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Heterotic supersymmetry, anomaly cancellation and equations of motion

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ABSTRACT

We show that the heterotic supersymmetry (Killing spinor equations) and the anomaly cancellation imply the heterotic equations of motion in dimensions five, six, seven, eight if and only if the connection on the tangent bundle is an instanton. For heterotic compactifications in dimension six this reduces the choice of that connection to the unique $SU(3)$ instanton on a manifold with stable tangent bundle of degree zero.

1. Introduction. Field and Killing-spinor equations

The bosonic fields of the ten-dimensional supergravity which arises as low energy effective theory of the heterotic string are the spacetime metric g, the NS three-form field strength H, the dilaton ϕ and the gauge connection A with curvature F^A. The bosonic geometry considered in this Letter is of the form $R^{1,9-d} \times M^d$ where the bosonic fields are non-trivial only on M^d, $d \leqslant 8$. One considers the two connections $\nabla^{\pm} = \nabla^g \pm \frac{1}{2}H$, where ∇^g is the Levi-Civita connection of the Riemannian metric g. Both connections preserve the metric, $\nabla^{\pm} g = 0$ and have totally skew-symmetric torsion $T^{\pm}_{ijk} = g_{sk}(T^{\pm})^s_{ij} = \pm H_{ijk}$, respectively.

The bosonic part of the ten-dimensional supergravity action in the string frame is [1]

$$[illegible] - \frac{\alpha'}{4}\left(\mathrm{Tr}\left|F^A\right|^2 - \mathrm{Tr}\left|R\right|^2\right)\Big], \tag{1.1}$$

where R is the curvature of a connection ∇ on the tangent bundle and F^A is the curvature of a connection A on a vector bundle E.

The string frame field equations (the equations of motion induced from the action (1.1)) of the heterotic string up to two-loops [2] in sigma model perturbation theory are (we use the notations in [3])

$$
\begin{aligned}
&Ric^g_{ij} - \frac{1}{4}H_{imn}H_j^{mn} + 2\nabla^g_i\nabla^g_j\phi \\
&\quad - \frac{\alpha'}{4}\left[\left(F^A\right)_{imns}\left(F^A\right)_j^{mns} - R_{imns}R_j^{mns}\right] = 0; \\
&\nabla^g_i\left(e^{-2\phi}H^i_{jk}\right) = 0; \\
&\nabla^+_i\left(e^{-2\phi}\left(F^A\right)^i_j\right) = 0.
\end{aligned} \tag{1.2}
$$

The field equation of the dilaton ϕ is implied from the first two equations above.

A heterotic geometry will preserve supersymmetry if and only if, in 10 dimensions, there exists at least one Majorana–Weyl spinor ϵ such that the supersymmetry variations of the fermionic fields vanish, i.e. the following Killing-spinor equations hold [4]

$$
\begin{aligned}
&\delta_{\lambda} = \nabla_m\epsilon = \left(\nabla^g_m + \frac{1}{4}H_{mnp}\Gamma^{np}\right)\epsilon = \nabla^+\epsilon = 0; \\
&\delta_{\Psi} = \left(\Gamma^m\partial_m\phi - \frac{1}{12}H_{mnp}\Gamma^{mnp}\right)\cdot\epsilon = \left(d\phi - \frac{1}{2}H\right)\cdot\epsilon = 0; \\
&\delta_{\xi} = F^A_{mn}\Gamma^{mn}\epsilon = F^A\cdot\epsilon = 0,
\end{aligned} \tag{1.3}
$$

where λ, Ψ, ξ are the gravitino, the dilatino and the gaugino fields, respectively and $\cdot$ means Clifford action of forms on spinors.

The instanton equation, the last equation in (1.3) means that the curvature 2-form F^A is contained in the Lie algebra of the Lie group which is the stabilizer of the spinor ϵ. It is known that in dimension 5, 6, 7 and 8 the stabilizer is the group $SU(2)$, $SU(3)$, G_2 and $Spin(7)$, respectively. An instanton (a solution to the last equation in (1.3)) in dimension 5, 6, 7 and 8 is a connection with curvature 2-from which is contained in the lie algebra $su(2)$, $su(3)$, g_2 and $spin(7)$, respectively [5,4,6–10].

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The Green–Schwarz anomaly cancellation mechanism requires that the three-form Bianchi identity receives an α' correction of the form

$$dH = \frac{\alpha'}{4}\big(\mathrm{Tr}(R \wedge R) - \mathrm{Tr}\big(F^A \wedge F^A\big)\big). \tag{1.4}$$

A class of heterotic-string backgrounds for which the Bianchi identity of the three-form H receives a correction of type (1.4) are those with $(2,0)$ world-volume supersymmetry. Such models were considered in [11]. The target-space geometry of $(2,0)$-supersymmetric sigma models has been extensively investigated in [11,4,12]. Recently, there is revived interest in these models [13–15,9,3] as string backgrounds and in connection to heterotic-string compactifications with fluxes [16–23].

In writing (1.4) there is a subtlety to the choice of connection ∇ on M^d since anomalies can be cancelled independently of the choice [24]. Different connections correspond to different regularization schemes in the two-dimensional worldsheet non-linear sigma model. Hence the background fields given for the particular choice of ∇ must be related to those for a different choice by a field redefinition [25]. Connections on M^d proposed to investigate the anomaly cancellation (1.4) are ∇^g [4,9], ∇^+ [14], ∇^- [1,16,3,26], Chern connection ∇^c when $d = 6$ [4,20–23].

It is known [27,15] ([3] for dimension $d = 6$), that the equations of motion of type I supergravity (1.2) with $R = 0$ are automatically satisfied if one imposes, in addition to the preserving supersymmetry equations (1.3), the three-form Bianchi identity (1.4) taken with respect to a flat connection on TM, $R = 0$.

