A Rational Congruence for a Standard Orbit Decomposition

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We consider the orbit decomposition given by a standard automorphism group of a point-divisible design (SRGDD in Bose and Connor [1] terminology), and we exhibit a rational congruence naturally associated to this decomposition.

1. INTRODUCTION

It is well known that the orbits of any automorphism group of a design \( D \) give rise to a tactical decomposition of \( D \) (see Hughes and Piper [6], 1.6). In this paper we study the orbit decomposition given by a standard automorphism group of a point-divisible design and we prove that there is a rational congruence which is naturally associated to the decomposition.

We note that it is possible to apply to this rational congruence the Hasse–Minkowski theory to obtain a non-existence theorem for standard automorphism groups of those designs (see [3] and [5] for the actual proof, simply sketched here in Section 3.1). That non-existence theorem simultaneously generalizes the famous Bruck–Ryser–Chowla, Hughes and Bose–Connor theorems (see Section 3).

To fix notation, we give in this section some definitions and the proof of a preliminary lemma (further details and all the definitions we omit can be found in [2], [3] or in Hughes and Piper [6]). The main theorem will be stated and proved in section 2.

DEFINITION 1.1. A point-divisible design (\( p \)-divisible, in short) is an incidence structure \( D = D(m, c, k, \lambda', \lambda) \) satisfying:

(i) \( D \) is a uniform 1-design of block size \( k \), for which the \( v \) points are partitioned into \( m \) classes of size \( c \) such that points have joining number \( \lambda' \) (respectively, \( \lambda \)) if they are in the same class (respectively, different classes);

(ii) \( D \) is square (i.e. with

\[
b = v = mc
\]  

blocks and thus \( k \) blocks through a point);

(iii) \( D \) is non-singular (i.e. it has a non-singular incidence matrix).

Let \( I_x \) and \( J_x \) be, as usual, the identity matrix and the all ones matrix of order \( x \), respectively; we denote by \( \tilde{A} \) an incidence matrix for \( D \) associated with the point-division (i.e. the first \( c \) rows of \( \tilde{A} \) correspond to the points of the first class \( \mathbb{P}_1 \), the next \( c \) rows to the points of the second class \( \mathbb{P}_2 \), etc.). By definition, \( \tilde{A} \) satisfies

\[
\tilde{A}\tilde{A}' = I_n \times C + \lambda J_v,
\]  

where \( \times \) denotes (as throughout this paper) the Kronecker product of matrices and

\[
C = (k - \lambda')I_c + (\lambda' - \lambda)J_c.
\]  

Furthermore,

\[
\tilde{A}J = J\tilde{A} = kJ.
\]  

When \( c = 1 \) or \( \lambda' = \lambda \), a \( p \)-divisible design is a symmetric \((v, k, \lambda)\). By analogy, we define the order of \( D \) to be the integer

\[
n = k - \lambda' + c(\lambda' = \lambda).
\]  

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Counting flags \((P, y)\) with \(P\) incident with a fixed block \(x \neq y\) we see that
\[
n = k^2 - v\lambda. \tag{1.6}
\]

The determinant of \(\tilde{A}\) is
\[
|\tilde{A}| = (k - \lambda')^{m(c - 1)/2}n^{(m-1)/2}k, \tag{1.7}
\]
where \(k > 0, k - \lambda' > 0\), by definition of \(D\) (see, for instance, [2], 2.1 and 2.2, and see (1.5) and (1.6) above). Hence, the non-singularity of \(D\) implies \(n > 0\); that is, \(v < k^2/\lambda\).

**Definition 1.2.** An automorphism group \(G\) of a \(p\)-divisible design acts *standardly* on points, blocks and point-classes if all orbits of \(G\) on those sets have length either 1 or \(|G|\); in the above case, \(G\) is said to be *standard*. Note that this implies that every non-identity element of \(G\) fixes the same sets of points, blocks and point classes. In particular, every group of prime order is standard. We note that an automorphism of a \(p\)-divisible design must preserve the partition of points.

From now on, \(D\) will be a \(p\)-divisible design with a standard automorphism group \(G\), of order \(\mu\), which fixes \(N\) points and thus has
\[
\nu' = \{(v - N)/\mu\} + N \tag{1.8}
\]
orbits \(P_1, \ldots, P_{\nu'}\) on points. If \(B_1, \ldots, B_{b'}\) are the orbits on blocks, the orbit theorem says that \(v' = b'\) (see Hughes and Piper, 1.46); since \(v = b\), this implies that \(N\) is also the number of fixed blocks.

