Magnetohydrodynamic unaxisymmetric stagnation-point flow and heat transfer of a viscous fluid on a stationary cylinder

Rasool Alizadeh\textsuperscript{a,\,*}, Asghar B. Rahimi\textsuperscript{b}, Mohammad Najafi\textsuperscript{c}

\textsuperscript{a} Department of Mechanical Engineering, Quchan Branch, Islamic Azad University, Quchan, Iran
\textsuperscript{b} Faculty of Engineering, Ferdowsi University of Mashhad, P.O. Box No. 91775-1111, Mashhad, Iran
\textsuperscript{c} Department of Mechanical and Aerospace Engineering, Tehran Science and Research Branch, Islamic Azad University, Tehran, Iran

Received 30 November 2015; revised 2 January 2016; accepted 16 January 2016
Available online 6 February 2016

\textbf{KEYWORDS}

Unaxisymmetric stagnation-point flow; 
Heat transfer; 
Stationary cylinder; 
Magnetohydrodynamic flow; 
Numerical solution; 
Non-uniform transpiration

\textbf{Abstract}

The steady-state viscous flow and heat transfer in the vicinity of an unaxisymmetric stagnation-point of an infinite stationary cylinder with non-uniform normal transpiration $U_0(\phi)$ and uniform transverse magnetic field and constant wall temperature are investigated. The impinging free-stream is steady and with a constant strain rate $\dot{\gamma}$. A reduction of Navier–Stokes and energy equations is obtained by use of appropriate similarity transformations. The semi-similar solution of the Navier–Stokes equations and energy equation has been obtained numerically using an implicit finite-difference scheme. All the solutions aforesaid are presented for Reynolds numbers, $Re = \dot{\gamma} a^2/2\nu$, ranging from 0.01 to 100 for different values of Prandtl number and magnetic parameter and for selected values of transpiration rate function, $S(\phi) = U_0(\phi)/\dot{\gamma} a$, where $a$ is cylinder radius and $\nu$ is kinematic viscosity of the fluid. Dimensionless shear-stresses corresponding to all the cases increase with the increase in Reynolds number and transpiration rate function while dimensionless shear-stresses decrease with the increase in magnetic parameter. The local coefficient of heat transfer (Nusselt number) increases with the increasing transpiration rate function and Prandtl number.

\textcopyright 2016 Faculty of Engineering, Alexandria University. Production and hosting by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

1. Introduction

There are many solutions of the Navier–Stokes and energy equations regarding the problem of stagnation-point flow and heat transfer in the vicinity of a flat plate or a cylinder. These studies were started by Hiemenz [1], who obtained an exact solution of the Navier–Stokes equations governing the two-dimensional stagnation-point flow on a flat plate, and were continued by Homann [2] with an analogous axisymmetric study and by Howarth [3] and Davey [4], whose results for stagnation-point flow against a flat plate for asymmetric cases were presented. Wang [5,6] was the first to find an exact solution for the problem of axisymmetric stagnation-point flow on an infinite stationary circular cylinder; this was continued by Gorla’s works [7–11], which are a series of steady and unsteady flows and heat transfer over a circular cylinder in the vicinity of the stagnation-point.
of the stagnation-point for the cases of constant axial movement and the special case of axial harmonic motion of a non-rotating cylinder. Cunning et al. [12] have considered the stagnation-point flow problem on a rotating circular cylinder with constant angular velocity; Grosch and Salwen [13] as well as Takhar et al. [14] studied special cases of unsteady viscous flow on an infinite circular cylinder. The most recent works of the same types are the ones by Saleh and Rahimi [15] and Rahimi and Saleh [16,17], which are exact solution studies of a stagnation-point flow and heat transfer on a circular cylinder with time-dependent axial and rotational movements, as well as studies by Abbasi and Rahimi [18–21], which are exact solutions of stagnation-point flow and heat transfer but on a flat plate. Some existing compressible flow studies but in the stagnation region of bodies and by using boundary layer equations include the study by Subhashini and Nath [22] as well as Kumari and Nath [23,24], which are in the stagnation region of a body, and work of Katz [25] as well as Afzal and Ahmad [26], Libby [27], and Gersten et al. [28], which are all general studies in the stagnation region of a body. Recently, Alizadeh et al. [29] have considered the unaxisymmetric stagnation-point flow and heat transfer of a viscous fluid on a moving cylinder with time-dependent axial velocity.

