Applications of Front Tracking to the Simulation of Resin Transfer Molding

Y. SONG
Department of Mathematics and Computer Science
Indiana University South Bend, South Bend, IN 46634, U.S.A.

W. CHUI, J. GLIMM, B. LINDQUIST AND F. TANGERMAN
Department of Applied Mathematics and Statistics
State University of New York at Stony Brook
Stony Brook, NY 11794-3600, U.S.A.

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Abstract—Resin Transfer Molding, as a method for the manufacture of advanced fiber reinforced composite materials, is attractive because it offers the possibility of lower manufacturing costs and more complex shapes than the traditional manufacturing methods. A major issue in this new manufacturing process is the elimination of void spaces in the resin fill operation, so that products with high quality are manufactured. In this paper, we present a two-phase, two-component air solubility model to study the formation and migration of the macro and micro-voids. The numerical solutions of the model are obtained through a front tracking code. The front tracking method has the distinguishing feature of preserving sharp interfaces throughout the simulation. The results demonstrate that the model proposed here has desirable qualitative agreement with experimental results. Based on these results, we make numerical predictions to show how modeling could improve the manufacturing process and, hence, enhance product quality.

Keywords—Resin transfer molding, Composite materials, Front tracking, Mathematical modeling, Porous media flow.

1. INTRODUCTION

Resin Transfer Molding (RTM) is a process for the manufacture of fiber-reinforced composites. The resulting light weight, high strength parts are attractive for many applications. Examples are consumer items such as chairs, automobile parts, and aircraft components. RTM is of particular interest to the aerospace industry, since it promises cost savings and performance improvements over traditional hand lay-up methods. In the RTM process, dry fiber reinforcement, or fiber preform, is packed into a mold cavity which has the shape of the desired part. The mold is then closed and resin is injected under pressure into the mold where it impregnates the preform. After the fill cycle, the cure cycle begins, during which the mold is heated and resin polymerizes to become rigid plastic.

Our work concentrates on the fill process, which produces air bubbles in the part. Bubbles, formed as the moving resin front entrap air, reduce the strength of the part and, important...
for some applications, degrade surface quality. The fiber preforms are mats, often woven, and are heterogeneous in nature. They have variations of porosity and permeability and consist of several distinct pore structures whose length scales vary by orders of magnitude. This hierarchical structure is responsible for the formation of air bubbles on each length scale. It is therefore important to have an understanding of air bubble formation and migration during the fill cycle and to develop strategies for void content reduction in the finished part. Process modeling is particularly useful in understanding, designing, and optimizing the process conditions to achieve this goal.

The RTM fill process is a typical example of porous media flow, which can be modeled by a set of partial differential equations, together with an appropriate set of boundary conditions, which should adequately describe the significant physical processes within the system. The numerical technique which we use to solve this system is the front tracking method. Front tracking is a numerical method designed to provide enhanced resolution computations of a set of distinguished waves in solutions to systems of hyperbolic partial differential equations. Front tracking can be applied to any problem for which the velocity of a tracked wave can be determined at each point on that wave. It has been used in a number of different applications, including shock wave interactions in one- and two-dimensional flows, gravity-driven chaotic mixing of two fluids, two-phase flow in porous media, and the propagation of elastic and plastic waves in solids. The current two-space-dimensional version of front tracking, similar to that originally proposed by Richtmyer and Morton [1] in the middle 1960s, has been highly developed by a group of researchers at New York University and the University at Stony Brook. For the details of the development and the structural aspects, as well as the applications, of the front tracking code, see [2-5].

