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Two-loop low-energy effective action in Abelian supersymmetric Chern–Simons matter models

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Abstract

We compute two-loop low-energy effective actions in Abelian Chern–Simons matter models with $\mathcal{N}=2$ and $\mathcal{N}=3$ supersymmetry up to four-derivative order. Calculations are performed with a slowly-varying gauge superfield background. Though the gauge superfield propagator depends on the gauge fixing parameter, it is shown that the obtained results are independent of this parameter. In the massless case the considered models are superconformal. We demonstrate that the superconformal symmetry strongly restricts the form of two-loop quantum corrections to the effective actions such that the obtained terms have simpler structure than the analogous ones in the effective action of three-dimensional supersymmetric electrodynamics (SQED) with vanishing topological mass.

Keywords: Effective action; Extended supersymmetry; Chern–Simons model; Quantum electrodynamics

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1. Introduction

Three-dimensional gauge field theories have one important difference from the four-dimensional ones: they allow for a gauge invariant topological mass term described by the Chern–Simons action. In supersymmetric gauge theories, the Chern–Simons term appears to be crucial in construction of $\mathcal{N}=8$ and $\mathcal{N}=6$ superconformal models, known as the BLG [1–6] and ABJM [7] ones, which are central objects in the AdS_4/CFT_3 correspondence. As is stressed in the recent paper by John Schwarz [8], it is important to study the low-energy effective action in these models to check the conjecture that it describes the dynamics of probe M2 brane in the AdS_4 background.

Leaving the issue of low-energy effective action in ABJM and BLG models for further studies, in the present paper we consider a simple problem: what is the dependence of low-energy effective action in three-dimensional supersymmetric models on the topological mass $\mathfrak{m}=\frac{kg^2}{2\pi}$, where g is the three-dimensional gauge coupling constant and k is the Chern–Simons level. There are two special cases, $g\to\infty$ with k finite and $k=0$ with g finite. The latter corresponds to the gauge theory without the Chern–Simons term (e.g., SQED or SQCD) while the former case describes a gauge theory with infinitely large topological mass. The aim of this paper is to compare the structure of low-energy effective actions in three-dimensional gauge theories in these two particular cases.

We address this question by considering low-energy effective action in Abelian $\mathcal{N}=2$ supersymmetric gauge theories with matter. In the recent paper [9] the two-loop low-energy effective action in $\mathcal{N}=2$, $d=3$ SQED (with vanishing topological mass) was computed, owing to the background field method in $\mathcal{N}=2$, $d=3$ superspace [10–12]. In the present paper we consider a similar model, but with the Chern–Simons kinetic term for the gauge superfield rather than the Maxwell one (i.e., infinitely large topological mass). We compute two-loop low-energy effective action in this model up to the four-derivative order and compare it with the similar terms in the effective action of $\mathcal{N}=2$, $d=3$ SQED with vanishing topological mass considered in [9]. To be more precise, we consider a part of the effective action which includes only the gauge superfield because these terms can be naturally compared with the ones studied in [9]. In general, the effective action involves also contributions with the chiral matter superfields which are not considered here. The study of such terms in the effective action is a separate problem.

The one-loop effective action in gauge superfield sector (supersymmetric one-loop Euler–Heisenberg effective action) originates from the loop of matter chiral superfields with external gauge superfield. It is independent of both couplings g and k. So, we have to consider the two-loop effective action to study the problem described above. In three-dimensions, the $\mathcal{N}=2$ gauge superfield V has not only Grassmann-odd superfield strengths W_α and $\bar{W}_\alpha$, but also the Grassmann-even scalar superfield strength G. Up to four-derivative order, the low-energy effective action for these superfields has the following structure (see Section 2.2 for a more detailed discussion):

$$\Gamma=\int d^7z\left[f_1(G)+f_2(G)W^\alpha\bar{W}^\beta N_{\alpha\beta}+f_3(G)W^2\bar{W}^2\right], \tag{1.1}$$

where $f_i(G)$ are some functions and $N_{\alpha\beta}=D_\alpha W_\beta$. In the present paper we find two-loop quantum contributions to the functions $f_i(G)$ and compare them with similar results in the $\mathcal{N}=2$, $d=3$ SQED without the Chern–Simons term.

The function $f_1(G)$ in (1.1) is the leading term in the low-energy effective action for the gauge superfield. In components, it is responsible for the F^2 terms and its supersymmetric completions, with F_{mn} being the Maxwell field strength. In the $\mathcal{N}=2$ SQED with the pure Maxwell kinetic term for the gauge superfield this function has a good geometrical interpretation: its second derivative defines the moduli space metric in Coulomb branch [13]. In particular, in [9] we computed two-loop quantum corrections to the moduli space in the $\mathcal{N}=2$ SQED. However, it is known [14] that the Coulomb branch is absent in three-dimensional gauge theories with non-trivial Chern–Simons term because the corresponding equations of motion do not have constant solutions for scalar fields in the gauge multiplet. In the present paper we show that the function $f_1(G)$ does not receive two-loop quantum corrections in the $\mathcal{N}=2$ Chern–Simons electrodynamics, but it has non-trivial one-loop contributions found in [10]. This one-loop contribution to $f_1(G)$ originates from the loop of chiral superfields with external gauge superfields and it is independent of whether we have the super-Maxwell or Chern–Simons propagator for the gauge superfield.

The functions $f_2(G)$ and $f_3(G)$ in (1.1) are responsible for the F^4 component term and its supersymmetric completions. This term is present in the effective action in both cases, when the gauge superfield is described by the Maxwell and Chern–Simons terms. Clearly, the form of these functions f_1 and f_2 should be different in these two cases. Indeed, the conventional three-dimensional SQED with the Maxwell kinetic term for the gauge superfield involves the dimensionful gauge coupling constant g, $[g^2]=1$, such that the model is not conformal. As a consequence, in the SQED with the Maxwell kinetic term the functions $f_i(G)$ in (1.1) are not restricted by the conformal invariance. On the contrary, the (massless) Chern–Simons matter theories are superconformal and the form of these functions is fixed, up to coefficients. We show that the superconformal invariance requires the vanishing of two-loop quantum corrections to f_1 and f_2 in the Chern–Simons matter models while f_3 is expressed in terms of superconformal invariants in the $\mathcal{N}=2$, $d=3$ superspace constructed in [10]. These results are also generalized to the Abelian $\mathcal{N}=2$ Chern–Simons theory with one chiral matter superfield (in Section 3.1) and to $\mathcal{N}=3$ Chern–Simons matter model (in Sect 3.2).

Our general conclusion about the Chern–Simons matter models is that the structure of low-energy effective action in such theories is strongly constrained by superconformal invariance. On the contrary, when the gauge superfield is described by non-conformal supersymmetric Maxwell term, many new non-conformal terms appear in the low-energy effective action.

Before starting the main part of the paper, one more comment is in order. In general, the off-shell effective action is known to be gauge dependent by construction.[2] It becomes gauge independent only for background fields satisfying the effective equations of motion. In the present paper we consider the low-energy effective action for slowly-varying gauge superfield background. The conditions determining such a background coincide with the $\mathcal{N}=2$ supersymmetric Maxwell equations, rather than the equations of motion in the Chern–Simons matter models under considerations. Hence, one can expect that, in general, the obtained effective action will be

[2] The gauge dependence should not be confused with the gauge invariance of the effective action. In general, the effective action in gauge theories depends on gauge fixing conditions which are used for quantization and correct definition of the path integral. The background field method is based on special class of gauge fixing conditions (the so-called background field gauges, see e.g. [15] and references therein). The background field gauges allow one to construct the effective action which is gauge invariant under the classical gauge transformations. However, there are infinitely many background field gauges, for example if χ is an admissible background field gauge then $\alpha\chi$ is also admissible background field gauge with arbitrary real parameter α. As a result, the gauge invariant effective action constructed in framework of background field method will depend on the parameter α. Therefore it is said that the effective action constructed in framework of background field method is gauge invariant but gauge dependent. However, the S-matrix computed on the basis of the effective action will be completely gauge independent. All these points are discussed, e.g., in [16].

gauge dependent. In particular, the effective action can depend on the gauge-fixing parameter appearing in the gauge superfield propagator. In our case, doing two-loop computation we use the gauge superfield propagator with arbitrary gauge fixing parameter and prove that the obtained low-energy results are independent of this parameter. This is a good evidence that the obtained two-loop contributions to the effective action are, in fact, gauge independent although they are derived with use of gauge superfield background which does not solve the classical (and effective) equations of motion.

Throughout this paper we use the $\mathcal{N}=2$, $d=3$ superspace notations and conventions introduced in earlier works [10,11].

2. $\mathcal{N}=2$ Chern–Simons electrodynamics

2.1. Classical action and propagators

The classical action of the considered model in $\mathcal{N}=2$, $d=3$ superspace reads

$$S=\frac{k}{2\pi}\int d^7z\,VG-\int d^7z\,(\bar{Q}_+e^{2V}Q_++\bar{Q}_-e^{-2V}Q_-) \\ -\left(m\int d^5z\,Q_+Q_-+c.c.\right), \tag{2.1}$$

where V is a gauge superfield with superfield strength $G=\frac{i}{2}\bar{D}^\alpha D_\alpha V$ and $Q_\pm$ are chiral matter superfields having opposite charges with respect to the gauge superfield. Here m is the mass of the chiral superfield and k is the Chern–Simons level. For $m=0$ this model is superconformal [10]. The classical action (2.1) describes $\mathcal{N}=2$, $d=3$ supersymmetric electrodynamics with Chern–Simons rather than Maxwell kinetic term for the photon.

To study the effective action in the gauge superfield sector it is convenient to use the background field method which was developed for field theories in the $\mathcal{N}=2$, $d=3$ superspace in [12,17]. We split the gauge superfield V into the background V and quantum v parts,[3]

$$V\to V+v. \tag{2.2}$$

Upon this splitting the Chern–Simons term in (2.1) changes as

$$\frac{k}{2\pi}\int d^7z\,VG\to\frac{k}{2\pi}\int d^7z\,VG+\frac{k}{\pi}\int d^7z\,vG+\frac{ik}{4\pi}\int d^7z\,vD^\alpha\bar{D}_\alpha v, \tag{2.3}$$

with the background superfields V and G in the r.h.s. The terms in (2.3) which are linear in v are irrelevant for quantum loop computations. The chiral superfields $Q_\pm$ are treated as purely quantum and should be integrated out in the functional integral.

The operator in the last term in (2.3) is degenerate and requires gauge fixing,

$$f=i\bar{D}^2v,\qquad \bar{f}=iD^2v, \tag{2.4}$$

where f is a fixed chiral superfield. This gauge is usually accounted by the following gauge fixing term [18–20]:

[3] Note that we denote the background gauge superfield by the same letter V as the original gauge superfield in the classical action (2.1). We hope that it will not lead to any confusions since after the background-quantum splitting (2.2) the original gauge superfield V never appears.

$$S_{\rm gf} = \frac{ik\alpha}{8\pi}\int d^7z\, v\big(D^2 + \bar{D}^2\big)v, \tag{2.5}$$

with α being a real parameter. Adding (2.5) to (2.1) we get the gauge fixed action for the quantum superfields corresponding to internal lines of Feynman supergraphs,

$$S_{\rm quant} = S_2 + S_{\rm int}, \tag{2.6}$$

$$S_2 = \int d^7z\left(\frac{ik}{4\pi}vHv - \bar{\mathcal{Q}}_+\mathcal{Q}_+ - \bar{\mathcal{Q}}_-\mathcal{Q}_-\right) - \left(m\int d^5z\,\mathcal{Q}_+\mathcal{Q}_- + c.c.\right), \tag{2.7}$$

$$S_{\rm int} = -2\int d^7z\big[(\bar{\mathcal{Q}}_+\mathcal{Q}_+ - \bar{\mathcal{Q}}_-\mathcal{Q}_-)v + (\bar{\mathcal{Q}}_+\mathcal{Q}_+ + \bar{\mathcal{Q}}_-\mathcal{Q}_-)v^2\big] + O\big(v^3\big), \tag{2.8}$$

where the operator H reads

$$H = D^\alpha\bar{D}_\alpha + \frac{\alpha}{2}\big(D^2 + \bar{D}^2\big). \tag{2.9}$$

In (2.7) and (2.8) we introduced the notations $\mathcal{Q}_\pm$ and $\bar{\mathcal{Q}}_\pm$ for covariantly (anti)chiral superfields with respect to the background gauge superfield,

$$\bar{\mathcal{Q}}_+ = \bar{Q}_+e^{2V}, \qquad \mathcal{Q}_+ = Q_+, \qquad \bar{\mathcal{Q}}_- = \bar{Q}_-e^{-2V}, \qquad \mathcal{Q}_- = Q_-. \tag{2.10}$$

Let us consider the propagator for the superfield v,

$$2i\big\langle v(z)v\big(z'\big)\big\rangle = G\big(z,z'\big), \tag{2.11}$$

where the Green's function $G(z,z')$ obeys the equation

$$\frac{ik}{4\pi}HG\big(z,z'\big) = -\delta^7\big(z-z'\big). \tag{2.12}$$

A formal solution to this equation reads

$$G\big(z,z'\big) = G_1\big(z,z'\big) + G_2\big(z,z'\big), \tag{2.13}$$

where

$$G_1\big(z,z'\big) = \frac{i\pi}{k}\frac{D^\alpha\bar{D}_\alpha}{\Box}\delta^7\big(z-z'\big) = -\frac{\pi}{k}D^\alpha\bar{D}_\alpha\int_0^\infty\frac{ds}{(4\pi is)^{3/2}}e^{\frac{i\xi^2}{4s}}\zeta^2\bar{\zeta}^2, \tag{2.14}$$

$$G_2\big(z,z'\big) = \frac{i\pi}{2k\alpha}\frac{D^2+\bar{D}^2}{\Box}\delta^7\big(z-z'\big) = -\frac{\pi}{2k\alpha}\big(D^2+\bar{D}^2\big)\int_0^\infty\frac{ds}{(4\pi is)^{3/2}}e^{\frac{i\xi^2}{4s}}\zeta^2\bar{\zeta}^2. \tag{2.15}$$

Here we applied the standard proper time representation for the inverse d'Alembertian operator in terms of the components of supersymmetric interval ξ^m and ζ's (see the details and references in Appendices A and B).