Studies of magnetohydrodynamic flow and heat transfer due to a stretching cylinder were performed by Ishak et al. [30] who obtained numerical solutions using the Keller-box method, Joneidi et al. [31] who obtained solutions using the homotopy analysis method, and Butt and Ali [32] who included the effects of entropy generation. Butt and Ali [32] generalized the results of Joneidi et al. by considering the cylinder to be embedded in a porous medium along with a partial slip boundary condition. Chauhan et al. [33] examined unsteady flow and heat transfer due to a stretching cylinder in consideration of two general types of thermal boundary conditions. Also, magnetohydrodynamic stagnation flow and heat transfer toward a stretching permeable cylinder have been analyzed in the recent studies conducted by Munawar et al. [34]. Uddin et al. [35] have considered the Group Analysis of Free Convection Flow of a Magnetic Nanofluid with Chemical Reaction. Studies of MHD axisymmetric flow of third grade fluid by a stretching cylinder were performed by Hayat et al. [36] who obtained solutions using the homotopy analysis method, Magnetohydrodynamic mixed convective slip flow over an inclined porous plate with viscous dissipation and Joule heating has been analyzed in the recent studies conducted by Das et al. [37]. Uddin et al. [38] have presented the hydromagnetic transport phenomena from a stretching or shrinking nonlinear nanomaterial sheet with Navier slip and convective heating. Adesanya and Falade [39] have studied a thermodynamic analysis of hydromagnetic third grade fluid flow through a channel filled with porous medium. Uddin et al. [40] studied the Group analysis and numerical computation of magneto-convective non-Newtonian nanofluid slip flow from a permeable stretching sheet and solved the governing problem by using Runge–Kutta–Fehlberg fourth-fifth order numerical method provided in the symbolic computer software Maple 14. Hayat and Nawaz [41] computed exact solution for the unsteady stagnation point flow of viscous fluid caused by an impulsively rotating disk. He noticed the effects of Hartman, Schmidt and mass Grashof numbers on the dimensionless velocity, temperature and concentration.

All the studies mentioned above are regarding the axisymmetric flow and heat transfer and none has considered the effect of flow being unaxisymmetric and magnetohydrodynamic. In the present analysis, the problem of magnetohydrodynamic unaxisymmetric stagnation-point flow and heat transfer of a viscous fluid on a stationary cylinder with non-uniform transpiration and constant wall temperature are considered for the first time. A reduction of Navier–Stokes and energy equations is obtained by use of appropriate similarity transformations. The semi-similar solution of these equations is obtained numerically using an implicit finite-difference scheme. All the solutions aforesaid are presented for Reynolds numbers, \( Re = ka^2/2 \), ranging from 0.01 to 100 for different values of Prandtl number and magnetic parameter and for

\[
\begin{align*}
\alpha & \quad \text{cylinder radius} \\
r & \quad \text{radial coordinate} \\
z & \quad \text{axial coordinate} \\
u, w & \quad \text{velocity components along (r, z)-axis} \\
B_0 & \quad \text{magnetic field} \\
T & \quad \text{temperature} \\
T_w & \quad \text{wall temperature} \\
T_\infty & \quad \text{freestream temperature} \\
k & \quad \text{thermal conductivity} \\
\dot{k} & \quad \text{freestream strain rate} \\
f(\eta, \varphi) & \quad \text{function related to u-component of velocity} \\
Nu & \quad \text{Nusselt number} \\
U_0(\varphi) & \quad \text{transpiration} \\
Re & \quad \text{Reynolds number} \\
Pr & \quad \text{Prandtl number} \\
M & \quad \text{magnetic parameter} \\
S(\varphi) & \quad \text{transpiration rate function} \\
P & \quad \text{non-dimensional fluid pressure} \\
p & \quad \text{fluid pressure} \\
h & \quad \text{heat transfer coefficient} \\
q_w & \quad \text{heat flow at wall} \\
\eta & \quad \text{similarity variable} \\
\varphi & \quad \text{angular coordinate} \\
\zeta & \quad \text{thermal diffusivity} \\
\rho & \quad \text{fluid density} \\
v & \quad \text{kinematic viscosity} \\
\sigma & \quad \text{fluid electrical conductivity} \\
\mu & \quad \text{dynamic viscosity} \\
\theta(\eta, \varphi) & \quad \text{non-dimensional temperature} \\
\sigma & \quad \text{shear stress}
\end{align*}
\]
selected values of transpiration rate function, 
\[ S(\phi) = \frac{U_0(\phi)}{ka}, \] 
where \( a \) is cylinder radius and \( \nu \) is kinematic viscosity of the fluid. Dimensionless shear-stresses corresponding to all the cases increase with the increase in Reynolds number and transpiration rate function while dimensionless shear-stresses decrease with the increase in magnetic parameter. The local coefficient of heat transfer (Nusselt number) increases with the increasing transpiration rate function and Prandtl number.

