

Contents lists available at [ScienceDirect](http://ScienceDirect)

## Physics Letters B

[www.elsevier.com/locate/physletb](http://www.elsevier.com/locate/physletb)The nucleon's octet axial-charge  $g_A^{(8)}$  with chiral correctionsSteven D. Bass<sup>a,\*</sup>, Anthony W. Thomas<sup>b</sup><sup>a</sup> Institute for Theoretical Physics, Universität Innsbruck, Technikerstrasse 25, Innsbruck, A 6020 Austria<sup>b</sup> CSSM, School of Chemistry and Physics, University of Adelaide, Adelaide SA 5005, Australia

## ARTICLE INFO

## Article history:

Received 10 December 2009

Received in revised form 5 January 2010

Accepted 7 January 2010

Available online 16 January 2010

Editor: G.F. Giudice

## Keywords:

Proton spin puzzle

SU(3) breaking

## ABSTRACT

The value of the nucleon's flavour-singlet axial-charge extracted from polarised deep inelastic scattering is sensitive to the value of the octet axial-charge  $g_A^{(8)}$  which is usually taken from an analysis of hyperon  $\beta$ -decays within the framework of SU(3) symmetry, namely  $0.58 \pm 0.03$ . Using the Cloudy Bag model we find that the value of  $g_A^{(8)}$  is reduced by as much as 20% below the usual phenomenological value. This increases the value of the flavour-singlet axial-charge ( $g_A^{(0)}|_{\text{inv}}$ ) derived from deep inelastic data and significantly reduces the difference between it and  $g_A^{(8)}$ .

© 2010 Elsevier B.V. All rights reserved.

## 1. Introduction

Polarised deep inelastic scattering experiments have revealed a small value for the nucleon's flavour-singlet axial-charge  $g_A^{(0)}|_{\text{pDIS}} \sim 0.3$  suggesting that the quarks' intrinsic spin contributes little of the proton's spin. The challenge to understand the spin structure of the proton [1–6] has inspired a vast programme of theoretical activity and new experiments. Why is the quark spin content  $g_A^{(0)}|_{\text{pDIS}}$  so small? How is the spin  $\frac{1}{2}$  of the proton built up from the spin and orbital angular momentum of the quarks and gluons inside?

The analysis which leads to  $g_A^{(0)}|_{\text{pDIS}}$  uses the value of the nucleon's octet axial-charge  $g_A^{(8)}$  which is commonly extracted from a 2 parameter fit to hyperon  $\beta$ -decays using SU(3):  $g_A^{(8)} = 0.58 \pm 0.03$  [7]. What separates the values of the octet and singlet axial-charges? In this Letter we examine the chiral corrections to  $g_A^{(8)}$ . We base our analysis on the Cloudy Bag model [8,9] which has the attractive feature that when pion cloud and quark mass effects are turned off the model reproduces the SU(3) analysis. We find that chiral corrections significantly reduce the value of  $g_A^{(8)}$ . This, in turn, has the effect of increasing the value of  $g_A^{(0)}|_{\text{pDIS}}$  and consequently reducing the absolute value of the “polarised strangeness” extracted from inclusive polarised deep inelastic scattering.

We start by recalling the  $g_1$  spin sum-rules, which are derived from the dispersion relation for polarised photon–nucleon scattering and, for deep inelastic scattering, the light-cone oper-

ator product expansion. One finds that the first moment of the  $g_1$  structure function is related to the scale-invariant axial charges of the target nucleon by

$$\int_0^1 dx g_1^p(x, Q^2) = \left( \frac{1}{12} g_A^{(3)} + \frac{1}{36} g_A^{(8)} \right) \left\{ 1 + \sum_{\ell \geq 1} c_{\text{NS}\ell} \alpha_s^\ell(Q) \right\} + \frac{1}{9} g_A^{(0)}|_{\text{inv}} \left\{ 1 + \sum_{\ell \geq 1} c_{\text{S}\ell} \alpha_s^\ell(Q) \right\} + \mathcal{O}\left(\frac{1}{Q^2}\right) + \beta_\infty. \quad (1)$$

Here  $g_A^{(3)}$ ,  $g_A^{(8)}$  and  $g_A^{(0)}|_{\text{inv}}$  are the isovector, SU(3) octet and scale-invariant flavour-singlet axial-charges, respectively. The flavour non-singlet  $c_{\text{NS}\ell}$  and singlet  $c_{\text{S}\ell}$  Wilson coefficients are calculable in  $\ell$ -loop perturbative QCD [10]. The term  $\beta_\infty$  represents a possible leading-twist subtraction constant from the circle at infinity when one closes the contour in the complex plane in the dispersion relation [1]. If finite, the subtraction constant affects just the first moment. The first moment of  $g_1$  plus the subtraction constant, if finite, is equal to the axial-charge contribution.