In Section 2, an overview of the RTM manufacturing process is given. The details of the multiscale fiber preform structure are described. Most of the modeling work in the RTM field [6-18] has been directed at the macroscopic flow behavior at large length scales through one-phase flow models. Some work [19,20] made use of two-phase flow models. In Section 3, we present a simple two-phase flow model developed in [21,22], which is sufficient for the modeling of the formation and migration of the macro-voids. The key part of the work [22] is the relative permeability model to determine the relationship between the relative permeabilities and other process parameters. A pressure dependent residual air saturation acts as the threshold for macro-void mobilization. Based on the relative permeability model, we will describe a two-phase, two-component air solubility model to study the formation and migration of the macro-voids. The major improvement for our air solubility model as distinguished from the two-phase model [22] is that the effect of air dissolved in resin can be modeled accurately. The air solubility model is also known as “black oil” in petroleum reservoir simulations. The front tracking algorithm for this “gas-oil” systems will be presented in Section 4. Numerical experiments and predictions are presented in Section 5, which demonstrate qualitative agreement with experimental results. The air solubility model is able to capture the effect of post-fill pressure cycles to reduce voids which the two-phase model cannot handle correctly.

2. MANUFACTURING PROCESS

The filling of the mold packed with fiber preform by the liquid resin is a typical example of flow through a porous medium. Other applications of porous media flow include ground water hydrology and petroleum reservoir simulation. As with other porous media flow problems, the resin filling process involves the interaction between the liquid phase (resin) and the porous medium (fiber preform) at a number of different length scales. The preforms of interest for aerospace applications are constructed from tows (yarn) of fiberglass or graphite that contain hundreds to thousands of filaments. These fiber filaments are intertwined and twisted into fiber tows, and the fiber tows are woven into fiber mats. The fiber mats are pressed in layers in the mold form. This structure promotes the development of a heterogeneous flow field that can
result in voids or air entrapment in the interstices of the fiber structure. To understand this heterogeneous flow behavior, it is therefore useful to consider the resin-fiber interaction on four distinct length scales, namely, the length scales of the mold, the mat, the tow, and the filament.

At the length scale of the mold, variations in local permeability, inappropriate inlet and outlet ports locations, or complicated geometry of the parts can result in nonuniform flow fronts and potentially large areas of dry preform after the fill process is complete. Such dry spots usually are unacceptable in products. Such dry spots can be reasonably well predicted based on macroscopic flow models [9,10].

Variation in permeability can also occur at the scale of the fiber mat. When the mats are pressed into the mold, they may be compressed, bent, or stretched. As a result the contact between the layers of fiber mats or that with the wall of the mold may not be uniform and result in variation of local permeability. The resin will flow rapidly in the high permeability regions, i.e., near the mold walls or between the fiber mats, and facilitates the formation of dry spots [13,23].

The third level of interaction occurs at the length scale of the fiber tow. At this length scale, the main issue is to study the flow properties of resin moving between and around the fiber tows. Heterogeneities at this length scale are responsible for the formation of macro-voids, or macro-bubbles, in the interstices of the tow structure [15,18,24]. These voids, spherical in shape, are trapped between fiber tows or across many fiber filaments and are known to reduce the mechanical strength of the finished part [25].

The next level of interaction occurs at the length scale of the fiber filaments. At this length scale the flow properties of resin flowing inside individual fiber tows and between fiber filaments are of main interest. In the case of fast filling, resin flows much faster in the pore space between or around the fiber tows than in the small interstitial space between the fiber filaments inside individual fiber tows. This often results in trapping of small cylindrical air bubbles, or micro-voids, inside the fiber tows after the tows are bypassed by the surrounding resin [20,22].

3. THE SIMPLE TWO-PHASE FLOW MODEL

3.1. Model Equations

Two-phase flow models take into account the interaction among resin, air, and fiber during the mold fill process, where the resin displaces the air. In our initial two-phase flow model of this process, we assume that there is no mass transfer between the two fluids, that capillary and gravitational effects can be ignored and that both phases are incompressible. We furthermore assume that all variables are constant in the thickness direction, and consider a two dimensional model. We combine the flow equations with mass conservation principles and Darcy's law [26-28]. Following the standard porous media flow notations, we let \( S_r \) and \( S_a \) denote the saturation of the resin phase and the air phase relative to the available pore space. We will use the subscripts \( r \) and \( a \) to represent the resin and air phase, respectively. Since the two phases fill the available pore space

\[ S_r + S_a = 1. \]  

(1)