Note that $G_2(z,z')$ depends on the gauge-fixing parameter α while $G_1(z,z')$ does not. We do not fix particular values of this parameter to keep control on gauge dependence of the effective action.

The action (2.8) is responsible for cubic and quartic interaction vertices while the terms in (2.7) give the propagators for the chiral matter superfields,

$$
\begin{aligned}
&i\langle \mathcal{Q}_+(z)\mathcal{Q}_-(z')\rangle = -mG_+(z,z'),\\
&i\langle \bar{\mathcal{Q}}_+(z)\bar{\mathcal{Q}}_-(z')\rangle = mG_-(z',z),\\
&i\langle \mathcal{Q}_+(z)\bar{\mathcal{Q}}_+(z')\rangle = G_{+-}(z,z') = G_{-+}(z',z),\\
&i\langle \bar{\mathcal{Q}}_-(z)\mathcal{Q}_-(z')\rangle = G_{-+}(z,z').
\end{aligned}
\tag{2.16}
$$

Properties of Green's functions in the r.h.s. of (2.16) were studied in [9,10]. Explicit expressions for them are given in Appendix B.

2.2. General structure of effective action

Our aim is to study the low-energy effective action in the model (2.1) in the gauge superfield sector. It can be written as

$$
\Gamma = S_{\rm cl} + \bar{\Gamma}, \tag{2.17}
$$

where $S_{\rm cl} = \frac{k}{2\pi}\int d^7z\, VG$ is the classical Chern–Simons term and $\bar{\Gamma}$ takes into account quantum corrections to the effective action. In what follows we will consider only $\bar{\Gamma}$ omitting 'bar' for brevity.

In general, Γ is a functional of superfield strengths G, W_α, $\bar{W}_\alpha$ and their derivatives, $N_{\alpha\beta} = D_\alpha W_\beta$, $\bar{N}_{\alpha\beta} = \bar{D}_\alpha \bar{W}_\beta$,

$$
\Gamma = \int d^7z\, \mathcal{L}(G, W_\alpha, \bar{W}_\alpha, N_{\alpha\beta}, \bar{N}_{\alpha\beta}, \ldots), \tag{2.18}
$$

where dots stand for higher-order derivatives of the superfield strengths. It is very difficult to find the effective action (2.18) taking into account all derivatives of the fields. Therefore, to simplify the problem, we restrict ourself to the terms with no more than four space–time derivatives of component fields. A typical bosonic representative in components is $f(\phi)(F^{mn}F_{mn})^2$, where F_{mn} is the Maxwell field strength and $f(\phi)$ is some function of the scalar field ϕ which is part of the $\mathcal{N}=2$, $d=3$ gauge multiplet. It is clear that to find this term in the effective action it is sufficient to consider constant fields F_{mn} and ϕ. In terms of superfields, such a background corresponds to the following constraints on the superfield strengths:

(i) Supersymmetric Maxwell equations,

$$
D^\alpha W_\alpha = 0, \qquad \bar{D}^\alpha \bar{W}_\alpha = 0; \tag{2.19}
$$

(ii) Superfield strengths are constant with respect to the space–time coordinates,

$$
\partial_m G = \partial_m W_\alpha = \partial_m \bar{W}_\alpha = 0. \tag{2.20}
$$

We emphasize that though Eqs. (2.19) are not the equations of motion in the theory under consideration, they, together with Eqs. (2.20), single out the slowly varying gauge superfield background. In components, such a background contains constant scalar ϕ, spinor λ_α, $\bar{\lambda}_\alpha$ and Maxwell F_{mn} fields while the auxiliary field D vanishes owing to (2.19). For the gauge superfield background constrained by (2.19) and (2.20) we can use the exact expressions for the chiral superfield propagators (B.7), (B.8) and (B.9) which were derived in [9].

Note that the superfields $N_{\alpha\beta}$ and $\bar{N}_{\alpha\beta}$ and not independent subject to the constraints (2.19) and (2.20),

$$
N_{\alpha\beta} = -\bar{N}_{\alpha\beta}. \tag{2.21}
$$

Hence, we keep only $N_{\alpha\beta}$ and discard $\bar{N}_{\alpha\beta}$ in what follows assuming that the latter is expressed from the former.

Under the constraints (2.19) and (2.20) the effective action (2.18) in components contains Maxwell field strength in arbitrary power and, so, involves arbitrary number of space–time derivatives. The superfield action which contains the terms with no more than four derivatives is given by

$$\Gamma = \int d^7z \left[f_1(G) + f_2(G)W^\alpha \bar{W}^\beta N_{\alpha\beta} + f_3(G)W^2\bar{W}^2\right], \tag{2.22}$$

with some functions $f_i(G)$, $i = 1, 2, 3$. Indeed, the full superspace measure d^7z involves the Grassmann-odd coordinate part $d^2\theta\, d^2\bar{\theta} \propto D^2\bar{D}^2$. Thus, it counts as two space–time derivatives. Next, $W^2\bar{W}^2$ also contain effectively four D's (which count as two ∂_m) because of $W_\alpha = \bar{D}_\alpha G$ and $\bar{W}_\alpha = D_\alpha G$. Hence, the first term in the r.h.s. of (2.22) is a two-derivative piece while the other terms are four-derivative ones.

In principle, one could include in (2.22) also the term of the form $\int d^7z\, f(G)W^\alpha\bar{W}_\alpha$, but it vanishes for the gauge superfield background subject to (2.19),

$$\begin{aligned}\int d^7z\, f(G)W^\alpha\bar{W}_\alpha &= -\frac{1}{2}\int d^5z \left(\bar{D}^\alpha f(G)\right)\left(\bar{D}_\alpha \bar{W}^\beta\right)W_\beta \\ &= -\frac{1}{2}\int d^5z\, W^\alpha N_\alpha^\beta W_\beta f'(G) = 0.\end{aligned} \tag{2.23}$$

Here we passed from the full superspace to the chiral measure and used the fact that N^α_β is traceless, $N^\alpha_\alpha = 0$, subject to (2.19).

Let us discuss the component structure of the effective action (2.22) in the bosonic sector. For this purpose it is sufficient to consider the gauge superfield V of the special form:

$$\hat{V} = i\theta^\alpha\bar{\theta}_\alpha\phi + \theta^\alpha\bar{\theta}^\beta\gamma^m_{\alpha\beta}A_m, \tag{2.24}$$

where ϕ is a constant scalar and A_m is a gauge vector field with constant Maxwell field strength, $F_{mn} = \partial_m A_n - \partial_n A_m$. The superfield strengths constructed with the use of this gauge superfield have the following component structure:

$$\hat{G} = -\phi - \frac{1}{2}\varepsilon^{mnp}(\gamma_p)^{\alpha\beta}\theta_\alpha\bar{\theta}_\beta F_{mn}, \tag{2.25}$$

$$\hat{W}_\alpha = \frac{1}{2}\varepsilon^{mnp}(\gamma_p)^\beta_\alpha\theta_\beta F_{mn}, \qquad \hat{\bar{W}}_\alpha = \frac{1}{2}\varepsilon^{mnp}(\gamma_p)^\beta_\alpha\bar{\theta}_\beta F_{mn}. \tag{2.26}$$

With these superfields, we find that the effective action (2.22) contains the following terms in its component field decomposition:

$$\Gamma = \frac{1}{8}\int d^3x \left\{f_1''(-\phi)F^{mn}F_{mn} + \left[2f_3(-\phi) - f_2'(-\phi)\right]\left(F^{mn}F_{mn}\right)^2\right\} + \cdots, \tag{2.27}$$

where dots stand for other components which are related with the given ones by $\mathcal{N} = 2$ supersymmetry. Eq. (2.27) shows that the first term in r.h.s. of (2.22) is responsible for the F^2 term while the terms with the functions f_2 and f_3 result in the F^4 term.

In the present paper we will perturbatively compute the functions f_i in (2.22) in the two-loop approximation,

$$f_i(G) = f_i^{(1)}(G) + f_i^{(2)}(G), \tag{2.28}$$

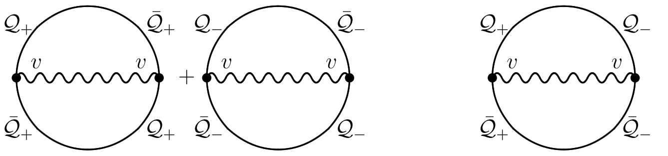

Fig. 1. Two-loop supergraphs in $\mathcal{N}=2$ supersymmetric electrodynamics.

where $f_i^{(1)}(G)$ and $f_i^{(2)}(G)$ correspond to one- and two-loop contributions, respectively. Note that at the one-loop order the effective action (2.22) receives contributions from the loop of (anti)chiral matter fields only. These contributions were calculated in [10][4]:

$$f_1^{(1)} = \frac{1}{2\pi}\left(G\ln\left(G+\sqrt{G^2+m^2}\right)-\sqrt{G^2+m^2}\right), \tag{2.29}$$

$$f_2^{(1)} = 0, \tag{2.30}$$

$$f_3^{(1)} = \frac{1}{128\pi}\frac{1}{(G^2+m^2)^{5/2}}. \tag{2.31}$$

Our aim now is to find the functions $f_i^{(2)}$ which take into account two-loop quantum contributions to the effective action (2.22).

The two-loop effective action is given by the following formal expression:

$$\Gamma^{(2)} = \Gamma_{\rm A} + \Gamma_{\rm B}, \tag{2.32}$$

$$\Gamma_{\rm A} = -2\int d^7z\, d^7z'\, G_{+-}(z,z')G_{-+}(z,z')G(z,z'), \tag{2.33}$$

$$\Gamma_{\rm B} = -2m^2\int d^7z\, d^7z'\, G_{+}(z,z')G_{-}(z,z')G(z,z'). \tag{2.34}$$

The two terms $\Gamma_{\rm A}$ and $\Gamma_{\rm B}$ are represented by corresponding Feynman graphs in Fig. 1.

Note that, in general, in the two-loop effective action the diagrams of topology "eight" are also present. Such diagrams involve either G_{+-} or G_+ propagator and the gauge superfield propagator (2.13) which should be considered at coincident superspace points. However, at coincident points the gauge superfield propagator (2.13) vanishes, $G(z,z)=0$. Hence, there are no contributions to the effective action from the graphs of topology "eight".

2.3. Independence of two-loop effective action of the gauge-fixing parameter

The part of the gauge superfield propagator which depends on the gauge fixing parameter α is given by (2.15). In this section we will demonstrate that the two-loop contributions to the low-energy effective action of the form (2.22) are independent of this parameter. To prove this, we check the vanishing of contributions to the two-loop effective actions (2.33) and (2.34) which correspond to the propagator (2.15).

[4] The function (2.29) was introduced for three-dimensional gauge theories in [21] in the study of non-linear sigma-models with extended supersymmetry. In four dimensions, analogous function corresponds to the Lagrangian of improved tensor multiplet (see e.g. [22]).

Consider first the part of the effective action (2.33). The propagator (2.15) contains the operator $D^2 + \bar{D}^2$ acting on the full superspace delta-function. With the use of integration by parts, the operator $\bar{D}^2$ hits the Green's function $G_{-+}(z,z')$ (similarly, D^2 hits $G_{+-}(z,z')$). According to (B.5), one gets two terms:

$$\frac{1}{4}\bar{\nabla}^2 G_{-+}(z,z') = -\delta_+(z,z') - m^2 G_+(z,z'). \tag{2.35}$$

The delta-function in (2.35) gives vanishing contribution to (2.33) since the expression (2.15) already contains the Grassmann delta-function $\zeta^2\bar{\zeta}^2 = \delta^2(\theta-\theta')\delta^2(\bar{\theta}-\bar{\theta}')$.

Consider the contributions to $\Gamma_{\rm A}$ from the last term in (2.35). With the use of the heat kernel representations of the propagators (B.7)–(B.8), the part of the effective action corresponding to the last term in (2.35) reads

$$\begin{aligned}&\int d^7z\, d^7z' \int_0^\infty \frac{ds\,dt\,du}{(4\pi i u)^{3/2}} \zeta^2\bar{\zeta}^2 e^{\frac{i\xi^2}{4u}} e^{i(s+t)m^2} K_+(z,z'|s) K_{+-}(z,z'|t) \\ &= \int d^7z\, d^3\xi \int_0^\infty \frac{ds\,dt\,du}{(4\pi i u)^{3/2}} e^{\frac{i\xi^2}{4u}} e^{i(s+t)m^2} K_+(z,z'|s) K_{+-}(z,z'|t)\big|.\end{aligned} \tag{2.36}$$

Here we integrated over one set of Grassmann variables using the delta-function. The symbol | in the second line of (2.36) means that this expression is considered at coincident Grassmann coordinates,

$$| \equiv |_{\theta=\theta',\,\bar{\theta}=\bar{\theta}'}. \tag{2.37}$$

Note that the bosonic coordinates x_m and x'_m remain different under this projection. We will employ the notation (2.37) throughout the present paper.

It is important to note that the heat kernel K_+ at coincident superspace points contains W^2, see (B.26). Hence, the result of calculation of the expression (2.36) can always be represented in the form

$$\int d^7z\, W^2 \mathcal{F}(G), \tag{2.38}$$

with some function $\mathcal{F}(G)$. One can easily see that the quantity (2.38) vanishes for the on-shell gauge superfield (2.19). Indeed, passing to the chiral subspace one gets

$$\int d^7z\, W^2\mathcal{F}(G) = -\frac{1}{4}\int d^5z\, W^2\bar{D}^2\mathcal{F}(G) = -\frac{1}{4}\int d^5z\, W^2W^2\mathcal{F}''(G) \equiv 0. \tag{2.39}$$

This expression vanishes as it contains too many Grassmann-odd superfields W_α.