In terms of the flavour dependent axial-charges

$$2Ms_\mu \Delta q = \langle p, s | \bar{q} \gamma_\mu \gamma_5 q | p, s \rangle \quad (2)$$

the isovector, octet and singlet axial-charges are:

$$\begin{aligned} g_A^{(3)} &= \Delta u - \Delta d, \\ g_A^{(8)} &= \Delta u + \Delta d - 2\Delta s, \\ g_A^{(0)}|_{\text{inv}}/E(\alpha_s) &\equiv g_A^{(0)} = \Delta u + \Delta d + \Delta s. \end{aligned} \quad (3)$$

\* Corresponding author.

E-mail address: [Steven.Bass@uibk.ac.at](mailto:Steven.Bass@uibk.ac.at) (S.D. Bass).

Here  $E(\alpha_s) = \exp \int_0^{\alpha_s} d\tilde{\alpha}_s \gamma(\tilde{\alpha}_s)/\beta(\tilde{\alpha}_s)$  is a renormalisation group factor which corrects for the (two loop) non-zero anomalous dimension  $\gamma(\alpha_s)$  of the singlet axial-vector current [11],  $J_{\mu 5} = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s$ , which is close to one and which goes to one in the limit  $Q^2 \rightarrow \infty$ ;  $\beta(\alpha_s)$  is the QCD beta function. The singlet axial-charge,  $g_A^{(0)}|_{\text{inv}}$ , is independent of the renormalisation scale  $\mu$  and corresponds to  $g_A^{(0)}(Q^2)$  evaluated in the limit  $Q^2 \rightarrow \infty$ .

If one assumes no twist-two subtraction constant ( $\beta_\infty = O(1/Q^2)$ ) the axial-charge contributions saturate the first moment at leading twist. The isovector axial-charge is measured independently in neutron  $\beta$ -decays ( $g_A^{(3)} = 1.270 \pm 0.003$  [12]) and the octet axial-charge is commonly taken to be the value extracted from hyperon  $\beta$ -decays assuming a 2-parameter SU(3) fit ( $g_A^{(8)} = 0.58 \pm 0.03$  [7]). The uncertainty quoted for  $g_A^{(8)}$  has been a matter of some debate. There is considerable evidence that SU(3) symmetry may be badly broken and some have suggested that the error on  $g_A^{(8)}$  should be as large as 25% [13].

Using the sum rule for the first moment of  $g_1$ , given in Eq. (1), polarised deep inelastic scattering experiments have been interpreted in terms of a small value for the flavour-singlet axial-charge. Inclusive  $g_1$  data with  $Q^2 > 1 \text{ GeV}^2$  give [14]

$$g_A^{(0)}|_{\text{pDIS}, Q^2 \rightarrow \infty} = 0.33 \pm 0.03(\text{stat.}) \pm 0.05(\text{syst.}) \quad (4)$$

– considerably smaller than the value of  $g_A^{(8)}$  quoted above.

In the naive parton model  $g_A^{(0)}|_{\text{pDIS}}$  is interpreted as the fraction of the proton's spin which is carried by the intrinsic spin of its quark and antiquark constituents. When combined with  $g_A^{(8)} = 0.58 \pm 0.03$  this value corresponds to a negative strange-quark polarisation

$$\begin{aligned} \Delta s_{Q^2 \rightarrow \infty} &= \frac{1}{3} (g_A^{(0)}|_{\text{pDIS}, Q^2 \rightarrow \infty} - g_A^{(8)}) \\ &= -0.08 \pm 0.01(\text{stat.}) \pm 0.02(\text{syst.}) \end{aligned} \quad (5)$$

– that is, polarised in the opposite direction to the spin of the proton. New fits have been performed which also include data from semi-inclusive polarised deep inelastic scattering as well as polarised proton–proton collisions at RHIC. De Florian et al. [15] take as input  $g_A^{(8)} = 0.59 \pm 0.03$  and find values  $g_A^{(0)} \sim 0.24$  and  $\Delta s \sim -0.12$ , with the “polarised strangeness” coming almost entirely from small values of  $x$  outside the measured kinematic region – i.e., for Bjorken  $x$  between 0 and 0.001.