According to no-go (vanishing) theorems (a consequence of the equations of motion [28,27]; a consequence of the supersymmetry [29,30] for $SU(n)$-case and [9] for the general case) there are no compact solutions with non-zero flux and non-constant dilaton satisfying simultaneously the supersymmetry equations (1.3) and the three-form Bianchi identity (1.4) if one takes flat connection on TM, more precisely a connection satisfying $\mathrm{Tr}(R \wedge R) = 0$. Therefore, in the compact case one necessarily has to have a non-zero term $\mathrm{Tr}(R \wedge R)$. However, under the presence of a non-zero curvature 4-form $\mathrm{Tr}(R \wedge R)$ the solution of the supersymmetry equations (1.3) and the anomaly cancellation condition (1.4) obeys the second and the third equations of motion but does not always satisfy the Einstein equation of motion (the first equation in (1.2)) [3]. A quadratic expression for R which is necessary and sufficient condition in order that (1.3) and (1.4) imply (1.2) in dimension five, six, seven and eight are presented in [31–33]. In particular, if R is an instanton the supersymmetry equations together with the anomaly cancellation condition imply the equations of motion.

In this note we show that the converse statement holds. The main goal of the Letter is to prove

Theorem 1.1. *The heterotic supersymmetry equations* (1.3) *together with the anomaly cancellation* (1.4) *imply the heterotic equations of motion* (1.2) *on a manifold in dimensions five, six, seven and eight if and only if the connection on the tangent bundle in* (1.4) *is an* $SU(2)$, $SU(3)$, G_2 *and* $Spin(7)$ *instanton in dimension five, six, seven and eight, respectively.*

In the compact case in dimension six, it is shown in [32, Theorem 1.1b] that the no-go theorems in [29,30] force the flux H to vanish and the dilaton ϕ to be a constant for any compact solution to the heterotic supersymmetry (1.3) such that the $(-)$-connection on the tangent bundle is an $SU(3)$-instanton, i.e. such a solution is a Calabi–Yau manifold. This result combined with Theorem 1.1 leads to

Corollary 1.2. *In dimension six, a compact solution to the heterotic supersymmetry equations* (1.3) *satisfying anomaly cancellation* (1.4) *taken with respect to the* $(-)$*-connection imply the heterotic equations of motion* (1.2) *if and only if the flux H is zero, i.e. the solution is a Calabi–Yau manifold.*

Remark 1.3. Theorem 1.1 states that the heterotic equations of motion (1.2) are consequences of the heterotic supersymmetry (1.3) and the anomaly cancellation (1.4) if and only if the connection on the tangent bundle is of instanton type. On a compact solution to the gravitino and dilatino Killing spinor equations in dimension six, i.e. on a compact conformally balanced hermitian six-manifold with a holomorphic complex volume form [4] if there exists an $SU(3)$-instanton it is unique. Indeed, the non-Kähler version of the Donaldson–Uhlenbeck–Yau theorem [34,35] established by Li–Yau [36] asserts via the Kobayashi–Hitchin correspondence that there exists a unique $SU(3)$-instanton (Yang–Mills connection) if and only if the holomorphic tangent bundle is stable of degree zero. Thus, Theorem 1.1 shows that the choice of the connection taken on the tangent bundle in (1.3) for compact supersymmetric heterotic solutions to (1.2) in dimension six is fixed with the unique $SU(3)$-instanton.

This suggests that in order to find compact heterotic supersymmetric solutions to the equations of motion (1.2) in dimension six one needs to start with a conformally balanced hermitian six manifold admitting holomorphic complex volume form with stable tangent bundle of degree zero and take the corresponding unique $SU(3)$-instanton in (1.4) and (1.1).

Six-dimensional compact supersymmetric solutions with non-zero flux H and constant dilaton of this kind are presented in [32].

In the context of perturbation theory the curvature R^- of the $(-)$-connection is a one-loop-instanton due to the well-known identity $R^+_{ijkl} - R^-_{klij} = \frac{1}{2} dT_{ijkl}$, the first equation in (1.3) and (1.4) taken with respect to the $(-)$-connection. We thank the referee reminding this point to us. In this case, according to Theorem 1.1, the supersymmetry (1.3) together with the anomaly cancellation (1.4) imply the heterotic equations of motion (1.2) up to two-loops. In fact the $SU(3)$ case in dimension six has originally been dealt in [3]. The G_2 case in dimension seven has been investigated in [37, Section 6] when the anomaly cancellation has no zeroth order terms in α'. Compact up to two-loops solutions in dimension six with non-zero flux H and non-constant dilaton involving the $(-)$-connection are constructed in [38].

If the anomaly cancellation has zeroth order term in α' (for example in heterotic near horizons associated with AdS_3 investigated in the very recent paper [39]) then R^- is no longer one-loop instanton. In particular, in dimension six, Corollary 1.2 and Remark 1.3 is applicable suggesting a possible lines for further investigations.

One can take the anomaly contribution which appears at order α' as exact. Suppose that (1.4) is exact in the first order in α'. Then, in dimension six Corollary 1.2 applies and arguments in Remark 1.3 could be helpful in further developments.

Conventions: We choose a local orthonormal frame $e_1, \ldots, e_d$, identifying it with the dual basis via the metric and write $e_{i_1 i_2 \ldots i_p}$ for the monomial $e_{i_1} \wedge e_{i_2} \wedge \cdots \wedge e_{i_p}$.

We rise and lower the indices with the metric and use the summation convention on repeated indices. For example, $B_{ijk}C^{ijk} = B_i^{jk}C^i_{jk} = B_{ijk}C_{ijk} = \sum_{ijk=1}^n B_{ijk}C_{ijk}$.

For a p-form β we have the convention $\beta = \frac{1}{p!}\beta_{i_1 i_2 \ldots i_p} e_{i_1 i_2 \ldots i_p}$.

The tensor norm is denoted with $\|.\|^2$. For example $\|B\|^2 = B_{ijk}B^{ijk} = B_i^{jk}B^i_{jk} = B_{ijk}B_{ijk}$.