Consider now the contributions to the effective action $\Gamma_{\rm B}$ from the propagator (2.15). Similarly as for $\Gamma_{\rm A}$, after integration by parts, the operator $\bar{D}^2$ hits K_- and produces K_{+-} because of the identity

$$K_{+-}(z,z'|s) = \frac{1}{4}\bar{\nabla}^2 K_-(z,z'|s). \tag{2.40}$$

Hence, the part of the effective action $\Gamma_{\rm B}$ gets the same form (2.36) and, thus, vanishes.

The present analysis was done for the operator $\bar{D}^2$ in (2.15). The operator D^2 can be considered in a similar way with the same conclusion. Thus, we proved that the two-loop contributions to the effective action (2.22) with the propagator (2.15) vanish. In other words, the considered low-energy effective action is independent of the gauge-fixing parameter α. In the following sections we will compute non-trivial contributions to the two-loop effective actions (2.33) and (2.34) coming from the gauge superfield propagator G_1 given by (2.14).

2.4. Two-loop graph A

Consider the part of the effective action (2.33) and represent all the Green's functions in terms of the corresponding heat kernels,

$$\Gamma_{\rm A} = -\frac{2\pi}{k}\int d^7z\, d^3\xi \int_0^\infty \frac{ds\, dt\, du}{(4\pi i u)^{3/2}} e^{\frac{i\xi^2}{4u}} e^{i(s+t)m^2} \nabla^\alpha K_{+-}\big(z,z'\big|s\big)\bar\nabla_\alpha K_{-+}\big(z,z'\big|t\big)\big|. \quad (2.41)$$

Here we integrated by pats the derivatives $D^\alpha \bar D_\alpha$ which come from the gauge superfield propagator (2.14). To find the effective action we need to compute the derivatives of the heat kernels, $\nabla_\alpha K_{+-}(z,z'|s)$ and $\bar\nabla_\alpha K_{-+}(z,z'|t)$. In general, this problem is very hard since the heat kernels themselves have very complicated form (B.20) and (B.21). However, we will take into account the following simplifications:

- Upon computing the derivative of the heat kernels we omit the terms which vanish in the limit $\theta=\theta'$, $\bar\theta=\bar\theta'$.
- Since we are interested in the low-energy effective action of the form (2.22), it is sufficient to consider only the terms which depend on superfield strengths G, W_α, $\bar W_\alpha$, but which contain $N_{\alpha\beta}$ at most in the first power. Terms with higher orders of $N_{\alpha\beta}$ should be systematically neglected.

For instance, the formulas (B.22) up to the first order in $N_{\alpha\beta}$ read

$$W^\alpha(s) \approx W^\alpha - sN^\alpha_\beta W^\beta, \qquad \bar W^\alpha(s) \approx \bar W^\alpha - sN^\alpha_\beta \bar W^\beta, \quad (2.42)$$

$$\zeta^\alpha(s) \approx \zeta^\alpha - sW^\alpha + \frac{1}{2}s^2 N^\alpha_\beta W^\beta, \quad (2.43)$$

$$\bar\zeta^\alpha(s) \approx \bar\zeta^\alpha - s\bar W^\alpha + \frac{1}{2}s^2 N^\alpha_\beta \bar W^\beta, \quad (2.44)$$

$$\xi^m(s) \approx \xi^m - i\big(\gamma^m\big)^{\alpha\beta}\bigg[s(W_\alpha\bar\zeta_\beta + \bar W_\alpha\zeta_\beta) - \frac{s^2}{2}N_{\alpha\gamma}\big(W^\gamma\bar\zeta_\beta + \bar W^\gamma\zeta_\beta\big) + \frac{s^3}{6}N_{\alpha\beta}W\bar W\bigg]. \quad (2.45)$$

Here and further the symbol "≈" means that the expressions are considered in the corresponding approximation up to the first order in $N_{\alpha\beta}$ and all terms of order $O(N^2)$ are omitted.

To compute the expression (2.41) we have to find $\nabla_\alpha K_{+-}(z,z'|s)|$ and $\bar\nabla_\alpha K_{-+}(z,z'|t)|$. Using (B.20) these quantities can be recast as

$$\nabla_\alpha K_{+-}\big(z,z'\big|s\big)\big| = M_\alpha(s)\cdot K_{+-}\big(z,z'\big|s\big)\big|,$$
$$\bar\nabla_\alpha K_{-+}\big(z,z'\big|t\big)\big| = \tilde M_\alpha(t)\cdot K_{-+}\big(z,z'\big|t\big)\big|, \quad (2.46)$$

where

$$M_\alpha(s)=\left[2isG\bar W_\alpha+\frac{i}{2}\big(F\coth(sF)\big)_{mn}\rho^m(s)\nabla_\alpha\rho^n(s)+\nabla_\alpha R\big(z,z'\big)+\nabla_\alpha I\big(z,z'\big)\right.$$
$$\left.+\int_0^s d\tau\,\nabla_\alpha\big(R'(\tau)+\Sigma(\tau)\big)\right]\Bigg|,\tag{2.47}$$
$$\tilde M_\alpha(t)=\left[2itGW_\alpha+\frac{i}{2}\big(F\coth(tF)\big)_{mn}\tilde\rho^m(t)\bar\nabla_\alpha\tilde\rho^n(t)+\bar\nabla_\alpha\tilde R\big(z,z'\big)+\bar\nabla_\alpha I\big(z,z'\big)\right.$$
$$\left.+\int_0^t d\tau\,\bar\nabla_\alpha\big(\tilde R'(\tau)+\Sigma(\tau)\big)\right]\Bigg|.\tag{2.48}$$

Here ρ^m and $\tilde\rho^m$ are versions of the bosonic interval with specific chirality properties (B.17). The two-point quantities $R(z,z')$, $\tilde R(z,z')$ and $\Sigma(z,z')$ are written down explicitly in (B.15), (B.16) and (B.23), respectively. Basic properties of the parallel transport propagator $I(z,z')$ are summarized in Appendix A.

Using Eqs. (2.45), (A.5), (B.15) and (B.24) we compute derivatives of various objects in (2.47) and (2.48),

$$\nabla_\alpha\rho^m(s)\big|\approx is^2\gamma^m_{\beta\gamma}N^\beta_\alpha\bar W^\gamma,\tag{2.49}$$
$$\nabla_\alpha R\big(z,z'\big)\big|\approx-\frac{1}{2}\xi_{\alpha\beta}\bar W^\beta,\tag{2.50}$$
$$\nabla_\alpha I\big(z,z'\big)\big|\approx\frac{1}{2}\xi_{\alpha\beta}\bar W^\beta I\big(z,z'\big),\tag{2.51}$$
$$\int_0^s d\tau\,\nabla_\alpha\big(R'(\tau)+\Sigma(\tau)\big)\big|\approx is^2GN_{\alpha\beta}\bar W^\beta+2is^2\bar W^2W_\alpha.\tag{2.52}$$

One can easily find similar expressions involving the derivative $\bar\nabla_\alpha$ in the l.h.s. Substituting (2.49)–(2.52) into (2.47) we get

$$\begin{aligned}M_\alpha(s)&\approx 2isG\bar W_\alpha+is^2GN_{\alpha\beta}\bar W^\beta+2is^2\bar W^2W_\alpha-\frac{s}{2}\xi_m\gamma^m_{\beta\gamma}N^\beta_\alpha\bar W^\gamma-\frac{3i}{4}s^3\bar W^2N_{\alpha\beta}W^\beta,\\ \tilde M_\alpha(t)&\approx 2itGW_\alpha+it^2GN_{\alpha\beta}W^\beta-2it^2W^2\bar W_\alpha+\frac{t}{2}\xi_m\gamma^m_{\beta\gamma}N^\gamma_\alpha W^\beta+\frac{3i}{4}t^3W^2N_{\alpha\beta}\bar W^\beta.\end{aligned}\tag{2.53}$$

Eqs. (2.46) include also the heat kernels K_{+-} and K_{-+} at coincident Grassmann points (B.27). We have to expand (B.27) up to the first order in $N_{\alpha\beta}$. In particular, the functions (B.28) in this approximation are

$$f_\alpha{}^\beta(s)\approx-s^2\delta^\beta_\alpha+\frac{1}{3}s^3N^\beta_\alpha,\tag{2.54}$$
$$f(s)\approx-\frac{7}{12}s^3,\tag{2.55}$$
$$f^m_{\alpha\beta}(s)\approx-\frac{s}{2}\gamma^m_{\alpha\beta}+\frac{1}{12}s^2\varepsilon_{\alpha\beta}\big(\gamma^m_{\rho\sigma}N^{\rho\sigma}\big)+\frac{3}{4}s^2\big(\gamma^m_{\beta\gamma}N^\gamma_\alpha+\gamma^m_{\alpha\gamma}N^\gamma_\beta\big).\tag{2.56}$$

Substituting these functions into (B.27) we find

$$K_{+-}\big(z,z'\big|s\big)\big| \approx -\frac{1}{(4i\pi s)^{3/2}} e^{\frac{i}{4s}\xi^2+isG^2} e^{X(\xi^m,s)}, \tag{2.57}$$

where

$$X\big(\xi^m,s\big) = is^2 G W^\alpha \bar{W}_\alpha - \frac{i}{3}s^3 G W^\alpha N_\alpha^\beta \bar{W}_\beta - \frac{s}{2}\xi_m \gamma^m_{\alpha\beta} W^\alpha \bar{W}^\beta + \frac{1}{12}s^2\xi_m\big(\gamma^m N\big)W^\alpha\bar{W}_\alpha + \frac{3}{2}s^2\xi_m\gamma^m_{\gamma(\alpha}N^\gamma_{\beta)}W^\alpha\bar{W}^\beta - \frac{7i}{24}s^3 W^2\bar{W}^2. \tag{2.58}$$

With the use of (2.46) and (2.57) the part of the effective action (2.41) can be recast as

$$\varGamma_{\mathrm{A}} = -\frac{2\pi}{k(4\pi i)^{9/2}} \int d^7z\, d^3\xi \int_0^\infty \frac{ds\,dt\,du}{(stu)^{3/2}} e^{\frac{i\xi^2}{4}(\frac{1}{s}+\frac{1}{t}+\frac{1}{u})} e^{i(s+t)(G^2+m^2)} \times M^\alpha(s)\tilde{M}_\alpha(t) e^{X(\xi^m,s)+X(-\xi^m,t)}. \tag{2.59}$$

The expression in the second line in (2.59) should be expanded in a series up to the first order in $N_{\alpha\beta}$,

$$\begin{aligned} &M^\alpha(s)\tilde{M}_\alpha(t)e^{X(\xi^m,s)+X(-\xi^m,t)} \\ &\quad\approx -4stG^2W^\alpha\bar{W}_\alpha + 2stG^2(s-t)\bar{W}^\alpha W^\beta N_{\alpha\beta} \\ &\qquad + 4st(t-s)GW^2\bar{W}^2 + 2ist\big(s^2+t^2\big)G^3W^2\bar{W}^2 \\ &\qquad + istG\xi_m\gamma^m_{\rho\sigma}N^\sigma_\alpha\big(\bar{W}^\alpha W^\rho + \bar{W}^\rho W^\alpha\big) \\ &\qquad + st\left[\frac{i}{2}(s+t) + \frac{G^2}{12}(s-t)(5s-t)\right]\xi_m\big(\gamma^m N\big)W^2\bar{W}^2. \end{aligned} \tag{2.60}$$

Here we used explicit forms of the quantities $M_\alpha(s)$ and $X(\xi^m,s)$ given in (2.53) and (2.58), respectively. The terms in the last two lines in (2.60) contain bosonic interval ξ^m in the first power. They do not contribute to the effective action because of the identity

$$\int d^3\xi\, \xi_m e^{\frac{i\xi^2}{4}(\frac{1}{s}+\frac{1}{t}+\frac{1}{u})} = 0. \tag{2.61}$$

For the terms in the first two lines in (2.60) the integration over $d^3\xi$ is simply Gaussian,

$$\int d^3\xi\, e^{\frac{i}{4}a\xi^2} = -\left(\frac{4i\pi}{a}\right)^{\frac{3}{2}}, \quad a = \frac{1}{s}+\frac{1}{t}+\frac{1}{u}. \tag{2.62}$$

Hence, after integration over du, the effective action (2.59) can be recast as

$$\varGamma_{\mathrm{A}} = \frac{i}{16\pi^2 k}\int d^7z \int_0^\infty ds\,dt \frac{\sqrt{st}}{s+t} e^{i(s+t)(G^2+m^2)}\big[-4G^2\bar{W}^\alpha W_\alpha + 2(s-t)G^2\bar{W}^\alpha W^\beta N_{\alpha\beta} - 4(s-t)GW^2\bar{W}^2 + 2i\big(s^2+t^2\big)G^3W^2\bar{W}^2\big]. \tag{2.63}$$

The expression (2.63) contains the term with $W^\alpha\bar{W}_\alpha$. This term vanishes on shell because of (2.23). There are also two terms in (2.63) containing $(s-t)$. These terms are also vanishing since they are odd under the change of integration variables $s \leftrightarrow t$. So, only the last term in (2.63) remains non-trivial for the considered gauge superfield background. Performing the integration over s and t in this term we get the final result for the effective action $\Gamma_{\rm A}$:

$$\Gamma_{\rm A} = -\frac{15}{256\pi k}\int d^7z\,\frac{G^3W^2\bar{W}^2}{(G^2+m^2)^4}. \tag{2.64}$$