There has been considerable theoretical effort to understand the flavour-singlet axial-charge in QCD. QCD theoretical analysis leads to the formula [1,16–19]

$$g_A^{(0)} = \left( \sum_q \Delta q - 3 \frac{\alpha_s}{2\pi} \Delta g \right)_{\text{partons}} + C_\infty. \quad (6)$$

Here  $\Delta g_{\text{partons}}$  is the amount of spin carried by polarised gluons in the polarised proton ( $\alpha_s \Delta g \sim \text{constant}$  as  $Q^2 \rightarrow \infty$  [16, 17]) and  $\Delta q_{\text{partons}}$  measures the spin carried by quarks and antiquarks carrying “soft” transverse momentum  $k_t^2 \sim P^2, m^2$  where  $P$  is a typical gluon virtuality and  $m$  is the light quark mass. The polarised gluon term is associated with events in polarised deep inelastic scattering where the hard photon strikes a quark or antiquark generated from photon–gluon fusion and carrying  $k_t^2 \sim Q^2$  [18,19].  $C_\infty$  denotes a potential non-perturbative gluon topological contribution which is associated with the possible subtraction constant in the dispersion relation for  $g_1$  and Bjorken  $x = 0$  [1]:  $g_A^{(0)}|_{\text{pDIS}} = g_A^{(0)} - C_\infty$ .

**Table 1**

$g_A/g_V$  from  $\beta$ -decays with  $F = 0.46$  and  $D = 0.80$ , together with the mathematical form predicted in the MIT Bag with effective colour-hyperfine interaction (see text and [22]).

Process	Measurement	SU(3) combination	Fit value	MIT + OGE
$n \rightarrow p$	$1.270 \pm 0.003$	$F + D$	1.26	$\frac{5}{3}B' + G$
$\Lambda^0 \rightarrow p$	$0.718 \pm 0.015$	$F + \frac{1}{3}D$	0.73	$B'$
$\Sigma^- \rightarrow n$	$-0.340 \pm 0.017$	$F - D$	-0.34	$-\frac{1}{3}B' - 2G$
$\Xi^- \rightarrow \Lambda^0$	$0.25 \pm 0.05$	$F - \frac{1}{3}D$	0.19	$\frac{1}{3}B' - G$
$\Xi^0 \rightarrow \Sigma^+$	$1.21 \pm 0.05$	$F + D$	1.26	$\frac{2}{3}B' + G$

There is presently a vigorous programme to disentangle the different contributions involving experiments in semi-inclusive polarised deep inelastic scattering and polarised proton–proton collisions [3,20,21]. These direct measurements show no evidence for negative polarised strangeness in the region  $x > 0.006$  and suggest  $|-3\frac{\alpha_s}{2\pi}\Delta g| < 0.06$  corresponding to  $|\Delta g| < 0.4$  with  $\alpha_s \sim 0.3$ . That is, they are not able to account for the difference ( $g_A^{(0)}|_{\text{pDIS}} - g_A^{(8)}) = -0.25(\pm 0.07)$  obtained in the analysis of [14], or  $\sim -0.35$  in [15].

## 2. SU(3) breaking and $g_A^{(8)}$

Given that the contributions to  $g_A^{(0)}$  from the measured distribution  $\Delta s$  and from  $-3\frac{\alpha_s}{2\pi}\Delta g$  are small, it is worthwhile to ask about the value of  $g_A^{(8)}$ . The canonical value of 0.58 is extracted from a 2 parameter fit to hyperon  $\beta$ -decays in terms of the SU(3) constants  $F = 0.46$  and  $D = 0.80$  [7] – see Table 1. The fit is good to  $\sim 20\%$  accuracy [13,23]. More sophisticated fits will also include chiral corrections. Calculations of non-singlet axial-charges in relativistic constituent quark models are sensitive to the confinement potential, effective colour-hyperfine interaction [24,25], pion and kaon clouds plus additional wavefunction corrections [8]. The latter are often treated phenomenologically and chosen to reproduce the physical value of  $g_A^{(3)}$ .