The curvature 2-forms R_{ij} of a connection ∇ are defined by $R_{ij} = [\nabla_i, \nabla_j] - \nabla_{[i,j]}$, $R_{ijkl} = R^s_{ijk} g_{ls}$.

The 4-form $\mathrm{Tr}(R \wedge R)$ reads $\mathrm{Tr}(R \wedge R)_{ijkl} = 2(R_{ijab}R_{klab} + R_{jkab}R_{ilab} + R_{kiab}R_{jlab})$.

The Hodge star operator on a d-dimensional manifold is denoted by $*_d$.

2. Geometry of the heterotic supersymmetry

Geometrically, the vanishing of the gravitino variation is equivalent to the existence of a non-trivial real spinor parallel with respect to the metric connection ∇^+ with totally skew-symmetric torsion $T = H$. The presence of ∇^+-parallel spinor leads to restriction of the holonomy group $Hol(\nabla^+)$ of the torsion connection ∇^+. Namely, $Hol(\nabla^+)$ has to be contained in $SU(2)$, $d = 5$ [40,41,31], $SU(3)$, $d = 6$ [4,29,30,9,42,14,17,18], the exceptional group G_2, $d = 7$ [40,13,9,43], the Lie group $Spin(7)$, $d = 8$ [13,44,9]. A detailed analysis of the induced geometries is carried out in [9] and all possible geometries (including non-compact stabilizers) are investigated in [45–48].

A consequence of the gravitino and dilatino Killing spinor equations is an expression of the Ricci tensor $Ric^+_{mn} = R^+_{imnj} g^{ij}$ of the $(+)$-connection, and therefore an expression of the Ricci tensor Ric^g of the Levi-Civita connection, in terms of the suitable trace of the torsion three-form $T = H$ (the Lee form) and the exterior derivative of the torsion form $dT = dH$ (see [40] for dimensions 5 and 7, [29] for dimension 6 (more precisely for any even dimension) and [44] for dimension 8 as well as [31–33]).

We recall that the Ricci tensors of ∇^g and ∇^+ are connected by (see e.g. [40,33])

$$
\begin{aligned}
&Ric^g_{mn} = \frac{1}{2}\big(Ric^+_{mn} + Ric^+_{nm}\big) + \frac{1}{4} T_{mpq} T_n^{\ pq}, \\
&Ric^+_{mn} - Ric^+_{nm} = (\delta T)_{mn} = -(*_d d *_d T)_{mn}.
\end{aligned} \tag{2.1}
$$

2.1. Dimension five. Proof of Theorem 1.1 in $d = 5$

The existence of ∇^+-parallel spinor in dimension 5 determines an almost contact metric structure whose properties as well as solutions to gravitino and dilatino Killing-spinor equations are investigated in [40,41,31].

We recall that an almost contact metric structure consists of an odd-dimensional manifold M^{2k+1} equipped with a Riemannian metric g, vector field ξ of length one, its dual 1-form η as well as an endomorphism ψ of the tangent bundle such that $\psi(\xi) = 0$, $\psi^2 = -id + \eta \otimes \xi$, $g(\psi.,\psi.) = g(.,.) - \eta \otimes \eta$. In local coordinates we also have $\psi^i_j \xi^j = 0$, $\psi^i_s \psi^s_j = -\delta^i_j + \eta_j \xi^i$, $g_{st}\psi^s_i \psi^t_j = g_{ij} - \eta_i \eta_j$. The Reeb vector field ξ is determined by the equations $\eta(\xi) = \eta_s \xi^s = 1$, $(\xi \lrcorner d\eta)_i = d\eta_{si}\xi^s = 0$, where $\lrcorner$ denotes the interior multiplication. The fundamental form F is defined by $F(.,.) = g(.,\psi.)$, $F_{ij} = g_{is}\psi^s_j$ and the Nijenhuis tensor N of an almost contact metric structure is given by $N = [\psi.,\psi.] + \psi^2[.,.] - \psi[\psi.,.] - \psi[.,\psi.] + d\eta \otimes \xi$.

An almost contact metric structure is called normal if $N = 0$; contact if $d\eta = 2F$; quasi-Sasaki if $N = dF = 0$; Sasaki if $N = 0$, $d\eta = 2F$. The Reeb vector field ξ is Killing in the last two cases [49].

An almost contact metric structure admits a linear connection ∇^+ with torsion 3-form preserving the structure, i.e. $\nabla^+ g = \nabla^+ \xi = \nabla^+ \psi = 0$, if and only if the Nijenhuis tensor is totally skew-symmetric, and the vector field ξ is a Killing vector field [40]. In fact, if the Nijenhuis tensor is totally skew-symmetric then ξ is a Killing vector field exactly when [41, Proposition 3.1], [31]

$$
(\xi \lrcorner dF)_{ij} = dF_{sij}\xi^s = 0 \quad \Leftrightarrow \quad (\xi \lrcorner N)_{ij} = N_{sij}\xi^s = 0. \tag{2.2}
$$

In this case the torsion connection is unique. The torsion T of ∇^+ is expressed by [40,41,31] $T = \eta \wedge d\eta + d^\psi F + N$, where $d^\psi F = -dF(\psi.,\psi.,\psi)$, $(d^\psi F)_{ijk} = -dF_{str}\psi^s_i \psi^t_j \psi^r_k$. In particular one has $d\eta_{ij} = (\xi \lrcorner T)_{ij} = T_{sij}\xi^s$, $(\xi \lrcorner d\eta)_i = T_{sti}\xi^s\xi^t = 0$, $d\eta(.,.) = d\eta(\psi.,\psi.)$, $d\eta_{ij} = d\eta_{st}\,\psi^s_i \psi^t_j$.