2.5. Two-loop graph B

Consider the part of the effective action (2.34) with the gauge superfield propagator (2.14),

$$\Gamma_{\rm B} = -\frac{2\pi m^2}{k}\int d^7z\,d^3\xi\int_0^\infty \frac{ds\,dt\,du}{(4\pi i u)^{3/2}}e^{\frac{i\xi^2}{4u}}e^{i(s+t)m^2}\nabla^\alpha K_+\big(z,z'\big|s\big)\bar{\nabla}_\alpha K_-\big(z,z'\big|t\big)\big|. \tag{2.65}$$

Here we integrated by parts the operator $D^\alpha\bar{D}_\alpha$ and integrated out one set of Grassmann variables using the delta-function. For computing this part of the effective action we need to find the derivatives of the heat kernels (B.18) and (B.19) at coincident Grassmann points,

$$\nabla_\alpha K_+\big(z,z'\big|s\big)\big| = \frac{1}{(4\pi i s)^{3/2}}P_\alpha(s)e^{Y(s)}e^{isG^2}e^{\frac{i\xi^2}{4s}}I\big(z,z'\big)\big|, \tag{2.66}$$

where

$$Y(s) = \frac{i}{4}\big(F\coth(sF)\big)_{mn}\xi^m(s)\xi^n(s) - \frac{i\xi^2}{4s} - \frac{1}{2}\bar{\zeta}^\beta(s)\xi_{\beta\gamma}(s)W^\gamma(s) + \int_0^s dt\,\Sigma\big(z,z'\big|t\big), \tag{2.67}$$

$$P_\alpha(s) = \nabla_\alpha\zeta^2(s) + \zeta^2(s)\nabla_\alpha Y(s). \tag{2.68}$$

It is sufficient to compute the derivatives of all objects in (2.68) up to the first order in $N_{\alpha\beta}$,

$$\nabla_\alpha\xi^m(s)\big| \approx is\gamma^m_{\alpha\beta}\bar{W}^\beta - \frac{is^2}{2}\gamma^m_{\alpha\beta}N^\beta_\gamma\bar{W}^\gamma, \tag{2.69}$$

$$\nabla_\alpha\zeta^2(s)\big| \approx -2sW_\alpha - s^2N_{\alpha\beta}W^\beta, \tag{2.70}$$

$$-\frac{1}{2}\nabla_\alpha\big(\bar{\zeta}^\beta(s)\xi_{\beta\gamma}(s)W^\gamma(s)\big)\big| \approx -\frac{s}{2}\bar{W}^\beta\xi_m\gamma^m_{\beta\gamma}N^\gamma_\alpha + \frac{3i}{4}\big(s^2\bar{W}^2W_\alpha - s^3\bar{W}^2N_{\alpha\beta}W^\beta\big), \tag{2.71}$$

$$\int_0^s dt\,\nabla_\alpha\Sigma\big(z,z'\big|t\big)\big| \approx -isG\bar{W}_\alpha + \frac{is^2}{2}GN_{\alpha\beta}\bar{W}^\beta - \frac{is^3}{6}W^2N_{\alpha\beta}W^\beta - \frac{s}{12}\xi_m\big(\gamma^mN\big)\bar{W}_\alpha. \tag{2.72}$$

Substituting these formulas to (2.68) and expanding up to the first order in $N_{\alpha\beta}$ we get

$$P_\alpha(s)e^{Y(s)}\big| \approx -2sW_\alpha - s^2 N_{\alpha\beta}W^\beta + is^3 G W^2 \bar{W}_\alpha - \frac{s^2}{2}\xi_m \gamma^m_{\alpha\beta}\bar{W}^\beta W^2 + \frac{3s^3}{4}\xi_m N_\alpha^\gamma \gamma^m_{\beta\gamma}\bar{W}^\beta W^2 - \frac{5s^3}{12}\xi_m\big(\gamma^m N\big)\bar{W}_\alpha W^2 + \frac{is^4}{6} G N_{\alpha\beta}\bar{W}^\beta W^2. \tag{2.73}$$

In a similar way we find

$$\bar{\nabla}_\alpha K_-\big(z,z'\big|s\big)\big| = \frac{1}{(4\pi i s)^{3/2}}\tilde{P}_\alpha(s)e^{\tilde{Y}(s)}e^{isG^2}e^{\frac{i}{4s}\xi^2}\big|, \tag{2.74}$$

$$\tilde{P}_\alpha(s)e^{\tilde{Y}(s)}\big| \approx 2s\bar{W}_\alpha + s^2 N_{\alpha\beta}\bar{W}^\beta - is^3 G\bar{W}^2 W_\alpha + \frac{s^2}{2}\xi_m\gamma^m_{\alpha\beta}W^\beta\bar{W}^2 - \frac{3s^3}{4}\xi_m N_\alpha^\gamma\gamma^m_{\gamma\beta}W^\beta\bar{W}^2 + \frac{5s^3}{12}\xi_m\big(\gamma^m N\big)W_\alpha\bar{W}^2 + \frac{is^4}{6}GN_{\alpha\beta}W^\beta\bar{W}^2. \tag{2.75}$$

Substituting (2.66) and (2.74) into (2.65) and using explicit form of the functions (2.73) and (2.75) we perform Gaussian integration over $d^3\xi$,

$$\Gamma_{\rm B} = \frac{im^2}{16\pi^2 k}\int d^7z \int_0^\infty \frac{ds\,dt\sqrt{st}}{(s+t)} e^{i(s+t)(G^2+m^2)} \times \big[-4W^\alpha\bar{W}_\alpha + 2(s-t)W^\alpha N_{\alpha\beta}\bar{W}^\beta + 2i\big(s^2+t^2\big)GW^2\bar{W}^2\big]. \tag{2.76}$$

Note that the term containing $W^\alpha\bar{W}_\alpha$ in (2.76) does not contribute to the effective action according to (2.23). The first term in the second line of (2.76) also vanishes since it is odd under the change of integration variables s and t. After computing the integrals over s and t in the last term in (2.76) we obtain

$$\Gamma_{\rm B} = -\frac{15m^2}{256\pi k}\int d^7z\,\frac{GW^2\bar{W}^2}{(G^2+m^2)^4}. \tag{2.77}$$

2.6. *Summary of two-loop computations*

The two-loop low-energy effective action is given by the sum of Eqs. (2.64) and (2.77),

$$\Gamma^{(2)} = -\frac{15}{256\pi k}\int d^7z\,\frac{GW^2\bar{W}^2}{(G^2+m^2)^3}. \tag{2.78}$$

This expression shows that the functions $f_1(G)$ and $f_2(G)$ in (2.22) receive no two-loop quantum corrections,

$$f_1^{(2)}(G) = f_2^{(2)}(G) = 0, \tag{2.79}$$

and only the function $f_3(G)$ gets non-trivial two-loop contribution,

$$f_3^{(2)}(G) = -\frac{15}{256\pi k}\frac{G}{(G^2+m^2)^3}. \tag{2.80}$$

It is instructive to compare the two-loop low-energy effective action (2.78) with analogous result in $\mathcal{N}=2$ SQED with vanishing topological mass considered in [9]. The latter is described by the classical action similar to (2.1), but in which the gauge superfield V has $\mathcal{N}=2$ supersymmetric Maxwell rather than the Chern–Simons term. The four-derivative low-energy effective action has the same form (2.22), but with the functions f_i given by (see Appendix C for details of derivation of these functions)

$$\tilde{f}_1^{(2)} = -\frac{g^2}{16\pi^2}\ln\left(G^2+m^2\right), \tag{2.81}$$

$$\tilde{f}_2^{(2)} = \frac{5g^2}{192\pi^2}\frac{G}{(G^2+m^2)^3}, \tag{2.82}$$

$$\tilde{f}_3^{(2)} = \frac{g^2}{\pi^2}\frac{98G^2-73m^2}{3072(G^2+m^2)^4}. \tag{2.83}$$

Here we put tilde on these functions to distinguish them from (2.79) and (2.80).

The obvious difference of the functions $\tilde{f}_i^{(2)}$ from $f_i^{(2)}$ is that they contain dimensionful gauge coupling constant g^2. Therefore, even in the massless limit $m=0$, the functions $\tilde{f}_i^{(2)}$ give non-conformal effective action while $f_i^{(2)}$ do.

Let us discuss conformal properties of the effective action (2.78). Of course, the model (2.1) is non-conformal as it explicitly involves the mass parameter m, but we can still get profit from the power of constraints of the superconformal group either by considering the corresponding massless theory, $m=0$, or by promoting the mass parameter to a chiral superfield. The latter option is closer to the $\mathcal{N}=3$ supersymmetric electrodynamics considered in Section 3.2, but here, for the sake of simplicity, we will discuss only the massless case,

$$\Gamma^{(2)}\big|_{m=0} = -\frac{15}{256\pi k}\int d^7z\,\frac{W^2\bar{W}^2}{G^5}. \tag{2.84}$$

Being scale invariant, this effective action is not $\mathcal{N}=2$ superconformal as the superfields W_α and $\bar{W}_\alpha$ are not quasi-primaries [10]. The latter means that these superfields do not have right transformation lows of superconformal spin-tensors of engineering dimension $3/2$.[5] Nevertheless, this does not imply any anomaly of the superconformal symmetry. Recall that the expression (2.84) was derived for the background gauge superfield obeying supersymmetric Maxwell equations (2.19). Now, one can add some terms with $D^\alpha W_\alpha$ or $\bar{D}^\alpha\bar{W}_\alpha$ to the action (2.84) to make it superconformal.[6]

In [10] it was shown that the object

$$\Psi = \frac{i}{G}D^\alpha\bar{D}_\alpha\ln G \tag{2.85}$$

transforms as a scalar with vanishing scaling dimension under $\mathcal{N}=2$, $d=3$ superconformal group. Up to a term proportional to the super Maxwell equations (2.19), this superfield has the following expression in terms of the superfield strengths W_α and $\bar{W}_\alpha$:

$$\Psi = -i\frac{W^\alpha\bar{W}_\alpha}{G^3}. \tag{2.86}$$

[5] The representation of superconformal group in $\mathcal{N}=2$, $d=3$ superspace was considered in [23].

[6] Similar procedure was applied in $\mathcal{N}=2$, $d=4$ superspace to construct superconformal off-shell effective action for gauge superfield [24].

Hence, the superconformal generalization of the action (2.84) reads

$$\Gamma^{(2)}\big|_{m=0} = \frac{15}{128\pi k}\int d^7z\,\frac{(D^\alpha \bar{D}_\alpha \ln G)^2}{G}. \tag{2.87}$$

Representing the action (2.84) in the superconformal form (2.87) has several important consequences. First, we point out that for the action (2.87) we can now relax the constraints (2.19) on the background gauge superfield which were used in the derivation of this result. Indeed, the superconformal invariance allows us to uniquely restore in the final answer the terms proportional to the supersymmetric Maxwell equations which were omitted in the intermediate steps of deriving Eq. (2.84).

Second, it is clear now that the function $f_2(G)$ in (2.22) should vanish as the corresponding term in the effective action does not have a superconformal generalization. The unique superconformal generalization of the four-derivative term is given by (2.87) which corresponds to the last term in (2.22).

Finally, it is clear now that it is the superconformal symmetry which forbids any higher-loop quantum corrections to the function $f_1(G)$ in (2.22). Indeed, the superconformal generalization of the two-derivative term in the effective action is given uniquely by $\int d^7z\, G\ln G$, which is nothing but the one-loop contribution (2.29) in the massless limit.

Thus, we conclude that the superconformal invariance imposes strong constraints on the structure of two-loop quantum corrections to the low-energy effective action (2.22) in the model (2.1). The similar model with the Maxwell term for the gauge superfields has no superconformal properties and the structure of its effective action is much reacher, as is seen in (2.81)–(2.83).

3. Generalizations to other Abelian Chern–Simons matter models

3.1. Two-loop effective action in supersymmetric electrodynamics with one chiral superfield

The results of the previous section can be easily extended to the Chern–Simons matter model with one chiral superfield,

$$S = \frac{k}{2\pi}\int d^7z\, VG - \int d^7z\, \bar{Q}e^{2V}Q. \tag{3.1}$$

This model is known to be superconformal [10], but has parity anomaly [25–27,14]. The parity anomaly manifests itself in the presence of the Chern–Simons term in the one-loop effective action [10],

$$\Gamma_{\rm odd} = \frac{1}{4\pi}\int d^7z\, VG. \tag{3.2}$$

The subscript "odd" here means that the induced Chern–Simons term is unique part of the effective action which is parity-odd. This induced Chern–Simons term gives half-integer shift to the classical value of the Chern–Simons level k,

$$k_{\rm eff} = k + \frac{1}{2}. \tag{3.3}$$

In quantum theory the effective coupling $k_{\rm eff}$ rather than k quantizes in integers, $k_{\rm eff} \in \mathbb{Z}$.

The rest of the effective action is parity-even and we denote it as $\Gamma_{\rm even}$. So, all the conclusions of Section 2.2 apply to it. Hence, its general structure should be the same as of (2.22). The one-loop contributions to the functions $f_i(G)$ for the model (3.1) were found in [10],

$$f_1^{(1)} = \frac{1}{4\pi} G \ln G, \qquad f_2^{(1)} = 0, \qquad f_3^{(1)} = \frac{1}{256\pi} \frac{1}{G^5}. \tag{3.4}$$

Our aim now is to compute two-loop corrections to this result, i.e., to find $f_i^{(2)}$.

The two-loop effective action in the model (3.1) is given by the formula

$$\Gamma^{(2)} = -\int d^7z\, d^7z'\, G_{+-}(z,z') G_{-+}(z,z') G(z,z'). \tag{3.5}$$

This effective action corresponds to the first graph in Fig. 1.

The expression (2.33) resembles from (3.5) by the factor 2. Hence, we can immediately borrow the result from Section 2.4: One should divide by two Eq. (2.64) and apply the massless limit $m \to 0$,

$$\Gamma^{(2)} = -\frac{15}{512\pi k_{\text{eff}}} \int d^7z\, \frac{W^2 \bar{W}^2}{G^5}. \tag{3.6}$$

Here we also used the effective Chern–Simons level k_{eff} which includes one-loop correction to the classical value, (3.3).