As for the action of \(G\) on the point classes \(\bar{P}_1, \ldots, \bar{P}_m\), it is not difficult to see that the following holds.

**Remarks 1.3.** (a) If a divisibility class \(\bar{P}_i\) contains a point orbit (in particular, a fixed point), then \(\bar{P}_i\) is a fixed class (i.e. it is union of orbits).

(b) A non-fixed class does not contain orbits (in particular, fixed points). Furthermore, the order of its intersection with an orbit is either 0 or 1.

Let \(\Phi\) be the *number of fixed classes*. We denote by
\[
h = (m - \Phi)/\mu = (v - \Phi c)/\mu c \tag{1.9}
\]
the number of orbits of non-fixed classes, and by
\[
u = h + \Phi, \tag{1.10}
\]
the number of orbits on classes.

If \(i\) is an integer, \(0 \leq i \leq c\), we define \(\Phi_i\) to be the *number of fixed classes containing exactly \(i\) fixed points* (and thus
\[
\nu_i = (c - i)/\mu \tag{1.11}
\]
orbits of length \(\mu\). We note that
\[
\Phi_i = 0 \quad \text{if } i \not\equiv c \pmod{\mu}. \tag{1.12}
\]

Clearly,
\[
\Phi = \sum_{i=0}^{c} \Phi_i, \quad N = \sum_{i=0}^{c} i \Phi_i. \tag{1.13}
\]

Counting orbits of non-fixed points contained in fixed classes, we obtain
\[
\sum_{i=0}^{c} \Phi_i \nu_i = v' - N - ch = (c\Phi - N)/\mu. \tag{1.14}
\]
Rational congruence for standard orbit decomposition

Hence
\[ \sum_{i=0}^{c} \Phi_i(v_i + i) = v' - ch, \quad \sum_{i=0}^{c} \Phi_i(v_i + i - 1) = v' - ch - \Phi. \]  
(1.15)

We observe that when \( \mu > c \) the situation is much simpler; then
\[ \Phi = \Phi_c = N/c. \]  
(1.16)

Also, if \( \mu \geq c \), then \( \Phi_i = 0 \) for every \( i \neq 0, c \), and \( \Phi_c = N/c \), so that
\[ \Phi = \Phi_0 + N/c. \]  
(1.17)

**Definition 1.4.** An incidence matrix for \( D \) associated with the orbit decomposition is a matrix in which the first \( |P_1| \) rows correspond to the points of the first point-orbit \( P_1 \), the next \( |P_2| \) rows to \( P_2 \) etc., and similarly for the columns, the first \( |B_1| \) correspond to the blocks of the first block-orbit \( B_1 \), etc.

Clearly, if \( A \) and \( \tilde{A} \) are incidence matrices for \( D \) associated with the orbit decomposition and with the point-division, respectively, there are permutation matrices \( \pi \) and \( \beta \) such that
\[ A = \pi \tilde{A} \beta. \]  
(1.18)

Therefore
\[ AA' = \pi \tilde{A} \tilde{A}' \pi' = \pi (I_m \times C) \pi' + \lambda I. \]  
(1.19)

**Lemma 1.5.** With the above notation, we can choose \( A \) and \( \tilde{A} \) such that the permutation matrix \( \pi \) is of the form
\[ \pi = (I_h \times \tau) \oplus I_{bc}, \]  
(1.20)

where \( \tau \) is the square matrix of order \( \mu c \) defined, for \( i = 1, \ldots, \mu \), \( r = 1, \ldots, c \), \( s = 1, \ldots, c \)
\[ \tau_{(i-1)c+r,s} = \delta_{s,(r-1)c+i}, \]  
(1.21)

where \( \delta \) is the Kronecker symbol.