### 2. Problem formulation

Flow is considered in cylindrical coordinates \((r, \phi, z)\) with corresponding velocity components \((u, v, w)\); see Fig. 1. An external axisymmetric radial stagnation-point flow of strain rate \( \dot{k} \).
impinges on the cylinder of radius $a$, centered at $r = 0$. Because of existence of a selected transpiration function shown below, a laminar steady incompressible flow and heat transfer of a viscous fluid in the neighborhood of an unaxisymmetric stagnation-point of a stationary infinite circular cylinder with uniform transverse magnetic field and constant wall temperature form.

The fundamental laws that are helpful for the description of present problems are \[41\]:

**Mass equation:**

$$\nabla \cdot \mathbf{V} = 0$$  \hspace{1cm} (1)

**Momentum equation:**

$$\frac{d}{dt} \rho \mathbf{V} = -\nabla p + \mu \nabla^2 \mathbf{V} + \mathbf{J} \times \mathbf{B}$$  \hspace{1cm} (2)

**Energy equation:**

$$\frac{dT}{dt} = \alpha \cdot \nabla^2 T$$  \hspace{1cm} (3)

**Current density vector:**

$$\mathbf{J} = \sigma \mathbf{V} \times \mathbf{B}$$  \hspace{1cm} (4)

in which $\frac{d}{dt}$ is the material derivative, $\mathbf{V}$ is the velocity vector, $\mu$ is the viscosity of the fluid, $\rho$ is the fluid density, $\alpha$ is the fluid thermal diffusivity, $T$ is the temperature inside the boundary layer, $\mathbf{J}$ is the current density vector, $\mathbf{B}(= [0, 0, B_0])$ magnetic induction vector, $\sigma$ is the electrical conductivity of the fluid and $p$ is the fluid pressure and after the impingement has occurred, respectively. The velocity, temperature and pressure fields for unaxisymmetric flow are of the following form:

$$\mathbf{V} = [u(r, \varphi), w(r, \varphi, z)]$$, \hspace{1cm} $T = T(r, \varphi)$, \hspace{1cm} $p = p(r, z)$  \hspace{1cm} (5)

Using above definitions of velocity, temperature and pressure fields in Eqs. (1)–(4), the steady Navier–Stokes and energy equations in cylindrical polar coordinates governing the unax-

---

**Figure 3** Variation of $f(\eta, \varphi)$ in terms of (a) $\eta$ and (b) $\varphi$ at $Re = 10$. $S(\varphi) = \cos(\varphi)$ and for different values of magnetic parameter.

**Figure 4** Variation of $f(\eta, \varphi)$ in terms of (a) $\eta$ and (b) $\varphi$ at $M = 1$. $S(\varphi) = \cos(\varphi)$ and for different values of Reynolds numbers.
isymmetric incompressible flow and heat transfer with magnetohydrodynamic effects are as follows. [7,30]

Mass:
\[
\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0
\]
(6)

\( r \) - Momentum:
\[
\frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{1}{r} \left[ \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \phi^2} \right]
\]
(7)

\( z \) - momentum:
\[
\frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{1}{r} \left[ \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{u}{r^2} \frac{\partial^2 w}{\partial \phi^2} + \frac{\partial^2 w}{\partial z^2} \right]
\]
\[- \frac{\sigma B_0^2}{\rho} \frac{\partial^2 w}{\partial z^2}\]
(8)

Energy:
\[
u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = 2 \left[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} \right]
\]
(9)

The boundary conditions for the velocity field are as follows:
\[
w = 0, \quad u = -U_0(\phi) \quad r = a:
\]
(10)

