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ORIGINAL ARTICLE

Testing NBU(2) class of life distribution based on goodness of fit approach

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Abstract In this paper, new testing procedures for exponentiality against the NBU(2) class is addressed based on the goodness of fit approach. It is shown that the proposed test has high relative efficiency for some commonly used alternative and enjoys a good power. The critical values of the proposed statistic are calculated and some applications are given to elucidate the use of the proposed test in reliability analysis.

1. Introduction

Many applications in reliability theory and biostatistics involve the modeling of lifetime data. In these applications the outcome of interest is the time T, until some event occurs. This event may be death, the appearance of a tumor, the development of some disease, recurrence of a disease, conception, cessation of smoking, and so forth. The basic quantity employed to describe time-to-event phenomena is the survival function. This function, also known as the survivor function or survivorship function, is the probability an individual survives beyond time t and it is defined as $\overline{F}(t) = \int_t^\infty f(u)du$.

An important characteristic of survival distribution is its ageing properties. There are a number of classes that have been suggested in the literature to categorize distributions. The first aging class is the class of new better than used (NBU) distribution. Here a life distribution has the NBU property if, and only if,

$$\overline{F}(x+t) \leqslant \overline{F}(x)\overline{F}(t), \quad \text{for all } t > 0.$$

The NBU class of life distributions has been widely studied in the theory of reliability and life-testing since it was introduced earlier (Barlow and Proschan, 1981). The class, which emphasizes that a new item has a stochastically larger life length than does a used one at age $t > 0$, arises in many situations in which a better maintenance policy is believed to exist. Because it is easy to make a valuable judgment when such a model really exists, and because incorporating this model into an inferential procedure increases its statistical efficiency, it is desirable to recognize the existence of the model and to select a corresponding maintenance policy.

A second aging class is the class of new better than used in expectation (NBUE) distribution. A non-negative random

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variable T is said to have NBUE distribution if, and only if,

$$\int_x^\infty \overline{F}(u)du \leqslant \mu\overline{F}(x), \quad \text{for all } x > 0,$$

where $\mu = E(T)$, assumed finite.

A third aging class is the class of new better than used in the increasing concave order (NBU(2)) distributions. A non-negative random variable T is said to have NBU(2) if

$$\int_0^y \overline{F}(x+t)dx \leqslant \overline{F}(t)\int_0^y \overline{F}(x)dx, \quad \text{for all } y,\ t > 0. \tag{1.1}$$

Inequality (1.1) has definite physical meaning: the life length of a new item is stochastically larger than that of a used one at age $t > 0$ in terms of increasing concave ordering. Probabilistic properties of the above three classes of aging distributions as well as many others have been extensively studied in the literature see e.g., Bryson and Siddique (1969), Marshall and Proschan (1972), Hollander and Proschan (1975), Deshpande et al. (1986), Hendi et al. (1999), Franco et al. (2001), Ahmad (2001), Hu and Xie (2002) and Lia and Xie (2006), among others. The relations between the above classes as easily seen are as follows:

$$\text{NBU} \subset \text{NBU}(2) \subset \text{NBUE}. \tag{1.2}$$

In the context of reliability and life testing, its well-known that no ageing property corresponds to the exponential distribution. Hence testing non-parametric classes is done by testing exponentiality versus some kind of ageing classes. In this paper, a new testing procedure for exponentiality against the NBU(2) class is addressed based on the goodness of fit approach showing that it is simpler than most earlier ones and holds high relative efficiency for some commonly used alternatives. In Section 2, we present a procedure to test that T is exponential against that it is NBU(2) and not exponential for non-censored data. The Pitman asymptotic relative efficiency with respect to Hendi et al. (1999) and Hollander and Proschan (1975) are discussed. Also, the power of the test for some commonly used distributions in reliability and the critical values of the proposed test are calculated. In Section 3, we deal with the right censored problem. As a consequence the critical percentiles of the proposed test are calculated and tabulated for sample size $n = 5(5)80$, 81, 85 and 86 based on 10,000 replications. Finally, in Section 4, some applications are presented for non-censored and censored data to illustrate the theoretical results.

2. Testing against NBU(2) class for non-censored data

In this section a test statistic based on the goodness of fit approach is developed to test H_0:F is fully specified exponential against H_1:F is NBU(2) and not exponential. Since under H_0, F is completely known, we take it's mean to be one.

