NUMERICAL SOLUTION OF A MICROPOLAR FLOW BETWEEN A ROTATING AND A STATIONARY DISC

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Abstract—The steady flow of a micropolar fluid between two infinite discs, when one is held at rest and the other rotating with constant angular velocity, is considered. The equations of motion are reduced to a system of ordinary differential equations, which in turn are solved numerically using the Gauss-Seidel iterative procedure and Simpson’s rule, for four different combinations of the seven parameters involved. Results are given both in tabular and graphical form, and compared with the known theoretical results.

1. INTRODUCTION

The theory of micropolar fluids[5] has received considerable attention in recent years, the extent of which can be judged from the comprehensive review by Smith[15]. Poiseuille flow has been considered by Eringen[5], Condiff and Dahler[6] and Aero, Bulygin and Kuvshinskii[4]. Couette flow by Condiff and Dahler[6]. The micropolar model has been used to describe the flow of liquid crystals[7]. Singh and Smith[8] investigated the steady porous plane Couette and Poiseuille flows of a micropolar fluid. They found, for certain ranges of the rate of suction and injection, that the velocity is composed of a linear combination of real exponential terms, whereas for other values products of exponentials and sinusoidal terms occur, provided the material constants satisfy certain inequalities.

Smith and Guram[10] considered the two-dimensional incompressible motions of a micropolar fluid, in which the vorticity and spin are constant along a streamline at any time, are shown to be one of two types, and particular solutions obtained in a number of cases.

Guram and Smith[11] investigated the flow of a micropolar fluid bounded by a rigid wall moving with speed proportional to $t^n$ parallel to itself, with injection proportional to $t^{-1/2}$. The first two terms of a perturbation expansion for speed and spin in powers of $t$, with coefficients which are functions of $\eta \propto \gamma^{-1/2}$, were obtained, when the material constants satisfy the relation $(\mu + K) = \gamma$.

Guram and Smith[12] established the uniqueness and existence of rectilinear flow of a micropolar fluid, through a pipe of arbitrary cross-section, with no slip at the boundary, and a perturbation expansion obtained for flow through an elliptic pipe. They suggested a method of estimating the material constants, when the coupling constant is small.

Smith and Guram[13] considered the flow of a micropolar fluid, which is steady relative to a frame of reference rotating with small uniform angular velocity, when the velocity and spin are two-dimensional and depend only on the depth, and the pressure is independent of the horizontal co-ordinates. They obtained the orientation of the velocity, spin, stress and couple-stress vectors on the surface and the flow is determined when the surface is free from the couple stress.

The problem of rotating disc for Newtonian fluids has been considered by Kármán[1] and Cochran[2]. Datta and Sastry[14] considered the flow of a micropolar fluid between two infinite slowly rotating disks and obtained the series solutions in terms of the Reynolds number of the motion.

In this paper, the flow of a micropolar fluid contained between two infinite discs, when one is held at rest and the other is rotating with constant angular velocity about their common axis, is considered. The specific equations of motion are stated in Section 2. In Section 3, the equations of motion are reduced to seven ordinary differential equations, involving seven parameters. In Section 4, the finite-difference equations are given. The computational procedure is presented in Section 5. In Section 6, the numerical results are presented both in tabular and graphical form, and compared with the known theoretical results[14].
2. EQUATIONS OF MOTION

The equations of motion of a micropolar fluid are [5]

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (2.1) \]

\[ (\lambda + 2\mu + \kappa) \nabla (\nabla \cdot \mathbf{v}) - (\mu + \kappa) \nabla \times \nabla \times \mathbf{v} + \kappa \nabla \times \mathbf{v} - \nabla p + \rho f = \rho \dot{\mathbf{v}}, \quad (2.2) \]

\[ (\alpha + \beta + \gamma) \nabla (\nabla \cdot \mathbf{v}) - \gamma (\nabla \times \nabla \times \mathbf{v}) + \kappa \nabla \times \mathbf{v} - 2\kappa \mathbf{v} + \rho J = \rho \dot{\mathbf{v}}, \quad (2.3) \]

where \( \rho \) is the density, \( \mathbf{v} \) the velocity, \( \nu \) the micro-rotation or spin, \( p \) the thermo-dynamic pressure, \( f \) and \( \dot{f} \) the body force and couple per unit mass, and \( j \) the micro-inertia; \( \alpha, \beta, \gamma, \lambda, \mu, \) and \( \kappa \) are material constants (viscosity coefficients); and the dot signifies material differentiation.