\( r \to \infty \): \( w = 2kz, \quad u = -k \left( r - \frac{a^2}{r} \right) \)
(11)

And the two boundary conditions with respect to \( \phi \) are as follows:
\[
u(r, 0) = u(r, 2\pi)
\]
\[
\frac{\partial u(r, 0)}{\partial \phi} = \frac{\partial u(r, 2\pi)}{\partial \phi}
\]
(12)
in which Eq. (10) represents no-slip conditions on the cylinder wall, and the relations of Eq. (11) show that the viscous flow solution approaches, in a manner analogous to the Hiemenz wall, and the relations of Eq.(11) show that the viscous flow firmed by starting from continuity equation as the following:

\[ u \approx u_0 \]  

where \( T_{CW} \) is the cylinder surface temperature and \( T_\infty \) is the free-stream temperature.

A reduction of the Navier–Stokes equations is obtained by applying the following similarity transformations [5]:

\[ u = - \frac{k_0}{\sqrt{\eta}} f(\eta, \varphi), \quad w = 2k f'(\eta, \varphi)z, \quad p = \rho k_0^2 a^2 P \]  

where \( \eta = \frac{x}{a} \) is dimensionless radial variable. Transformations (15) satisfy Eq. (6) automatically and their insertion into Eqs. (7) and (8) yields a coupled system of differential equations in terms of \( f(\eta, \varphi) \), and an expression for the pressure:

\[ \eta f'' + f' + \frac{1}{4n} \frac{\partial^2 f}{\partial \varphi^2} + Re \left[ 1 + f'' - (f')^2 - M f' \right] = 0 \]  

\[ P - P_0 = - \frac{1}{2} \left( \frac{P_0}{\eta} \right) \left( \frac{f}{Re} \right) - \frac{1}{4Re} \int_0^\infty \frac{1}{\eta} \frac{\partial^2 f}{\partial \varphi^2} \, d\eta - 2 \left( \frac{x}{a} \right)^2 \]  

And the two boundary conditions with respect to \( \varphi \) are as follows:

\[ T(r, 0) = T(r, 2\pi) \]  

\[ \frac{\partial T(r, 0)}{\partial \varphi} = \frac{\partial T(r, 2\pi)}{\partial \varphi} \]  

for different values of \( \text{Re} = 1 \), \( \text{Re} = 10 \), \( \text{Re} = 50 \), \( \text{Re} = 100 \).
where $Re = \frac{r^2}{\nu}$ is the Reynolds number, $M = \frac{B}{\mu}$ is the magnetic parameter and prime indicates differentiation with respect to $\eta$. From conditions (10)–(12) the boundary conditions for Eqs. (16) and (17) are as follows:

\begin{align}
\eta & = 1 : \quad f(1, \varphi) = 0, \quad f(1, \varphi) = S(\varphi) \\
\eta & \rightarrow \infty : \quad f(\infty, \varphi) = 1
\end{align}

(18)

(19)

\begin{equation}
f(\eta, 0) = f(\eta, 2\pi), \quad \frac{\partial f(\eta, 0)}{\partial \varphi} = \frac{\partial f(\eta, 2\pi)}{\partial \varphi}
\end{equation}

(20)

in which, $S(\varphi) = \frac{\nu}{\mu} \frac{r^2}{\nu}$ is the transpiration rate function. Note that Eqs. (16) and (17) are the complete form of Eqs. (9) and (11) in Ref. [15]. These equations are the same if transpiration rate is constant.

For the sake of brevity, only results for selected values of $S(\varphi) = \cos(\varphi)$ and $S(\varphi) = \ln(\varphi)$ are shown in this paper.

To transform the energy equation into a non-dimensional form, we introduce

\begin{equation}
\theta(\eta, \varphi) = \frac{T(\eta, \varphi) - T_{\infty}}{T_w - T_{\infty}}
\end{equation}

(21)

Making use of Eqs. (15) and (21) in the energy equation and with neglecting the small dissipation terms, we have:

\begin{equation}
\eta \theta'' + \theta' + \frac{1}{4\eta} \frac{\partial^2 \theta}{\partial \varphi^2} + Re \cdot Pr (\theta')' = 0
\end{equation}

(22)

With boundary conditions as

\begin{align}
\eta & = 1 : \quad \theta(1, \varphi) = 1 \\
\eta & \rightarrow \infty : \quad \theta(\infty, \varphi) = 0
\end{align}

(23)

\begin{equation}
\theta(r, 0) = \theta(r, 2\pi), \quad \frac{\partial \theta(r, 0)}{\partial \varphi} = \frac{\partial \theta(r, 2\pi)}{\partial \varphi}
\end{equation}

(24)
Note that Eqs. (22) is the complete form of Eq. (14) in Ref. [15]. These equations are the same if transpiration rate is constant.