2.1. Testing procedure

The following lemma offers a measure of departure from H_0 in favor of H_1. Hence it could be used to offer a testing procedure.

Lemma 2.1. *If F is NBU(2) then a measure of deviation from the null hypothesis H_0 is $\Delta_F > 0$, where*

$$\Delta_F = E\left(Xe^{-X}\right) + E\left(e^{-X}\right)^2 - E\left(e^{-X}\right). \tag{2.1}$$

Proof. First, in (1.1) set

$$v(x) = \int_0^x \overline{F}(u)du.$$

Thus, $X \in \text{NBU}(2)$ iff,

$$v(x+t) - v(t) \leqslant \overline{F}(t)v(x), \quad \text{for all } x \geqslant 0.$$

Define a measure of departure from H_0 as

$$\Delta_F = \int_0^\infty v(x)dF_0(x)\left[1 + \int_0^\infty \overline{F}(t)dF_0(t)\right] - \int_0^\infty \int_0^\infty v(x+t)dF_0(x)dF_0(t).$$

Set,

$$\begin{aligned} I_1 &= E\left(\int_0^X dF_0(x)\right), \\ &= 1 - E\left(e^{-X}\right), \end{aligned} \tag{2.2}$$

$$\begin{aligned} I_2 &= \int_0^\infty v(x)dF_0(x), \\ &= \mu - E\left(\int_0^X F_0(x)dx\right), \\ &= 1 - E\left(e^{-X}\right), \end{aligned} \tag{2.3}$$

and

$$\begin{aligned} I_3 &= \int_0^\infty \int_0^\infty v(x+t)dF_0(x)dF_0(t), \\ &= \int_0^\infty \int_0^w v(w)f_0(w-z)f_0(z)dzdw, \\ &= \int_0^\infty v(x)dF_0^{(2)}(x), \\ &= \mu - E\left(\int_0^X F_0^{(2)}(x)dx\right), \\ &= 2 - 2E(e^{-X}) - E(xe^{-X}). \end{aligned} \tag{2.4}$$

From (2.2)–(2.4), the result follows. □

Based on a random sample $X_1, X_2, \ldots, X_n$ from a distribution F, a direct empirical estimate of the measure Δ_F in (2.1) is:

$$\begin{aligned} \hat{\Delta}_F &= \frac{1}{n}\sum_{i=1}^n X_i e^{-X_i} + \left(\frac{1}{n}\sum_{i=1}^n e^{-X_i}\right)^2 - \frac{1}{n}\sum_{i=1}^n e^{-X_i}, \\ &= \frac{1}{n(n-1)}\sum_{i=1}^n \sum_{j=1}^n \left[X_i e^{-X_i} + e^{-X_i - X_j} - e^{-X_i}\right]. \end{aligned}$$

Set

$$\phi(X_1, X_2) = X_1 e^{-X_1} + e^{-X_1 - X_2} - e^{-X_1},$$

then under H_0

$$E[\phi(X_1, X_2)|X_1] = X_1 e^{-X_1} - \frac{1}{2}e^{-X_1},$$

and

$$E[\phi(X_1, X_2)|X_2] = \frac{1}{2}e^{-X_2} - \frac{1}{4},$$

and thus

$$\psi(X) = Xe^{-X} - \frac{1}{4}.$$

Hence by the central limit theory we have that $\sqrt{n}(\hat{\Delta}_F - \Delta_F) \to N(0, \sigma^2)$. Under H_0, $\sigma_0^2 = \frac{5}{432}$. Reject H_0 when $\sqrt{\frac{432n}{5}}\hat{\Delta}_F \geqslant Z_\alpha$, Z_α is the standard normal variate. Note that if under the null hypothesis, the exponential has a mean different than one and is unknown, we start by estimating it by $\overline{X}$. Then set the observations as: $\frac{X_1}{\overline{X}}, \ldots, \frac{X_n}{\overline{X}}$. Invoking Theorem 2.13 of Randles (1982), the limiting distribution of $\hat{\Delta}_F$ based on the standardize observations $\left(\frac{X_i}{\overline{X}}\right)$, $i = 1, 2, \ldots, n$ remain unaffected and so are the efficiencies.