**Proof.** An ordering for \( D \) is defined as follows. Let \( (1, g_1, \ldots, g_{\mu-1}) \) be a fixed ordering for the elements of \( G \). We first consider the \( m - \Phi \) non-fixed classes of \( D \) and we order them as
\[ \tilde{P}_{j_1}, \tilde{P}_{j_2}, \ldots, \tilde{P}_{j_{\mu-1}}, \quad j_1 = 1, \ldots, h. \]  
(1.22)

Then, for every fixed \( j \), if \( \tilde{P}_j = (P_1, \ldots, P_c) \), we fix, for the points of \( \tilde{P}_{j_s} \), the following order;
\[ \tilde{P}_{j_s} = (P_{j_s^1}, \ldots, P_{j_s^c}), \quad s = 1, \ldots, \mu - 1. \]  
(1.23)

We next consider for each \( i, 0 \leq i \leq c \), the \( \Phi_i \) fixed classes with exactly \( i \) fixed points. In each of these classes we order the points so that we have first the orbits of length \( \mu \), and then the \( i \) fixed points. In this way, the last \( c \) points of \( D \) are in an order suitable for an orbit tactical decomposition, and the result is immediate. \( \Box \)

2. **The Main Result**

In this section we will prove the following theorem;

**Theorem 2.1.** Let \( D = D(m, c, k, \lambda', \lambda) \) be a \( p \)-divisible design. If \( D \) has a standard automorphism group \( G \) of order \( \mu \), which fixes \( N \) points and \( \Phi \) divisibility classes (and
thus has \( v' = \{(v - N)/\mu\} + N \) orbits on points and blocks), then the diagonal \( v' \) by \( v' \) matrix

\[
I_{v' - N} \oplus \mu^{-1}I_N
\]  

(2.1)
is rationally congruent to the matrix

\[
Z = (I_h \times C) \oplus K + \mu \lambda I_{v'},
\]

(2.2)
where \( C \) is as in (1.3) (or, as we also say, is a \((k - \lambda', \lambda' - \lambda, c)\)-matrix), \( h \) is the number of orbits of non-fixed classes, and

\[
K = \bigoplus_{i=0}^{c} (I_{\Phi_i} \times K_i)
\]  

(2.3)
with

\[
K_i = (k - \lambda')(I_{v_i} \oplus \mu I_i) + \mu (\lambda' - \lambda) I_{v_i + i} \quad (v_i = (c - i)/\mu, i = 0, \ldots, c).
\]

(2.4)
The proof of Theorem 2.1 will be deduced from the following general results on decomposition of matrices.

**Notation 2.2.** Let \( A = (a_{ij}) \), \( r = 1, \ldots, v \), \( c = 1, \ldots, b \) be a \( v \) by \( b \) matrix over a field \( F \). We denote by \( \Delta \) any decomposition of \( A \) (i.e. a partition \( P_1, \ldots, P_v \) of the rows of \( A \) and a partition \( B_1, \ldots, B_b \) of the columns of \( A \)).

Let \( X \) be the \( v' \) by \( v \) matrix the \( i \)th row of which consists of the values of the characteristic function of the set \( P_i \) \((i = 1, \ldots, v')\). Similarly, let \( Y \) be the \( b' \) by \( b \) matrix the \( j \)th row of which gives the values of the characteristic function of \( B_j \) \((j = 1, \ldots, b')\). We say that the diagonal matrices

\[
V = \text{diag}(|P_1|, \ldots, |P_v|),
\]

(2.5)
and

\[
B = \text{diag}(|B_1|, \ldots, |B_b|),
\]

(2.6)
are the diagonal matrices of the decomposition \( \Delta \). Clearly,

\[
V = XX' \quad \text{and} \quad B = YY'.
\]

(2.7)

With the above notation, we have the following lemma:

**Lemma 2.3.** If \( \Delta \) is a column-tactical decomposition of the matrix \( A \) and \( L \) denotes the matrix of column sums of \( \Delta \), then

\[
XAA'X' = LBL'.
\]

(2.8)

**Proof.** If \( \Delta \) is column tactical, we have, by definition (see Hughes and Piper [6], 1.6)

\[
XA = LY,
\]

(2.9)
and, by transposition of matrices,

\[
A'X' = Y'L',
\]