Here we discuss these effects first in the MIT Bag and then in an extended Cloudy Bag model calculation [9], where chiral corrections are in-built. We focus on  $g_A^{(3)}$  and  $g_A^{(8)}$ . The Cloudy Bag was designed to model confinement and spontaneous chiral symmetry breaking, taking into account pion physics and the manifest breakdown of chiral symmetry at the bag surface in the MIT bag. If we wish to describe proton spin data including matrix elements of  $J_{\mu 5}^3, J_{\mu 5}^8$  and  $J_{\mu 5}$ , then we would like to know that the model versions of these currents satisfy the relevant Ward identities. For the non-singlet axial-charges  $g_A^{(3)}$  and  $g_A^{(8)}$ , corresponding to the matrix elements of partially conserved currents, the model is well designed to make a solid prediction. For the singlet axial-charge the situation is less clear since one has first to make an ansatz about the relationship between the (partially conserved) semi-classical model current and the QCD current which includes the QCD axial anomaly, including the possible topological contribution [1].<sup>1</sup>

We start with the MIT Bag model. There are a number of issues associated with the calculation. First one must account for the effect of the confining potential; then the colour-hyperfine in-

<sup>1</sup> That is, should the model be describing  $g_A^{(0)}$  at some low scale or one of the scale independent quantities  $\Delta q_{\text{partons}}, E(\alpha_s)g_A^{(0)}$ ? How should the topological contribution (if finite) be included in the model current? There are also gauge dependence issues if one extrapolates matrix elements of the partially conserved QCD singlet axial-vector current away from the forward direction to look at generalised parton distributions and the spin dependence of deeply virtual Compton scattering [1,13,26].

teraction; next one must evaluate the corrections associated with spurious centre of mass motion (CM) and recoil effects. When we turn to the chiral corrections we shall also have to choose the chiral representation, in particular whether the original surface coupling or the later volume coupling [27] version. Since the latter tends to reproduce the empirical value of  $g_A^{(3)}$  without any CM or recoil corrections [9] and those are not really on a solid theoretical foundation and therefore treated phenomenologically, the choice of whether or not to include CM and recoil corrections depends on which chiral representation is eventually to be used.

For the MIT Bag, the nucleon matrix element of the axial-vector current is [28]

$$\begin{aligned} \int d^3x \sum_i \langle ps | \bar{q}_i \vec{\gamma} \gamma_5 q_i | ps \rangle &= \int_{\text{bag}} d^3x \psi_i^\dagger(x) \gamma_0 \vec{\gamma} \gamma_5 \psi(x) \\ &= N^2 \int_0^R dx x^2 \left\{ j_0^2\left(\frac{\omega x}{R}\right) - \frac{1}{3} j_1^2\left(\frac{\omega x}{R}\right) \right\} \\ &= 1 - \frac{1}{3} \left( \frac{2\omega - 3}{\omega - 1} \right) = 0.65 \end{aligned} \quad (7)$$

when we substitute for the MIT Bag wavefunction  $\psi(x)$ . (Here  $\omega = 2.04$ ,  $R$  is the bag radius and  $N$  is the wavefunction normalisation.) This factor (0.65 for massless quarks, 0.67 for quarks with mass  $\sim 10$  MeV) is the crucial difference from non-relativistic constituent quark models.

The effective colour-hyperfine interaction has the quantum numbers of one-gluon exchange (OGE). In models of hadron spectroscopy this interaction plays an important role in the nucleon- $\Delta$  and  $\Sigma - \Lambda$  mass differences, as well as the nucleon magnetic moments [25] and the spin and flavor dependence of parton distribution functions [29]. It shifts total angular-momentum between spin and orbital contributions and, therefore, also contributes to model calculations of the octet axial-charges [24]. We denote this contribution, which has been evaluated to be 0.0373 [24,30] in the MIT Bag (without centre of mass corrections), as  $\mathcal{G}$ .

One also has to include additional wavefunction corrections associated with the well-known issue that, for the MIT and Cloudy Bag models, the nucleon wavefunction is not translationally invariant and the centre of mass is not fixed. Corrections to  $g_A^{(3)}$  arising from these effects have been estimated to be as large as 15–20% [31,32]. Including such a correction on the original MIT prediction for  $g_A^{(3)}$  yields a value which is in excellent agreement with experiment. To compare the model results with experiment we take the view [8] that, in principle, the model – with corrections – should give the experimental value of  $g_A^{(3)}$ . We therefore choose the centre-of-mass factor,  $Z_{\text{MIT}}$ , phenomenologically to give the experimental value of  $g_A^{(3)}$ . This then fixes the parameters of the model and allows us to use it to make a model prediction for  $g_A^{(8)}$ .