The constitutive equations, giving the stress tensor \( t_{kl} \) and the couple stress tensor \( m_{kl} \), are (in curvilinear co-ordinates)

\[ t_{kl} = t_{kl0} + g_{kl} + \kappa (\nu_{k} - \nu), \quad (2.4) \]

\[ m_{kl} = \alpha \nu_{k} + \beta \nu_{k} + \gamma \nu_{k}, \quad (2.5) \]

where \( g_{kl} \) and \( \varepsilon_{kl} \) are the metric tensor and the covariant \( \varepsilon \)-symbol, respectively. The semi-colon denotes the covariant partial differentiation with respect to a space co-ordinate and repeated indices are summed. The stress vector \( t \) and couple stress vector \( m \) at a point on a surface with normal \( n \) are given by \( t_{kl} n_{k} = t_{k} \) and \( m_{kl} n_{k} = m_{k} \). The material constants must satisfy the following inequalities, derived from the Clausius-Duhem inequality:

\[ 3\lambda + 2\mu + \kappa \geq 0, 2\mu + \kappa \geq 0, \kappa \geq 0, 3\alpha + \beta + \gamma \geq 0, \gamma \geq |\beta|. \quad (2.6) \]

The boundary conditions assumed are \( \mathbf{v}(x, t) = \mathbf{v} B, \mathbf{v}(x, t) = \mathbf{v} B \) where \( x = 0 \) is a point on a rigid boundary with prescribed velocity \( v_{B} \) and spin \( \omega_{B} \). That is, we assume neither slip nor spin at a fixed boundary. Other possible boundary conditions are discussed in [4] and [6].

3. FLOW BETWEEN A STATIONARY AND A ROTATING DISC

We consider the flow of a micropolar fluid between two infinite co-axial discs, at a finite distance \( h \) apart from each other. The fluid motion is induced due to the motion of the lower disc which is rotating with constant angular velocity \( \Omega \). We use the cylindrical polar co-ordinate \( r, \theta, z \) system, where \( r = 0 \) is the axis of the rotation of the plane of the lower disc. We assume that the material constants of the micropolar fluid are independent of position and neglect body forces and body couples. We, also, assume that the flow is steady, incompressible and axially-symmetric, and so we look for a solution for which

\[ \mathbf{v} = (u(r, z), v(r, z), w(r, z)), \quad (3.1) \]

\[ \nu = (\nu_{1}(r, z), \nu_{2}(r, z), \nu_{3}(r, z)). \]

Substituting (3.1) in (2.1), (2.2) and (2.3), we obtain

\[ 1 \frac{\partial}{\partial r} (ru) + \frac{\partial w}{\partial z} = 0, \quad (3.2) \]

\[ (\mu + \kappa) \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} \frac{u}{r^2} \right) - \kappa \frac{\partial \nu_2}{\partial z} \frac{\partial p}{\partial r} = \rho \left( \frac{w \partial u}{\partial z} \frac{u}{r} + \frac{u \partial u}{\partial r} \right), \quad (3.3) \]

\[ (\mu + \kappa) \left( \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} \frac{v}{r^2} \right) + \kappa \left( \frac{\partial \nu_1}{\partial z} \frac{\partial u}{\partial r} + \frac{\partial \nu_3}{\partial r} \frac{\partial u}{\partial r} \right) = \rho \left( \frac{w \partial v}{\partial z} \frac{v}{r} + \frac{u \partial v}{\partial r} \right). \quad (3.4) \]
Numerical solution of a micropolar flow between a rotating and a stationary disc

\[
(\mu + \kappa) \left( \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right) + \kappa \frac{\partial}{\partial r} \left( \rho \frac{\partial v}{\partial r} \right) - \frac{\partial p}{\partial z} = \rho \left( \frac{u \partial w}{\partial r} + w \frac{\partial w}{\partial z} \right),
\]

(3.5)

\[
(\alpha + \beta + \gamma) \frac{\partial}{\partial r} \left( \frac{\partial v_1}{\partial r} + \frac{\partial v_3}{\partial r} \right) - \frac{\partial}{\partial z} \left( \frac{\partial v_2}{\partial z} - \frac{\partial v_3}{\partial z} \right) = \kappa \frac{\partial}{\partial z} - 2k \nu_1 = \rho \left( \frac{u \partial v_1}{\partial r} + w \frac{\partial v_1}{\partial z} \right),
\]

(3.6)

\[
The boundary conditions are
\]

\[
\text{when } z = 0: \quad u = 0, v = r \Omega, w = 0,
\]

\[
\nu_1 = 0, \nu_2 = 0, \nu_3 = 0, \rho = 0,
\]

(3.9)

and when \( z = h \): \( u = 0, v = 0, \nu_1 = \nu_2 = \nu_3 = 0 \).