Since $\nabla^+\xi = 0$ the restricted holonomy group $Hol(\nabla^+)$ of ∇^+ contains in $U(k)$ and $Hol(\nabla^+) \subset SU(k)$ is equivalent to the following curvature condition found in [40, Proposition 9.1]

$$
\begin{aligned}
&R^+_{ijkl}F^{kl} = 0 \quad \Leftrightarrow \quad R^+_{kijk} = Ric^+_{ij} = -\nabla^+_i \theta^5_j - \frac{1}{4}\psi^s_j\, dT_{islm}F^{lm}, \\
&\theta^5_i = \frac{1}{2}\psi^s_i T_{skl}F^{kl} = \frac{1}{2} dF_{ikl}F^{kl},
\end{aligned} \tag{2.3}
$$

where θ^5 is the Lee form defined in [41]. Consequently, $\theta^5(\xi) = 0$.

In dimension five the Nijenhuis tensor is totally skew-symmetric exactly when it vanishes [50]. In this case ξ is a Killing vector field [49], the Lee form determines completely the three form dF due to (2.2), $dF = \theta^5 \wedge F$. The dilatino equation admits a solution if and only if ([41], Proposition 5.5)

$$
2\,d\phi = \theta^5, \qquad *_{\mathbb{H}} d\eta = -d\eta, \tag{2.4}
$$

where $*_{\mathbb{H}}$ denote the Hodge operator acting in the four-dimensional orthogonal complement $\mathbb{H}$ of the vector ξ, $\mathbb{H} = Ker\eta$. In particular, there is no solution on any Sasaki 5-manifold.

The torsion (the NS three-form H) of a solution to gravitino and dilatino Killing spinor equations in dimension five is given by [40,31]

$$
H = T = \eta \wedge d\eta + 2d^\psi \phi \wedge F. \tag{2.5}
$$

An equivalent formulation is presented in [31]. The gravitino Killing spinor equation defines a reduction of the structure group $SO(5)$ to $SU(2)$ which is described in terms of forms by Conti and Salamon in [51] (see also [52]) as follows: an $SU(2)$-structure on five-dimensional manifold M^5 is $(\eta, \omega_1, \omega_2, \omega_3)$, where η is a 1-form and $\omega_1, \omega_2, \omega_3$ are 2-forms on M satisfying $\omega_q \wedge \omega_r = \delta_{qr} v$, $q, r = 1, 2, 3$, $v \wedge \eta \neq 0$, for some 4-form v, and $X \lrcorner \omega_1 = Y \lrcorner \omega_2 \Rightarrow \omega_3(X, Y) \geqslant 0$.

The gravitino and dilatino Killing-spinor equations have a solution exactly when there exists an $SU(2)$-structure $(\eta, \omega_1, \omega_2, \omega_3)$ satisfying [31] $d\omega_p = \theta^5 \wedge \omega_p$, $\theta^5(\xi) = 0$, $\theta^5 = 2\,d\phi$, $*_{\mathbb{H}}\, d\eta = -d\eta$. This means that the 'conformal' structure $\bar\eta = \eta$, $\bar\omega_p = e^{-2\phi}\omega_p$ is quasi Sasaki with $*_{\mathbb{H}} d\eta = -d\eta$.

In addition to these equations, the vanishing of the gaugino variation requires the 2-form F^A to be of instanton type [5,4,6–10]. In dimension five, an $SU(2)$-instanton is a connection A with curvature two form $F^A \in su(2)$. The $SU(2)$-instanton condition reads

$$
\begin{aligned}
&\big(\xi \lrcorner F^A\big)_n = \xi^s F^A_{sn} = 0, \quad F^A(e_i, \psi e_i) = F^A_{st}F^{st} = 0, \\
&\psi^s_m \psi^t_n F^A_{mn} - F^A_{st} = 0.
\end{aligned} \tag{2.6}
$$

2.1.1. Theorem 1.1 in dimension 5

Proof. We have to investigate only the Einstein equation of motion in dimension 5. First we observe that $dd^\psi\phi(\xi, X) = -\xi\psi X\phi + \psi[\xi, X]\phi = 0$, where we applied to the dilaton ϕ the identity $0 = (\mathbb{L}_\xi \psi)X = [\xi, \psi X] - \psi[\xi, X]$, $\mathbb{L}$ is the Lie derivative, valid on any normal almost contact manifold [49], and use $\xi(\phi) = 0$. Then we calculate from (2.5) that $\psi^s_j\, dT_{islm}F^{lm} = -4\,d\eta_{si}\,d\eta_{sj} + [(2dd^\psi\phi)_{st}F^{st} - 8\|d\phi\|^2]g_{ij}$ which implies $\psi^s_j\, dT_{islm}\,F^{lm} = \psi^s_i\, dT_{jslm}\,F^{lm}$. Use the latter identity, substitute (2.4) into the second equation of (2.3) and the obtained equality insert into (2.1) using $2\nabla^g = 2\nabla^+ - T$ to get [31]

$$Ric^g_{ij} = -2\nabla^g_i d\phi_j - \frac{1}{4}\psi^s_j dT_{islm} F^{lm} + \frac{1}{4} T_{mpq} T^{pq}_n. \tag{2.7}$$

Substitute (1.4) into (2.7), use (2.6) and compare the result with the first equation in (1.2) to conclude that the supersymmetry equations (1.3) together with the anomaly cancellation (1.4) imply the first equation in (1.2) if and only if the next equality holds [31]

$$R_{mstr} R^{str}_n = \frac{1}{2}[R_{msij}R_{trij} + R_{mtij}R_{rsij} + R_{mrij}R_{stij}] F^{tr}\psi^s_n. \tag{2.8}$$

Multiplying (2.8) with $\xi^m\xi^n$ we obtain $\|\xi^m R_{mijk}\|^2 = 0$. Hence, $\xi \lrcorner R = 0$ which implies the first equation in (2.6). Thus, the curvature 2-form R_{ij} is defined on $\mathbb{H}$. The restriction of ψ on $\mathbb{H}$, $\psi|_{\mathbb{H}}$ is an almost complex structure on $\mathbb{H}$. The curvature two-form R_{ij} decomposes into two orthogonal parts R' and R'' under the action of ψ as follows

$$R'_{ij} = \frac{1}{2}\left(R_{ij} + \psi^s_i\psi^t_j R_{st}\right), \qquad R''_{ij} = \frac{1}{2}\left(R_{ij} - \psi^s_i\psi^t_j R_{st}\right),$$
$$\psi^s_i\psi^t_j R'_{st} = R'_{ij}, \qquad \psi^s_i\psi^t_j R''_{st} = -R''_{ij}. \tag{2.9}$$

