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ORIGINAL ARTICLE

Investigation of the viscoelastic flow and species diffusion in a porous channel with high permeability

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Abstract In this study, effect of mass transfer on laminar flow of viscoelastic fluid in a porous channel with high permeability medium is investigated. The viscoelastic model used in this work is the upper convected Maxwell (UCM) model. Applying the similarity transformation, the governing partial equations are converted to ordinary differential equations. The problem is studied by a hybrid technique based on Differential Transformation Method (DTM) and iterative Newton's method (INM). Also a numerical solution is done to validate the present analytical method. The effects of active parameters such as Darcy number (*Da*), transpiration Reynolds number (Re_T) Deborah number (*De*) and Schmidt number (*Sc*) on the both velocity components and concentration function are discussed in this work. The results indicate that the stream function increases for large Deborah and Darcy numbers. The axial velocity is initially decreased by increasing the Deborah number but then increased while approaching the upper channel wall.

1. Introduction

In the current decade, the examination on the behavior of viscoelastic flows has attracted considerable interest due to their wide range of applications such as chemical process industries, food processing and biological systems. Viscoelastic fluids show both viscous and elastic properties, so because of this complexity of such fluids there is no specific model that indicates all of their properties, simultaneously. Some models have been proposed for such fluids which exhibit the non-linear relationship between stress and the rate of strain include second-grade model, Walters-B model, the Oldroyd model and upper convected Maxwell model (UCM). Many researchers studied the flow of all of these models. Fan et al. [1] utilized finite volume method to represent the viscoelastic flow which includes UCM and Oldroyd-B fluid in curved pipes. Sadeghi and Sharifi [2] studied the boundary layer flow of second-grade viscoelastic fluid above a moving plate. Nandeppanavar et al. [3] investigated the flow of Walters-B liquid fluid over an impermeable stretching sheet with the presence of non-uniform heat source/sink. The second-grade model is not suitable for flows of highly elastic fluids which occur at high Deborah number. Some studies show that the use of this kind of fluid is suitable only for slow flows with small level of elasticity [4,5] while there are some practical cases in which the Deborah

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Nomenclature

a_i	unknown parameter ($i = 1,2,3$)
C	concentration of species of fluid
C_H	concentration of channel center
C_W	concentration of channel wall
D	diffusion coefficient of the diffusing spices
Da	Darcy number
De	Deborah number
DTM	differential transformation method
$f(y)$	dimensionless normal velocity component
$f'(y)$	dimensionless axial velocity component
H	channel width
K	permeability of the porous medium
Re	Reynolds number
R_i	residual value ($i = 1,2,3$)
Sc	Schmidt number
U	fluid velocity along x-direction
V	fluid velocity along y-direction
X	coordinate along the channel
Y	coordinate perpendicular to the channel

Greek symbols

λ	relaxation time of the UCM fluid
ρ	density of the fluid
μ	dynamic viscosity of the fluid
ν	Kinetic viscosity of the fluid
ϕ	dimensionless concentration function

number is high [6]. So, the upper convected Maxwell model is proposed in simulating highly elastic fluids because it can predict the effects of stress relaxation. Khayat [7] analyzed the viscoelastic flow of UCM fluid between two parallel plates with moving free boundaries using perturbation method. Kumari and Nath [8] examined steady mixed convection flow of UCM fluids in the area of two-dimensional stagnation point with magnetic field by using boundary layer theory and finite difference method. The results are opposite of those reported for second-grade fluids. Frey et al. [9] utilized finite element method to study the flow of UCM fluids around a cylinder, and the model is approximated by a Galerkin least-squares formulation in extra-stress, pressure and velocity. Hayat et al. [10] employed homotopy analysis method (HAM) to investigate the influence of mass transfer on the two-dimensional stagnation point flow of UCM fluid over a stretching surface. Li et al. [11] studied viscoelasticity on lubricant thin film under the assumption that fluid belongs to the UCM model. The results show that the viscoelasticity that increases the lubricant pressure field has an influent effect on the lubrication performance. Abel et al. [12] used similarity transformation to investigate magnetohydrodynamic flow and heat transfer in a boundary layer of UCM flow over a stretching sheet applying numerical solution. Renardy and Wang [13] studied boundary layers arising in the high Weissenberg number limit of viscoelastic UCM flows using two mechanisms for the formation of viscoelastic boundary layer.