Mass conservation for the resin and air phases is described by

\[ \phi \partial_t S_r + \nabla \cdot q_r = 0, \]  

(2)

\[ \phi \partial_t S_a + \nabla \cdot q_a = 0. \]  

(3)

In these equations, \( \phi \) denotes the fiber volume fraction, also known as preform porosity, and \( q_r \) and \( q_a \) denote the phase volumetric velocities. We assume that these volumetric velocities are described by Darcy's law,

\[ q_r = -\frac{k_{rel, r}}{\mu_r} K \nabla P, \]  

(4)

\[ q_a = -\frac{k_{rel, a}}{\mu_a} K \nabla P, \]  

(5)
where $\mu_r$ and $\mu_a$ denote the phase viscosities, $k_{rel,r}$ and $k_{rel,a}$ denote relative phase permeabilities, $P$ denotes the pressure, and $K$ is the absolute preform permeability.

To solve this system numerically through, one rewrites the model equations by introducing the total volumetric velocity

$$q = q_r + q_a,$$

and the fractional flow function of resin

$$f_r = \frac{k_{rel,r}/\mu_r}{(k_{rel,r}/\mu_r) + (k_{rel,a}/\mu_a)}.$$  

The resulting system of equations is

$$\frac{\partial}{\partial t} S_r + \nabla \cdot (f_r q) = 0,$$

$$\nabla \cdot q = 0,$$

$$q = -\left( \frac{k_{rel,r}}{\mu_r} + \frac{k_{rel,a}}{\mu_a} \right) K \nabla P.$$

This system is hyperbolic in the saturation variable and elliptic in the pressure and is known as the Buckley-Leverett system. We solve this system numerically using Front Tracking [2,3], a method which preserves sharp interfaces throughout the simulation. The model equations imply that at any time there will be a sharp discontinuity, i.e., a shock, in the resin saturation at the resin front. The resin saturation ahead of this front is zero, while the resin saturation immediately behind the resin front is determined by conservation of resin volume [26]. The resin saturation behind the front increases continuously to 1 as the distance from the front increases, i.e., a rarefaction wave is formed behind the front.

Appropriate initial conditions are initial resin and air saturations. For the boundary conditions, a Dirichlet boundary condition (pressure held constant) is imposed at the inlet and outlet ports, whereas Neumann boundary condition (no flow) is assumed at the other boundaries. The fraction of resin (normally $S_r = 1$) at the injection port is an additional boundary condition.

### 3.2. Relative Permeability Model

For successful modeling of the formation and migration of the macro-voids, the modeling of the relative permeabilities $k_{rel,r}$ and $k_{rel,a}$, which take values between 0 and 1, is critical. These relative permeabilities depend strongly on the saturations $S_r$ and $S_a$. The functional dependency between the relative permeabilities and the saturations, however, are not known for general porous media. Experimental and modeling efforts are therefore necessary to determine the relationship of these parameters to measurable process conditions. A relative permeability model to determine the relationship between the relative permeabilities and other process parameters is described in [22].

In the RTM process, resin saturations are typically high, i.e., greater than 90%, and vary over only a small range. A sensible assessment for the resin phase relative permeability $k_{rel,r}$ is that it is close to one in this range.

To model $k_{rel,a}$, we assume that macro-bubbles are created during mold filling at the front at outlet pressure $P_{out}$. Assume also that the macro-bubbles have, at origination, a constant volume $V_o$, which is determined by the preform geometry, but which is independent of outlet pressure $P_{out}$. If the bubble is large enough, it will stay in the same place but shrink in size as the resin pressure increases. In this simple model, it is assumed that the bubble becomes mobile when its size decreases to a critical volume $V_c$, which is a resin/air/fiber-preform property and independent of pressure. Assuming that surface tension can be ignored, so that the pressure in the bubble equals the pressure in the surrounding resin, and that the air in the macro-bubble is an ideal gas, we obtain for a bubble at position $x$ and time $t$:

$$P(x,t) V(x,t) = P_{out} V_o,$$

(11)
where \( P(x, t) \) and \( V(x, t) \) denote the local pressure and bubble volume, respectively. Therefore, there is a critical pressure \( P_c = P_{\text{out}}(V_o/V_c) \) required for the bubble to become mobile. In particular, this critical pressure is linear in the outlet pressure. It follows that there is an immobile zone behind the resin front in which the bubbles have not yet moved. At the boundary of this zone, in the resin-rich region, the pressure equals the critical pressure.

