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Long range forces and limits on unparticle interactions

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Abstract

Couplings between standard model particles and unparticles from a nontrivial scale invariant sector can lead to long range forces. If the forces couple to quantities such as baryon or lepton (electron) number, stringent limits result from tests of the gravitational inverse square law. These limits are much stronger than from collider phenomenology and astrophysics.

1. Introduction

Georgi [1,2] has proposed that unparticles—a nontrivial scale invariant sector [3]—might couple to standard model particles, leading to novel phenomenological signatures. This idea has been further developed by a number of authors [4–48]. In this Letter, we discuss the possibility of long range forces resulting from such interactions. We assume strict scale invariance down to low energies, so that the unparticle propagator necessarily has a zero momentum pole.

Our analysis is quite similar to that of Goldberg and Nath [27], who considered the possibility that (exactly scale invariant) unparticles might couple to the ordinary energy-momentum tensor $T_{\mu\nu}$. Here, we consider couplings between unparticles and currents such as $J_\mu = \bar{e}\gamma_\mu e$ or $\bar{q}\gamma_\mu q$. Interestingly, the electron number current appears in Georgi's $e^+e^- \to \mu^+\mu^-$ example in [2]. We find extremely strong limits on such couplings, much stronger than can be obtained by collider experiments and even astrophysics.

2. Long range force due to vector unparticle

We consider first the baryon current B_μ, which in terms of quark fields is

$$B_\mu = \frac{1}{3}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d + \cdots). \tag{1}$$

We assume that at UV scales the current B_μ interacts with operator $\mathcal{O}^\mu_{\mathrm{UV}}$ by exchange of massive particles to give an interaction $\sim M_{\mathrm{UV}}^{1-d_{\mathrm{UV}}} B_\mu \mathcal{O}^\mu_{\mathrm{UV}}$. Let Λ_U be the energy scale at which scale invariance emerges from the UV sector (i.e., where the unparticle sector couplings reach an approximate fixed point), and let $\mathcal{O}_U$ be the unparticle operator corresponding to $\mathcal{O}_{\mathrm{UV}}$. Then, we get an effective interaction

$$\mathcal{L} = c_B \left(\frac{\Lambda_U}{M_{\mathrm{UV}}}\right)^{d_{\mathrm{UV}}-1} \Lambda_U^{1-d_U} B_\mu \mathcal{O}^\mu_U. \tag{2}$$

Following Georgi [1], we define a scale invariant coupling

$$\lambda_B = c_B \left(\frac{\Lambda_U}{M_{\mathrm{UV}}}\right)^{d_{\mathrm{UV}}-1} \tag{3}$$

and rewrite the interaction as

$$\mathcal{L} = \lambda_B \Lambda_U^{1-d_U} B_\mu \mathcal{O}^\mu_U. \tag{4}$$

Note λ_B is a dimensionless coupling constant and is in general very small compared to c_B. We shall later take $\Lambda_U = 1$ TeV and obtain constraints on λ_B. These can be converted into constraints on c_B if the scale M_{UV} and dimension d_{UV} are known.

In the static limit the interaction generates the potential

$$V_U = \lambda_B^2 \Lambda_U^{2-2d_U} \frac{1}{4\pi^2} \frac{1}{r^{2d_U-1}} A_{d_U} \Gamma(2d_U - 2) B_1 B_2, \tag{5}$$

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where

$$A_{d_U} = \frac{16\pi^{\frac{5}{2}}}{(2\pi)^{2d_U}} \frac{\Gamma(d_U + \frac{1}{2})}{\Gamma(d_U - 1)\Gamma(2d_U)} \tag{6}$$

is the coefficient associated with the transverse four-vector unparticle propagator [2] and $B_{1,2}$ are the baryon numbers of the two interacting masses. Using a relationship involving Gamma functions, V_U can be simplified into

$$V_U = \frac{1}{2\pi^{2d_U}} \lambda_B^2 \Lambda_U^{2-2d_U} \frac{\Gamma(d_U + \frac{1}{2})\Gamma(d_U - \frac{1}{2})}{\Gamma(2d_U)} \times \frac{1}{r^{2d_U - 1}} B_1 B_2. \tag{7}$$

Note that for $d_U = 1$, this will produce a $1/r$ repulsive potential

$$V_U = \frac{1}{4\pi} \lambda_B^2 \frac{1}{r} B_1 B_2. \tag{8}$$

Similarly, the unparticle operator can couple to lepton currents with coupling strength λ_L and the above can be directly applied as well. For numerical results, we exhibit three examples: $\lambda_B = \lambda \neq 0$ and $\lambda_L = 0$; $\lambda_B = 0$ and $\lambda_L = \lambda \neq 0$; and $\lambda_B = -\lambda_L = \lambda$. In these cases, the unparticle operator couples to B, L and B–L currents, respectively.