Eqs. (16) and (22), along with boundary conditions (18), (19), (20), (23) and (24), have been solved numerically by an implicit, iterative tri-diagonal finite difference method similar to that discussed by Blottner [42]. To assess the grid independence of the numerical scheme, the distributions of the $f(\eta, \varphi)$ function against $\eta$ on the cylinder were initially tested with different $(\eta, \varphi)$ mesh sizes of $51 \times 18, 92 \times 32, 166 \times 58, 299 \times 104$ and $538 \times 187$ in Fig. 2(a). In this set of mesh sizes, as can be seen, the coefficient of 1.8 was used in each test to increase the number of mesh grids in both directions of $\eta$ and $\varphi$. It was found that the variations of the $f(\eta, \varphi)$ function distributions on the cylinder were not significant, between $(\eta, \varphi)$ and mesh sizes of $(166 \times 58)$, $(299 \times 104)$ and $(538 \times 187)$. Hence, a $(299 \times 104)$ grid in $\eta - \varphi$ directions was applied for the computational domain in the cylinder. Fine, non-uniform grid spacing is used in the $\eta$-direction to capture the rapid changes, such as grid lines, being closer packed near the walls. On the other hand, a uniform mesh was implemented in the $\varphi$-direction. Fig. 2(b) illustrates a sample of computational meshes used in this investigation.

Validation in numerical study is very important to ensure accuracy, consistency and reliability of the numerical results obtained. For this reason, comparisons between present and previous results available for various values of $\eta$ and $Re$ are presented in Tables 1 and 2 accordingly.

3. Shear-stress

The shear-stress at the cylinder surface is calculated from the following:

$$\sigma = \mu \left[ \frac{\partial u}{\partial r} \right]_{r=a}$$  (25)
where \( \mu \) is the fluid viscosity. Using definition (15), the shear stress at the cylinder surface for semi-similar solutions becomes.

\[
\sigma = \mu \frac{2}{a} \left[ 2kz f'(1, \varphi) \right] \Rightarrow \frac{\sigma a}{4kz} = f'(1, \varphi)
\]  

(26)

Results for \( \sigma a/4kz \) against \( \varphi \) for different values of Reynolds numbers and transpiration rate function are presented later.

4. Heat transfer coefficient

The local heat transfer coefficient and rate of heat transfer for defined wall temperature case are given by the following:

\[
h = q_w \frac{T_u - T_\infty}{T_u - T_\infty} = -\frac{k}{T_u - T_\infty} \frac{\partial \theta(1, \varphi)}{\partial \eta} = -\frac{2k}{a} \frac{\partial \theta(1, \varphi)}{\partial \eta}
\]  

(27)

where

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure13}
\caption{Variation of Nusselt number in terms of \( \varphi \) (a) for different values of Prandtl number (b) for different values of transpiration rate function.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure14}
\caption{Schematic diagram of the transpiration function \( S(\varphi) = \ln(\varphi) \).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure15}
\caption{Variation of \( f(\eta, \varphi) \) in terms of (a) \( \eta \) and (b) \( \varphi \) at \( Pr = 1.0, S(\varphi) = \ln(\varphi) \) and for different values of magnetic parameter.}
\end{figure}
In terms of Nusselt number
\[ Nu = \frac{ha}{2k} = -\theta'(1, \varphi) \]  

(28)

5. Presentation of results

In this section solution of semi-similar Eqs. (16) and (22) along with surface shear-stresses for prescribed values of surface temperature for selected values of Reynolds and Prandtl numbers, Magnetic parameter and transpiration rate function, is presented. For the sake of brevity, only results for selected values of \( S(\varphi) = \cos(\varphi) \), \( S(\varphi) = \ln(\varphi) \) are shown in this paper.