The physical model implies that when the mold is filled and the resin is cured at a constant pressure \( P_{\text{cure}} \), the bubbles have not moved in a primary zone behind the front. Therefore, the void fraction in this zone will be constant. However, since the bubbles are compressed by the cure pressure (typically different from the outlet pressure), the final void fraction in this zone is then proportional to the outlet pressure. The length \( l \) of this primary zone satisfies the equation

\[
\frac{l}{L} = \frac{P_{\text{out}}}{|P|} \left( \frac{V_o}{V_c} - 1 \right),
\]

where \( L \) denotes the mold length and \( |P| \) denotes the pressure drop \( P_{\text{in}} - P_{\text{out}} \), and is proportional to the outlet pressure \( P_{\text{out}} \) and inversely proportional to the pressure drop \( |P| \). The after-cure void fraction in the primary zone is proportional to \( (P_{\text{out}}/P_{\text{cure}})V_o \) and is linear in the outlet pressure.

These considerations are key to the modeling of \( k_{\text{rel}, a} \) and its functional dependencies. We propose a residual air saturation \( S_{a, \text{resid}}(P) \), so that the mobility of the air phase is determined by \( S_a - S_{a, \text{resid}}(P) \). \( S_{a, \text{resid}}(P) \) can be thought of as the saturation of the air located in the immobile bubbles. Assuming that \( S_{a, \text{resid}} \) is a function of the rescaled pressure, \( \bar{P} = P/P_{\text{out}} \), the reduced saturation is defined as

\[
S_{a, \text{red}} = \frac{S_a - S_{a, \text{resid}}(\bar{P})}{1 - S_{a, \text{resid}}(\bar{P})}.
\]

We now assume that \( k_{\text{rel}, a} \) depends only on air saturation and rescaled pressure:

\[
k_{\text{rel}, a} = k_{\text{rel}, a}(S_a, \bar{P}) = k_{\text{rel}, a}(S_{a, \text{red}}).
\]

Standard choices for \( k_{\text{rel}, a} \) are either linear (\( k_{\text{rel}, a} = S_{a, \text{red}} \)) or quadratic (\( k_{\text{rel}, a} = S_{a, \text{red}}^2 \)) in the reduced saturation. Equation (11) asserts that \( k_{\text{rel}, a} \) depends on the reduced saturation alone and not on additional process conditions, such as pressure drop and outlet pressure. The residual air saturation \( S_{a, \text{resid}}(\bar{P}) \) is relatively easy to measure from steady-state conditions or post-mortem analyses of the produced part, see [22] for the details.

4. THE AIR SOLUBILITY MODEL

4.1. Model Equations

Micro-voids are the air bubbles trapped within the fiber tow. They are thin and cylindrical in shape, oriented parallel to the tow direction. Since the intra-tow permeability is very low, the flow velocity is low and insufficient to dislodge these bubbles. Air has some degree of solubility in the resin, and this solubility will in general be pressure dependent. As the local pressure increases, the macro-bubbles within the tow dissolve gradually into the slowly moving resin surrounding them inside the tow, and this air-rich resin then flows into the inter-tow space and eventually to the exit ports. To model this mass transfer between phases, a component model is needed [28]. The component model also offers improved modeling of the micro-voids, due to the inclusion of solubility effects. Because there are only two fluid phases, and only one component is involved in the mass transfer, the black-oil model [26–30] developed for petroleum reservoir and specialized to an oil-gas system is sufficient for the RTM process.
In the following, we present the mathematical formulation of a two-phase, two-component air solubility model which takes into account the mass transfer between different phases. Let the two phases be liquid and gaseous, denoted by lower-case subscripts $r$ and $a$ and the two components be resin and air, denoted by capital letter subscripts $R$ and $A$. We assume that air may dissolve in the liquid phase, and that resin does not evaporate into gaseous phase. Thus, the gaseous phase contains only the air component, while the liquid phase contains both resin and air components.