2.2. Asymptotic relative efficiency

Let T_{n_1} and T_{n_2} be two test statistic for test $H_0 : F_\theta \in \{F_{\theta_n}\}$, $\theta_n = \theta + cn^{-\frac{1}{2}}$ with c an arbitrary constant, then the asymptotic relative efficiency of T_{n_1} relative to T_{n_2} is defined by

$$e(T_{n_1}, T_{n_2}) = \frac{[\mu_1'(\theta_0)/\sigma_1(\theta_0)]}{[\mu_2'(\theta_0)/\sigma_2(\theta_0)]},$$

where

$$\mu_i'(\theta_0) = \lim_{n\to\infty}\left(\frac{\partial}{\partial\theta}E(T_{in})\right)_{\theta\to\theta_0},$$

and

$$\sigma_i^2(\theta_0) = \lim_{n\to\infty} var(T_{n_i}), \quad i = 1, 2,$$

is the null variance. Its well-known that the pitman asymptotic efficiency (PAE) is defined by:

$$\text{PAE} = \left.\frac{\partial\Delta}{\partial\theta}\right|_{\theta\to\theta_0} \Big/ \sigma_0, s.$$

To assess the quality of this procedure, we evaluate its PAE for four alternatives in the class. These are:

(i) *Linear failure rate family:*

$$\overline{F}_1(x) = e^{-x-\frac{\theta}{2}x^2}, \quad x \geqslant 0, \ \theta \geqslant 0;$$

(ii) *Makham family:*

$$\overline{F}_2(x) = e^{-x-\theta(x+e^{-x}+1)}, \quad x \geqslant 0, \ \theta \geqslant 0;$$

(iii) *Weibull family:*

$$\overline{F}_3(x) = e^{-x^\theta}, \quad x \geqslant 0, \ \theta \geqslant 0;$$

(iv) *Gamma family:*

$$\overline{F}_4(x) = \int_x^\infty e^{-u}u^{\theta-1}du/\Gamma(\theta), \quad x \geqslant 0, \ \theta \geqslant 0.$$

Since

$$\Delta_{F_\theta} = E_\theta(Xe^{-X}) + E_\theta(e^{-X})^2 - E_\theta(e^{-X}),$$

then

$$\left.\frac{\partial\Delta F_\theta}{\partial\theta}\right|_{\theta\to\theta_0} = \int_0^\infty xe^{-x}f_{\theta_0}'(x)dx.$$

Note also that under $\theta = \theta_0$, linear failure rate, Makhem, Weibull and Gamma distributions reduce to the exponential distribution. Direct calculations of the asymptotic efficiencies of the NBU(2) test above are given in Table 2.1.

Table 2.1

	LFR	Makeham	Weibull	Gamma
Efficiency	0.5809	0.2582	2.3238	5.1131

Hendi et al. (1999) called the NBU(2) class by NBUA. As a result, we compere our test U_n with Hendi et al. (1999) test U^* for NBUA class and with Hollander and Proschan (1975) test K^* for NBUE class. Table 2.2 present the asymptotic relative efficiencies of the U_n test of NBU(2) with respect to U^*, K^* for NBUA and NBUE, respectively.

Table 2.2

	LFR	Makeham	Weibul	Gamma
$E(U_n, U^*)$	0.9998	1.0008	0.4401	1.5603
$E(U_n, K^*)$	0.6689	0.8937	1.9365	7.0633

2.3. Monte Carlo null distribution critical values

In practice, simulated percentiles for small samples are commonly used by applied statisticians and reliability analysts. We have simulated the upper percentile values for 95, 98 and 99. Table 2.3 presents these percentile values of the statistic $\hat{\Delta}_F$ and the calculations are based on 10,000 simulated samples of sizes $n = 5(5)100$.

