

# Viscous dark fluid

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## Abstract

An unified dark energy and dark matter model within the framework of a model of a continuous medium with bulk viscosity (dark fluid) is considered. It is supposed that a bulk viscosity coefficient is an arbitrary function of the Hubble parameter. The choice of this function is carried out under the requirement to satisfy the observational data from recombination ( $z \approx 1000$ ) to present time.

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## 1. Introduction

Modelling of the accelerated expansion of the present Universe lies on the way of creation of phenomenological models which may explain the observational data. Following which one tries to obtain some predictions that do not follow from observations directly. For example, a theoretical model adapts for the correct description of the acceleration of the Universe in an accessible interval  $z$ . Further, the results of the modelling are extrapolated for large  $z$  that are not accessible for observation yet. The corresponding cosmological scenario defines growth of the large scale structure which determines the present-day fluctuations of the microwave background radiation. Certainly, the models under consideration should not contradict the available observational data within the framework of general relativity in a field of its applicability. A number of cosmological models which were successfully applied in the theory of the early Universe is used for the purposes mentioned above, including cosmological models with various scalar and non-scalar fields

filling the space together with cold dark matter (see, e.g., the reviews [1]). Cosmological models of the present accelerated Universe within the framework of high-order theories of gravity (HOTG) are also quite popular [2].

A number of models which describe the present Universe with use of models of a continuous medium in the presence of bulk viscosity have recently been suggested [3–5]. Consideration of effects of viscosity within the framework of HOTG was also carried out [6]. Note that such models were well known in the theory of the early Universe (see, for example, [7,8]). In particular, in Ref. [8] a few exact solutions with the constant bulk viscosity coefficient and with the bulk viscosity being an arbitrary power function of energy density were obtained. In Ref. [3] the model of viscous dark fluid was considered. The main result of this Letter is the model with the constant bulk viscosity coefficient. The model satisfies the observational data on luminosity with adequate accuracy. In Ref. [4] the models both with the constant bulk viscosity coefficient and the bulk viscosity linearly proportional to the Hubble parameter are examined. The question about the effect of the viscosity on the presence of a singularity in the future (the so-called Big Rip) is investigated.

In this Letter we consider a model of “viscous dark fluid” with the bulk viscosity coefficient  $\mu(H)$  which depends on the

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Hubble parameter arbitrarily. In contrast to Ref. [3], comparison of our model with observations is not restricted to the observational data on luminosity. The model is being compared with observations of change of the deceleration parameter  $q$  and values of the Hubble parameter in the range  $2 > z > 0$ . It will be shown below that the model with the constant bulk viscosity coefficient does not provide a good description for  $q(z)$  and  $H(z)$  which follow from observations. We propose such a dependence  $\mu(H)$  which is adequate to the mentioned observations. The proposed model can be extrapolated for  $z$  beyond the mentioned range  $2 > z > 0$ .

## 2. Equations and solutions

The flat metric can be taken in the form:

$$ds^2 = c^2 dt^2 - a(t)^2(dx^2 + dy^2 + dz^2). \quad (1)$$

The corresponding 0–0 component of the Einstein equations is

$$H^2 = \frac{1}{a^2} \left( \frac{da}{dt} \right)^2 = \frac{8\pi G}{3c^2} \varepsilon, \quad (2)$$

where  $H$  is the Hubble parameter,  $\varepsilon$  is the energy density of matter. Introducing the dimensionless energy density

$$\delta = \frac{\varepsilon}{\varepsilon_*},$$

where the critical density  $\varepsilon_* = 3c^2 H_0^2 / 8\pi G$  (the subscript 0 indicates the value of the parameter at the present time), it can be derived from (2) that

$$\delta = \frac{H^2}{H_0^2} = h^2. \quad (3)$$

Here the dimensionless Hubble parameter  $h$  is expressed in units of its present value  $H_0$ .

The corresponding energy conservation law for the viscous dark fluid can be obtained from the equation

$$[T_i^k + \tau_i^k]_{;k} = 0, \quad (4)$$

where the energy–momentum tensor of matter is

$$T_i^k = (\varepsilon + p)u_i u^k - \delta_i^k p,$$

and the tensor of viscosity is

$$\tau_i^k = \mu u_{;l}^l (\delta_i^k - u_i u^k)$$

with the bulk viscosity coefficient  $\mu$ . By carrying out covariant differentiation in (4) with taking into account the metric (1), one can obtain the following equation:

$$\frac{d\delta}{d\theta} + 3h\delta = 9\lambda h^2. \quad (5)$$

Here the dimensionless time  $\theta = H_0 t$  is introduced, and the bulk viscosity coefficient rewritten via the dimensionless parameter  $\lambda$  as

$$\mu = \varepsilon_* \lambda / H_0.$$

Similarly to [3], let us suppose that the viscous medium has the pressure  $p = 0$ . Then Eqs. (3) and (5) imply one equation