Several dimensionless numbers need to be defined. Air solubility, $R_{so}$ (also called dissolved air-resin ratio), is defined as the volume of air (measured at standard conditions) dissolved at given pressure $P$ in a unit volume of resin component, i.e., $R_{so}(P) = V_{AS}/V_{RS}$, where capital subscript $S$ is used to indicate the standard condition. The resin volume factor $B_r$ is defined as the ratio of the volume of resin plus its dissolved air (measured at given $P$) to the volume of the resin component measured at standard conditions, i.e., $B_r(P) = V_r(P)/V_{RS}$. Furthermore, the air volume factor $B_a$ is defined as the ratio of the volume of free air (all of which is air component), measured at given $P$, to the volume of the same air measured at standard conditions, i.e., $B_a(P) = V_a(P)/V_{AS}$. Note that the temperature $T$ is assumed constant throughout the calculation, otherwise, these ratios are also functions of temperature.

The equations that describe the mass conservation of each component are given by

$$\phi \partial_t \left( \frac{1}{B_r} S_r \right) + \nabla \cdot \left( \frac{1}{B_r} q_r \right) = 0, \quad (15)$$

$$\phi \partial_t \left( \frac{R_{so}}{B_r} S_r + \frac{1}{B_a} S_a \right) + \nabla \cdot \left( \frac{R_{so}}{B_r} q_r + \frac{1}{B_a} q_a \right) = 0, \quad (16)$$

where $\phi$ denotes preform porosity, $q_r$ and $q_a$ denote the phase volumetric velocities, and $S_r$ and $S_a$ denote the phase saturations, with $0 \leq S_r \leq 1$, $S_a \leq 1$, and

$$S_r + S_a \leq 1. \quad (17)$$

The inequality sign arises only for undersaturated resin case represented by $S_a < 0$, which will be discussed in detail in the next section. The phase velocities are given by Darcy's law

$$q_r = - \frac{k_{rel,r}}{\mu_r} K \nabla P, \quad (18)$$

$$q_a = - \frac{k_{rel,a}}{\mu_a} K \nabla P, \quad (19)$$

where $\mu_r, \mu_a, k_{rel,r}, k_{rel,a}, P,$ and $K$ are defined in Section 3. As a physical modeling assumption, we consider incompressible resin, with no solubility volume swelling, so that $B_r = 1$, while $B_a$ and $R_{so}$ have significant dependence on $P$.

4.2. Numerical Methods

We propose a sequential method [23,26,27], and first rewrite the system in the form similar to (8) and (9). We will see that their mathematical structure is different from the previous two-phase flow models, since it is parabolic, rather than elliptic, in the pressure variable.

Besides introducing the total volumetric velocity $q = q_r + q_a$ and the fractional flow function of air $f_a$, we also introduce the total mass density for the air component

$$U_a = U_b + S_a (U_{max} - U_b), \quad (20)$$

with

$$U_b = \rho_{AS} R_{so}, \quad (21)$$

$$U_{max} = \frac{\rho_{AS}}{B_a}, \quad (22)$$
where $U_b$ is the bubble point mass density, i.e., the maximum mass density of air which can be dissolved in the resin at given $P$, $U_{\text{max}}$ is the maximum mass density the gas can achieve, i.e., the mass density of the gas phase at the given $P$, and $\rho_{AS}$ is the density of the air component measured at standard conditions. Since $B_r = 1$ and eliminating the $\partial_t S_a$ and $\partial_t S_r$ terms in (15) and (16), the resulting system of equations is

$$
\alpha \frac{\partial P}{\partial t} + \beta \cdot \nabla P + \nabla \cdot q = 0, 
$$
(23)

$$
\phi \partial_t U_a + \nabla \cdot (f_a q) = 0,
$$
(24)