5.1. \( S(\varphi) = \cos(\varphi) \)

Sample profiles of the \( f(\eta, \varphi) \) function against \( \eta \) and \( \varphi \) at \( Pr = 1.0 \), \( S(\varphi) = \ln(\varphi) \) and for different values of magnetic parameter.

\[ q_w = \frac{-2k}{a} \frac{\partial \theta(1, \varphi)}{\partial \eta} (T_w - T_\infty) \]

In terms of Nusselt number

\[ Nu = \frac{ha}{2k} = -\theta'(1, \varphi) \]  

Sample profiles of the \( f(\eta, \varphi) \) function against \( \eta \) and \( \varphi \) at \( Pr = 1.0 \), \( S(\varphi) = \ln(\varphi) \) and for different values of magnetic parameter.

5.1. \( S(\varphi) = \cos(\varphi) \)

Sample profiles of the \( f(\eta, \varphi) \) function against \( \eta \) and \( \varphi \) for \( Pr = 1.0 \), and for selected values of Magnetic parameter are presented in Fig. 3. As Magnetic parameter increases, the depth of diffusion of the fluid velocity field in radial and angular direction decreases. The presence of the transverse magnetic field produces a resistive force for the fluid flow. This force is called the Lorentz force, which leads to slow down the motion of electrically conducting fluid, which tends to increase the temperature.

Sample profiles of the \( f(\eta, \varphi) \) function against \( \eta \) and \( \varphi \) for \( Pr = 1.0 \), and for selected values of Reynolds numbers are presented in Fig. 4. As Reynolds number increases, the depth of diffusion of the fluid velocity field in radial and angular direction increases.

Figure 16 Variation of \( f(\eta, \varphi) \) in terms of (a) \( \eta \) and (b) \( \varphi \) at \( Pr = 1.0 \), \( S(\varphi) = \ln(\varphi) \) and for different values of magnetic parameter.

Figure 17 Variation of \( \theta(\eta, \varphi) \) in terms of (a) \( \eta \) and (b) \( \varphi \) at \( Pr = 1.0 \), \( S(\varphi) = \ln(\varphi) \) and for different values of magnetic parameter.
Effects of transpiration rate function on $f(\eta, \phi)$ function against $\eta$ and $\phi$ for $Pr = 1.0$ and selected value of Reynolds number $Re = 1.0$ are shown in Fig. 5. In this figure negative $S(\phi)$ is blowing rate and positive $S(\phi)$ is the suction rate. It is evident from this figure that, as transpiration rate function increases, the $f$ function increases and if $S(\phi)$ decreases, the $f$ function decreases. It is interesting to note that, as $S(\phi)$ increases, the depth of diffusion of the fluid velocity field in radial and angular direction increases. Sample profiles of the $f(\eta, \phi)$ functions in terms of $\eta$ and $\phi$ are depicted in Figs. 6–8, for selected values of Reynolds number, Magnetic parameter and transpiration rate function. From this figures, the initial slope of the $f$ function increases with increasing Reynolds number and transpiration rate function while the initial slope of the $f$ function decreases with increasing Magnetic parameter. In fact the increase in transpiration rate function and Reynolds number decreases the thickness of the boundary layer.

Effect of variation of Magnetic parameter and Reynolds numbers on $\theta(\eta, \phi)$ function in terms of $\eta$ and $\phi$ for the case of constant surface temperature and $S(\phi) = \cos(\phi)$ is shown in Figs. 9,10. As Magnetic parameter increases, the depth of diffusion of the thermal boundary layer increases, and therefore the heat-transfer coefficient decreases while with increases in Reynolds number, the depth of diffusion of the thermal boundary layer decreases, and therefore the heat-transfer coefficient increases.

Sample profiles of the $\theta(\eta, \phi)$ function in terms of $\eta$ and $\phi$ for $Re = 10$, $Pr = 1.0$ are presented in Fig. 11, for selected values of transpiration rate function. It is seen that as the rate of transpiration rate function increases, the depth of diffusion of the temperature field decreases and thus the heat transfer coefficient increases.

Sample profiles of surface shear stress against $\phi$ for $Re = 10$, $Pr = 1.0$ and $S(\phi) = \cos(\phi)$ are shown in Fig. 12, for selected values of Magnetic parameter and Reynolds numbers.
bers. The increase in Reynolds Number and decrease in Magnetic parameter increase the wall shear-stress in \( \varphi \) direction and on the other hand cause that the value of fluid velocity in this direction approaches its value in inviscid flow, rapidly. In fact the increase in Reynolds number and decrease in Magnetic parameter decrease the thickness of the boundary layer.