The effective action (3.6) corresponds to the following values of the functions $f_i^{(2)}$ in (2.22):

$$f_1^{(2)} = f_2^{(2)} = 0, \qquad f_3^{(2)} = -\frac{15}{512\pi k_{\text{eff}}} \frac{1}{G^5}. \tag{3.7}$$

Since the model (3.1) is superconformal, the two-loop effective action (3.6) can be represented in a superconformal form. Similarly as for the action (2.84), by adding the terms with $D^\alpha W_\alpha$ and $\bar{D}^\alpha \bar{W}_\alpha$, the quantity (3.6) can be recast as follows:

$$\Gamma^{(2)} = \frac{15}{256\pi k_{\text{eff}}} \int d^7z\, \frac{(D^\alpha \bar{D}_\alpha \ln G)^2}{G}. \tag{3.8}$$

Summarizing now one- and two-loop results, we get the parity-even part of the two-loop effective action in the superconformal form,

$$\begin{aligned}\Gamma_{\text{even}} &= \Gamma^{(1)} + \Gamma^{(2)} \\ &= \frac{1}{4\pi} \int d^7z\, G \ln G + \frac{1}{128\pi} \left(\frac{15}{2k_{\text{eff}}} - 1 \right) \int d^7z\, \frac{(D^\alpha \bar{D}_\alpha \ln G)^2}{G}. \end{aligned} \tag{3.9}$$

As is explained in Section 2.6, once the effective action is represented in the superconformal form (3.9), the constraint (2.19) can be relaxed. Eq. (3.9) represents the parity-even part of the low-energy effective action in the model (3.1) up to the four-derivative order.

The two-loop effective actions obtained in this and previous sections can be easily generalized to Abelian $\mathcal{N} = 2$ Chern–Simons matter models with arbitrary number of chiral matter superfields.

3.2. $\mathcal{N} = 3$ Chern–Simons electrodynamics

The classical action of $\mathcal{N} = 3$ Chern–Simons electrodynamics reads

$$S_{\mathcal{N}=3} = S^{\text{CS}}_{\mathcal{N}=3} + S_{\text{hyper}}, \tag{3.10}$$

$$S^{\text{CS}}_{\mathcal{N}=3} = \frac{k}{2\pi} \int d^7z\, VG - \frac{ik}{4\pi} \int d^5z\, \Phi^2 - \frac{ik}{4\pi} \int d^5\bar{z}\, \bar{\Phi}^2, \tag{3.11}$$

$$S_{\text{hyper}} = -\int d^7z\, \left(\bar{Q}_+ e^{2V} Q_+ + \bar{Q}_- e^{-2V} Q_- \right) - \left(\int d^5z\, \Phi Q_+ Q_- + c.c. \right), \tag{3.12}$$

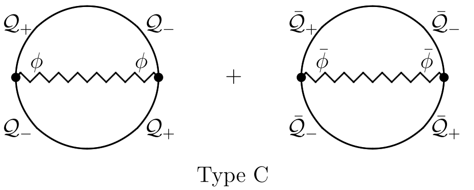

Fig. 2. Two-loop supergraphs in $\mathcal{N}=3$ supersymmetric electrodynamics which involve (anti)chiral propagators $\langle\phi\phi\rangle$ and $\langle\bar{\phi}\bar{\phi}\rangle$.

where Φ is a chiral superfield which is part of the $\mathcal{N}=3$ gauge multiplet (V,Φ). Note that this model reduces to (2.1) for $\Phi=m$. However, in contrast to (2.1), the action of the $\mathcal{N}=3$ Chern–Simons electrodynamics (3.10) is superconformal.

Let us make the background-quantum splitting for the $\mathcal{N}=3$ gauge multiplet,

$$(V,\Phi)\to(V,\Phi)+(v,\phi), \tag{3.13}$$

where the superfields (V,Φ) in the r.h.s. are treated as background while (v,ϕ) as the quantum ones. Under this splitting the part of the $\mathcal{N}=3$ Chern–Simons action which is quadratic with respect to the quantum superfields reads

$$S^{\rm CS}_{\mathcal{N}=3}=\frac{ik}{4\pi}\left(\int d^7z\, vD^\alpha\bar{D}_\alpha v-\int d^5z\,\phi^2-\int d^5\bar{z}\,\bar{\phi}^2\right)+\cdots, \tag{3.14}$$

where dots stand for the linear terms for the quantum superfields which are irrelevant in quantum loop computations. Note that the superfield ϕ is gauge invariant since the gauge group is Abelian. Hence, to fix the gauge freedom it is sufficient to add to (3.14) the same gauge fixing term (2.5) as in the $\mathcal{N}=2$ case. This yields the following action for quantum superfields up to quartic order:

$$S_{\rm quant}=S_2+S_{\rm int}, \tag{3.15}$$

$$S_2=\int d^7z\left(\frac{ik}{4\pi}vHv-\bar{\mathcal{Q}}_+\mathcal{Q}_+-\bar{\mathcal{Q}}_-\mathcal{Q}_-\right)-\int d^5z\left(\frac{ik}{4\pi}\phi^2+\Phi\mathcal{Q}_+\mathcal{Q}_-\right)$$
$$-\int d^5\bar{z}\left(\frac{ik}{4\pi}\bar{\phi}^2-\bar{\Phi}\bar{\mathcal{Q}}_+\bar{\mathcal{Q}}_-\right), \tag{3.16}$$

$$S_{\rm int}=-2\int d^7z\left[(\bar{\mathcal{Q}}_+\mathcal{Q}_+-\bar{\mathcal{Q}}_-\mathcal{Q}_-)v+(\bar{\mathcal{Q}}_+\mathcal{Q}_++\bar{\mathcal{Q}}_-\mathcal{Q}_-)v^2\right]-\int d^5z\,\phi\mathcal{Q}_+\mathcal{Q}_-$$
$$+\int d^5\bar{z}\,\bar{\phi}\bar{\mathcal{Q}}_+\bar{\mathcal{Q}}_-+O\left(v^3\right). \tag{3.17}$$

The action $S_{\rm int}$ is responsible for the interaction vertices while S_2 gives propagators for the quantum superfields. As compared with the $\mathcal{N}=2$ electrodynamics, there is a new vertex $\phi\mathcal{Q}_+\mathcal{Q}_-$ (and its conjugate) and the propagators $\langle\phi\phi\rangle$, $\langle\bar{\phi}\bar{\phi}\rangle$,

$$\left\langle\phi(z)\phi\left(z'\right)\right\rangle=-\frac{2\pi}{k}\delta_+\left(z,z'\right),\qquad\left\langle\bar{\phi}(z)\bar{\phi}\left(z'\right)\right\rangle=-\frac{2\pi}{k}\delta_-\left(z,z'\right). \tag{3.18}$$

Hence, apart from the graphs in Fig. 1, there are two extra Feynman graphs in the $\mathcal{N}=3$ SQED with these propagators which are represented in Fig. 2. Correspondingly, two-loop effective action is given by the following formal expression:

$$\Gamma^{(2)}_{\mathcal{N}=3} = \Gamma_{\mathrm{A}} + \Gamma_{\mathrm{B}} + \Gamma_{\mathrm{C}}, \tag{3.19}$$

$$\Gamma_{\mathrm{A}} = -2\int d^7z\, d^7z'\, G_{+-}(z,z')G_{-+}(z,z')G(z,z'), \tag{3.20}$$

$$\Gamma_{\mathrm{B}} = -2\int d^7z\, d^7z'\, \Phi\bar{\Phi} G_{+}(z,z')G_{-}(z,z')G(z,z'), \tag{3.21}$$

$$\Gamma_{\mathrm{C}} = \frac{\pi i}{k}\int d^5z\, G_{+}(z,z')G_{+}(z',z)\delta_{+}(z,z') + c.c. \tag{3.22}$$

The chiral delta-function in the expression (3.22) originates from the propagators (3.18).

Recall that the background gauge superfield V is constrained by (2.19) and (2.20). Analogous constraints for Φ,

$$D_\alpha \Phi = 0, \qquad \bar{D}_\alpha \bar{\Phi} = 0, \tag{3.23}$$

just mean that this superfield is simply a constant. For such a background the heat kernels for the propagators G_{+-} and G_+ are given in Appendix B. In particular, Eq. (B.26) shows that at coincident superspace points the heat kernel K_+ is proportional to W^2,

$$K_+(z,z|s) \propto W^2. \tag{3.24}$$

The quantity (3.22) contains two chiral propagators G_+ at coincident superspace points after integration over dz' with the help of chiral delta-function. Hence, Γ_{C} vanishes as it contains too many W's,

$$\Gamma_{\mathrm{C}} = 0. \tag{3.25}$$

It is clear that for the constant chiral superfield background (3.23) computations of the contributions Γ_{A} and Γ_{B} to the two-loop effective action are absolutely identical to the ones given in Sections 2.4 and 2.5. Hence, we can borrow the result (2.78) just by promoting the mass parameter to the chiral superfield,

$$\Gamma^{(2)}_{\mathcal{N}=3} = -\frac{15}{256\pi k}\int d^7z\, \frac{G W^2 \bar{W}^2}{(G^2 + \Phi\bar{\Phi})^3}. \tag{3.26}$$

The effective action (3.26) is scale invariant, but is not superconformal similarly as the effective action (3.6) obtained in the previous section. To construct a superconformal generalization of (3.26) we use a version of the quasi-primary superfield (2.85) which involves the chiral superfield Φ [10],

$$\boldsymbol{\Psi} = \frac{i}{G} D^\alpha \bar{D}_\alpha \ln\big(G + \sqrt{G^2 + \Phi\bar{\Phi}}\,\big). \tag{3.27}$$

Up to a term proportional to the super Maxwell equations (2.19), this superfield reads

$$\boldsymbol{\Psi} = -i\frac{W^\alpha \bar{W}_\alpha}{(G^2 + \Phi\bar{\Phi})^{3/2}}. \tag{3.28}$$

Hence, the superconformal generalization of (3.26) is given by

$$\begin{aligned}\Gamma^{(2)}_{\mathcal{N}=3} &= -\frac{15}{128\pi k}\int d^7z\, G\boldsymbol{\Psi}^2 \\ &= \frac{15}{128\pi k}\int d^7z\, \frac{1}{G}\big[D^\alpha \bar{D}_\alpha \ln\big(G + \sqrt{G^2 + \Phi\bar{\Phi}}\,\big)\big]^2.\end{aligned} \tag{3.29}$$

The representation of the effective actions (3.29) in superconformal form allows us to relax the constraint (2.19) which was used for deriving this result.

For completeness, we present here the four-derivative part of one-loop effective action in the model (3.10) which was found in [10]:

$$\Gamma^{(1)}_{\mathcal{N}=3} = \frac{1}{64\pi}\int d^7z\, \boldsymbol{\Psi}^2\sqrt{G^2+\Phi\bar{\Phi}} = -\frac{1}{64\pi}\int d^7z\,\frac{\sqrt{G^2+\Phi\bar{\Phi}}}{G^2}\big[D^\alpha\bar{D}_\alpha \ln\big(G+\sqrt{G^2+\Phi\bar{\Phi}}\big)\big]^2. \tag{3.30}$$

It is interesting to note that the expressions (3.29) and (3.30) have slightly different functional structure. This is explained by the fact that the two-loop effective action (3.26) was obtained in the gauge (2.4) which is only $\mathcal{N}=2$ supersymmetric. As a consequence, the two-loop result (3.26) does not respect full $\mathcal{N}=3$ superconformal group and requires $\mathcal{N}=3$ supersymmetrization. The issue of finding $\mathcal{N}=3$ supersymmeteric versions of the actions (3.29) and (3.30) deserves a separate study.

The most natural way to obtain the effective action in the model (3.10) in explicitly $\mathcal{N}=3$ supersymmetric form is by using the $\mathcal{N}=3$, $d=3$ harmonic superspace [28,29]. Quantum aspects of supersymmetric gauge theories in this superspace were studied in [30]. It would be interesting to explore the low-energy effective action in $\mathcal{N}=3$ gauge theories using this approach.

4. Conclusions

Recently, we computed two-loop low-energy effective actions in the $\mathcal{N}=2$ and $\mathcal{N}=4$ SQED theories [9] with vanishing topological mass. In the present paper we considered similar models in which the gauge superfield is described by the Chern–Simons rather than the supersymmetric Maxwell action. In these models we computed two-loop low-energy effective actions up to four-derivative order in the gauge superfield sector and compared them with similar results in the SQED theories considered in [9]. In the massless case these Chern–Simons matter models are superconformal. We demonstrated that the superconformal invariance imposes strong restrictions on the structure of two-loop effective actions forbidding a number of superfield structures (described by the functions f_1 and f_2 in (2.22)) which are non-trivial in similar SQED theories with vanishing topological mass. Note that any superconformal effective action for the $\mathcal{N}=2$ gauge superfield can be expressed in terms of superconformal invariants classified in [10]. So, the quantum loop computations performed in the present paper only fix numerical coefficients in the decomposition of the effective action over these invariants.

The low-energy effective action in the $\mathcal{N}=3$ Chern–Simons electrodynamics is also expressed in terms of $\mathcal{N}=2$ superconformal invariants. However, the full $\mathcal{N}=3$ supersymmetry is not explicit as the two-loop effective action is computed in the $\mathcal{N}=2$ supersymmetric gauge. The most natural way of recasting this effective action in the $\mathcal{N}=3$ supersymmetric form is based on the $\mathcal{N}=3$, $d=3$ harmonic superspace [28,29]. Some quantum aspects of supersymmetric gauge theories in this superspace were studied in [30]. It would be interesting to explore the low-energy effective action in $\mathcal{N}=3$ gauge theories using this approach.