(2.10)
hence \( XAA'X' = L(YY')L' \), which, by (2.7), gives (2.8). □

Equation (2.8) is particularly interesting when \( F = \mathbb{Q} \) and \( v' = b' \) is equal to the rank of \( L \) (or, equivalently, \( A \) has rank \( v \) and \( v' = b' \)). In this case (2.8) establishes a rational congruence between the diagonal matrix \( B \) and the matrix \( XAA'X' \).
From now on, $\Delta$ will be the orbit decomposition given by a standard automorphism group of a $p$-divisible design. Hence $\Delta$ is tactical and $v' = b'$ (see Hughes and Piper [6], 1.45, 1.46). In this case Lemma 2.1 and (1.19) give

$$LBL' = X\pi(I_m \times C)\pi'X' + \lambda XJ_vX'.$$

Since

$$XJ_vX' = (|P_1| \cdot |P'|) = VJ_vV,$$

from (2.11) we obtain

$$(V^{-1}L)B(V^{-1}L)' = D + \lambda J_v,$$

where

$$D = (V^{-1}X)\pi(I_m \times C)\pi'(V^{-1}X').$$

**Notation 2.4.** We fix an ordering for our design as in the proof of Lemma 1.5 so that Lemma 1.5 holds.

If $r$ and $s$ are positive integers, then we denote by $L(r, sr)$ the ladder $r$ by $(sr)$ matrix obtained from $I_r$ by replacing the $j$th column with $s$ of its copies. We point out that, if $M$ is any $(sr)$ by $(sr)$ matrix and we consider the partition of

$$M = (M_{\alpha\beta})$$

into submatrices of order $s$, then

$$L(r, sr)ML(r, sr)' = (\sigma(M_{\alpha\beta})),$$

where $\sigma(M_{\alpha\beta})$ denotes the sum of all entries in the matrix $M_{\alpha\beta}$.

For each $i = 0, \ldots, c$, we denote by $C(i)$ the $i$ by $i$ matrix

$$C(i) = (k - \lambda')I_i + (\lambda' - \lambda)I_i,$$

with the convention that, when $i = 0$, the parts of our matrices containing $C(i)$ must be suppressed. We also use the convention that, if $x = 0$, the terms containing $I_x$ must be suppressed in a direct sum of matrices.

With the above ordering and notation, we have for the matrices $X$ and $V$ of the orbit tactical decomposition

$$X = L(ch, \mu ch) \oplus \left\{ \bigoplus_{i=0}^{c} I_{\Phi_i} \times [L(v_i, \mu v_i) \oplus I_i] \right\},$$

$$V = XX' = \mu I_{hc} \oplus \left\{ \bigoplus_{i} I_{\Phi_i} \times [\mu I_{v_i} \oplus I_i] \right\}.$$  

Hence,

$$V^{-1}X = \mu^{-1}L(ch, \mu ch) \oplus \left\{ \bigoplus_{i} I_{\Phi_i} \times [\mu^{-1}L(v_i, \mu v_i) \oplus I_i] \right\}.$$  

Since $m = h \mu + \Phi = h \mu + \Sigma_i \Phi_i$, we can write

$$I_m \times C = \{I_h \times (I_\mu \times C)\} \oplus \left\{ \bigoplus_{i} (I_{\Phi_i} \times C) \right\},$$

so that, by (1.20), we obtain

$$\pi(I_m \times C) \pi' = \{I_h \times \tau(I_\mu \times C)\} \tau' \oplus \left\{ \bigoplus_{i} (I_{\Phi_i} \times C) \right\}.$$  

Keeping the definition of $C$ and $\tau$ in mind, it is not too difficult to prove the following statement.
Lemma 2.5. With the above notation, we have
\[ \tau(I_\mu \times C)^t = E = (E_{\gamma \delta}), \quad \gamma, \delta = 1, \ldots, c, \] (2.23)
where \( E_{\gamma \delta} \) is the \( \mu \) by \( \mu \) matrix
\[ E_{\gamma \delta} = \begin{cases} (k - \lambda)I_\mu, & \text{if } \gamma = \delta; \\ (\lambda' - \lambda)I_\mu, & \text{if } \gamma \neq \delta. \end{cases} \] (2.24)
Thus,
\[ \sigma(E_{\gamma \delta}) = \begin{cases} \mu(k - \lambda), & \text{if } \gamma = \delta; \\ \mu(\lambda' - \lambda), & \text{if } \gamma \neq \delta. \end{cases} \] (2.25)
This implies
\[ \mu C = (\sigma(E_{\gamma \delta})). \] (2.26)

Now we are able to prove the following lemma, which is a slightly different restatement of Theorem 2.1.