(3.10)

Using the dimensional analysis, the velocity, the micro-rotation and the pressure are assumed to be of the form:

\[
u_1 = \frac{r \Omega}{h_1} G_1(\eta), \quad \nu_2 = \frac{r \Omega}{h_1} G_2(\eta), \quad \nu_3 = \Omega G_3(\eta),
\]

(3.11)

\[
p = p(z) = - (\mu + \kappa) \Omega \rho(\eta),
\]

where

\[
\eta = \frac{z}{h_1}.
\]

(3.12)

If we substitute (3.11) into the eqns (3.2)-(3.8), then after some calculations we obtain the following seven ordinary differential equations:

\[
2F_1 + F_3 = 0,
\]

(3.13)

\[
F_1^* - \eta_1 G_2 = \eta_2 (F_1 F_3 - F_2^2 + F_3^2),
\]

(3.14)

\[
F_2^* + \eta_1 G_1 = \eta_3 (F_2 F_3 + 2F_1 F_2),
\]

(3.15)

\[
F_3^* + 2 \eta_1 G_2 + P' = \eta_2 F_2 F_3',
\]

(3.16)

\[
G_1^* - \eta_3 F_2^* - 2 \eta_1 G_1 = \eta_4 (F_1 G_1 - F_2 G_3 + F_3 G_1),
\]

(3.17)

\[
G_2^* + \eta_2 F_1^* - 2 \eta_2 G_2 = \eta_4 (F_1 G_2 + F_2 G_1 + F_3 G_2),
\]

(3.18)
\( G_2^2 + 2\eta_1 G_1^1 + 2\eta_0 (F_2 - G_3) = \eta_1 F_1 G_2^1, \)  

(3.19)

where the prime denotes differentiation with respect to \( \eta \), and

\[
\begin{align*}
\eta_1 &= \frac{\kappa}{\mu + \kappa}, \\
\eta_2 &= \frac{\rho \Omega h_0^2}{\mu + \kappa}, \\
\eta_3 &= \frac{\kappa h_0^2}{\gamma}, \\
\eta_4 &= \frac{\rho \Omega h_0^2}{\alpha + \beta}, \\
\eta_5 &= \frac{\alpha + \beta}{\alpha + \beta + \gamma}, \\
\eta_6 &= \frac{\kappa h_0^2}{\alpha + \beta + \gamma},
\end{align*}
\]

(3.20)

and we note that all of these are dimensionless. The dimensions of the above parameters are as follows:

\[
\begin{align*}
[\mu, \kappa] &= ML^{-1}T^{-1}, \\
[\alpha, \beta, \gamma] &= MLT^{-1}, \\
[j] &= L^2, \\
[\Omega] &= T^{-1}, \\
[\eta_0] &= L, \\
[p] &= ML^{-3},
\end{align*}
\]

(3.21)

The boundary conditions (3.9) and (3.10), in the new independent variable become

\[
\begin{align*}
\eta = 0: & \quad F_1 = 0, F_2 = 1, F_3 = 0, G_1 = G_2 = G_3 = 0, P = 0, \\
\eta - 1: & \quad F_1 - F_2 = 0, G_1 - G_2 - G_3 = 0.
\end{align*}
\]

(3.22)

4. FINITE-DIFFERENCE EQUATIONS

The interval \([0, 1]\) considered is divided into grid points with uniform spacing \( h \). A typical grid point is denoted by \( \eta_i \). The eqns (3.14), (3.15), (3.17)-(3.19) are approximated by replacing all derivatives by central-difference approximations. The resulting finite-difference equations are the following:

\[
F_i^{(1)} - 2F_i^{(0)} + F_{i+1}^{(0)} = \frac{\eta_i h}{2}(G_i^{(1)} - G_{i+1}^{(2)}) + \eta_i \left[ \frac{1}{2} F_{i+1}^{(0)} h (F_{i+1}^{(0)} - F_i^{(1)}) \right. \\
- h^2 F_{i+1}^{(0)} F_i^{(1)} + h^2 F_i^{(0)} F_{i+1}^{(0)} \right],
\]