The model predictions with the centre of mass correction chosen so that the final answer for  $g_A^{(3)}$  is normalised to its physical value are

$$\begin{aligned} g_A^{(3)}|_{\text{MIT}} &= (g_A^{(3)}|_{\text{bare}} + \mathcal{G}) \times Z_{\text{MIT}}, \\ g_A^{(8)}|_{\text{MIT}} &= (g_A^{(8)}|_{\text{bare}} - 3\mathcal{G}) \times Z_{\text{MIT}}. \end{aligned} \quad (8)$$

The step by step MIT Bag calculation is shown in Table 2 for massless quarks. Note that at the level of Table 1 without additional physics input, e.g. pion chiral corrections, there is a simple algebraic relation between the SU(3) parameters  $F$  and  $D$ , the bag parameter  $B'$  and the OGE correction  $G$ :

**Table 2**  
MIT Bag model calculation.

	$g_A^{(3)}$	$S_z$ (singlet axial-charge)
Non-relativistic	+1.66	+1.00
Relativistic factor	+1.09	+0.65
+ OGE ( $\mathcal{G}$ factor)	+1.13	+0.54
+ centre of mass	+1.27	+0.61

**Table 3**

Bag model calculation with pions included,  $Z = 0.7$ ,  $P_{N\pi} = 0.20$ ,  $P_{\Delta\pi} = 0.10$ .

	$g_A^{(3)}$	$S_z$ (singlet axial-charge)
Non-relativistic	+1.66	+1.00
Relativistic	+1.09	+0.65
+ OGE	+1.13	+0.54
+ Pions	+1.06	+0.43
+ centre of mass	+1.27	+0.52

$$F = \frac{2}{3}B' - \frac{1}{2}G,$$

$$D = B' + \frac{3}{2}G. \quad (9)$$

Substituting the values  $F = 0.46$  and  $D = 0.80$  gives  $B' = 0.73$  and  $G = 0.05$ . The values  $G = Z_{\text{MIT}}\mathcal{G} = 0.042$  and  $B' = Z_{\text{MIT}} \times 0.65 = 0.73$  are in very good agreement with the values extracted from the SU(3) fit to hyperon  $\beta$ -decays [22].

The pion cloud of the nucleon also renormalises the nucleon's axial-charges by shifting intrinsic spin into orbital angular momentum [24,4]. In the Cloudy Bag Model (CBM) [33], the nucleon wavefunction is written as a Fock expansion in terms of a bare MIT nucleon,  $|N\rangle$ , and baryon-pion,  $|N\pi\rangle$  and  $|\Delta\pi\rangle$ , Fock states. The expansion converges rapidly and we may safely truncate the Fock expansion at the one pion level. The CBM axial-charges are [8]:

$$\begin{aligned} g_A^{(3)} &= g_A^{(3)}|_{\text{MIT}} \times Z_{\text{CBM}} \times \left( 1 - \frac{8}{9}P_{N\pi} - \frac{4}{9}P_{\Delta\pi} + \frac{8}{15}P_{N\Delta\pi} \right), \\ g_A^{(8)} &= g_A^{(8)}|_{\text{MIT}} \times Z_{\text{CBM}} \times \left( 1 - \frac{4}{3}P_{N\pi} + \frac{2}{3}P_{\Delta\pi} \right). \end{aligned} \quad (10)$$

Here,  $Z_{\text{CBM}}$  is the phenomenological CM correction factor chosen to preserve the physical value of  $g_A^{(3)}$  after the chiral correction associated with the pion cloud has been included. The coefficients  $P_{N\pi}$  and  $P_{\Delta\pi}$  denote the probabilities to find the physical nucleon in the  $|N\pi\rangle$  and  $|\Delta\pi\rangle$  Fock states, respectively and  $P_{N\Delta\pi}$  is the interference term. There is a wealth of phenomenological information to suggest that  $P_{N\pi}$  is between 20 and 25%, while  $P_{\Delta\pi}$  is in the range 5–10% [34]. For the interference term we simply follow the calculation of Ref. [8] and set  $P_{N\Delta\pi} = 0.30$ . If we initially take  $(P_{N\pi}, P_{\Delta\pi}) = (0.20, 0.10)$ , the bracketed pion cloud renormalisation factors in Eq. (10) are 0.94 for  $g_A^{(3)}$  and 0.8 for  $g_A^{(8)}$ . With these parameters, the Cloudy Bag prediction for the axial-charges is shown in Table 3.

