |
| | B 80 | 476 | 22.7 | 16.29 | 3 | 0.05 | 23 | 208 | 16.48 | 42.76 | 6.08 |
| | B 100 | 475 | 27.9 | 15.43 | 3.1 | 0.04 | 22 | 209 | 15.64 | 40.52 | 5.61 |
| | <i>f_{ij}²</i> | Diesel | 251001 | 259.21 | 321.1264 | 9 | 0.0049 | 1024 | 38416 | 312.5824 | 2225.952 |
| B 20 | | 255025 | 349.69 | 350.0641 | 6.76 | 0.0036 | 841 | 39601 | 297.5625 | 2072.07 | 55.9504 |
| B 40 | | 245025 | 479.61 | 309.0564 | 7.29 | 0.0036 | 676 | 40804 | 289.6804 | 2028.602 | 47.3344 |
| B 60 | | 233289 | 388.09 | 275.8921 | 8.41 | 0.0025 | 529 | 42025 | 286.2864 | 1989.16 | 40.3225 |
| B 80 | | 226576 | 515.29 | 265.3641 | 9 | 0.0025 | 529 | 43264 | 271.5904 | 1828.418 | 36.9664 |
| B 100 | | 225625 | 778.41 | 238.0849 | 9.61 | 0.0016 | 484 | 43681 | 244.6096 | 1641.87 | 31.4721 |
| $\frac{\sum_{j=1}^J f_{ij}^2}{\sqrt{\sum_{j=1}^J f_{ij}^2}}$ | | | 1436541 | 2770.3 | 1759.588 | 50.07 | 0.0187 | 4083 | 247791 | 1702.312 | 11786.07 |
| | | 1198.558 | 52.63364 | 41.94744 | 7.076016 | 0.136748 | 63.89836 | 497.7861 | 41.25908 | 108.5637 | 16.59061 |
| <i>r_{ij}</i> | Diesel | 0.418002 | 0.305888 | 0.427201 | 0.423967 | 0.511891 | 0.500795 | 0.393743 | 0.428512 | 0.434584 | 0.479187 |
| | B 20 | 0.42134 | 0.355286 | 0.446034 | 0.367438 | 0.438763 | 0.453846 | 0.39977 | 0.41809 | 0.419293 | 0.450857 |
| | B 40 | 0.412996 | 0.416084 | 0.419096 | 0.381571 | 0.438763 | 0.406896 | 0.405797 | 0.412515 | 0.414872 | 0.414692 |
| | B 60 | 0.402984 | 0.374285 | 0.395972 | 0.409835 | 0.365636 | 0.359947 | 0.411823 | 0.410092 | 0.410819 | 0.382747 |
| | B 80 | 0.397144 | 0.431283 | 0.388343 | 0.423967 | 0.365636 | 0.359947 | 0.41785 | 0.399427 | 0.39387 | 0.366472 |
| | B 100 | 0.39631 | 0.530079 | 0.367841 | 0.4381 | 0.292509 | 0.344297 | 0.419859 | 0.379068 | 0.373237 | 0.338143 |

Step 2: Construction of the weighted normalized decision matrix:

$$v_{ij} = w_i * r_{ij}, \quad j = 1, 2, 3, \dots, J; \quad i = 1, 2, 3, \dots, n. \tag{2}$$

where w_i is given by $\sum_{i=1}^n w_i = 1$

| Load (%) | Criteria | | | | | | | | | | |
|----------------------------|----------|-----------------|----------|----------|-----------------|----------|----------|----------|----------|----------|----------|
| | Blends | NO _x | Smoke | BTE | CO ₂ | CO | HC | EGT | ID | CD | MRPR |
| Criteria weights (w_i) | | 0.26567 | 0.18213 | 0.17497 | 0.10653 | 0.08647 | 0.06591 | 0.03821 | 0.03871 | 0.02704 | 0.01436 |
| <i>v_{ij}</i> | Diesel | 0.11105 | 0.055711 | 0.074747 | 0.045165 | 0.044263 | 0.033007 | 0.015045 | 0.016588 | 0.011751 | 0.006881 |
| | B 20 | 0.11194 | 0.064708 | 0.078043 | 0.039143 | 0.03794 | 0.029913 | 0.015275 | 0.016184 | 0.011338 | 0.006474 |
| | B 40 | 0.10972 | 0.075781 | 0.073329 | 0.040649 | 0.03794 | 0.026819 | 0.015505 | 0.015968 | 0.011218 | 0.005955 |
| | B 60 | 0.10706 | 0.068169 | 0.069283 | 0.04366 | 0.031617 | 0.023724 | 0.015736 | 0.015875 | 0.011109 | 0.005496 |
| | B 80 | 0.10551 | 0.07855 | 0.067948 | 0.045165 | 0.031617 | 0.023724 | 0.015966 | 0.015462 | 0.01065 | 0.005263 |
| | B 100 | 0.10529 | 0.096543 | 0.064361 | 0.046671 | 0.025293 | 0.022693 | 0.016043 | 0.014674 | 0.010092 | 0.004856 |