and where, as before $f_a$ denotes the fractional flow function for the air component. The parameters $\alpha$ and $\beta$ are defined as

$$
\alpha = c_a S_a + c_s S_r,
$$
(25)

$$
\beta = c_a q_a + c_s q_r,
$$
(26)

where

$$
c_a = -\frac{1}{B_a} \frac{dB_a}{dP},
$$
(27)

$$
c_s = B_a \frac{dR_{\infty}}{dP},
$$
(28)

$$
S_a = \frac{U_a - U_b}{U_{\text{max}} - U_b},
$$
(29)

$$
S_r = \frac{U_{\text{max}} - U_a}{U_{\text{max}} - U_b},
$$
(30)

Although the air solubility model involves two components, this does not necessarily mean that both of the two phases are always present. For example, if air is allowed to dissolve in liquid, then for sufficiently high pressures, it is possible for all the air to dissolve in the liquid phase, and for the gaseous phase to disappear. In such a case, the liquid is said to be undersaturated. For the case of undersaturated resin, where $S_a < 0$, the conservation laws (15) and (16) reduce to

$$
\phi \partial_t (S_r) + \nabla \cdot (q_r) = 0,
$$
(31)

$$
\phi \partial_t (R_s S_r) + \nabla \cdot (R_s q_r) = 0,
$$
(32)

where $R_s$ is the volume solubility of air in undersaturated resin with $R_s < R_{\infty}$. In addition, we have $U_a = \rho_{AS} R_s$ and $q_r = q$, so the coefficients defining the equations (23) and (24) reduce to

$$
f_a = U_a,
$$
(33)

$$
q = -\left(\frac{k_{\text{rel},r}}{\mu_r}\right) K \nabla P,
$$
(34)

$$
\alpha = c_s,
$$
(35)

$$
\beta = c_s q_r,
$$
(36)

while $R_s$ has disappeared from the calculation.

The front tracking oil-gas code is then designed to solve equations (23) and (24) including the undersaturated resin case. The sequential method solves the pressure equation first, then uses the result to calculate the velocity, so that the conservation law can be solved for each time step.

Appropriate initial conditions are initial air mass density plus initial pressures for the pressure parabolic equation. To provide a smooth transition in the pressure solutions, in our model, equation (23) is solved with $\alpha = 0$ initially at $t = 0$, then this pressure solution will be used as the initial pressure condition for the system. For the boundary conditions, a Dirichlet boundary condition (pressure held constant) is imposed at the inlet and outlet ports, whereas Neumann boundary condition (no flow) is assumed at the other boundaries.
5. NUMERICAL RESULTS

5.1. Numerical Experiments

The experiments described in [31] were performed in rectangular molds, with inlet and outlet ports at either end. Resin was injected at the inlet port, and inlet and outlet pressures were held constant during the experiment. The experiment was stopped just when the resin front was about to leave the fiber preform (at breakthrough). Care was taken that the resin did not cure during fill. The resin was cured at atmospheric pressure, and a void measurement was made at various cross sections in the mold.

Besides $S_a$, residual described in Section 3, the simulation model requires measurements of preform porosity $\phi$, preform permeability $K$, inlet and outlet pressures, resin viscosity $\mu_r$, air viscosity $\mu_a$, and relative permeabilities $k_{rel,a}$ and $k_{rel,r}$. Preform porosity, inlet and outlet pressures, and resin viscosity were obtained through reported measurements. The preform permeability $K$ determines the time scale for the numerical simulations and was not reported in [31]. We choose the relative permeability of the resin phase to be quadratic in the resin saturation, and that of the air phase to be quadratic in the reduced air saturation as well.

Another model parameter is the ratio of viscosities $\mu_r/\mu_a$. This quantity controls the extent to which the numerical void fraction profile approximates the one given by the residual saturation as a function of pressure. If this ratio is large, the rarefaction is hardly noticeable. In their numerical fill process [22], the initial inlet resin saturation is chosen as $S_r = 1$ and is kept at this value during the fill process.