Table 2.3 The upper percentile of $\hat{\Delta}_F$ with 10,000 replications.

n	95%	98%	99%
5	0.1200	0.1441	0.1563
10	0.0711	0.0855	0.0969
15	0.0562	0.0709	0.0798
20	0.0461	0.0558	0.0642
25	0.0407	0.0515	0.0574
30	0.0361	0.0454	0.0509
35	0.0338	0.0413	0.0461
40	0.0314	0.0394	0.0445
45	0.0293	0.0362	0.0404
50	0.0274	0.0341	0.0381
55	0.0268	0.0325	0.0366
60	0.0246	0.0307	0.0353
65	0.0238	0.0300	0.0338
70	0.0228	0.0282	0.0314
75	0.0214	0.0273	0.0312
80	0.0212	0.0258	0.0296
85	0.0208	0.0258	0.0286
90	0.0200	0.0248	0.0282
95	0.0192	0.0240	0.0271
100	0.0188	0.0234	0.0263

2.4. The power estimates

The power of the proposed test at a significance level α using simulated number of sample 10,000 for sample size ($n = 10$, 20 and 30) and $\theta = 1$, 2 and 3 with respect to the alternatives

Table 2.4 Alternative distributions: LFR, Makhem, Weibull, Gamma.

n	θ	LFR	Makhem	Weibull	Gamma
10	1	0.99950	0.99490	0.95350	0.95350
	2	1.00000	0.998400	1.00000	0.99900
	3	1.00000	1.00000	1.00000	1.00000
20	1	0.99990	0.99810	0.94370	0.94370
	2	1.00000	0.99990	1.00000	1.00000
	3	1.00000	0.99990	1.00000	1.00000
30	1	0.99990	0.99880	0.95000	0.95000
	2	1.00000	0.99990	1.00000	1.00000
	3	1.00000	1.00000	1.00000	1.00000

F_1, F_2, F_3 and F_4. Table 2.4 shows the power of the test at different values of θ and significance level $\alpha = 0.05$.

From the above table, it is noted that the power of the test increases by increasing the values of the parameter θ and sample size n, and it is clear that our test has good powers.

3. Testing against NBU(2) class for censored data

In this section, a test statistic is proposed to test H_0 versus H_1 with randomly right-censored data. Such a censored data is usually the only information available in a life-testing model or in a clinical study where patients may be lost (censored) before the completion of a study. This experimental situation can formally be modeled as follows.

Suppose n objects are put on test, and $X_1, X_2, \ldots, X_n$ denote their true life time. We assume that $X_1, X_2, \ldots, X_n$ be independent, identically distributed (i.i.d.) according to a continuous life distribution F. Let $Y_1, Y_2, \ldots, Y_n$ be (i.i.d.) according to a continuous life distribution G and assume that X's and Y's are independent. In the randomly right-censored model, we observe the pairs (Z_j, δ_j), $j = 1, \ldots, n$, where $Z_j = \min(X_j, Y_j)$ and

$$\delta_j = \begin{cases} 1, & \text{if} \quad Z_j = X_j \ (j\text{th observation is uncensored}) \\ 0, & \text{if} \quad Z_j = Y_j \ (j\text{th observation is censored}). \end{cases}$$

Let $Z(0) = 0 < Z(1) < Z(2) < \cdots < Z(n)$ denote the ordered Z's and $\delta_{(j)}$ is the δ_j the corresponding to $Z_{(j)}$. Using the censored data (Z_j, δ_j), $j = 1, \ldots, n$. Kaplan and Meier (1958) proposed the product limit estimator,

$$\overline{F}_n(X) = 1 - F_n(X),$$
$$= \prod_{j:Z_{(j)} \leqslant 1} \left[\frac{n-j}{n-j+1}\right]^{\delta_{(j)}}, \quad X \in [0, Z_{(n)}].$$

Now for testing $H_0{:}\Delta_F = 0$, against $H_1{:}\Delta_F > 0$, using randomly right-censored data, we propose the following test statistic:

$$\hat{\Delta} = \int_0^\infty \int_0^\infty e^{-x} e^{-t} v(x) \overline{F}(t) dx dt + \int_0^\infty e^{-x} v(x) dx - \int_0^\infty \int_0^\infty e^{-x} e^{-t} v(x+t) dx dt$$

Table 3.1 The upper percentile of $\hat{\Delta}_c$ with 5000 replications.