The study of heat and mass through a porous media is of special interest in many engineering fields such as chemical engineering, solar collectors processing and nuclear reactors. The phenomena of porous media in viscoelastic flow were investigated in some studies. Sivaraj and Kumar [14] investigated unsteady, MHD and chemically reacting dusty viscoelastic (Walter's liquid-B model) fluid Couette flow in a porous channel with convecting cooling and varying mass diffusion. Srinivas et al. [15] have studied the effects of chemical reaction and mass transfer on the flow of viscoelastic fluid in a porous channel with moving or stationary walls using HAM.

Most problems in the investigation of the flow of viscoelastic fluid are nonlinear. All these problems are modeled by partial or ordinary nonlinear equation. Hayat and Abbas [16] studied the flow of UCM fluid in a porous channel with chemical reaction, while Beg and Makinde [17] extended their study with considering species diffusion in a Darcian porous medium channel only using numerical solution. According to the above description, the main gain of this paper is to apply DTM to find the approximation solution of nonlinear differential equations governing the problem of flow of the UCM fluid in a porous channel with high-permeability. The main motivation of this study is to investigate both component of axial and normal velocity of the flow by using analytical solution. The influences of various parameters on velocity components and species concentration field are discussed.

In this paper, a new hybrid technique is used for solving the governing equations of the problem. The procedure of solution is based on the DTM and Newton's iterative method. Here, we can obtain the approximate solution of the problem using the proposed technique. Differential transform method is an iterative technique to obtain the semi analytical solution for differential equations by computing the components of Taylor series. Zhou [18] first introduced DTM for solving the linear and nonlinear initial value problems. He used this method to derive the semi analytical solution for the electrical circuit analysis. A considerable amount of researches have been done using DTM to investigate the solution of linear differential algebraic equations [19], nonlinear ordinary differential equations [20–24], partial differential equations [25], fractional differential equations [26] and integral equations [27]. DTM is a powerful and simple technique which is well known as a high accurate technique for solving the differential equations. Recently, most engineering problems have been analyzed using the analytical and approximate methods [28–35].

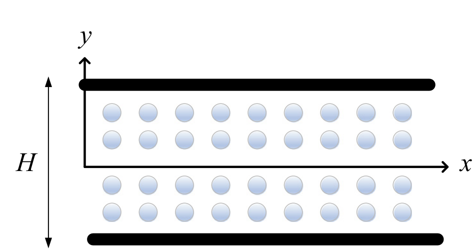

Figure 1 Schematic of the flow geometry in a channel with isotropic homogenous porous medium.

2. Description of the problem

Consider the steady, laminar and incompressible flow of UCM fluid in a parallel plate channel having porous walls of high permeability. The physical model is shown in Fig. 1. The x direction is parallel of channel which is set to the motion of flow and the y direction is perpendicular to x. Following the assumption of the above mentioned studies [16,17], the flow is symmetric about both axes in which the channel has the same boundary conditions at the walls. The flow and species diffusion take place in the channel with the fluid suction or injection through the porous walls with velocity $V/2$, where $V > 0$ represents the suction and $V < 0$ corresponds to injection. It is assumed that the width of the channel is much higher than the height of the channel, so this study represents the tow-dimensional model. Taking into consideration of these assumptions, the equations of mass, momentum and the concentration filed are as follows [17]:

$$\frac{\partial u}{\partial X}+\frac{\partial u}{\partial Y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial X}+v\frac{\partial u}{\partial Y}+\lambda\left[u^2\frac{\partial^2 u}{\partial X^2}+v^2\frac{\partial^2 u}{\partial Y^2}+2uv\frac{\partial^2 u}{\partial X\partial Y}\right]=v\frac{\partial^2 u}{\partial Y^2}-v\frac{u}{k}, \tag{2}$$

$$u\frac{\partial C}{\partial X}+v\frac{\partial C}{\partial Y}=D\frac{\partial^2 C}{\partial Y^2}. \tag{3}$$

where u and v are the velocity components in x and y directions, respectively. λ is relaxation time, D is the mass diffusion, C is the concentration field and K exhibits the permeability of the porous media.