In order to estimate the model dependent variation, we repeat this calculation using the values  $Z = 0.66$ ,  $P_{N\pi} = 0.24$ ,  $P_{\Delta\pi} = 0.10$  and  $Z = 0.70$ ,  $P_{N\pi} = 0.24$ ,  $P_{\Delta\pi} = 0.06$  [34]. For these parameter choices, the value of  $S_z$  in Table 3 reduces to 0.50 and 0.48, respectively. Thus the bag model, including OGE and pion loop corrections and with the CM correction adjusted to give the physical value of  $g_A^{(3)}$ , yields a light quark spin content  $S_z = 0.50 \pm 0.02$ . As long as we do not include strange quarks, this is also the value of  $g_A^{(8)}$  in the model.

However, having found significant effects from the pion cloud, it is also reasonable to ask about the effect of the kaon cloud, in particular the  $K\Lambda$  Fock component of the nucleon wave function. This term, which corresponds to a probability of order 5%

**Table 4**

Volume coupling version of the Cloudy Bag model, including pions and kaons, with the pion parameters as in Table 3. Following Ref. [9], in the last line  $g_A^{(3)}$  has been rescaled to match the experimental value and  $g_A^{(8)}$  and  $g_A^{(0)}$  have been rescaled by the same factor.

	$g_A^{(3)}$	$g_A^{(8)}$	$g_A^{(0)}$
Non-relativistic	+1.66	+1.00	+1.00
Relativistic	+1.09	+0.65	+0.65
+ OGE	+1.13	+0.54	+0.54
Volume CBM	+1.29	+0.45	+0.40
Rescale	+1.27	+0.44	+0.39

or less and naturally explains the measured strange electric and magnetic form factors of the proton [35,36], generates a small  $\Delta s \sim -0.01$  [9]. This would increase  $g_A^{(8)}$  by 0.02, however, the corresponding wave function renormalisation reduces the non-strange contribution by about 5%, leaving the combined value of  $g_A^{(8)} \equiv \Delta u + \Delta d - 2\Delta s$  with final model prediction between 0.47 and 0.51. That is, pion and kaon cloud chiral corrections have the potential to reduce  $g_A^{(8)}$  from the SU(3) value  $3F - D$  to  $0.49 \pm 0.02$ , which is still within the 20% variation found in the SU(3) fit in Table 1. (The corresponding semi-classical model value for  $g_A^{(0)}$  is  $0.46 \pm 0.02$  – a little higher than reported in Ref. [24] because of our requirement that the same model reproduce the phenomenological value of  $g_A^{(3)}$ .)

### 2.1. Volume coupling CBM

Although the volume coupling version of the CBM was derived by a unitary transformation on the original CBM and must therefore be equivalent, it is well known that because practical calculations are of necessity carried out only to a finite order, it is simpler to understand some physical phenomena in one version or the other. In particular, low energy theorems such as the Weinberg–Tomozawa relation for s-wave pion scattering are trivial to derive within the volume coupling version [27]. Within this representation there is an additional correction to  $g_A^{(3)}$  [27]. Indeed, in the SU(3) case this correction to the pure quark term has the form

$$\delta A_i^\lambda = -\frac{1}{2f_\pi} f_{ijk} \bar{q} \gamma^\lambda \lambda_j q \phi^k \theta_V, \quad (11)$$

where  $f_{ijk}$  are the usual SU(3) structure constants,  $\phi^k$ ,  $k \in (1, 8)$  are the octet Goldstone boson fields and  $q$  are the quark fields confined in the bag volume  $V$ . For  $g_A^{(3)}$  this term yields an increase of order 15%, which means that one has essentially no phenomenological need for the CM correction in order to reproduce the physical value [9]. On the other hand, this additional term does not effect the flavour singlet spin content. For  $g_A^{(8)}$  the meson loop generated by the additional term in Eq. (11) is only of order 3% [9], being suppressed relative to that for  $g_A^{(3)}$  because it involves a kaon rather than a pion. Thus, in this case the model yields values for  $S_z$  which more or less correspond to the “+ Pions” line of Table 3.

The full results of the volume coupling CBM calculation are summarised in Table 4. Once one allows for the variation in the pion–baryon Fock components considered earlier this leads to  $g_A^{(0)} = 0.37 \pm 0.02$ , while  $g_A^{(8)} = 0.42 \pm 0.02$ . Once again, +0.03 of the difference has the same origin, while the correction term  $\delta A_g$  (cf. Eq. (11)) yields the extra +0.02.<sup>2</sup>

<sup>2</sup> These results agree well with the results found in Ref. [9], namely  $g_A^{(8)} = 0.47$  and  $g_A^{(0)} = 0.41$ .