An application of (2.9) to (2.8) yields

$$2\left(\|R'\|^2 + \|R''\|^2\right) = 2R_{mstr}R^{str}_m = -\|R_{mstr}F^{ms}\|^2 + 2\|R'\|^2 - 2\|R''\|^2.$$

Consequently, $\|R_{mstr}F^{ms}\|^2 + 4\|R''\|^2 = 0$ which is equivalent to the second and the third equalities in (2.6). Hence, R is an $SU(2)$-instanton. □

2.2. Dimension six. Proof of Theorem 1.1 in $d = 6$

The necessary and sufficient condition for the existence of solutions to the first two equations in (1.3) in an even dimension were derived by Strominger [4] and investigated by many authors since then. Solutions are complex conformally balanced manifold with non-vanishing holomorphic volume form satisfying an additional condition.

In dimension six any solution to the gravitino Killing spinor equation reduces the holonomy group $Hol(\nabla^+) \subset SU(3)$. This defines an almost hermitian structure (g, J) with non-vanishing complex volume form [4] which is preserved by the torsion connection. We adopt for the Kähler form $\Omega_{ij} = g_{is}J^s_j$. The Lee form θ^6 is defined by $\theta^6_i = -(*_6 d *_6 \Omega)_s J^s_i = \frac{1}{2} d\Omega_{ist}\Omega^{st}$.

An almost hermitian structure admits a (unique) linear connection ∇^+ with torsion 3-form preserving the structure, i.e. $\nabla^+ g = \nabla^+ J = 0$, if and only if the Nijenhuis tensor is totally skew-symmetric [40].

In addition, the dilatino equation forces the almost complex structure to be integrable and the Lee form to be exact determined by the dilaton. The torsion (the NS three-form H) is given by [4]

$$H_{ijk} = T_{ijk} = -J^s_i J^t_j J^r_k d\Omega_{str}, \qquad \theta^6_i = 2\,d\phi_i = \frac{1}{2} J^k_i T_{kst}\Omega^{st}. \tag{2.10}$$

Since $\nabla^+ g = \nabla^+ J = 0$ the restricted holonomy group $Hol(\nabla^+)$ of ∇^+ contains in $U(k)$ and $Hol(\nabla^+) \subset SU(k)$ is equivalent to the next curvature condition found in [29, Proposition 3.1]

$$R^+_{ijkl}\Omega^{kl} = 0 \quad \Leftrightarrow \quad Ric^+_{ij} = -\nabla^+_i \theta^6_j - \frac{1}{4} J^s_j dT_{islm}\Omega^{lm}. \tag{2.11}$$

In addition to these equations, the vanishing of the gaugino variation requires the 2-form F^A to be of instanton type. In dimension six, an $SU(3)$-instanton (or a hermitian-Yang-Mils connection) is a connection A with curvature two form $F^A \in su(3)$. The $SU(3)$-instanton condition is

$$F^A(e_i, Je_i) = F^A_{st}\Omega^{st} = 0, \qquad J^s_m J^t_n F^A_{mn} - F^A_{st} = 0. \tag{2.12}$$

In complex coordinates the condition (2.12) reads $F^A_{\mu\nu} = F^A_{\bar\mu\bar\nu} = 0$, $F^A_{\mu\bar\nu}\Omega^{\mu\bar\nu} = 0$ which is the well-known Donaldson–Uhlenbeck–Yau instanton.

2.2.1. Theorem 1.1 in dimension 6

Proof. We need to investigate the Einstein equation of motion in dimension 6. Substitute the second equation of (2.10) into (2.11) and the obtained equality insert into (2.1) and use $2\nabla^g = \nabla^+ - T$ to get [29]

$$Ric^g_{ij} = -2\nabla^g_i d\phi_j - \frac{1}{4} J^s_j dT_{islm}\Omega^{lm} + \frac{1}{4} T_{mpq} T^{pq}_n, \tag{2.13}$$

where we used that on a complex manifold $dT = 2\sqrt{-1}\partial\bar\partial\Omega$ is a $(2,2)$-form and therefore $J^s_j dT_{islm}\Omega^{lm}$ is symmetric in i and j.

Substitute (1.4) into (2.13), use (2.12) and compare the result with the first equation in (1.2) to conclude that the supersymmetry equations (1.3) together with the anomaly cancellation (1.4) imply the first equation in (1.2) if and only if the next equality holds [32]

$$R_{mstr}R^{str}_n = \frac{1}{2}[R_{msij}R_{trij} + R_{mtij}R_{rsij} + R_{mrij}R_{stij}]\Omega^{tr} J^s_n. \tag{2.14}$$

The two-form R_{ij} decomposes into two orthogonal parts R' and R'' under the action of J as follows

$$R'_{ij} = \frac{1}{2}\left(R_{ij} + J^s_i J^t_j R_{st}\right), \qquad R''_{ij} = \frac{1}{2}\left(R_{ij} - J^s_i J^t_j R_{st}\right),$$
$$J^s_i J^t_j R'_{st} = R'_{ij}, \qquad J^s_i J^t_j R''_{st} = -R''_{ij}. \tag{2.15}$$

We derive from (2.14) and (2.15) that

$$2\left(\|R'\|^2 + \|R''\|^2\right) = 2R_{mstr}R^{str}_m = -\|R_{mstr}\Omega^{ms}\|^2 + 2\|R'\|^2 - 2\|R''\|^2.$$

Hence, $|R_{mstr}\Omega^{ms}\|^2 + 4\|R''\|^2 = 0$ which is precisely the $SU(3)$-instanton condition (2.12) for R. □

Remark 2.1. Note that Theorem 1.1, Corollary 1.2 and Remark 1.3 are valid for any even dimension.