The relevant boundary conditions for the flow are as follows:

$$\frac{\partial u}{\partial Y}=v=0; \quad C=C_w \quad \text{at} \quad Y=0, \tag{4}$$

$$u=0, v=\frac{V}{2}; \quad C=C_H \quad \text{at} \quad Y=\frac{H}{2}. \tag{5}$$

Defining the following transformation as introduced in [16]:

$$x=\frac{X}{H},\ \ y=\frac{Y}{H},\ \ u=-Vxf'(y),\ \ v=Vf(y),\ \ \phi=\frac{C-C_H}{C_w-C_H}. \tag{6}$$

Substituting the above variables, the governing partial equations are converted to ordinary differential equations. So the momentum equation (Eq. (2)) and concentration equation (Eq. (3)) can be written in non-dimensional form as:

$$f'''(Y)+Re_T\left[(f')^2-ff''\right]+De\left(2ff'f''-f^2f'''\right)+\frac{1}{Da}f'=0, \tag{7}$$

$$\phi''-Re_T\ Scf\ \phi'=0. \tag{8}$$

Therefore, the transformed boundary conditions become:

$$\begin{aligned} &f=0, \quad \phi=1 \text{ at } y=0,\\ &f=0.5,\ f'=0, \quad \phi=0 \text{ at } y=0.5. \end{aligned} \tag{9}$$

The dimensionless parameters of Da, De, Re, Sc are the Darcy, Deborah, Reynolds and Schmidt numbers, respectively. They are defined as follows:

$$Da=\frac{K}{H^2}, De=\frac{\lambda V^2}{v}, Re=\frac{\rho VH}{\mu}, Sc=\frac{v}{D}. \tag{10}$$

3. Differential transform method

The differential transform is defined as follows:

$$X(k)=\frac{1}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=t_0}. \tag{11}$$

where $x(t)$ is an arbitrary function, and $X(k)$ is the transformed function. The inverse transformation is as follows:

$$x(t)=\sum_{k=0}^{\infty}X(k)(t-t_0)^k. \tag{12}$$

Substituting Eq. (11) into Eq. (12), we have:

$$x(t)=\sum_{k=0}^{\infty}\frac{(t-t_0)^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=t_0}. \tag{13}$$

The function $x(t)$ is usually considered as a series with limited terms and Eq. (12), can be rewritten as follows:

$$x(t)\approx\sum_{k=0}^{m}X(k)(t-t_0)^k. \tag{14}$$

where m represents the number of Taylor series' components. Usually, through elevating this value, we can increase the accuracy of the solution.

Some of the properties of DTM are shown in Table 1. These properties are extracted from Eqs. (11) and (12).

4. Solution of the problem

This section tries to obtain a solution for Eqs. (7) and (8) using a new hybrid technique. The solution procedure has two steps, first by applying DTM, the Taylor series of solution is found. Then, the iterative Newton's method will be used to obtain the unknown parameters of the solution.

4.1. Applying DTM

The solution of the Eqs. (7) and (8) is considered as the Taylor series at $y = 0$ in the following form:

$$\begin{aligned} f(y)&=\sum_{k=0}^{m}F(k)y^k, \quad 0\leqslant y\leqslant 0.5,\\ \phi(y)&=\sum_{k=0}^{m}\Phi(k)y^k, \quad 0\leqslant y\leqslant 0.5. \end{aligned} \tag{15}$$

The system of BVPs (Eqs. (7) and (8)) can be transformed to initial value problems with the replacement of the unknown initial conditions instead of the boundary conditions:

Table 1 The properties of the DTM.