### 3. Concluding remarks

We have shown that once one takes care to consistently reproduce the experimental value of the proton’s axial-charge, the bag model with exchange current corrections arising from gluon exchange plus the chiral corrections associated primarily with the pion cloud lead to a substantial reduction of  $g_A^{(8)}$  below the value commonly used in the analysis of spin structure functions. The extent of the reduction depends upon the version of the CBM used, lying in the range  $0.49 \pm 0.02$  for the original CBM and  $0.42 \pm 0.02$  for the volume coupling version. These changes alone raise the value of  $g_A^{(0)}|_{\text{pDIS}, Q^2 \rightarrow \infty}$  derived from the experimental data from  $0.33 \pm 0.03(\text{stat.}) \pm 0.05(\text{syst.})$  to  $0.35 \pm 0.03(\text{stat.}) \pm 0.05(\text{syst.})$  and  $0.37 \pm 0.03(\text{stat.}) \pm 0.05(\text{syst.})$ , respectively. Both of these values have the effect of reducing the level of OZI violation associated with the difference  $g_A^{(0)}|_{\text{pDIS}} - g_A^{(8)}$  from  $0.25 \pm 0.07$  to just  $0.14 \pm 0.06$  and  $0.05 \pm 0.06$ , respectively.<sup>3</sup> It is this OZI violation which eventually needs to be explained in terms of singlet degrees of freedom: effects associated with polarised glue and/or a topological effect associated with  $\chi = 0$ .

As we have explained, the remaining uncertainty in this model calculation lies in the small ambiguity between the two chiral representations that one can choose. In order to quote an overall value that properly encompasses these possibilities we follow the Particle Data Group procedure [37] for combining data that may not be compatible to estimate the overall error, finding a combined value of  $g_A^{(8)} = 0.46 \pm 0.05$  (with the corresponding semi-classical singlet axial-charge or spin fraction being  $0.42 \pm 0.07$ ).<sup>4</sup> With this final value for  $g_A^{(8)}$  the corresponding experimental value of  $g_A^{(0)}|_{\text{pDIS}}$  would increase to  $g_A^{(0)}|_{\text{pDIS}} = 0.36 \pm 0.03 \pm 0.05$ .

### Acknowledgements

We would like to acknowledge helpful communications from F. Myhrer and K. Tsushima. The research of SDB is supported by the Austrian Science Fund, FWF, through grant P20436, while AWT is supported by the Australian Research Council through an Australian Laureate Fellowship and by the University of Adelaide.

### References

- [1] S.D. Bass, Rev. Mod. Phys. 77 (2005) 1257.
- [2] S.D. Bass, The Spin Structure of the Proton, World Scientific, 2008.
- [3] S.D. Bass, Mod. Phys. Lett. A 24 (2009) 1087.
- [4] A.W. Thomas, Prog. Part. Nucl. Phys. 61 (2008) 219, arXiv:0805.4437 [hep-ph].
- [5] A.W. Thomas, Phys. Rev. Lett. 101 (2008) 102003, arXiv:0803.2775 [hep-ph].
- [6] M. Anselmino, A. Efremov, E. Leader, Phys. Rep. 261 (1995) 1; H.-Y. Cheng, Int. J. Mod. Phys. A 11 (1996) 5109; J. Ellis, M. Karliner, arXiv:hep-ph/9601280; G. Altarelli, R.D. Ball, S. Forte, G. Ridolfi, Acta Phys. Pol. B 29 (1998) 1145; G.M. Shore, arXiv:hep-ph/9812355; G.M. Shore, B. Lampe, E. Reya, Phys. Rep. 332 (2000) 1; B.W. Filippone, X. Ji, Adv. Nucl. Phys. 26 (2001) 1; S.J. Brodsky, Int. J. Mod. Phys. A 18 (2003) 1531; S.E. Kuhn, J.-P. Chen, E. Leader, Prog. Part. Nucl. Phys. 63 (2009) 1.
- [7] F.E. Close, R.G. Roberts, Phys. Lett. B 316 (1993) 165.
- [8] A.W. Schreiber, A.W. Thomas, Phys. Lett. B 215 (1988) 141.
- [9] T. Yamaguchi, K. Tsushima, Y. Kohyama, K. Kubodera, Nucl. Phys. A 500 (1989) 429;

<sup>3</sup> It would be interesting to consider the effect of using the values of  $g_A^{(8)}$  calculated here on QCD global fits to polarised deep inelastic as well as proton–proton collision data [15] where the octet axial-charge enters non-linearly.