3. Numerical results

In the limit of unbroken scale invariance, the force between B_1 and B_2 is a long range force similar to gravity, but it may have different $1/r$ power dependence. To obtain numerical results, we make the approximation $B_{1,2} \approx m_{1,2}/u$, where $m_{1,2}$ are the masses of the two interacting bodies and u is the atomic mass unit. Combined with the gravitational potential, the potential between two objects of mass m_1 and m_2 is

$$V = -G \frac{m_1 m_2}{r} + \lambda_B^2 f_{d_U} \frac{1}{u^2} \frac{m_1 m_2}{r^{2d_U - 1}}, \tag{9}$$

where Newton's constant $G = 6.7 \times 10^{-39}\ \text{GeV}^{-2}$ and

$$f_{d_U} = \frac{1}{2\pi^{2d_U}} \Lambda_U^{2-2d_U} \frac{\Gamma(d_U + \frac{1}{2})\Gamma(d_U - \frac{1}{2})}{\Gamma(2d_U)} \tag{10}$$

captures the d_U dependent part of the coefficient. V can then be written as

$$V = -G \frac{m_1 m_2}{r} + G \frac{m_1 m_2}{r} \frac{1}{u^2} \frac{\lambda_B^2}{G} f_{d_U} \left(\frac{1}{r}\right)^{2d_U - 2}. \tag{11}$$

We compare the second term on the right-hand side to the power-law potential in Ref. [49],

$$V_{12}^k(r) = -G \frac{m_1 m_2}{r} \beta_k \left(\frac{1\ \text{mm}}{r}\right)^{k-1}, \tag{12}$$

and derive limits on the coupling λ, for $k = 2$ and $k = 3$. Ref. [49] also discussed constraints on Yukawa potentials generated by exchange a scalar or a vector boson. For small enough boson masses m, the experimental distance r is much less than the Compton wavelength $\hbar/mc$. In this limit, the Yukawa potential is approximately $1/r$, and we obtain a limit on the case $k \approx 1$. For non-integer value of k, we simply interpolate the limit linearly (the precise bound will be of the same order of magnitude). The range of d_U we are interested in is $1 \leqslant d_U \leqslant 2$ and it is related to k through equation $2d_U - 2 = k - 1$. Unitarity imposes constraints on dimensions of conformal fields, for example requiring $d_U \geqslant 2$ for vector fields [38]. For scale invariant (but not conformal) theories this bound need not apply, and after dimensional transmutation there is no simple relationship between IR and UV dimensionalities.

Table 1
The 68% CL constraints on the coupling λ for the B current. SN refers to supernova constraint on B current coupling

d_U	$\lvert\beta_k\rvert$	$\lvert\lambda\rvert$	SN
1	1.8×10^{-2}	3.9×10^{-20}	3.0×10^{-11}
1.25	9.1×10^{-3}	7.5×10^{-17}	4.1×10^{-10}
1.5	4.5×10^{-4}	4.3×10^{-14}	5.5×10^{-9}
1.75	2.9×10^{-4}	8.8×10^{-11}	7.4×10^{-8}
2.0	1.3×10^{-4}	1.5×10^{-7}	1.0×10^{-6}

The limits on the scalar or vector coupling can be converted into limits on an effective coefficient β_1 for the case of a $1/r$ potential. Note that when we consider the B, L and B–L cases, the effective couplings will be multiplied by factors of $(Z + N)/A \approx 1$, Z/A and N/A respectively, where Z is the number of protons, A the atomic number, and N is the number of neutrons. For the molybdenum pendulum considered in Ref. [49], $Z/A = 0.438$ and $N/A = 0.563$. If λ_B and λ_L happen to satisfy

$$\frac{\lambda_B}{\lambda_B + \lambda_L} = -\frac{Z}{N}, \tag{13}$$

the effective charge becomes zero and the torsion-balance experiment becomes insensitive to λ, so no limit is obtained. However, the experiment [50] was performed with an aluminum pendulum and copper attractor and arrived at similar bounds, covering as well the exceptional parameter space (13).

We should also note that forces coupling to almost any linear combination of B and L which extend over truly macroscopic (e.g., solar system scale) distances are even more tightly constrained if they deviate from $1/r$, since they would affect Newtonian orbits.

Table 1 shows the 68% confidence level (CL) constraints on $|\beta_k|$ (first column) and the derived constraints on λ for the case of B current (second column). The results for L currents are only different for $d_U = 1$, $|\lambda| < 2.5 \times 10^{-20}$ and for $d_U = 1.25$, $|\lambda| < 5.0 \times 10^{-17}$. The results for the B–L current are almost identical to that for the L current by accident. These limits are proportional to $\Lambda_U^{2-2d_U}$. The values in the table are for $\Lambda_U = 1$ TeV and limits for other values of Λ_U can be easily calculated by simple scaling.

For comparison purposes, Table 1 also displays results from supernova constraints (last column, taking $\Lambda_U = 1$ TeV). Unparticles can induce too-rapid cooling of supernovae [15,47]. A constraint on λ can be deduced, yielding

$$\lambda \left(\frac{\Lambda_U}{30\ \text{MeV}}\right)^{1-d_U} < 3 \times 10^{-11}. \tag{14}$$

Note that for our limits to apply scale invariance must hold to length scales as long as a fraction of a millimeter. If scale invariance is broken at a scale intermediate between a millimeter and the thermal wavelength of a supernova (roughly, inverse MeV), the supernova constraints will still apply, while ours will not.

4. Conclusions

A sector of particle physics which exhibits nontrivial scale invariance would be an exciting discovery. If, however, the scale invariance is exact, long range forces may result which are already strongly constrained by measurements of the gravitational inverse square law. Given the limits derived here, perhaps the most likely unparticle scenario is one in which the scale invariance is only approximate—i.e., it is broken below some energy scale,[1] thereby screening the long range forces. However, in this case the new particle sector, while exhibiting novel dynamics, is not really an *unparticle* sector!

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[1] Of course, the breaking of scale invariance in the standard model sector will feed into the unparticle sector via the interactions assumed here. However, this is a one loop effect, and will alter the unparticle beta function (which exhibits the approximate fixed point) only slightly. Unless some of the less constrained couplings between the standard model and unparticles (e.g., via the top quark) are very large, we expect these quantum effects to be sufficiently small that they do not induce screening of long range forces on the millimeter scale.