Original function	Transformed function
$f(t)=g(t)\pm h(t)$	$F(k)=G(k)\pm H(k)$
$f(t)=cg(t)$	$F(k)=cG(k)$
$f(t)=\frac{d^n g(t)}{dt^n}$	$F(k)=\frac{(k+n)!}{k!}G(k+n)$
$f(t)=g(t)h(t)$	$F(k)=\sum_{r=0}^{k}G(r)H(k-r)$
$f(t)=t^n$	$F(k)=\delta(k-n)=\begin{cases}1 & \text{if } k=n\\ 0 & \text{if } k\neq n\end{cases}$

Table 2 Comparison of the present results and numerical solution for $Da = 1$, $Re_T = 1$, $De = 1$ and $Sc = 1$.

y	$f(y)$			$\phi(y)$		
	Present (Eq. (22))	Numerical solution	Error	Present (Eq. (23))	Numerical solution	Error
0	0	0	0	1	1	0
0.05	0.09269	0.09285	1.59E−4	0.90604	0.90605	8.13E−6
0.1	0.17658	0.17687	2.88E−4	0.81165	0.81167	1.54E−5
0.15	0.25123	0.25167	4.40E−4	0.71643	0.71645	2.10E−5
0.2	0.31636	0.31684	4.84E−4	0.62001	0.62003	2.47E−5
0.25	0.37180	0.37227	4.77E−4	0.52205	0.52208	2.61E−5
0.30	0.41744	0.41786	4.22E−4	0.42226	0.42229	2.51E−5
0.35	0.45321	0.45353	3.20E−4	0.32037	0.32039	2.18E−5
0.4	0.47903	0.47920	1.71E−4	0.21615	0.21617	1.66E−5
0.45	0.49477	0.49475	2.29E−5	0.10941	0.10941	9.44E−6
0.5	0.5	0.5	0	0	0	0

$$f(0) = 0, \quad f'(0) = a_1, \quad f''(0) = a_2, \\ \phi(0) = 1, \quad \phi'(0) = a_3. \tag{16}$$

where a_1 to a_3 are the unknown parameters of initial conditions. After applying DTM on Eqs. (7) and (8) at $y = 0$ and simplification, the following recursive equations are obtained to calculate the series solutions' components

$$\begin{aligned} F(k+3) = &\frac{1}{(k+1)(k+2)(k+3)} \\ &\times \left\{ -Re\sum_{r=0}^{k}(r+1)(k-r+1)F(r+1)F(k-r+1) \right. \\ &+ Re\sum_{r=0}^{k}(r+1)(r+2)F(r+2)F(k-r) \\ &- \frac{1}{Da}(k+1)F(k+1) \\ &- 2De\sum_{p=0}^{k}\sum_{r=0}^{p}(r+1)(r+2)(r+3)F(r+3)F(p-r)F(k-p) \\ &\left. + De\sum_{p=0}^{k-1}\sum_{r=0}^{p}(r+1)(r+2)(r+3)F(r+3)F(p-r)F(k-p) \right\}. \end{aligned} \tag{17}$$

$$\Phi(k+2) = \frac{1}{(k+1)(k+2)} Re \cdot Sc \sum_{r=0}^{k}(r+1)\Phi(r+1)F(k-r). \tag{18}$$

The differential transform of the conditions in Eq. (16) is as follows:

$$F(0) = 0, \quad F(1) = a_1, \quad F(2) = \frac{a_2}{2}, \\ \Phi(0) = 1, \quad \Phi(1) = a_3. \tag{19}$$

4.2. Applying iterative Newton's method

Now, we should compute the unknown parameters (a_1 to a_3) from the boundary conditions at the end domain ($y = 0$) in Eq. (9). For this reason, we will try to minimize the following residual functions for obtaining the unknown parameters:

$$\begin{aligned} R_1 &= f(0.5, a_1, a_2, a_3) - f(0.5) = \sum_{k=0}^{m} F(k)(0.5)^k - 0.5, \\ R_2 &= f'(0.5, a_1, a_2, a_3) - f'(0.5) = \sum_{k=1}^{m} kF(k)(0.5)^{k-1}, \\ R_3 &= \phi(0.5, a_1, a_2, a_3) - \phi(0.5) = \sum_{k=0}^{m} \Phi(k)(0.5)^k. \end{aligned} \tag{20}$$