Step 3: Determination of the positive and negative ideal solutions:

$$V_i^* = \{v_1^*, \dots, v_i^*\} = \left\{ \left(\max_j v_{ij} | i \in I' \right), \left(\min_j v_{ij} | i \in I'' \right) \right\}, \tag{3}$$

$$V_i^- = \{v_1^-, \dots, v_i^-\} = \left\{ \left(\min_j v_{ij} | i \in I' \right), \left(\max_j v_{ij} | i \in I'' \right) \right\}. \tag{4}$$

| Load (%) | Criteria | | | | | | | | | | |
|-----------------------------|----------|-----------------|----------|----------|-----------------|----------|----------|----------|----------|----------|----------|
| | Blends | NO _x | Smoke | BTE | CO ₂ | CO | HC | EGT | ID | CD | MRPR |
| v _{ij} | Diesel | 0.11105 | 0.055711 | 0.074747 | 0.045165 | 0.044263 | 0.033007 | 0.015045 | 0.016588 | 0.011751 | 0.006881 |
| | B 20 | 0.11194 | 0.064708 | 0.078043 | 0.039143 | 0.03794 | 0.029913 | 0.015275 | 0.016184 | 0.011338 | 0.006474 |
| | B 40 | 0.10972 | 0.075781 | 0.073329 | 0.040649 | 0.03794 | 0.026819 | 0.015505 | 0.015968 | 0.011218 | 0.005955 |
| | B 60 | 0.10706 | 0.068169 | 0.069283 | 0.04366 | 0.031617 | 0.023724 | 0.015736 | 0.015875 | 0.011109 | 0.005496 |
| | B 80 | 0.10551 | 0.07855 | 0.067948 | 0.045165 | 0.031617 | 0.023724 | 0.015966 | 0.015462 | 0.01065 | 0.005263 |
| | B 100 | 0.10529 | 0.096543 | 0.064361 | 0.046671 | 0.025293 | 0.022693 | 0.016043 | 0.014674 | 0.010092 | 0.004856 |
| v _i [*] | | 0.10529 | 0.05571 | 0.07804 | 0.03914 | 0.02529 | 0.022693 | 0.01504 | 0.01467 | 0.01009 | 0.00688 |
| v _i ⁻ | | 0.11194 | 0.07855 | 0.06795 | 0.04517 | 0.04426 | 0.033007 | 0.01597 | 0.01659 | 0.01175 | 0.00526 |

Step 4: Calculation of the separation measure:

$$D_j^* = \sqrt{\sum_{i=1}^n (v_{ij_i} - v_i^*)^2}, \quad j = 1, 2, 3, \dots, J. \tag{5}$$

| Load (%) | Criteria | | | | | | | | | | |
|---|----------|-----------------|----------|----------|-----------------|---------|----------|----------|----------|----------|----------|
| | Blends | NO _x | Smoke | BTE | CO ₂ | CO | HC | EGT | ID | CD | MRPR |
| (v _{ij_i} - v _i [*]) ² | Diesel | 3.32E-05 | 0 | 1.09E-05 | 3.63E-05 | 0.00036 | 0.000106 | 0 | 3.66E-06 | 2.75E-06 | 0 |
| | B 20 | 4.42E-05 | 8.09E-05 | 0 | 0 | 0.00016 | 5.21E-05 | 5.3E-08 | 2.28E-06 | 1.55E-06 | 1.65E-07 |
| | B 40 | 1.97E-05 | 0.000403 | 2.22E-05 | 2.27E-06 | 0.00016 | 1.7E-05 | 2.12E-07 | 1.68E-06 | 1.27E-06 | 8.58E-07 |
| | B 60 | 3.14E-06 | 0.000155 | 7.67E-05 | 2.04E-05 | 4E-05 | 1.06E-06 | 4.77E-07 | 1.44E-06 | 1.03E-06 | 1.92E-06 |
| | B 80 | 4.91E-08 | 0.000522 | 0.000102 | 3.63E-05 | 4E-05 | 1.06E-06 | 8.48E-07 | 6.21E-07 | 3.11E-07 | 2.62E-06 |
| | B 100 | 0 | 0.001667 | 0.000187 | 5.67E-05 | 0 | 0 | 9.96E-07 | 0 | 0 | 4.1E-06 |