The results of the present paper, together with similar results of [9], give the structure of low-energy effective actions in Abelian three-dimensional $\mathcal{N}=2$ and $\mathcal{N}=3$ supersymmetric gauge theories in two particular cases, when the gauge superfield is described either by Chern–Simons or by pure super Maxwell action. The latter corresponds to vanishing topological mass while the former describes gauge superfield with infinitely large topological mass. It would be interesting to consider more general case of the supersymmetric gauge theories with a finite value of the topological mass. The effective actions in such models should interpolate between the results of the present paper and those of [9]. Another natural generalization could be a computation of two-loop quantum corrections to low-energy effective actions in non-Abelian gauge theories in the $\mathcal{N}=2$, $d=3$ superspace considered, e.g., in [11].

In the present paper we studied the effective action in the gauge superfield sector. It is interesting to consider also the part of the effective action for (anti)chiral superfields and, in particular, to study two-loop effective Kähler potential. In components, such an effective action is responsible, in particular, for the effective scalar potential. This problem was studied for $\mathcal{N}=1$, $d=3$ superfield models in [31–34] and for pure $\mathcal{N}=2$, $d=3$ Wess–Zumino model in [35]. It is natural to extend the results of the latter work to models of $\mathcal{N}=2$ and $\mathcal{N}=3$ SQED considered in the present paper and compare them with analogous results for the $\mathcal{N}=1$ models. In non-supersymmetric three-dimensional scalar electrodynamics the two-loop effective potential was studied in [36,37].

Finally, it is very tempting to study the structure of low-energy effective actions in the BLG and ABJM models. This problem becomes very hot in the light of recent discussion in [8] where the relations of such an effective action to the dynamics of M2 branes was proposed. We expect that the techniques of quantum computations in the $\mathcal{N}=2$, $d=3$ superspace developed in [10–12,9] and in the present paper might be useful for studying this issue. Alternatively, the $\mathcal{N}=3$ harmonic superspace formulation [38] of the ABJM and BLG models can be employed.

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Appendix A. Parallel displacement propagator in $\mathcal{N}=2$, $d=3$ superspace

The technique of gauge-covariant multiloop quantum computations in $\mathcal{N}=1$, $d=4$ superspace was developed in [39]. Its power was demonstrated in studying two-loop effective actions in the $\mathcal{N}=1$ and $\mathcal{N}=2$, four-dimensional SQEDs in [40,41] and other gauge theories with extended supersymmetry in $\mathcal{N}=1$ superspace, [42–45].

The key ingredient of this technique is the parallel displacement propagator $I(z,z')$ which relates gauge-covariant objects in different superspace points. In the $\mathcal{N}=2$, $d=3$ superspace the parallel displacement propagator was considered in [9]. Here we review basic properties of this object which are necessary for two-loop quantum computations in the $\mathcal{N}=2$ Chern–Simons matter model studied in this paper.

The parallel displacement propagator $I(z,z')$ is a two-point superspace function taking its values in the gauge group and depending on the gauge superfields with the following properties:

(i) Under gauge transformations it changes as

$$I\bigl(z,z'\bigr) \to e^{i\tau(z)} I\bigl(z,z'\bigr) e^{-i\tau(z')}, \tag{A.1}$$

with $\tau(z)$ being a real gauge superfield parameter;

(ii) It obeys the equation

$$\zeta^A \nabla_A I\bigl(z,z'\bigr) = \zeta^A \bigl(D_A + V_A(z)\bigr) I\bigl(z,z'\bigr) = 0, \tag{A.2}$$

where V^A are gauge connections for D^A and $\zeta^A = (\xi^m, \zeta^\alpha, \bar\zeta_\alpha)$ is the $\mathcal{N}=2$ supersymmetric interval,

$$\zeta^\alpha = \bigl(\theta - \theta'\bigr)^\alpha, \qquad \bar\zeta^\alpha = \bigl(\bar\theta - \bar\theta'\bigr)^\alpha,$$
$$\xi^m = \bigl(x - x'\bigr)^m - i\gamma^m_{\alpha\beta}\zeta^\alpha\bar\theta'^\beta + i\gamma^m_{\alpha\beta}\theta'^\alpha\bar\zeta^\beta; \tag{A.3}$$

(iii) For coincident superspace points $z = z'$ it reduces to the identity operator in the gauge group,

$$I(z,z) = 1. \tag{A.4}$$

The properties (A.1)–(A.4) allow one to express the derivatives of the parallel transport propagator in terms of the parallel transport propagator itself and gauge-covariant superfield strengths. In particular, the following equations hold [9]:

$$\nabla_\beta I\bigl(z,z'\bigr) = \biggl[-i\bar\zeta_\beta G + \frac{1}{2}\xi_{\alpha\beta}\bar W^\alpha - \frac{i}{12}\bar\zeta^2 W_\beta + \frac{i}{6}\bar\zeta_\beta\zeta^\alpha\bar W_\alpha - \frac{i}{3}\bar\zeta^\alpha\zeta_\alpha\bar W_\beta$$
$$+ \frac{1}{12}\bar\zeta^\alpha\xi_{\beta\gamma}\bar\nabla_\alpha\bar W^\gamma - \frac{1}{12}\bar\zeta^\alpha\xi_{\alpha\gamma}\bar\nabla^\gamma\bar W_\beta - \frac{i}{12}\bar\zeta^2\zeta_\beta\bar\nabla^\alpha\bar W_\alpha\biggr] I\bigl(z,z'\bigr), \tag{A.5}$$

$$\bar\nabla^\beta I\bigl(z,z'\bigr) = \biggl[-i\zeta^\beta G - \frac{1}{2}\xi^\beta_\alpha W^\alpha + \frac{i}{12}\zeta^2\bar W^\beta - \frac{i}{6}\zeta^\beta\bar\zeta^\alpha W_\alpha + \frac{i}{3}\zeta^\alpha\bar\zeta_\alpha W^\beta$$
$$+ \frac{1}{12}\zeta_\alpha\xi^{\beta\gamma}\nabla^\alpha W_\gamma - \frac{1}{12}\zeta_\alpha\xi^{\alpha\gamma}\nabla_\gamma W^\beta - \frac{i}{12}\zeta^2\bar\zeta^\beta\nabla^\alpha W_\alpha\biggr] I\bigl(z,z'\bigr). \tag{A.6}$$

Appendix B. Green's functions in $\mathcal{N}=2$, $d=3$ superspace

Consider a covariantly chiral superfield Φ, $\bar\nabla_\alpha\Phi = 0$, where ∇_α and $\bar\nabla_\alpha$ are gauge-covariant spinor derivatives. There are two types of Green's functions for this superfield: $G_+(z,z')$ which is chiral with respect to both arguments and $G_{+-}(z,z')$ which is chiral with respect to z and is antichiral with respect to z',

$$i\bigl\langle\Phi(z)\Phi\bigl(z'\bigr)\bigr\rangle \equiv mG_+\bigl(z,z'\bigr), \qquad i\bigl\langle\Phi(z)\bar\Phi\bigl(z'\bigr)\bigr\rangle \equiv G_{+-}\bigl(z,z'\bigr). \tag{B.1}$$

By definition, they obey the following equations:

$$\bigl(\Box_+ + m^2\bigr)G_+\bigl(z,z'\bigr) = -\delta_+\bigl(z,z'\bigr), \tag{B.2}$$

$$\bigl(\Box_- + m^2\bigr)G_-\bigl(z,z'\bigr) = -\delta_-\bigl(z,z'\bigr), \tag{B.3}$$

$$\frac{1}{4}\bar\nabla^2 G_{-+}\bigl(z,z'\bigr) + m^2 G_+\bigl(z,z'\bigr) = -\delta_+\bigl(z,z'\bigr), \tag{B.4}$$

$$\frac{1}{4}\nabla^2 G_{+-}\bigl(z,z'\bigr) + m^2 G_-\bigl(z,z'\bigr) = -\delta_-\bigl(z,z'\bigr), \tag{B.5}$$

where $\delta_\pm(z,z')$ are (anti)chiral delta-functions and the operators $\Box_\pm$ are given by

$$\Box_+ = \nabla^m\nabla_m + G^2 + \frac{i}{2}\big(\nabla^\alpha W_\alpha\big) + iW^\alpha\nabla_\alpha,$$
$$\Box_- = \nabla^m\nabla_m + G^2 - \frac{i}{2}\big(\bar\nabla^\alpha \bar W_\alpha\big) - i\bar W^\alpha\bar\nabla_\alpha. \tag{B.6}$$

It is convenient to express the Green's functions in terms of corresponding heat kernels,

$$G_\pm\big(z,z'\big) = i\int_0^\infty ds\, K_\pm\big(z,z'\big|s\big)e^{ism^2}, \tag{B.7}$$

$$G_{+-}\big(z,z'\big) = i\int_0^\infty ds\, K_{+-}\big(z,z'\big|s\big)e^{ism^2}, \tag{B.8}$$

$$G_{-+}\big(z,z'\big) = i\int_0^\infty ds\, K_{-+}\big(z,z'\big|s\big)e^{ism^2}. \tag{B.9}$$

Explicit expressions for these heat kernels were found in [9]:

$$K_+\big(z,z'\big|s\big) = \frac{1}{8(i\pi s)^{3/2}}\frac{sB}{\sinh(sB)}e^{isG^2}\mathcal{O}(s)e^{\frac{i}{4}(F\coth(sF))_{mn}\xi^m\xi^n - \frac12\bar\zeta^\beta\xi_{\beta\gamma}W^\gamma}\zeta^2 I\big(z,z'\big), \tag{B.10}$$

$$K_-\big(z,z'\big|s\big) = \frac{1}{8(i\pi s)^{3/2}}\frac{sB}{\sinh(sB)}e^{isG^2}\mathcal{O}(s)e^{\frac{i}{4}(F\coth(sF))_{mn}\xi^m\xi^n - \frac12\zeta^\beta\xi_{\beta\gamma}\bar W^\gamma}\bar\zeta^2 I\big(z,z'\big), \tag{B.11}$$

$$K_{+-}\big(z,z'\big|s\big) = -\frac{1}{8(i\pi s)^{3/2}}\frac{sB}{\sinh(sB)}e^{isG^2}\mathcal{O}(s)e^{\frac{i}{4}(F\coth(sF))_{mn}\rho^m\rho^n + R(z,z')}I\big(z,z'\big), \tag{B.12}$$

$$K_{-+}\big(z,z'\big|s\big) = -\frac{1}{8(i\pi s)^{3/2}}\frac{sB}{\sinh(sB)}e^{isG^2}\mathcal{O}(s)e^{\frac{i}{4}(F\coth(sF))_{mn}\tilde\rho^m\tilde\rho^n + \tilde R(z,z')}I\big(z,z'\big). \tag{B.13}$$

Here $B^2 = \frac12 N^\alpha_\beta N^\beta_\alpha$ and $\mathcal{O}(s)$ is the operator of the form

$$\mathcal{O}(s) = e^{s(\bar W^\alpha\bar\nabla_\alpha - W^\alpha\nabla_\alpha)}. \tag{B.14}$$

The functions $R(z,z')$ and $\tilde R(z,z')$ read

$$R\big(z,z'\big) = -i\zeta\bar\zeta G + \frac{7i}{12}\bar\zeta^2\zeta W + \frac{i}{12}\zeta^2\bar\zeta\bar W - \frac12\bar\zeta^\alpha\rho_{\alpha\beta}W^\beta - \frac12\zeta^\alpha\rho_{\alpha\beta}\bar W^\beta$$
$$+ \frac{1}{12}\zeta^\alpha\bar\zeta^\beta\big[\rho^\gamma_\beta D_\alpha W_\gamma - 7\rho^\gamma_\alpha D_\gamma W_\beta\big], \tag{B.15}$$

$$\tilde R\big(z,z'\big) = i\zeta\bar\zeta G + \frac{i}{12}\bar\zeta^2\zeta W + \frac{7i}{12}\zeta^2\bar\zeta\bar W - \frac12\bar\zeta^\alpha\tilde\rho_{\alpha\beta}W^\beta - \frac12\zeta^\alpha\tilde\rho_{\alpha\beta}\bar W^\beta$$
$$+ \frac{1}{12}\zeta^\alpha\bar\zeta^\beta\big[7\tilde\rho_{\beta\gamma}\bar D^\gamma\bar W_\alpha - \tilde\rho_{\alpha\gamma}\bar D_\beta\bar W^\gamma\big]. \tag{B.16}$$

The objects ρ^m and $\tilde\rho^m$ are versions of bosonic interval ξ^m with specific chirality properties:

$$\rho^m = \xi^m + i\zeta^\alpha \gamma^m_{\alpha\beta}\bar\zeta^\beta, \qquad D'_\alpha \rho^m = \bar D_\alpha \rho^m = 0,$$
$$\tilde\rho^m = \xi^m - i\zeta^\alpha \gamma^m_{\alpha\beta}\bar\zeta^\beta, \qquad \bar D'_\alpha \tilde\rho^m = D_\alpha \tilde\rho^m = 0. \tag{B.17}$$