To obtain the roots of the Eq. (20), we can use the following multi-variable iterative Newoton's method:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}_{n+1} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}_{n} - \begin{bmatrix} \frac{\partial R_1}{\partial a_1} & \frac{\partial R_1}{\partial a_2} & \frac{\partial R_1}{\partial a_3} \\ \frac{\partial R_2}{\partial a_1} & \frac{\partial R_2}{\partial a_1} & \frac{\partial R_2}{\partial a_1} \\ \frac{\partial R_3}{\partial a_1} & \frac{\partial R_3}{\partial a_1} & \frac{\partial R_3}{\partial a_1} \end{bmatrix}^{-1} \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}_{n}, \quad n = 0, 1, 2, \ldots \tag{21}$$

where n shows the number of iteration. After guessing the initial values for a_1 to a_3, we should calculate the residual vector (R) and Jacobian Matrix ($\frac{\partial R_i}{\partial a_j}$). The residual vector can be obtained by substituting $(a_1, a_2, a_3)^n$ in Eq. (20). The components of the Jacobian matrix in Eq. (21) can be computed by differentiating analytically with respect to a_1 to a_3 and then substituting $(a_1, a_2, a_3)^n$ in that equation.

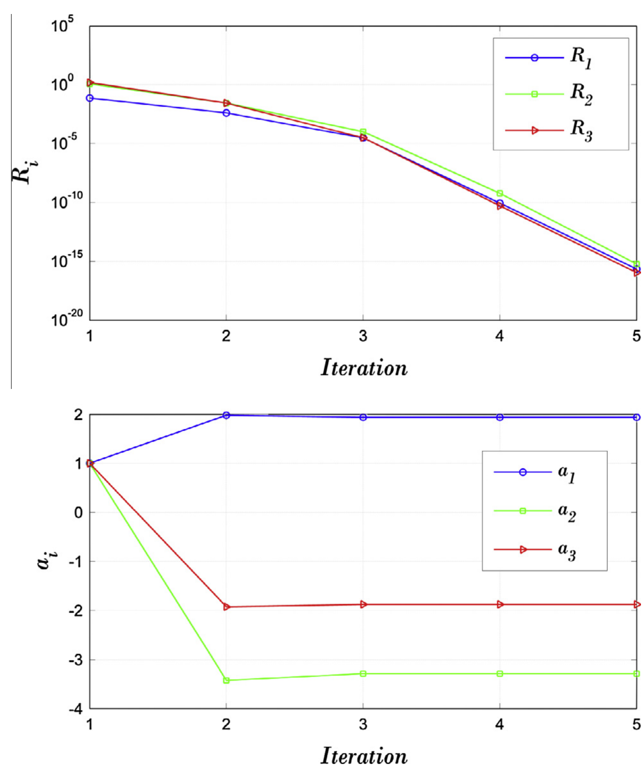

Figure 2 The convergence history of the iterative Newton's method when $Da = 1$, $Re_T = 1$, $De = 1$ and $Sc = 1$ for (a) residual functions and (b) unknown parameters.

4.3. Accuracy and convergence history of the solution

The accuracy for computing a_1 to a_3 by Newton's method is chosen 10^{-9} and all of the initial guesses for a_1 to a_3 are considered one. The number of Taylor series components is considered $m = 8$.

The approximate solution of the problem is in the following form for $Da = 1$, $Re_T = 1$, $De = 1$ and $Sc = 1$:

$$f(y) = 1.9383y - 1.6429y^2 - 0.9492y^3 + 1.4310y^4 - 0.7322y^5, \quad (22)$$

$$\phi(y) = 1 - 1.8776y - 0.6066y^3 + 0.2571y^4 - 0.0872y^5. \quad (23)$$

For validating the present solution of the problem and finding the accuracy, we will compare results of our procedure and numerical solution. Numerical solution of the problem is done with the Maple package. The available methods in this software are a combination of the base scheme (midpoint or trapezoid), and a method enhancement scheme (deferred corrections or Richardson extrapolation). This technique is capable of handling both linear and nonlinear BVPs with fixed, periodic and even nonlinear boundary conditions.