The main additional model parameters in our air solubility model are the air solubility $R_{so}$ and the air formation volume factor $B_a$, both functions of pressure. The ratios $R_{so}$ and $B_a$ must be nonnegative functions. We also require that if $R_{so}$ is positive, its derivative is strictly positive for all pressures. Over the range of pressure considered, a simple linear solubility model

$$R_{so}(p) = R_{so,1} + R_{so,2}p,$$

is believed to be sufficient, where $p$ is the dimensionless pressure with $0 \leq p \leq 1$ converted from the pressure $P$ in the model equations, $R_{so,1} = 0.06$ and $R_{so,2} = 0.06$ are determined from data $0.06 \leq R_{so} \leq 0.12$ in [32].

For the choice of $B_a$, we take the air component to obey the ideal gas law

$$\rho_a = \frac{M_a}{R} \frac{P}{Z_a},$$

where $R$ is the air constant, $M_a$ is a molar mass of air, and $Z_a$ is a pressure dependent factor to describe deviations from ideal gas behavior. The data in [33] shows that air behaves as an ideal gas in a reasonable range of temperature and pressure, i.e., $Z_a = 1$. We then have the air compressibility coefficient

$$c_a = \frac{1}{\rho_a} \frac{d\rho_a}{dP} = \frac{1}{P},$$

and by (29), we obtain

$$B_a = \frac{P_S}{P},$$

where $P_S$ is the pressure at standard conditions, in our calculation $P_S = 1$ atmosphere.

To model the undersaturated resin, we have to choose the initial inlet saturation conditions as $S_r = 1$ and $S_a < 0$. Using $S_a = (U_a - U_b)/(U_{max} - U_b)$ with the condition $0 \leq U_a \leq U_b$ at the inlet, we then have $- (U_b)/(U_{max} - U_b) \leq S_a \leq 0$ at the inlet. Since the inlet saturation of dissolved air is 0 in our numerical experiments and approximately zero in the physical experiments, $S_a$ is chosen to be $- (U_b)/(U_{max} - U_b)$. The major improvement for our air solubility model as
distinguished from the two-phase model in [22] is that the effect of air dissolved in resin can be modeled accurately.

In Figure 1, we plot the numerical results from the two-phase model developed in [22], and the results by our air solubility model. In these experiments, the pressure difference between inlet and outlet was held at 5 atmospheres and the outlet pressure varied at 1, 0.5, and 0.3 atmosphere. The sum of fit error squares for three experiments are 12.44, 3.88, 4.11 using the simple two-phase model versus 11.38, 3.04, 2.74 using the air solubility model, respectively, which demonstrate a slight improvement in fit for the air solubility model.

5.2. Numerical Predictions

In the previous section, the numerical solutions demonstrated the satisfactory results for our air solubility model. Before we apply the model to make some numerical predictions, we first study the accuracy of the model using mesh refinement. Our computation domain is \((0,0.137) \times (0,1)\), the mesh grid size for the hyperbolic solver is \((m,n) = (4,30)\), the mesh side for the parabolic solver is \((m_1,n_1) = (8,60)\), i.e., together, the mesh sizes are \([4,30] \times [8,60]\). The mesh refinement sequence is \([3,22] \times [6,45]\), \([6,45] \times [12,90]\), and \([12,90] \times [24,180]\). The results by the different mesh size are presented in Figure 2, which demonstrate the convergence of the solutions. The range of the normalized distance to outlet is chosen to be \(0 \rightarrow 0.5\) in Figure 2, for better plotting results, since the void fractions are zero for \(x \geq 0.5\). Similar plotting meshes are also used in Figure 4 and 6.
Figure 2. Comparison of void fraction profiles for the first experiment $P_{\text{out}} = 1$ from [31] using air solubility model for different mesh size. Mesh$_1 = [(3, 22) \times (6, 45)]$, Mesh$_2 = [(6, 45) \times (12, 90)]$, and Mesh$_3 = [(12, 90) \times (24, 180)]$.