<sup>4</sup> The error  $\pm 0.02$  on  $g_A^{(8)}$  quoted for each version of the model follows from varying over the phenomenological range of possible pion parameters within first order perturbation theory. In terms of analogy to experimental errors, the  $\pm 0.02$  is like a statistical error and the final  $\pm 0.05$  error includes systematic effects.

- K. Tsushima, T. Yamaguchi, Y. Kohyama, K. Kubodera, *Nucl. Phys. A* 489 (1988) 557;  
K. Kubodera, Y. Kohyama, K. Tsushima, T. Yamaguchi, in: M. Morita, H. Ejiri, H. Ohtsubo, T. Sato (Eds.), *Proc. XXII Yamada Conference on Nuclear Processes and Nuclear Structure*, June 1989, Osaka, Japan, World Scientific, 1989, pp. 408–413.
- [10] S.A. Larin, T. van Ritbergen, J.A.M. Vermaseren, *Phys. Lett. B* 404 (1997) 153.  
[11] J. Kodaira, *Nucl. Phys. B* 165 (1980) 129.  
[12] Particle Data Group, C. Amsler, et al., *Phys. Lett. B* 667 (2008) 1.  
[13] R.L. Jaffe, A. Manohar, *Nucl. Phys. B* 337 (1990) 509.  
[14] COMPASS Collaboration, V.Yu. Alexakhin, et al., *Phys. Lett. B* 647 (2007) 8.  
[15] D. de Florian, R. Sassot, M. Stratmann, W. Vogelsang, *Phys. Rev. Lett.* 101 (2008) 072001, *Phys. Rev. D* 80 (2009) 034030.  
[16] G. Altarelli, G.G. Ross, *Phys. Lett. B* 212 (1988) 391.  
[17] A.V. Efremov, O. Teryaev, JINR Report No. E2-88-287.  
[18] R.D. Carlitz, J.C. Collins, A. Mueller, *Phys. Lett. B* 214 (1988) 229.  
[19] S.D. Bass, B.L. Ioffe, N.N. Nikolaev, A.W. Thomas, *J. Moscow Phys. Soc.* 1 (1991) 317.  
[20] G.K. Mallot, arXiv:hep-ex/0612055.  
[21] HERMES Collaboration, A. Airapetian, et al., *Phys. Rev. Lett.* 92 (2004) 012005; COMPASS Collaboration, M. Alekseev, et al., *Phys. Lett. B* 680 (2009) 217.  
[22] H. Hogaasen, F. Myhrer, *Z. Phys. C* 48 (1990) 295.  
[23] E. Leader, D.B. Stamenov, *Phys. Rev. D* 67 (2003) 037503.  
[24] F. Myhrer, A.W. Thomas, *Phys. Rev. D* 38 (1988) 1633, *Phys. Lett. B* 663 (2008) 302.  
[25] F.E. Close, *An Introduction to Quarks and Partons*, Academic, NY, 1978.  
[26] S.D. Bass, *Phys. Rev. D* 65 (2002) 074025.  
[27] A.W. Thomas, *J. Phys. G* 7 (1981) L283; M.A. Morgan, G.A. Miller, A.W. Thomas, *Phys. Rev. D* 33 (1986) 817.  
[28] J.F. Donoghue, E. Golowich, B.R. Holstein, *Phys. Rev. D* 12 (1975) 2875.  
[29] F.E. Close, A.W. Thomas, *Phys. Lett. B* 212 (1988) 227.  
[30] K. Ushio, H. Konashi, *Phys. Lett. B* 135 (1984) 468.  
[31] J.F. Donoghue, K. Johnson, *Phys. Rev. D* 21 (1980) 1975.  
[32] C.W. Wong, *Phys. Rev. D* 24 (1981) 1416.  
[33] A.W. Thomas, *Adv. Nucl. Phys.* 13 (1984) 1.  
[34] A.W. Thomas, *Prog. Theor. Phys.* 168 (2007) 614, arXiv:0711.2259 [nucl-th].  
[35] A.W. Thomas, R.D. Young, D.B. Leinweber, arXiv:nucl-th/0509082.  
[36] R.D. Young, J. Roche, R.D. Carlini, A.W. Thomas, *Phys. Rev. Lett.* 97 (2006) 102002, arXiv:nucl-ex/0604010.  
[37] C. Amsler, et al., Particle Data Group, *Phys. Lett. B* 667 (2008) 1, see Section 5.2.2.