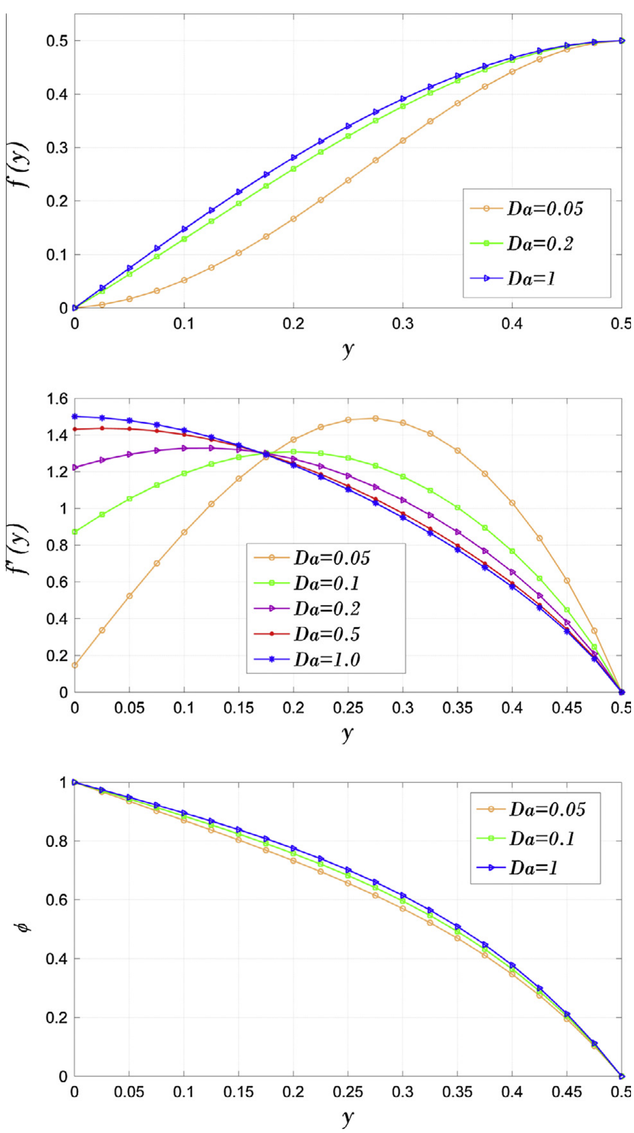

Figure 3 The profiles of $f(y)$, $f'(y)$ and $\phi(y)$ when $Re_T = 5$, $De = 3$, $Sc = 2$ for different Darcy numbers (Da).

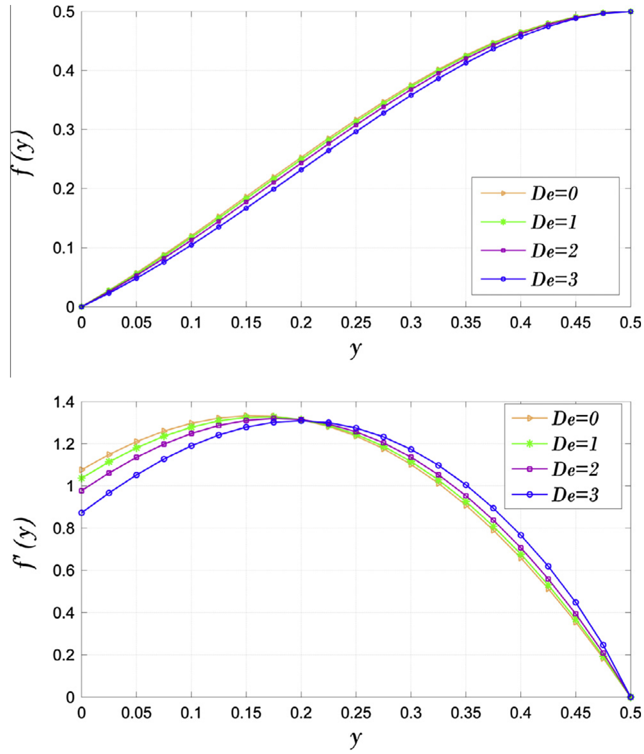

Figure 4 The profiles of $f(y)$ and $f'(y)$ when $Re_T = 5$, $Da = 0.1$, $Sc = 0.5$ for different Deborah numbers (De).

A good agreement between the present hybrid method and numerical solution is observed in Table 2 for a special case ($Da = 1$, $Re_T = 1$, $De = 1$ and $Sc = 1$), which confirms the validity of the proposed method. As it can be seen, error of the method is in order of 1E−6 to 1E−4.