(4.1)

\[
F_i^{(2)} - 2F_i^{(1)} + F_{i+1}^{(1)} = -\frac{1}{2} \eta_i h (G_i^{(1)} - G_{i+1}^{(2)}) + \eta_i \left[ \frac{1}{2} F_{i+1}^{(1)} h (F_{i+1}^{(1)} - F_i^{(2)}) \right. \\
+ 2h^2 F_i^{(1)} F_{i+1}^{(1)} \right],
\]

(4.2)

\[
G_i^{(1)} - 2(1 + h^2 \eta_3) G_i^{(0)} + G_{i+1}^{(0)} = \eta_i \frac{h}{2} (F_{i+1}^{(0)} - F_i^{(1)}) + \eta_i \left[ h^2 F_{i+1}^{(0)} G_i^{(0)} - h^2 F_i^{(1)} G_{i+1}^{(0)} \right. \\
+ \frac{h}{2} F_{i+1}^{(0)} (G_{i+1}^{(0)} - G_i^{(1)}) \right],
\]

(4.3)

\[
G_i^{(2)} - 2(1 + \eta_3 h^2) G_i^{(1)} + G_{i+1}^{(1)} = -\frac{\eta_i h}{2} (F_{i+1}^{(1)} - F_i^{(2)}) + \eta_i \left[ h^2 F_{i+1}^{(1)} G_i^{(1)} + h^2 F_i^{(2)} G_{i+1}^{(1)} \right. \\
+ \frac{h}{2} F_{i+1}^{(1)} (G_{i+1}^{(1)} - G_i^{(2)}) \right],
\]

(4.4)
and
\[
G^{(3)}_i - 2(1 + \eta h)G^{(1)}_i + G^{(3)}_i = -\eta h(G^{(1)}_i - G^{(1)}_{i-1}) - 2h^2 \eta \zeta F^{(3)}_i + \frac{1}{2} h \eta \zeta F^{(3)}_i (G^{(3)}_i - G^{(3)}_{i-1}),
\]

(4.5)

which are solved for all grid points interior to the interval \([0, 1]\) with conditions (3.22) on \(F_1, F_2, G_1, G_2\) and \(G_3\) at the end points. The first-order eqns (3.13) and (3.16) are integrated using the prescribed conditions (3.22) on \(F_3\) and \(P\).

5. COMPUTATIONAL PROCEDURE

Now, we need to solve the above system of finite-difference equations at each required grid point. Each set of equations is solved in turn, subject to the appropriate boundary conditions, using the Gauss-Seidel iterative procedure [9, pp. 154–156], whereas the equations (3.13) and (3.16) are integrated using the Simpson's rule [9, pp. 67–69] with the formula given in [3, pp. 48] and the conditions on \(F_3\) and \(P\).

The iterative sequence is as follows:

(i) The equations (4.1) are solved subject to \(F_1 = 0\) when \(\eta = 0\) and \(\eta = 1\), using the most recently available values for \(F_1, F_2, F_3,\) and \(G_2\) to calculate the elements of the matrix associated with \(F_1\). Then the eqn (3.13) is solved by the Simpson's rule with \(F_3 = 0\) when \(\eta = 0\);

(ii) the computed solution for \(F_1\) and \(F_3\) is introduced into (4.2) and these equations are solved subject to \(F_2 = 1\) at \(\eta = 0\) and \(F_2 = 0\) at \(\eta = 1\). The most recently available information for \(F_1, F_2, F_3\) and \(G_1\) is again used for calculating the values of \(F_2\);

(iii) the solutions for \(F_1, F_2\) and \(F_3\) are introduced into (4.3)–(4.5), and these sets of equations are solved for \(G_1, G_2\) and \(G_3\) respectively, subject to \(G_1 = G_2 = G_3 = 0\) when \(\eta = 0\), and \(\eta = 1\), using the recent values for \(F_1, F_2, F_3, G_1, G_2, G_3\). The solutions for these equations provide updated information for the subsequent application of (i) and (ii).

The above procedure is repeated until all the solutions have approached to some required criterion of accuracy. The criterion

\[
|F_i^{(n+1)} - F_i^{(n)}| < 10^{-6}, |G_i^{(n+1)} - G_i^{(n)}| < 10^{-6},
\]

where \(i = 1, 2, 3\), was adopted as a terminating condition, where here the subscript denotes the number of a function and the superscript the iteration.