Now we want to make some numerical predictions. In our next numerical experiment, we will fix the inlet pressure, and vary the outlet pressure after breakthrough ($t = 0.9$), so that we can induce additional bubble motion, and hence, reduce the overall void content.

Figure 3 is the pressure history for the inlet and the outlet after the breakthrough, while the pressure difference between the inlet and the outlet is held at 5 atmosphere (the first experiment in [31]) before the breakthrough time $t = 0.9$. We let the numerical fill process continue to $t = 1.0$ after the breakthrough, and then increase the outlet pressure linearly from 1 atm at time $t = 1.0$ to 3 atm at time $t = 1.1$, and let the outlet pressure stay at 3 atm until $t = 1.2$. After that the outlet pressure will drop back to 1 atm in the same time interval linearly. A similar outlet pressure cycling procedure is repeated thereafter.

In Figure 4, we obtained the void content for various times as a result of varying the outlet pressure. We can see that the void fraction content is decreasing from $t = 0.9$ to $t = 2.1$. But this void fraction decrease is not monotone in time. To understand the total effects of the outlet pressure cycling, we present an alternative measure of the changes of void content in Figure 5. We plot the void fraction curve at different locations (in the normalized distance to the outlet), for the pressure cycling times period from $t = 1.0$ to $t = 2.1$. We plot for $x$ between 0.050 and 0.217, only because these are the places where the major reduction of void content takes place. The closer $x$ is to the outlet, the faster the void content reduces. The level of reduction in the void content also becomes more significant as the location is closer to the outlet, since local pressure is the main factor to mobilize the bubbles.

To examine both Figure 4 and Figure 5 together, we see that the void fraction content is decreasing from $t = 0.9$ to $t = 1.2$. The void fraction content starts to increase from $t = 1.2$, until $t = 1.4$ due to the release of the outlet pressure to atmosphere, but the void content level
Figure 3. The pressure history after the breakthrough.

Figure 4. Void fraction vs. distance for times: \( t = 0.9, 1.3, 1.7, \) and \( 2.1 \); using air solubility model.
Figure 5. Void fraction vs. time at different normalized distance to outlet: $x = 0.05, 0.083, 0.117, 0.150, 0.183, \text{ and } 0.217, \text{ using air solubility model.}$

at time $t = 1.4$ is below its previous level at time $t = 1.0$, i.e., some air after being dissolved in the surrounding resin, has mobilized, and has been removed by outflow from the mold. This void fraction cycle repeats with the cycling of the outlet pressure. After each pressure cycle, the void content is reduced compared to its previous values at corresponding times (at the same pressure values). For example, comparison of the void content for times $t = 0.9 \rightarrow 1.3 \rightarrow 1.7 \rightarrow 2.1$, in both Figure 4 and Figure 5, show the steady decrease in void fraction.

As a comparison, in Figure 6, we plot the results (similar to Figure 4) using the two-phase flow model [22]. The effect on void saturation of varying the outlet pressure was not observed to nearly the same extent in this model, as we see that the void fraction is not decreasing markedly despite of the cycling of the outlet pressure.

It is clear that by varying the outlet pressure, the sizes of the bubbles (both macro and micro) reduce due to the compression initially and then by dissolution of the air into the resin. The reduced bubbles then become small enough to move as does the air dissolved in the resin so that the total void content decreases by this bubble motion and dissolved air. This effect has remarkable qualitative agreement with Northrup-Grumman’s experimental results. Future study should focus on a quantitative comparison through model parameters once we have such experimental data.

6. CONCLUSIONS

We have presented a two-phase, two-component flow model, which combines the model developed in [22] with the air solubility model, for modeling the RTM filling process. We apply the model to study the formation and migration of the macro- and micro-voids. A pressure dependent residual air saturation which acts as the threshold of macro-void mobilization introduced in [22], plus the air solubility presented here, provides a new way to improve RTM product quality after
the fill process has been completed. Numerical solutions are obtained using the front tracking oil-gas code. We predict in our numerical study, the extent to which an increase of fluid pressure in the exit side after the fill is complete, but before sealing the vents reduces the overall void content.

REFERENCES