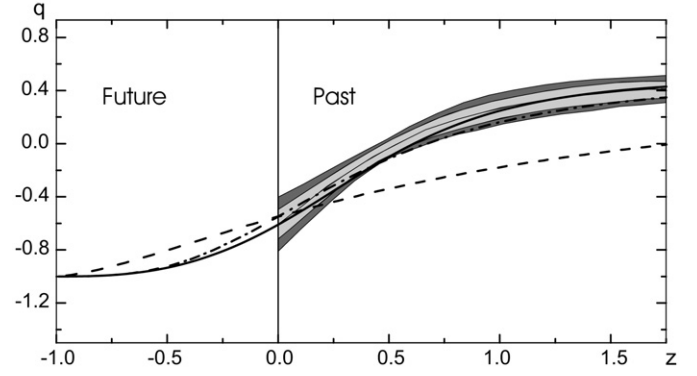


Fig. 1. The deceleration parameter for our model (thick solid line), the model with the constant bulk viscosity from [3] (dashed line), the  $\Lambda$ CDM model (dashed-dot line). The central solid thin line represents the best-fit, the light grey contours represent the  $1\sigma$  confidence level, and the dark grey contours represent the  $2\sigma$  confidence level (the data taken from [9]).

for the dimensionless Hubble parameter  $h$ . The case  $\lambda = \text{const}$  is equivalent to the Murphy's model [7]. As a matter of fact this model was used in [3] for comparison with observations. The models of the present Universe mentioned in Introduction use the fact that the deceleration parameter  $q \approx 0.5$  in the past at  $z \gg 1$  (cold dark matter), and at the present time the Universe expands with acceleration. For this reason all the models (both with scalar fields and in HOTG) are being created in such a way that the present inflation appears for rather small values of the Hubble parameter, or, that is the same thing, for the small average density of matter. That is why we will consider the model (5) in which the dimensionless bulk viscosity of the dark fluid  $\lambda$  is not a constant but an arbitrary function of the parameter  $h$ . In this Letter we chose this function as

$$9\lambda = 3 \tanh \left( \frac{b}{h^n} \right), \quad (6)$$

where  $b, n$  are arbitrary constants which will be defined from the observational data later.

For comparison with the observational data, it is convenient to rewrite Eqs. (3) and (5) via the cosmological redshift  $z = 1/a - 1$ . Then the equation for  $h$  takes the form

$$-2(z+1) \frac{dh}{dz} + 3h = 3 \tanh \left( \frac{b}{h^n} \right), \quad (7)$$

and the deceleration parameter  $q$  will be

$$q = - \left( 1 - \frac{(z+1)}{h} \frac{dh}{dz} \right).$$

The parameters  $b, n$  are being chosen from the requirement that at the present time (when  $h = 1$ ) the deceleration parameter  $q$  should be close to the observable value  $q \approx -0.6$ . This value of  $q$  can be obtained at  $b = 0.95$  and  $n = 2$ . Using these parameters, we have compared the model under consideration with the observational data in the range  $2 > z > 0$ . The results are presented in Fig. 1. For comparison with our model, the  $\Lambda$ CDM model with the same “initial conditions” is also shown in Fig. 1. The model with the constant bulk viscosity from [3] is presented as well.

It follows from Fig. 1 that the model with the constant bulk viscosity deviates from observations considerably. The model under consideration with  $\lambda(h)$  lies within the confidence levels. Note that our model close to the  $\Lambda$ CDM model (with  $\Omega_m = 0.3$  and  $\Omega_\Lambda = 0.7$ ), although there is no any special  $\Lambda$ -term in the model. Outside the interval of observations (in fact, at  $z \gtrsim 2$ ) our model, the  $\Lambda$ CDM model and extrapolation of the observable data from [9] are close to each other and give  $q \approx 0.5$ . The model with constant  $\lambda$  has much less value of  $q$  and reaches  $q \approx 0.5$  at  $z \gtrsim 40$ .

An extrapolation to the future for all three models is also shown in Fig. 1. Asymptotically (at  $z \rightarrow -1$ ) all three models tend to the de Sitter model.

### 3. Conclusion

The bulk viscosity in our model is an example of a dynamic  $\Lambda$ -term. However, our model is close to the  $\Lambda$ CDM model. As it was rightly noted in [3], the model with the viscosity does not give possibility to divide the true dust-like matter in the Universe and dark matter generated by the bulk viscosity. That is why it is difficult to introduce a phenomenological equation of state  $p = w\varepsilon$  which is often being used for interpretation of the observational data. The bulk viscosity can play a significant role in the formation of the large scale structure of the Universe. This question demands a special examination. But taking into account that the viscosity is being “involved” at rather small

$z \approx 2$ , it is possible to expect its influence on dynamics of clusters of galaxies only at later non-linear stage in the evolution of the Universe.

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