To make the heat kernels (B.10) and (B.12) more useful for loop quantum computations one has to push the operator $\mathcal{O}(s)$ on the right and act with it on the parallel transport propagator. The result of this procedure is [9]

$$K_+\big(z,z'\big|s\big) = \frac{1}{8(i\pi s)^{3/2}}\frac{sB}{\sinh(sB)} e^{isG^2} e^{\frac{i}{4}(F\coth(sF))_{mn}\xi^m(s)\xi^n(s) - \frac12\bar\zeta^\beta(s)\xi_{\beta\gamma}(s)W^\gamma(s)}$$
$$\times e^{\int_0^s dt\, \Sigma(z,z'|t)} \zeta^2(s) I\big(z,z'\big), \tag{B.18}$$
$$K_-\big(z,z'\big|s\big) = \frac{1}{8(i\pi s)^{3/2}}\frac{sB}{\sinh(sB)} e^{isG^2} e^{\frac{i}{4}(F\coth(sF))_{mn}\xi^m(s)\xi^n(s) - \frac12\zeta^\beta(s)\xi_{\beta\gamma}(s)\bar W^\gamma(s)}$$
$$\times e^{\int_0^s dt\, \Sigma(z,z'|t)} \bar\zeta^2(s) I\big(z,z'\big), \tag{B.19}$$
$$K_{+-}\big(z,z'\big|s\big) = -\frac{1}{8(i\pi s)^{3/2}}\frac{sB}{\sinh(sB)} e^{isG^2}$$
$$\times e^{\frac{i}{4}(F\coth(sF))_{mn}\rho^m(s)\rho^n(s) + R(z,z') + \int_0^s dt\,(R'(t)+\Sigma(t))} I\big(z,z'\big), \tag{B.20}$$
$$K_{-+}\big(z,z'\big|s\big) = -\frac{1}{8(i\pi s)^{3/2}}\frac{sB}{\sinh(sB)} e^{isG^2}$$
$$\times e^{\frac{i}{4}(F\coth(sF))_{mn}\tilde\rho^m(s)\tilde\rho^n(s) + \tilde R(z,z') + \int_0^s dt\,(\tilde R'(t)+\Sigma(t))} I\big(z,z'\big). \tag{B.21}$$

All s-dependent objects in these expressions are defined by the rule $X(s) = \mathcal{O}(s)X\mathcal{O}(-s)$, e.g.

$$W^\alpha(s) \equiv \mathcal{O}(s)W^\alpha\mathcal{O}(-s) = W^\beta\big(e^{-sN}\big)_\beta{}^\alpha,$$
$$\zeta^\alpha(s) \equiv \mathcal{O}(s)\zeta^\alpha\mathcal{O}(-s) = \zeta^\alpha + W^\beta\big(\big(e^{-sN}-1\big)N^{-1}\big)_\beta{}^\alpha,$$
$$\bar\zeta^\alpha(s) \equiv \mathcal{O}(s)\bar\zeta^\alpha\mathcal{O}(-s) = \bar\zeta^\alpha + \bar W^\beta\big(\big(e^{-sN}-1\big)N^{-1}\big)_\beta{}^\alpha,$$
$$\xi^m(s) \equiv \mathcal{O}(s)\xi^m\mathcal{O}(-s) = \xi^m - i\big(\gamma^m\big)^{\alpha\beta}\int_0^s dt\,\big(W_\alpha(t)\bar\zeta_\beta(t) + \bar W_\alpha(t)\zeta_\beta(t)\big). \tag{B.22}$$

The quantities $\Sigma(z,z')$ and $R'(z,z') + \Sigma(z,z')$ in (B.18)–(B.21) are given by

$$\Sigma\big(z,z'\big) = -i\big(\bar W^\beta\zeta_\beta - W^\beta\bar\zeta_\beta\big)G - \frac{i}{3}\zeta^\alpha\bar\zeta^\beta W_\beta\bar W_\alpha + \frac{2i}{3}\zeta^\alpha\bar\zeta_\alpha W^\beta\bar W_\beta$$
$$+ \frac{i}{12}\zeta^2\big[\bar W^2 - \bar\zeta^\alpha\bar W_\alpha D^\beta W_\beta\big] + \frac{i}{12}\bar\zeta^2\big[W^2 + \zeta^\alpha W_\alpha\bar D^\beta\bar W_\beta\big]$$
$$+ \frac{1}{12}\big(\zeta^\alpha\bar W^\beta - \bar\zeta^\beta W^\alpha\big)\big[\xi_{\alpha\gamma}D^\gamma W_\beta + \xi_{\beta\gamma}\bar D^\gamma\bar W_\alpha\big], \tag{B.23}$$
$$R' + \Sigma = 2i\bar\zeta WG + 2i(\zeta\bar\zeta\, W\bar W - \zeta W\,\bar\zeta\bar W) + i\bar\zeta^2\big[W^2 - \zeta^\alpha W^\beta D_\alpha W_\beta\big]$$
$$- \frac12\bar\zeta^\beta W^\alpha\big[\rho_{\beta\gamma}\bar D^\gamma\bar W_\beta - \rho_{\alpha\gamma}D^\gamma W_\beta\big], \tag{B.24}$$
$$\tilde R' + \Sigma = -2i\zeta\bar WG + 2i(\zeta\bar\zeta W\bar W - \zeta W\bar\zeta\bar W) + i\zeta^2\big[\bar W^2 + \bar\zeta^\alpha\bar W^\beta\bar D_\alpha\bar W_\beta\big]$$
$$- \frac12\zeta^\beta\bar W^\alpha\big[\tilde\rho_{\alpha\gamma}\bar D^\gamma\bar W_\beta + \tilde\rho_{\beta\gamma}\bar D_\alpha\bar W^\gamma\big]. \tag{B.25}$$

The heat kernels (B.10) and (B.12) at coincident Grassmann superspace points reduce to the following expressions [9]:

$$K_+\big(z,z'|s\big)\big| = \frac{1}{4(i\pi s)^{3/2}}\frac{sW^2}{B}\tanh\frac{sB}{2}e^{isG^2}e^{\frac{i}{4}(F\coth(sF))_{mn}\xi^m\xi^n}, \tag{B.26}$$

$$K_{+-}\big(z,z'|s\big)\big| = -\frac{1}{8(i\pi s)^{3/2}}\frac{sB}{\sinh(sB)}e^{isG^2}\exp\bigg\{\frac{i}{4}\big(F\coth(sF)\big)_{mn}\rho^m\rho^n - iGW^\alpha f_\alpha{}^\beta(s)\bar W_\beta + W^\alpha\rho_m f^m_{\alpha\beta}(s)\bar W^\beta + \frac{i}{2}W^2\bar W^2 f(s)\bigg\}, \tag{B.27}$$

where

$$f_\alpha{}^\beta(s) = 2B^{-2}\big(1 - sN - e^{-sN}\big)_\alpha{}^\beta,$$

$$f(s) = \frac{1}{sB^4}\big[(sB)^2 - 4\sinh^2(sB/2)\big(1 + sB\tanh(sB/2)\big)\big],$$

$$f^m_{\alpha\beta}(s) = \frac{1}{2}B^{-2}\big(\cosh(sB)-1\big)\big[\big(e^{-sN}\big)_\beta{}^\gamma N_\alpha{}^\delta\big(\gamma^m\big)_{\gamma\delta} + \big(N\big(e^{-sN}\big)\big)_\beta{}^\delta\big(\gamma^m\big)_{\alpha\delta}\big] - \frac{1}{2}\big(F\coth(sF)\big)^m{}_n\gamma^n_{\gamma\delta}\bigg[\bigg(\frac{e^{-sN}-1}{N}\bigg)_\alpha{}^\gamma\bigg(\frac{e^{-sN}-1}{N}\bigg)_\beta{}^\delta + \frac{\varepsilon_{\alpha\beta}N^{\gamma\delta}}{B^3}\big(sB - \sinh(sB)\big)\bigg]. \tag{B.28}$$

Appendix C. Two-loop effective action in $\mathcal{N}=2$ SQED up to four-derivative order

Classical action of $\mathcal{N}=2$ SQED has the form similar to (2.1), but the gauge superfield is described by supersymmetric Maxwell rather than the Chern–Simons term. The two-loop Euler–Heisenberg effective action in this model was studied in [9]. In components, such an action contains all powers of the Maxwell field strength. Here we wish to consider only the superfield terms up to four-derivative order, F^4, to compare them with the similar ones in the model (2.1) studied in Section 2. In principle, these terms can be extracted from the results obtained in [9] which include all powers of F_{mn} in components. However, we give here some details of deriving these terms "from scratch", following the same procedure as in Section 2 for similar Chern–Simons matter model (2.1).

Two-loop effective action in the $\mathcal{N}=2$ SQED has the structure analogous to (2.32), but with Γ_A and Γ_B given by

$$\Gamma_A = -2g^2\int d^7z\,d^7z'\,G_{+-}\big(z,z'\big)G_{-+}\big(z,z'\big)G_0\big(z,z'\big), \tag{C.1}$$

$$\Gamma_B = -2g^2m^2\int d^7z\,d^7z'\,G_{+}\big(z,z'\big)G_{-}\big(z,z'\big)G_0\big(z,z'\big), \tag{C.2}$$

where g^2 is the gauge coupling constant and $G_0(z,z')$ is the gauge superfield propagator,

$$G_0\big(z,z'\big) = \frac{1}{\Box}\delta^7\big(z-z'\big) = i\int_0^\infty \frac{ds}{(4\pi is)^{3/2}}e^{\frac{i\xi^2}{4s}}\zeta^2\bar\zeta^2. \tag{C.3}$$

Using this propagator and the heat kernels (B.10)–(B.13), the two-loop contributions (C.1) and (C.2) to the effective action can be recast as

$$\Gamma_{\rm A} = 2ig^2 \int d^7z\, d^3\xi \int_0^\infty \frac{ds\,dt\,du}{(4i\pi u)^{3/2}} e^{i(s+t)m^2} e^{\frac{i\xi^2}{4u}} K_{+-}\big(z,z'\big|s\big) K_{+-}\big(z',z\big|t\big)\big|, \tag{C.4}$$

$$\Gamma_{\rm B} = 2ig^2m^2 \int d^7z\, d^3\xi \int_0^\infty \frac{ds\,dt\,du}{(4i\pi u)^{3/2}} e^{i(s+t)m^2} e^{\frac{i\xi^2}{4u}} K_{+}\big(z,z'\big|s\big) K_{-}\big(z',z\big|t\big)\big|. \tag{C.5}$$

Consider first the details of computations of (C.4).

For studying the low-energy effective action up to the four-derivative order, it is sufficient to consider the heat kernel K_{+-} in the approximation (2.57),

$$\Gamma_{\rm A} \approx \frac{2ig^2}{(4i\pi)^{9/2}} \int d^7z\, d^3\xi \int_0^\infty \frac{ds\,dt\,du}{(stu)^{3/2}} e^{i(s+t)(m^2+G^2)} e^{\frac{i\xi^2}{4}(\frac{1}{s}+\frac{1}{t}+\frac{1}{u})} e^{X(\xi^m,s)+X(-\xi^m,t)}. \tag{C.6}$$

Using the explicit form of the function $X(\xi^m,s)$ in (2.58), we expand $e^{X(\xi^m,s)+X(-\xi^m,t)}$ in a series up to the first order in $N_{\alpha\beta}$,

$$\begin{aligned} e^{X(\xi^m,s)+X(-\xi^m,t)} = {} & 1 + i\big(s^2+t^2\big)GW^\alpha \bar W_\alpha + \frac{i}{3}\big(s^3+t^3\big)GW^\alpha N_{\alpha\beta}\bar W^\beta \\ & - \frac{s-t}{2}\xi_m\gamma^m_{\alpha\beta}W^\alpha\bar W^\beta + \frac{1}{2}\big(s^2-t^2\big)\xi_m\big(\gamma^m N\big)W^\alpha\bar W_\alpha \\ & + \frac{3}{2}\big(s^2-t^2\big)\xi_m\gamma^m_{\gamma(\alpha}N^\gamma_{\beta)}W^\alpha\bar W^\beta - \frac{7i}{24}\big(s^3+t^3\big)W^2\bar W^2 \\ & + \frac{1}{4}G^2\big(s^2+t^2\big)^2 W^2\bar W^2 - \frac{(s-t)^2}{16}\xi^m\xi_m W^2\bar W^2. \end{aligned} \tag{C.7}$$

Note that some of the terms in (C.7) give no contributions to (C.6). Indeed, the term with $GW^\alpha\bar W_\alpha$ in the r.h.s. of (C.7) gives vanishing contribution for considered gauge superfield background because of (2.23). The terms in (C.7) linear with respect to ξ_m also give vanishing contribution after integration over $d^3\xi$ because of (2.61). For the remaining terms in (C.7) we have

$$\begin{aligned} \Gamma_{\rm A} \approx {} & \frac{2ig^2}{(4i\pi)^{9/2}} \int d^7z\, d^3\xi \int_0^\infty \frac{ds\,dt\,du}{(stu)^{3/2}} e^{i(s+t)(m^2+G^2)} e^{\frac{i\xi^2}{4}(\frac{1}{s}+\frac{1}{t}+\frac{1}{u})} \\ & \times \bigg\{1 + \frac{i}{3}\big(s^3+t^3\big)GW^\alpha N_{\alpha\beta}\bar W^\beta \\ & + \frac{W^2\bar W^2}{4}\bigg[G^2\big(s^2+t^2\big)^2 - \frac{7i}{6}\big(s^3+t^3\big) - \frac{(s-t)^2}{4}\xi^2\bigg]\bigg\}. \end{aligned} \tag{C.8}$$

The integration over $d^3\xi$ is done using (2.62) and

$$\int d^3\xi\, \xi^2 e^{\frac{i}{4}a\xi^2} = -\frac{3}{2\pi}\left(\frac{4i\pi}{a}\right)^{5/2}. \tag{C.9}$$

Then the expression (C.8) reads

$$\Gamma_{\rm A} \approx \frac{g^2}{32\pi^3}\int d^7z\int_0^\infty \frac{ds\,dt\,du}{(st+su+tu)^{3/2}}e^{i(s+t)(m^2+G^2)}\bigg\{1+\frac{i}{3}\big(s^3+t^3\big)GW^\alpha N_{\alpha\beta}\bar W^\beta + \frac{W^2\bar W^2}{4}\bigg[G^2\big(s^2+t^2\big)^2-\frac{7i}{6}\big(s^3+t^3\big)-\frac{3i}{2}\frac{(s-t)^2stu}{st+su+tu}\bigg]\bigg\}. \tag{C.10}$$