The convergence history of the unknown parameters and residual is shown in Fig. 2 for a special case. As we can see in Fig. 2 the problem converged rapidly with only 5 iterations. This is because the Jacobian matrix obtained by differentiating analytically with respect to a_1 to a_3.

5. Results and discussion

In this section, Figs. 3–6 represent the effects of Darcy number (Da), Deborah number (De), Reynolds number (Re_T) and Schmidt number (Sc) on the dimensional velocity components ($f(y)$, $f'(y)$) and concentration field (ϕ). In order to clarify the dependency of viscoelastic flow on the permeability and fluid elasticity, stream function of flow and axial velocity are plotted versus Deborah number and Darcy number.

Fig. 3 illustrates the variation in Darcy number on the velocity and concentration filed. The value of parameters used in this simulating is as follows: $Re_T = 5$, $De = 3$, $Sc = 2$. It is observed that the normal velocity ($f(y)$) increases for large Da. The axial velocity ($f'(y)$) is increased initially but then decreased for the value of y between 0.18 and 0.5 with an increasing Darcy number. At higher value for Darcy number, the effect of viscous force is more and the Darcian drag will decrease which cause the acceleration in the flow while approaching the upper channel wall will reduce the fluid

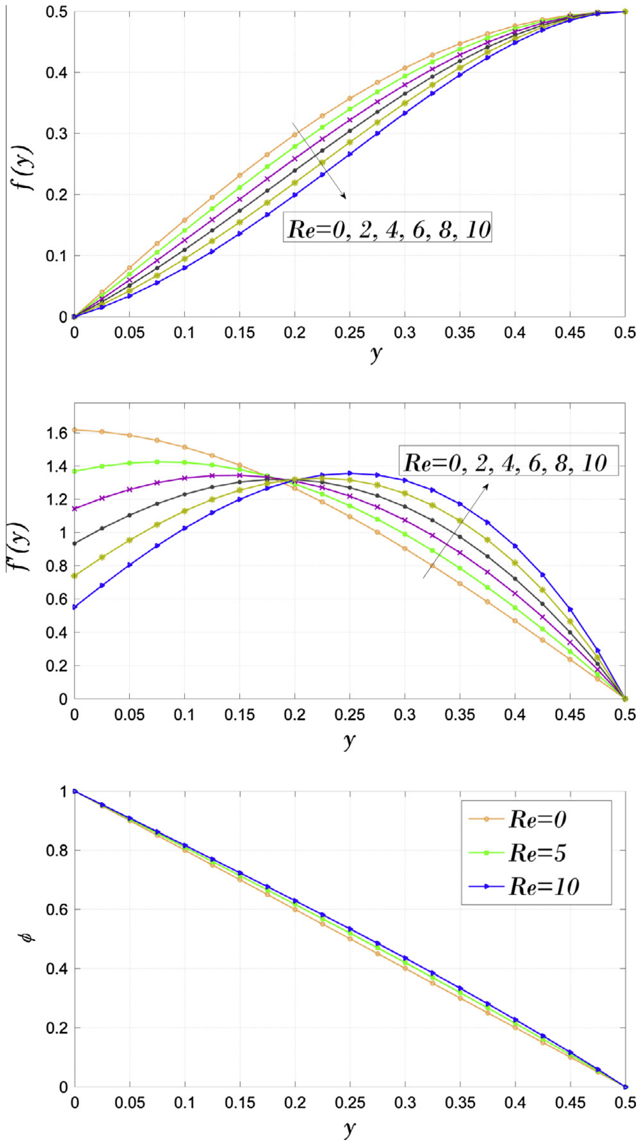

Figure 5 The profiles of $f(y)$, $f'(y)$ and $\phi(y)$ when $Da = 0.1$, $De = 1$, $Sc = 0.2$ for different Reynolds numbers (Re_T).

motion and cause the velocity to decrease with increasing permeability. Also it can be seen that the concentration distribution varies a little by varying the Darcy number, because there are no buoyancy forces, it is anticipated that permeability does not make considerable influence on the diffusion of the spices.