2.3. Dimension seven. Proof of Theorem 1.1 in $d = 7$

The existence of ∇^+-parallel spinor in dimension 7 determines a G_2 structure whose properties as well as solutions to gravitino and dilatino Killing-spinor equations are investigated in [40,13,43,15,9,33].

We briefly recall the notion of a G_2 structure. Consider the three-form Θ on $\mathbb{R}^7$ given by

$$\Theta = e_{127} - e_{236} + e_{347} + e_{567} - e_{146} - e_{245} + e_{135}.$$

The subgroup of $GL(7, \mathbb{R})$ fixing Θ is the Lie group G_2 of dimension 14. The 3-form Θ corresponds to a real spinor and therefore, G_2 can be identified as the isotropy group of a non-trivial real spinor.

The Hodge star operator supplies the 4-form $*_7\Theta$ given by

$$*_7\Theta = e_{3456} + e_{1457} + e_{1256} + e_{1234} + e_{2357} + e_{1367} - e_{2467}.$$

We have the well-known formula (see e.g. [53,9,54,55])

$$*_7\Theta_{ijpq} *_7 \Theta_{klpq} = 4\delta_{ik}\delta_{jl} - 4\delta_{il}\delta_{jk} + 2 *_7 \Theta_{ijkl}. \tag{2.16}$$

A seven-dimensional Riemannian manifold M is called a G_2-manifold if its structure group reduces to the exceptional Lie group G_2. The existence of a G_2-structure is equivalent to the existence of a global non-degenerate three-form which can be locally written as (2.16).

If $\nabla^g\Theta = 0$ then the Riemannian holonomy group is contained in G_2. It was shown by Gray [56] (see also [57–59]) that this condition is equivalent to $d\Theta = d * \Theta = 0$. The Lee form θ^7 is defined by [60] $\theta^7 = -\frac{1}{3} *_7 (*_7 d\Theta \wedge \Theta) = \frac{1}{3} *_7 (*_7 d *_7 \Theta \wedge *_7\Theta)$.

The precise conditions to have a solution to the gravitino Killing spinor equation in dimension 7 were found in [40]. Namely, there exists a non-trivial parallel spinor with respect to a G_2-connection with torsion 3-form T if and only if there exists a G_2-structure Θ satisfying $d *_7 \Theta = \theta^7 \wedge *_7\Theta$. In this case, the torsion connection ∇^+ is unique and the torsion 3-form T is given by $T = \frac{1}{6}(d\Theta, *_7\Theta)\,\Theta - *_7 d\Theta + *_7(\theta^7 \wedge \Theta)$. Applying Theorem 4.8 in [40] and the identity $*_7(\theta^7 \wedge \Theta) = -(\theta^7 \lrcorner *_7 \Theta)$ we can write

$$\theta^7_s = -\frac{1}{18}\big((*_7 d\Theta)_{ijk} *_7 \Theta_{sijk}\big), \qquad T_{ijk} *_7 \Theta_{sijk} = -6\theta^7_s. \tag{2.17}$$

The necessary conditions to have a solution to the system of dilatino and gravitino Killing spinor equations were derived in [13,40,43], and the sufficiency was proved in [40,43]. The general result [40,43] states that there exists a non-trivial solution to both dilatino and gravitino Killing spinor equations in dimension 7 if and only if there exists a G_2-structure Θ satisfying the equations $d *_7 \Theta = \theta^7 \wedge *_7\Theta$, $d\Theta \wedge \Theta = 0$, $\theta^7 = 2\,d\phi$, i.e. the conformal G_2-structure ($\bar\Theta = e^{-\frac{3}{2}\phi}\Theta$, $\bar g = e^{-\phi} g$) obeys the equations $d\bar{*}\bar\Theta = d\bar\Theta \wedge \bar\Theta = 0$.

The flux H of a solution to the gravitino and dilatino killing spinor equations is [13,40,43]

$$H = T = - *_7 d\Theta + 2 *_7 (d\phi \wedge \Theta). \tag{2.18}$$

The Ricci tensor of the torsion connection was calculated in [40] (see also [33])

$$Ric^+_{mn} = \frac{1}{12} dT_{mjkl} *_7 \Theta_{njkl} + \frac{1}{6}\nabla^+_m T_{jkl} *_7 \Theta_{njkl}. \tag{2.19}$$

Using the special expression of the torsion (2.18) and (2.17), Eq. (2.19) takes the form

$$\begin{aligned} Ric^+_{mn} &= \frac{1}{12} dT_{mjkl} *_7 \Theta_{njkl} - 2\nabla^+_m d\phi_n \\ &= \frac{1}{12} dT_{mjkl} *_7 \Theta_{njkl} - 2\nabla^g_m d\phi_n + d\phi_s T^s_{mn}. \end{aligned} \tag{2.20}$$

In addition to these equations, the vanishing of the gaugino variation requires the 2-form F^A to be of instanton type [5,4,6–10]. A G_2-instanton in dimension seven is a G_2-connection A with curvature $F^A \in \mathfrak{g}_2$. The latter can be expressed in any of the next two equivalent ways

$$F^A_{mn}\Theta^{mn}{}_p = 0 \quad \Leftrightarrow \quad F^A_{mn} = -\frac{1}{2} F^A_{pq}(*_7\Theta)^{pq}{}_{mn}. \tag{2.21}$$

2.3.1. Theorem 1.1 in dimension 7

Proof. We have to investigate the Einstein equation of motion in dimension 7. First we show that

$$dT_{mjkl} *_7 \Theta_{njkl} = dT_{njkl} *_7 \Theta_{mjkl}. \tag{2.22}$$

Indeed, the second identity in (2.1) and (2.18) yield

$$\begin{aligned} Ric^+_{mn} - Ric^+_{nm} &= (*_7 d *_7 T)_{mn} = -2\big(*_7(d\phi \wedge d\Theta)\big)_{mn} \\ &= 2\big(*_7(d\phi \wedge *_7 T)\big)_{mn} = 2\,d\phi_s T^s_{mn} \end{aligned} \tag{2.23}$$

which compared with the skew-symmetric part of (2.20) gives (2.22). In particular, (2.23) gives a proof of the second equality in (1.2) in dimension seven.