Lemma 2.6. With the above notation, if a \( p \)-divisible design has a standard automorphism group \( G \), then we have a rational congruence
\[ (\mu I_{v', -N}) \oplus I_N = D + J_{v'}, \] (2.27)
with
\[ D = \mu^{-1}(I_h \times C) \oplus \{ \oplus_i (I_{\Phi_i} \times D_i) \}, \] (2.28)
where
\[ D_i = (k - \lambda')(\mu^{-1}L_{v_i} \oplus I_i) + (\lambda' - \lambda)J_{v_i + i}, \quad (v_i = (c - i)/\mu, i = 0, \ldots, c). \] (2.29)

Proof. The ordering fixed for our design in the proof of Lemma 1.5 implies
\[ B = V = \mu I_{v', -N} \oplus I_N, \] (2.30)
and this, together with (2.13), implies (2.27) with the matrix \( D \) defined in (2.14). From (2.20), (2.22), Lemma 2.5 and (2.16), we obtain (2.28) where
\[ D_i = \{ \mu^{-1}L(v_i, \mu v_i) \oplus I_i \} \oplus \mu^{-1}L(v_i, \mu v_i \oplus I_i). \]
With the use of Notation 2.4,
\[ C = \begin{bmatrix} C(c - i) & (\lambda' - \lambda)J \\ (\lambda' - \lambda)J & C(i) \end{bmatrix}. \]
Hence
\[ D_i = \begin{bmatrix} (D_i)_{11} & (D_i)_{12} \\ (D_i)_{21} & C(i) \end{bmatrix}, \] (2.31)
and
\[ (D_i)_{12} = (D_i)_{21} = \mu^{-1}(\lambda' - \lambda)L(v_i, \mu v_i)J = (\lambda' - \lambda)J_{\{v_i, i\}}, \] (2.32)
(where \( J_{\{v_i, i\}} \) is the all ones \( v_i \) by \( i \) matrix), while
\[ (D_i)_{11} = \mu^{-1}L(v_i, \mu v_i)C(c - i)L(v_i, \mu v_i)^t. \]
Now, we partition \( C(c - i) = (F_{\alpha \beta}) \) into submatrices \( F_{\alpha \beta} \) of order \( \mu \) \((\alpha, \beta = 1, \ldots, v_i)\); since
\[ F_{\alpha \beta} = \begin{cases} C(\mu), & \text{if } \alpha = \beta, \\ (\lambda' - \lambda)J, & \text{if } \alpha \neq \beta, \end{cases} \]
our Corollary 2.2 in [2] yields
\[ \sigma(F_{\alpha\beta}) = \begin{cases} \mu((k - \lambda') + \mu(\lambda' - \lambda)), & \text{if } \alpha = \beta; \\ \mu^2(\lambda' - \lambda) & \text{if } \alpha \neq \beta. \end{cases} \]
Thus, by (2.16) we have
\[ (D_i)_{11} = \mu^{-1}(k - \lambda')J_{\lambda'} + (\lambda' - \lambda)J_{\lambda'}. \quad (2.33) \]
From (2.32) and (2.33) it follows that \( D_i \) has the form it takes in (2.29) and the lemma is proved. \( \square \)

**Proof of Theorem 2.1.** This now follows from Notation 2.2, Lemma 2.3, Notation 2.4, and Lemmas 2.5 and 2.6, just multiplying (2.13) by \( \mu \) and considering instead of \( V^{-1}L \) the matrix
\[ H = \mu V^{-1}L. \quad (2.34) \]
Restating (2.13) in terms of \( H \) gives
\[ H(\mu^{-1}B)H^t = \mu D + \mu \lambda J_{\lambda'}, \quad (2.35) \]
where \( D \) is given in (2.28). Since \( B = V = \mu I_{v'-N} \oplus I_N \), (2.35) yields that \( L_{v'-N} \oplus \mu^{-1}I_N \) is rationally congruent to the matrix \( Z \) of (2.2), which completes the proof of Theorem 2.1 \( \square \)

A reason for considering (2.35) instead of (2.13) (and so to give the statement 2.1 instead of 2.6 for our theorem) is that it is actually easier to compute the Hasse invariants of \( I_h \times C \) than those of \( \mu^{-1}(I_h \times C) \); this remark will be made clear in Section 3 (see also [5]).