Fig. 4 is plotted to show the influence of viscoelastic material parameter (Deborah number) on dimensionless velocity components. The terms $De(2ff'f'' - f^2f''')$ show the viscoelastic effects in the momentum equation which are the nonlinear terms. The Deborah number is often used to characterize the fluidity of materials which is the ratio of the time-scale of a flow to the relaxation time. It is noticed that the variation in the Deborah number has very little effect on normal velocity. As shown in Fig. 4, the component f increases with increasing De but the increment is very small. Here the axial velocity ($f'(y)$) initially decreases by increasing De and then increases after $y = 0.225$. With the increasing values of the Deborah number, the degree of strain hardening is enhanced and the elasticity effect will be more than the viscosity effects.

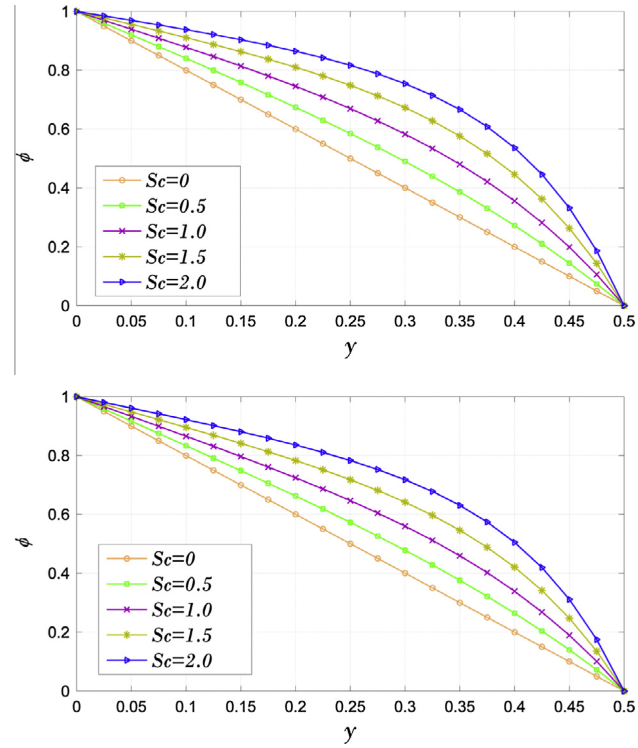

Figure 6 The profiles of $\phi(y)$ for different Schmidt numbers (Sc) when $Da = 0.1$, $Re_T = 10$ (a) $De = 1$ and (b) $De = 3$.

Fig. 5 demonstrates the velocity components and species distributions for various Reynolds numbers in case of suction flow. It is observed that for a given increase of Re, there is a decrement in the axial velocity, whereas $f'(y)$ decreased at the first and then increased after $y = 0.2$ for larger Re. In this case Re is considered positive because of contemplating only the suction flow. Also, in this figure it is seen that concentration distribution is an increasing function of Reynolds number but the increment is very small.

Fig. 6 indicates the effect of Schmidt number on dimensionless concentration distribution for different Deborah numbers. It shows that with an increase in Schmidt number, the concentration values increase highly throughout the upper semi-channel region. Also Fig. 6 represents that for different Schmidt numbers the concentration field has the same behavior in both Deborah numbers ($De = 1$, $De = 3$) since the viscoelasticity will not affect the diffusion of spices.

6. Conclusion

In the present article, a new hybrid technique based on the differential transform method (DTM) and iterative Newton's method (INM) has been successfully applied to find the solution to the viscoelastic flow of upper convected Maxwell fluid in a porous channel with high permeability. It is observed that the result of the present analytical method is in an excellent agreement with the numerical one, so it can be powerful and highly efficiency technique for finding analytical solutions in non-linear equation of viscoelastic flow problems. The results show that the velocity component of $f(y)$ (normal velocity) increases for large De and Da. The velocity component of $f'(y)$ (axial velocity) is initially increased with an increase in

Darcy number then decreased, but it has the opposite behavior by increasing Deborah number. With increasing of Reynolds number there is a decrease in the normal velocity but the velocity component of $f'(y)$ is increased near the walls. The concentration distribution (ϕ) is an increasing function of Schmidt number when Sc is increased. Also, it can be seen that for different species diffusing the concentration field has the same behavior in various Deborah number because the viscoelasticity will not affect the diffusion of spices.

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