| Blends | D _j [*] = √(∑ _{i=1} ⁿ (v _{ij_i} - v _i [*]) ²) |
|--------|---|
| Diesel | 0.023516 |
| B 20 | 0.018474 |
| B 40 | 0.025058 |
| B 60 | 0.01736 |
| B 80 | 0.026556 |
| B 100 | 0.043774 |

$$D_j^- = \sqrt{\sum_{i=1}^n (v_{ij_i} - v_i^-)^2}, \quad j = 1, 2, 3, \dots, J. \tag{6'}$$

| Load (%) | Criteria | | | | | | | | | | |
|---|----------|-----------------|----------|----------|-----------------|---------|----------|----------|----------|----------|----------|
| | Blends | NO _x | Smoke | BTE | CO ₂ | CO | HC | EGT | ID | CD | MRPR |
| (v _{ij_i} - v _i ⁻) ² | Diesel | 7.86E-07 | 0.001667 | 0.000108 | 2.27E-06 | 0 | 0 | 9.96E-07 | 0 | 0 | 4.1E-06 |
| | B 20 | 0 | 0.001013 | 0.000187 | 5.67E-05 | 4E-05 | 9.58E-06 | 5.89E-07 | 1.63E-07 | 1.71E-07 | 2.62E-06 |
| | B 40 | 4.91E-06 | 0.000431 | 8.04E-05 | 3.63E-05 | 4E-05 | 3.83E-05 | 2.89E-07 | 3.83E-07 | 2.84E-07 | 1.21E-06 |
| | B 60 | 2.38E-05 | 0.000805 | 2.42E-05 | 9.07E-06 | 0.00016 | 8.62E-05 | 9.43E-08 | 5.08E-07 | 4.13E-07 | 4.1E-07 |
| | B 80 | 4.13E-05 | 0.000324 | 1.29E-05 | 2.27E-06 | 0.00016 | 8.62E-05 | 5.89E-09 | 1.27E-06 | 1.21E-06 | 1.65E-07 |
| | B 100 | 4.42E-05 | 0 | 0 | 0 | 0.00036 | 0.000106 | 0 | 3.66E-06 | 2.75E-06 | 0 |

| Blends | D _j ⁻ = √(∑ _{i=1} ⁿ (v _{ij_i} - v _i ⁻) ²) |
|--------|---|
| Diesel | 0.042229 |
| B 20 | 0.0362 |
| B 40 | 0.025162 |
| B 60 | 0.033313 |
| B 80 | 0.02508 |
| B 100 | 0.022735 |

Step 5: Calculation of the relative closeness to the ideal solution

$$CC_j^* = \frac{D_j^-}{D_j^* + D_j^-}, \quad j = 1, 2, 3, \dots, J. \quad (7)$$

| Blends | CC_j^* |
|--------|----------|
| Diesel | 0.642313 |
| B 20 | 0.662106 |
| B 40 | 0.501032 |
| B 60 | 0.657409 |
| B 80 | 0.485703 |
| B 100 | 0.341833 |

Step 6: Ranking the priority: a set of alternatives then can be preference ranked according to the descending order of CC_j^* .

| Blends | Relative closeness to ideal solution and its ranking | |
|--------|--|------|
| | CC_j^* | Rank |
| Diesel | 0.642 | 3 |
| B 20 | 0.662 | 1 |
| B 40 | 0.501 | 4 |
| B 60 | 0.657 | 2 |
| B 80 | 0.486 | 5 |
| B 100 | 0.342 | 6 |

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