Insert (2.20) into the first equality in (2.1) and use (2.22) to get

$$Ric^g_{ij} = -2\nabla^g_i d\phi_j - \frac{1}{12} dT_{mjkl} *_7 \Theta_{njkl} + \frac{1}{4} T_{mpq} T_n^{pq}. \tag{2.24}$$

Substitute (1.4) into (2.24), use (2.21) and compare the result with the first equation in (1.2) to conclude that the supersymmetry equations (1.3) together with the anomaly cancellation (1.4) imply the first equation in (1.2) if and only if the next equality holds [33]

$$R_{mstr} R^{str}_n = -\frac{1}{6}[R_{msij} R_{trij} + R_{mtij} R_{rsij} + R_{mrij} R_{stij}] *_7 \Theta_{nstr}. \tag{2.25}$$

The twenty-one-dimensional space of two forms $\Lambda^2(\mathbb{R}^7)$ decomposes into two parts, a seven-dimensional part Λ^2_7 and a fourteen-dimensional part Λ^2_{14}, $\Lambda^2(\mathbb{R}^7) = \Lambda^2_7 \oplus \Lambda^2_{14}$. The Lie algebra $\mathfrak{g}_2$ of G_2 is isomorphic to the two-forms satisfying 7 linear equations, namely $\mathfrak{g}_2 \cong \Lambda^2_{14}(\mathbb{R}^7) = \{\beta \in \Lambda^2(\mathbb{R}^7) \mid *_7 (\beta \wedge \Theta) = -\beta\}$. The space $\Lambda^2_{14}(\mathbb{R}^7)$ can also be described as the subspace of 2-forms β which annihilate $*_7\Theta$, i.e. $\beta \wedge *_7\Theta = 0$.

For the curvature 2-form R we have the orthogonal splitting $R = R_7 \oplus R_{14}$, where

$$\begin{aligned} (R_7)_{ij} &= \frac{1}{6}(2R_{ij} + R_{kl} *_7 \Theta_{ijkl}); \\ (R_{14})_{ij} &= \frac{1}{6}(4R_{ij} - R_{kl} *_7 \Theta_{ijkl}). \end{aligned} \tag{2.26}$$

The equality (2.16) and (2.26) imply

$$(R_7)_{kl} *_7 \Theta_{klij} = 4(R_7)_{ij}, \qquad (R_{14})_{kl} *_7 \Theta_{klij} = -2(R_{14})_{ij}. \tag{2.27}$$

Using (2.27), we get from (2.25) that

$$6\big(\|R_7\|^2 + |R_{14}\|^2\big) = 6R_{mstr}R^{str}_m = -12\|R_7\|^2 + 6\|R_{14}\|^2. \tag{2.28}$$

Consequently, (2.28) yields $\|R_7\|^2 = 0$. Compare with the first equality in (2.26) to conclude that $R_7 = 0$ is equivalent to the G_2-instanton condition (the second equality in (2.21)), i.e. R is a G_2-instanton. □

2.4. Dimension eight. Proof of Theorem 1.1 in $d = 8$

The existence of ∇^+-parallel spinor in dimension 8 determines a $Spin(7)$ structure whose properties as well as solutions to gravitino and dilatino Killing-spinor equations are investigated in [44,13,9,33].

We briefly recall the notion of a $Spin(7)$ structure. Consider $\mathbb{R}^8$ endowed with an orientation and its standard inner product. Consider the 4-form Φ on $\mathbb{R}^8$ given by

$$\begin{aligned} \Phi = {} & e_{0127} - e_{0236} + e_{0347} + e_{0567} - e_{0146} - e_{0245} + e_{0135} \\ & + e_{3456} + e_{1457} + e_{1256} + e_{1234} + e_{2357} + e_{1367} - e_{2467}. \end{aligned} \tag{2.29}$$

The 4-form Φ is self-dual and the 8-form $\Phi \wedge \Phi$ coincides with the volume form of $\mathbb{R}^8$. The subgroup of $GL(8,\mathbb{R})$ which fixes Φ is isomorphic to the double covering $Spin(7)$ of $SO(7)$. The 4-form Φ corresponds to a real spinor and therefore, $Spin(7)$ can be identified as the isotropy group of a non-trivial real spinor.

We have the well-known formula (see e.g. [9])

$$\Phi_{ijpq}\Phi_{klpq} = 6\delta_{ik}\delta_{jl} - 6\delta_{il}\delta_{jk} + 4\Phi_{ijkl}. \tag{2.30}$$

A *Spin*(7)*-structure* on an 8-manifold M is by definition a reduction of the structure group of the tangent bundle to *Spin*(7). This can be described geometrically by saying that there exists a nowhere vanishing global differential 4-form Φ on M which can be locally written as (2.29).

If $\nabla^g\Phi = 0$ then the holonomy of the metric $Hol(g)$ is a subgroup of $Spin(7)$ and $Hol(g) \subset Spin(7)$ if and only if $d\Phi = 0$ [61] (see also [58,59]). The Lee form θ^8 is defined by [62] $\theta^8 = -\frac{1}{7} *_8 (*_8 d\Phi \wedge \Phi) = \frac{1}{7} *_8 (\delta\Phi \wedge \Phi)$.