Another reason is that the matrix \( H = (h_{ij}) = \mu V^{-1}L \), called the **generalized incidence matrix** ([2]), has an interesting geometrical meaning. Namely, if \( P_i \) is a fixed base point in \( P_i \) and \( x_j \) is a fixed base block in \( B_j \), then
\[ h_{ij} = |\{g \in G \mid P_i^g \text{ is incident with } x_j\}|. \quad (2.36) \]

**3. Applications**

In this section we outline how Theorem 2.1 can be applied to obtain non-existence theorems, using the Hasse–Minkowski theory on the equivalence of rational quadratic forms.

From 1.3 of [2] it is not difficult to deduce that the main results announced by the author in [2] (namely, 4.6 and 5.1 of [2]) can be stated as follows:

**Theorem 3.1.** If a point-divisible design \( D = D(m, c, k, \lambda', \lambda) \) exists with a standard automorphism group \( G \) of order \( \mu \), which fixes \( N \) points and \( \Phi \) divisibility classes (and so has \( u' = \{(v - N)/\mu\} + N \) orbits on points and \( u = \{(m - \Phi)/\mu\} + \Phi \) orbits on classes) and if \( n = (k - \lambda') + c(\lambda' - \lambda) \), then \( n = k^2 - v\lambda \) and:
(i) \((k - \lambda')^{-1}u^{-1}n^{-1} \) is a square;
(ii) the diophantine equations
\[ (k - \lambda')x^2 + (-1)^{(v'-u)(v'-u+1)/2}c^u\mu^{N+\Phi}y^2 = z^2 \quad (3.1) \]
and
\[ nx^2 + (-1)^{(u(u+1)/2)+1}c^u\lambda^{\Phi+1}y^2 = z^2 \quad (3.2) \]
are equivalent (i.e. (3.1) has a non-trivial solution in integers iff (3.2) has such a solution).

Sketch of the Proof (see [3] and [5] for the full proof, which is only very briefly sketched here). We know from Theorem 2.1 that the matrix

\[ Z = (I_h \times C) \oplus K \times \mu\lambda_{v'}, \]

(3.3)

where \( C, h \) and \( K \) are as in Theorem 2.1, is rationally congruent to the diagonal \( v' \) by \( v' \) matrix

\[ I_{v'-N} \oplus \mu^{-1}I_N. \]

(3.4)

Precisely, if \( H \) is the generalized incidence matrix of Section 2, then

\[ H(I_{v'-N} \oplus \mu^{-1}I_N)H^T = Z = \mu D + \mu\lambda_{v'}, \]

(3.5)

where \( D \) is given by (2.28).

Taking determinants, we obtain

\[ |H|^2 = |Z| \mu^N, \]

(3.6)

hence

\[ |Z| \mu^N \text{ is a square}, \]

(3.7)

which after a repeated application of Lemma 2.1 of [2] gives (i).

By the Hasse–Minkowski theorem, (3.5) implies that for all primes \( p \) (including \( p = \infty \)), the two matrices \( Z \) and \( I_{v'-N} \oplus \mu^{-1}I_N \) have the same Hasse invariants. From this, using the Hasse–Minkowski theory and the above-mentioned lemma of [2] it is possible to deduce condition (ii). \( \Box \)

It is not difficult to verify (see [5]) that when \( c = 1 \) (i.e. the design is a symmetric \((v, k, \lambda)\) and \( v = m, \Phi = N \), so that \( v' = u \)), then Theorem 3.1 gives the well known Hughes theorem on the non-existence of standard automorphism groups for symmetric designs.

Hence, if \( \mu = c = 1 \), Theorem 3.1 gives, as a particular case, the Bruck–Ryser–Chowla theorem.

For \( \mu = 1 \) and \( c \neq 1 \), Theorem 3.1 yields another well known non-existence theorem, the Bose–Connor theorem on the non-existence of point-divisible designs.

Thus, as a consequence of Theorem 2.1, we obtain not only a simultaneous generalization, but also a unified proof of the above theorems.

Furthermore, Theorem 3.1 can be applied to obtain other non-existence conditions for \( p \)-divisible designs with a Singer Group (or equivalently, for Relative Difference sets) (see [4]). Obviously, these conditions for \( c = 1 \) give back the well known Hall–Ryser theorem for symmetric designs with a Singer group (or equivalently, for difference sets in finite groups).

For other applications of Theorem 3.1, and thus of Theorem 2.1, the reader is referred to [3].

References


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