Sample profiles of the Nusselt number (local heat transfer coefficient) for \( Re = 10, Pr = 1.0 \) and \( S(\varphi) = \cos(\varphi) \) are shown in Fig. 13, for selected values of Prandtl number and transpiration rate function. Nusselt number increases as Prandtl number and transpiration rate function increase (see Fig. 14).

5.2. \( S(\varphi) = \ln(\varphi) \)

Sample profiles of the \( f(\eta, \varphi) \) function against \( \eta \) and \( \varphi \) for \( Pr = 1.0, S(\varphi) = \ln(\varphi) \) and for selected values of Magnetic parameter are presented in Fig. 15. As Magnetic parameter increases, the depth of diffusion of the fluid velocity field in radial and angular direction decreases. Sample profiles of the \( f(\eta, \varphi) \) functions in terms of \( \eta \) and \( \varphi \) are depicted in Fig. 16, for selected values of Magnetic parameter. From these figures, the initial slope of the \( f \) function decreases with increasing Magnetic parameter. In fact the increase in Magnetic parameter increases the thickness of the boundary layer.

Effect of variation of constant Prandtl number and Magnetic parameter on \( \theta(\eta, \varphi) \) function in terms of \( \eta \) and \( \varphi \) for the case of constant surface temperature and \( S(\varphi) = \ln(\varphi) \) is shown in Figs. 17 and 18. As Prandtl number and Magnetic parameter increase, the depth of diffusion of the thermal boundary layer decreases, and therefore the heat-transfer coefficient increases.

Sample profiles of surface shear stress against \( \varphi \) for \( Pr = 1.0 \) and \( S(\varphi) = \ln(\varphi) \) are shown in Fig. 19, for selected values of Magnetic parameter. The increase in Magnetic parameter decreases the wall shear stress in \( \varphi \) direction and on the other hand causes that the value of fluid velocity in this direction approaches its value in inviscid flow, rapidly. In fact the increase in Magnetic parameter increases the thickness of the boundary layer.

Sample profiles of the Nusselt number (local heat transfer coefficient) for \( Re = 10 \) and \( S(\varphi) = \ln(\varphi) \) are shown in Figs. 20 and 21, for selected values of Prandtl number and Magnetic parameter. Nusselt number increases as Prandtl number increases while Nusselt number decreases with increase in Magnetic parameter.

6. Conclusions

A numerical solution of the Navier–Stokes equations and energy equation has been obtained for the problem of unaxi-symmetric stagnation-point flow on a stationary circular cylinder with non-uniform normal transpiration \( U_0(\varphi) \) and uniform transverse magnetic field and constant wall temperature. A reduction of these equations has been obtained by use of appropriate similarity transformations. The semi-similar solution of the Navier–Stokes equations and energy equation has been obtained numerically using an implicit finite-difference scheme. All the solutions aforesaid have been presented for Reynolds numbers ranging from 0.01 to 100 for different values of Prandtl number and magnetic parameter and for selected values of transpiration rate function. For all transpiration rate functions with increase in Reynolds numbers both components of the velocity field increase and for all transpiration rate function with increase in Prandtl number the temperature field decreases. Dimensionless shear-stresses corresponding to all the cases increase with the increase in Reynolds number and transpiration rate function while dimensionless shear-stresses decrease with the increase in magnetic parameter. The local coefficient of heat transfer (Nusselt number) increases with the increasing transpiration rate function and Prandtl number. For the case of axisymmetric stagnation-point flow, \( f = f(\eta), f = f(\eta) \) and \( S(\varphi) = 0, M = 0 \) or \( \frac{\partial f}{\partial \varphi} = 0, \frac{\partial^2 f}{\partial \varphi^2} = 0, \frac{\partial^2 \theta}{\partial \varphi^2} = 0 \) and similarity variables and component of velocity by Wang [5], as well as energy equation by Gorla [7] and Saleh and Rahimi [15] are reached.

References

Magnetohydrodynamic unaxisymmetric stagnation-point flow and heat transfer of a viscous fluid