After integration over du we find

$$\Gamma_{\rm A} \approx \frac{g^2}{16\pi^3}\int d^7z\int_0^\infty \frac{ds\,dt}{\sqrt{st}(s+t)}e^{i(s+t)(m^2+G^2)}\bigg\{1+\frac{i}{3}\big(s^3+t^3\big)GW^\alpha N_{\alpha\beta}\bar W^\beta + \frac{W^2\bar W^2}{4}\bigg[G^2\big(s^2+t^2\big)^2-\frac{7i}{6}\big(s^3+t^3\big)-i\frac{(s-t)^2st}{s+t}\bigg]\bigg\}. \tag{C.11}$$

The remaining integrations over s and t can be done with the use of the following formulas:

$$\int_0^\infty \frac{ds\,dt}{\sqrt{st}(s+t)}e^{i(s+t)(G^2+m^2)} = -\pi\ln\big(G^2+m^2\big), \tag{C.12}$$

$$\int_0^\infty \frac{ds\,dt}{\sqrt{st}(s+t)}\big(s^3+t^3\big)e^{i(s+t)(G^2+m^2)} = -\frac{5i\pi}{4(G^2+m^2)^3}, \tag{C.13}$$

$$\int_0^\infty \frac{ds\,dt}{\sqrt{st}(s+t)}\big(s^2+t^2\big)^2e^{i(s+t)(G^2+m^2)} = \frac{57\pi}{16(G^2+m^2)^4}, \tag{C.14}$$

$$\int_0^\infty \frac{ds\,dt}{(s+t)^2}\sqrt{st}(s-t)^2e^{i(s+t)(G^2+m^2)} = -\frac{i\pi}{16(G^2+m^2)^3}. \tag{C.15}$$

As a result, we get

$$\Gamma_{\rm A} \approx \frac{g^2}{16\pi^2}\int d^7z\bigg[-\ln\big(G^2+m^2\big)+\frac{5}{12}\frac{N_{\alpha\beta}W^\alpha\bar W^\beta}{(G^2+m^2)^3} + \frac{W^2\bar W^2}{96}\bigg(\frac{49G^2}{(G^2+m^2)^4}-\frac{73}{2}\frac{m^2}{(G^2+m^2)^4}\bigg)\bigg]. \tag{C.16}$$

For computing the part of the effective action $\Gamma_{\rm B}$ up to the four-derivative order it is sufficient to approximate the heat kernel K_+ in (B.26) as

$$K_+\big(z,z'\big|s\big)\big| \approx \frac{1}{(4i\pi s)^{3/2}}s^2W^2e^{isG^2}e^{\frac{i\xi^2}{4s}}. \tag{C.17}$$

Substituting (C.17) into (C.2) and computing the integrals over $d^3\xi$ and du with the help of (2.62) one finds

$$\Gamma_{\rm B} \approx \frac{g^2m^2}{16\pi^3}\int d^7z\,W^2\bar W^2\int_0^\infty \frac{ds\,dt}{s+t}(st)^{3/2}e^{i(s+t)(G^2+m^2)}. \tag{C.18}$$

The integral over the remaining parameters reads

$$\int_0^\infty \frac{ds\,dt}{s+t}(st)^{3/2}e^{i(s+t)(G^2+m^2)} = \frac{9\pi}{64}\frac{1}{(G^2+m^2)^4}. \tag{C.19}$$

As a result,

$$\Gamma_{\mathrm{B}} \approx \frac{9g^2m^2}{1024\pi^2}\int d^7z\,\frac{W^2\bar{W}^2}{(G^2+m^2)^4}. \tag{C.20}$$

The four-derivative two-loop effective action is given by the sum of (C.16) and (C.20). It can be represented in the form (2.22) with the functions $f_i^{(2)}$ given by

$$f_1^{(2)} = -\frac{g^2}{16\pi^2}\ln\bigl(G^2+m^2\bigr), \tag{C.21}$$

$$f_2^{(2)} = \frac{5g^2}{192\pi^2}\frac{G}{(G^2+m^2)^3}, \tag{C.22}$$

$$f_3^{(2)} = \frac{g^2}{\pi^2}\frac{98G^2-73m^2}{3072(G^2+m^2)^4}. \tag{C.23}$$

In Section 2.6 we denote these functions as $\tilde{f}_i^{(2)}$ to distinguish them form the similar functions in the $\mathcal{N}=2$ Chern–Simons electrodynamics.

References

[1] J. Bagger, N. Lambert, Modeling multiple M2's, Phys. Rev. D 75 (2007) 04502, arXiv:hep-th/0611108.
[2] J. Bagger, N. Lambert, Gauge symmetry and supersymmetry of multiple M2-branes, Phys. Rev. D 77 (2008) 065008, arXiv:0711.0955 [hep-th].
[3] J. Bagger, N. Lambert, Comments on multiple M2-branes, J. High Energy Phys. 0802 (2008) 105, arXiv:0712.3738 [hep-th].
[4] J. Bagger, N. Lambert, Three-algebras and $\mathcal{N}=6$ Chern–Simons gauge theories, Phys. Rev. D 79 (2009) 025002, arXiv:0807.0163 [hep-th].
[5] A. Gustavsson, Algebraic structures on parallel M2-branes, Nucl. Phys. B 811 (2009) 66, arXiv:0709.1260 [hep-th].
[6] A. Gustavsson, Selfdual strings and loop space Nahm equations, J. High Energy Phys. 0804 (2008) 083, arXiv:0802.3456 [hep-th].
[7] O. Aharony, O. Bergman, D.L. Jafferis, J. Maldacena, $\mathcal{N}=6$ superconformal Chern–Simons-matter theories, M2-branes and their gravity duals, J. High Energy Phys. 0810 (2008) 091, arXiv:0806.1218 [hep-th].
[8] J.H. Schwarz, Highly effective actions, arXiv:1311.0305 [hep-th].
[9] I.L. Buchbinder, B.S. Merzlikin, I.B. Samsonov, Two-loop low-energy effective actions in $N=2$ and $N=4$ three-dimensional SQED, J. High Energy Phys. 1307 (2013) 012, arXiv:1305.4815 [hep-th].
[10] I.L. Buchbinder, N.G. Pletnev, I.B. Samsonov, Effective action of three-dimensional extended supersymmetric matter on gauge superfield background, J. High Energy Phys. 1004 (2010) 124, arXiv:1003.4806 [hep-th].
[11] I.L. Buchbinder, N.G. Pletnev, I.B. Samsonov, Low-energy effective actions in three-dimensional extended SYM theories, J. High Energy Phys. 1101 (2011) 121, arXiv:1010.4967 [hep-th].
[12] I.L. Buchbinder, N.G. Pletnev, I.B. Samsonov, Background field formalism and construction of effective action for $N=2$, $d=3$ supersymmetric gauge theories, Phys. Part. Nucl. 44 (2013) 234, arXiv:1206.5711 [hep-th].
[13] J. de Boer, K. Hori, Y. Oz, Dynamics of $N=2$ supersymmetric gauge theories in three dimensions, Nucl. Phys. B 500 (1997) 163, arXiv:hep-th/9703100.
[14] O. Aharony, A. Hanany, K.A. Intriligator, N. Seiberg, M.J. Strassler, Aspects of $N=2$ supersymmetric gauge theories in three-dimensions, Nucl. Phys. B 499 (1997) 67, arXiv:hep-th/9703110.
[15] S. Weinberg, The Quantum Theory of Fields, vol. II, Sections 17.4, 17.5, Cambridge University Press, 1996.
[16] G.A. Vilkovisky, The unique effective action in quantum field theory, Nucl. Phys. B 234 (1984) 125.
[17] I.L. Buchbinder, N.G. Pletnev, The background field method for $N=2$, d3 super Chern–Simons-matter theories, J. High Energy Phys. 1111 (2011) 085, arXiv:1108.2966 [hep-th].

[18] L.V. Avdeev, G.V. Grigorev, D.I. Kazakov, Renormalizations in Abelian Chern–Simons field theories with matter, Nucl. Phys. B 382 (1992) 561.
[19] L.V. Avdeev, D.I. Kazakov, I.N. Kondrashuk, Renormalizations in supersymmetric and nonsupersymmetric non-abelian Chern–Simons field theories with matter, Nucl. Phys. B 391 (1993) 333.
[20] S.J. Gates Jr., H. Nishino, Remarks on $N = 2$ supersymmetric Chern–Simons theories, Phys. Lett. B 281 (1992) 72.
[21] N.J. Hitchin, A. Karlhede, U. Lindstrom, M. Rocek, Hyperkahler metrics and supersymmetry, Commun. Math. Phys. 108 (1987) 535.
[22] I.L. Buchbinder, S.M. Kuzenko, Ideas and methods of supersymmetry and supergravity or a walk through superspace, IOP, Bristol, UK, 1998, 656 p.
[23] J.-H. Park, Superconformal symmetry in three-dimensions, J. Math. Phys. 41 (2000) 7129, arXiv:hep-th/9910199.
[24] I.L. Buchbinder, S.M. Kuzenko, A.A. Tseytlin, On low-energy effective actions in $N = 2$, $N = 4$ superconformal theories in four-dimensions, Phys. Rev. D 62 (2000) 045001, arXiv:hep-th/9911221.
[25] A.J. Niemi, G.W. Semenoff, Axial anomaly induced fermion fractionization and effective gauge theory actions in odd dimensional space-times, Phys. Rev. Lett. 51 (1983) 2077.
[26] A.N. Redlich, Gauge noninvariance and parity violation of three-dimensional fermions, Phys. Rev. Lett. 52 (1984) 18.
[27] A.N. Redlich, Parity violation and gauge noninvariance of the effective gauge field action in three dimensions, Phys. Rev. D 29 (1984) 2366.
[28] B.M. Zupnik, D.V. Khetselius, Three-dimensional extended supersymmetry in harmonic superspace, Yad. Fiz. 47 (1988) 1147, Sov. J. Nucl. Phys. 47 (1988) 730.
[29] B.M. Zupnik, Harmonic superspaces for three-dimensional theories, in: J. Wess, E. Ivanov (Eds.), Supersymmetries and Quantum Symmetries, in: Lect. Notes Phys., vol. 524, Springer, 1998, p. 116, arXiv:hep-th/9804167.
[30] I.L. Buchbinder, E.A. Ivanov, O. Lechtenfeld, N.G. Pletnev, I.B. Samsonov, B.M. Zupnik, Quantum $N = 3$, $d = 3$ Chern–Simons matter theories in harmonic superspace, J. High Energy Phys. 0910 (2009) 075, arXiv:0909.2970 [hep-th].
[31] A.C. Lehum, Dynamical generation of mass in the $D = (2 + 1)$ Wess–Zumino model, Phys. Rev. D 77 (2008) 067701, arXiv:0801.0411 [hep-th].
[32] A.F. Ferrari, M. Gomes, A.C. Lehum, J.R. Nascimento, A.Yu. Petrov, E.O. Silva, A.J. da Silva, On the superfield effective potential in three dimensions, Phys. Lett. B 678 (2009) 500, arXiv:0901.0679 [hep-th].
[33] A.C. Lehum, $D = (2 + 1)$ O(N) Wess–Zumino model in a large N limit, Phys. Rev. D 84 (2011) 107701, arXiv:1110.3332 [hep-th].
[34] M. Gomes, A.C. Lehum, J.R. Nascimento, A.Yu. Petrov, A.J. da Silva, On the effective superpotential in the supersymmetric Chern–Simons theory with matter, Phys. Rev. D 87 (2013) 027701, arXiv:1210.6863 [hep-th].
[35] I.L. Buchbinder, B.S. Merzlikin, I.B. Samsonov, Two-loop effective potentials in general $N = 2$, $d = 3$ chiral superfield model, Nucl. Phys. B 860 (2012) 87, arXiv:1201.5579 [hep-th].
[36] P.-N. Tan, B. Tekin, Y. Hosotani, Spontaneous symmetry breaking at two loop in 3-D massless scalar electrodynamics, Phys. Lett. B 388 (1996) 611, arXiv:hep-th/9607233.
[37] P.-N. Tan, B. Tekin, Y. Hosotani, Maxwell Chern–Simons scalar electrodynamics at two loop, Nucl. Phys. B 502 (1997) 483, arXiv:hep-th/9703121.
[38] I.L. Buchbinder, E.A. Ivanov, O. Lechtenfeld, N.G. Pletnev, I.B. Samsonov, B.M. Zupnik, ABJM models in $N = 3$ harmonic superspace, J. High Energy Phys. 0903 (2009) 096, arXiv:0811.4774 [hep-th].
[39] S.M. Kuzenko, I.N. McArthur, On the background field method beyond one loop: A manifestly covariant derivative expansion in super Yang–Mills theories, J. High Energy Phys. 0305 (2003) 015, arXiv:hep-th/0302205.
[40] S.M. Kuzenko, I.N. McArthur, Low-energy dynamics in $N = 2$ super QED: Two loop approximation, J. High Energy Phys. 0310 (2003) 029, arXiv:hep-th/0308136.
[41] S.M. Kuzenko, S.J. Tyler, Supersymmetric Euler–Heisenberg effective action: Two-loop results, J. High Energy Phys. 0705 (2007) 081, arXiv:hep-th/0703269.
[42] S.M. Kuzenko, I.N. McArthur, Effective action of beta-deformed $N = 4$ SYM theory: Farewell to two-loop BPS diagrams, Nucl. Phys. B 778 (2007) 159, arXiv:hep-th/0703126 [hep-th].
[43] S.M. Kuzenko, I.N. McArthur, Relaxed superselfduality and $N = 4$ SYM at two loops, Nucl. Phys. B 697 (2004) 89, arXiv:hep-th/0403240.
[44] S.M. Kuzenko, I.N. McArthur, Relaxed superselfduality and effective action, Phys. Lett. B 591 (2004) 304, arXiv:hep-th/0403082.
[45] S.M. Kuzenko, I.N. McArthur, On the two loop four derivative quantum corrections in 4-D $N = 2$ superconformal field theories, Nucl. Phys. B 683 (2004) 3, arXiv:hep-th/0310025.