It is shown in [44] that the gravitino Killing spinor equation always has a solution in dimension 8, i.e. any *Spin*(7)-structure admits a unique *Spin*(7)-connection with totally skew-symmetric torsion $T = *_8 d\Phi - \frac{7}{6} *_8 (\theta^8 \wedge \Phi)$. Applying [44, Corollary 6.18] and the identity $*_8(\theta^8 \wedge \Phi) = (\theta^8 \lrcorner \Phi)$ we can also write

$$\theta^8_s = \frac{1}{42}\big((*_8 d\Phi)_{ijk}\Phi_{sijk}\big) = -\frac{1}{42}(\delta\Phi_{ijk}\Phi_{sijk}),$$
$$T_{ijk}\Phi_{sijk} = -7\theta^8_s. \tag{2.31}$$

The necessary conditions to have a solution to the system of dilatino and gravitino Killing spinor equations were derived in [13, 44], and the sufficiency was proved in [44]. The general result [44] states that there exists a non-trivial solution to both dilatino and gravitino Killing spinor equations in dimension 8 if and only if there exists a *Spin*(7)-structure (Φ, g) with an exact Lee form which is equivalent to the statement that the conformal *Spin*(7)-structure $(\bar\Phi = e^{-\frac{12}{7}\phi}\Phi, \bar g = e^{-\frac{6}{7}\phi} g)$ has zero Lee form, $\bar\theta^8 = 0$.

The torsion 3-form (the flux H) and the Lee form of a solution to the gravitino and dilatino equations in dimension eight are given by [13,44]

$$H = T = *_8 d\Phi - 2 *_8 (d\phi \wedge \Phi), \qquad \theta^8 = \frac{12}{7}d\phi. \tag{2.32}$$

The Ricci tensor of the torsion connection is calculated in [44] (see also [33])

$$Ric^+_{mn} = \frac{1}{12}dT_{mjkl}\,\Phi_{njkl} + \frac{1}{6}\nabla^+_m T_{jkl}\Phi_{njkl}. \tag{2.33}$$

Using the special expression of the torsion (2.32) and (2.31), Eq. (2.33) takes the form

$$Ric^+_{mn} = \frac{1}{12}dT_{mjkl}\,\Phi_{njkl} - 2\nabla^+_m d\phi_n$$
$$= \frac{1}{12}dT_{mjkl}\,\Phi_{njkl} - 2\nabla^g_m d\phi_n + d\phi_s\, T^s_{mn}. \tag{2.34}$$

In addition to these equations, the vanishing of the gaugino variation requires the 2-form F^A to be of instanton type [5,4,6–10]. A *Spin*(7)-instanton in dimension eight is a *Spin*(7)-connection A with curvature 2-form $F^A \in \mathfrak{spin}(7)$. The latter is equivalent to

$$F^A_{mn} = -\frac{1}{2}F^A_{pq}\Phi^{pq}{}_{mn}. \tag{2.35}$$

2.4.1. Theorem 1.1 in dimension 8

Proof. It is sufficient to investigate only the Einstein equation of motion. First we show that

$$dT_{mjkl}\,\Phi_{njkl} = dT_{njkl}\,\Phi_{mjkl}. \tag{2.36}$$

Indeed, the second identity in (2.1) and (2.32) yield

$$Ric^+_{mn} - Ric^+_{nm} = (*_8 d *_8 T)_{mn} = 2\big(*_8(d\phi \wedge d\Theta)\big)_{mn}$$
$$= 2\big(*_8(d\phi \wedge *_8 T)\big)_{mn} = 2\,d\phi_s\,T^s_{mn} \tag{2.37}$$

which compared with the skew-symmetric part of (2.34) gives (2.36). In particular, (2.37) supplies a proof of the second equality in (1.2) in dimension eight.

Substitute (2.34) into (2.1) and use (2.36) to get

$$Ric^g_{ij} = -2\nabla^g_i d\phi_j - \frac{1}{12}dT_{mjkl}\Phi_{njkl} + \frac{1}{4}T_{mpq}T^{pq}_n. \tag{2.38}$$

Insert (1.4) into (2.38), use (2.35) and compare the result with the first equation in (1.2) to conclude that the supersymmetry equations (1.3) together with the anomaly cancellation (1.4) imply the first equation in (1.2) in dimension eight if and only if the next equality holds [33]

$$R_{mstr}R^{str}_n = -\frac{1}{6}[R_{msij}R_{trij} + R_{mtij}R_{rsij} + R_{mrij}R_{stij}]\Phi_{nstr}. \tag{2.39}$$

The twenty-eight-dimensional space of two forms $\Lambda^2(\mathbb{R}^8)$ decomposes into two parts, a seven-dimensional part Λ^2_7 and a twenty one-dimensional part Λ^2_{21}, $\Lambda^2(\mathbb{R}^8) = \Lambda^2_7 \oplus \Lambda^2_{21}$. The Lie algebra $\mathfrak{spin}(7)$ of *Spin*(7) is isomorphic to the two-forms satisfying 7 linear equations, namely $\mathfrak{spin}(7) \cong \{\beta \in \Lambda^2(\mathbb{R}^8) \mid *_8(\beta \wedge \Phi) = -\beta\}$.

For the curvature 2-form R we have the splitting $R = R_7 \oplus R_{21}$, where

$$(R_7)_{ij} = \frac{1}{8}(2R_{ij} + R_{kl}\Phi_{ijkl});$$
$$(R_{21})_{ij} = \frac{1}{8}(6R_{ij} - R_{kl}\Phi_{ijkl}). \tag{2.40}$$

The equality (2.30) and (2.40) imply

$$(R_7)_{kl}\Phi_{klij} = 6(R_7)_{ij}, \qquad (R_{21})_{kl}\Phi_{klij} = -2(R_{14})_{ij}. \tag{2.41}$$

Using (2.41), we get from (2.39) that

$$6\big(\|R_7\|^2 + \|R_{14}\|^2\big) = 6R_{mstr}R^{str}_m = -18\|R_7\|^2 + 6\|R_{14}\|^2. \tag{2.42}$$

Consequently, (2.42) yields $\|R_7\|^2 = 0$. Compare with the first equality in (2.40) to conclude that $R_7 = 0$ is equivalent the *Spin*(7)-instanton condition (2.35), i.e. R is a *Spin*(7)-instanton.

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