The computed solutions of \(F_1, F_3, G_2\) were introduced to solve (3.16) to calculate the pressure with \(P = 0\) when \(\eta = 0\).

6. RESULTS AND DISCUSSION

The results have been obtained for the following four cases:

<table>
<thead>
<tr>
<th>(\eta_1)</th>
<th>(\eta_2)</th>
<th>(\eta_3)</th>
<th>(\eta_4)</th>
<th>(\eta_5)</th>
<th>(\eta_6)</th>
<th>(\eta_7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0.3</td>
<td>0.8</td>
<td>0.2</td>
<td>0.02</td>
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<td>0.08</td>
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<td>0.45</td>
<td>1.4</td>
<td>0.3</td>
<td>0.03</td>
<td>0.85</td>
<td>0.12</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.6</td>
<td>1.6</td>
<td>0.4</td>
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<tr>
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<td>0.5</td>
<td>0.05</td>
<td>1.417</td>
<td>2.4</td>
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</table>

The calculations were carried out for two different grid sizes, namely (a) \(h = 0.025\); (b) \(h = 0.0125\). The accuracy of the results may be checked by comparing the results on the different grid sizes. For this comparison, in Tables 1–4, the values of \(F_1, F_2, F_3, G_1, G_2, G_3\) and \(P\) are given for the cases considered. The general trend of these results as the grid size is decreased tends to indicate that the solutions for \(h = 0.0125\) are accurate to within a 0.03% tolerance for \(F_1, F_2, F_3, G_1, G_2, G_3\) and 0.33% for \(P\).

Typical sets of solutions, showing the effects of the parameters \(\eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6\) and \(\eta_7\), are given in graphical form in Figs. 1–7. For all the four cases considered, it was found that as
### Table 1.

<table>
<thead>
<tr>
<th>Grid Size</th>
<th>$\eta$</th>
<th>$F_1$</th>
<th>$F_2$</th>
<th>$F_3$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$P$</th>
</tr>
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<tbody>
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<td>0.0</td>
<td>1.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
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<tr>
<td>0.1</td>
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<td>0.8987063</td>
<td>-0.0012296</td>
<td>0.0007350</td>
<td>0.0002138</td>
<td>0.0015623</td>
<td>0.0320491</td>
<td></td>
</tr>
<tr>
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<td>0.7974170</td>
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<td>0.0114781</td>
<td>0.000750</td>
<td>0.0090536</td>
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<tr>
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### Table 2.

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<th>Grid Size</th>
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<th>$F_3$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$P$</th>
</tr>
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<tbody>
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<td>0.0</td>
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Numerical solution of a micropolar flow between a rotating and a stationary disc

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Fig. 1.

Fig. 2.
the parameters increase
(1) the profiles of $F_1$, $G_1$, $G_2$ and $P$ were raised,
(2) the profiles of $F_2$ and $F_3$ were lowered,
(3) the profile of $G_2$ was raised between $\eta = 0$ and $\eta = 0.3$, and lowered between $\eta = 0.3$ and $\eta = 1.0$.

Figure 1 shows that $F_1$ is positive throughout between the two discs, showing thereby there is a radial outwards flow from the lower disc which is maximum at the plane $\eta = 0.4$ and starts approaching to zero towards the upper plate.

It is worthwhile to note here that in [14], $F_1$ is positive near the lower disc and negative near the upper disc because of the fact that in [14] the form of the pressure was taken to be dependent on two independent variables $r$ and $z$, whereas in the present paper we followed the form as in [2]. Also in the present investigation, the terms involving micro inertia are not neglected but it is noticed by comparing with the results in [14] that their effect is negligible as is assumed in [14].
Fig. 5.

Fig. 6.
The micro-rotation (or spin) distributions are shown in Figs. 4–6. In Fig. 4, the symmetry of the spin component $G_1$ about the plane $\eta = 0.5$, where it has maximum value confirms the power series solution obtained by Datta and Sastry[14]. Figure 6 shows that the spin component $G_1$ is positive everywhere implying that it is in the same direction in which the disc rotates, also confirming the result given in [14]. Figure 7 shows the profile of the pressure which is positive everywhere indicates that it is maximum at the plane $\eta = 0.4$ and approaches to zero towards the upper plate.

REFERENCES

15. A. C. Smith, Some problems in generalised fluid dynamics, Third Canadian Symposium on Fluid Dynamics, University of Toronto, (5-8 June 1978).