n	95%	98%	99%
5	0.8792	1.0659	1.2541
10	1.0000	1.2177	1.3547
15	0.9995	1.1814	1.3292
20	1.0252	1.1904	1.2973
25	1.0267	1.2089	1.3560
30	1.0349	1.2001	1.3233
35	1.0429	1.2158	1.3188
40	1.0347	1.1910	1.2888
45	1.0378	1.1780	1.12671
50	1.0540	1.1929	1.2758
55	1.0438	1.1984	1.3110
60	1.0687	1.2097	1.3848
65	1.0674	1.2052	1.2983
70	1.0709	1.2205	1.3254
75	1.0739	1.2213	1.3070
80	1.0752	1.2280	1.3215
81	1.0845	1.2310	1.3522
85	1.0771	1.2110	1.3089
86	1.0738	1.2155	1.3181

$$\hat{\Delta}^c_F = \left\{ \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{j-1} e^{-z_j} e^{-z_i} \left(\prod_{m=1}^{i-1} C_m^{\delta_m} \prod_{m=1}^{j-1} C_m^{\delta_m} \right) \times \left(z_{(k)} - z_{(k-1)}\right)\left(z_{(j)} - z_{(j-1)}\right)\left(z_{(i)} - z_{(i-1)}\right) \right\}$$
$$+ \sum_{j=1}^{n} \sum_{k=1}^{j-1} e^{-z_j} \left(\prod_{m=1}^{k-1} C_m^{\delta_m} \right) \left(z_{(k)} - z_{(k-1)}\right)\left(z_{(j)} - z_{(j-1)}\right)$$
$$- \left\{ \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{\ell} e^{-z_j} e^{-z_i} \left(\prod_{m=1}^{k-1} C_m^{\delta_m} \right) \times \left(z_{(k)} - z_{(k-1)}\right)\left(z_{(j)} - z_{(j-1)}\right)\left(z_{(i)} - z_{(i-1)}\right) \right\},$$

where $C_k = n - m/n - m + 1$, and

$$\ell = \begin{cases} \#Z\text{'s} \leqslant Z_{(i)} + Z_{(j)}, & \text{if } Z_{(i)} + Z_{(j)} < Z_{(n)} \\ 0, & \text{if } Z_{(i)} + Z_{(j)} \geqslant Z_{(n)} \end{cases}.$$

Table 3.1 gives the critical percentiles of $\hat{\Delta}^c$ test for sample sizes $n = 5(5)80$, 81, 85 and 86 based on 5000 replications.

From the above table, it is noted that the critical values of the test increases by increasing of the sample size n.

4. Some applications

In this section, we apply the test on some data-sets to elucidate the applications of the NBU(2) in the both non-censored and censored data at 95% confidence level.

4.1. Non-censored data

Example 4.1. Consider the following data in Abouammoh et al. (1994). These data represent set of 40 patients suffering from blood cancer (Leukemia) from one of ministry of health hospital in Saudi Arabia and the ordered values (in days) are:

115, 181, 255, 418, 441, 461, 516, 739, 743, 789, 807, 865, 924, 983, 1024, 1062, 1063, 1165, 1191, 1222, 1222, 1251, 1277, 1290, 1357, 1369, 1408, 1455, 1478, 1549, 1578, 1578, 15999, 1603, 1605, 1696, 1735, 1799, 1815, 1852.

It was found that $\hat{\Delta}_F = 0.0892$ which is greater than the critical value of the Table 2.4. Then we reject the null hypothesis of exponentiality and accept H_1 which states that the data set has NBU(2) property.

Example 4.2. In an experiment at Florida state university to study the effect of methyl mercury poisoning on the life lengths of fish goldfish were subjected to various dosages of methyl mercury (Kochar (1985)). At one dosage level the ordered times to death in week are:

6.000, 6.143, 7.286, 8.714, 9.429, 9.857, 10.143, 11.571, 11.714, 11.714.

It was found that $\hat{\Delta}_F = 0.01367$ which is less than the critical value of the Table 2.4. Then we accept the null hypothesis of exponentiality property.

4.2. Censored data

Example 4.3. Consider the data from Susarla and Vanryzin (1978), which represent 81 survival times (in months) of patients of melanoma. Out of these 46 represents non-censored data, and order values:

3.25, 3.5, 4.75, 4.75, 5, 5.25, 5.75, 5.75, 6.25, 6.5, 6.5, 6.75, 6.75, 7.78, 8, 8.5, 8.5, 9.25, 9.5, 9.5, 10, 11.5, 12.5, 13.25, 13.5, 14.25, 14.5, 14.75, 15, 16.25, 16.25, 16.5, 17.5, 21.75, 22.5, 24.5, 25.5, 25.75, 27.5, 29.5, 31, 32.5, 34, 34.5, 35.25, 58.5.

The order censored data are:

4, 5.25, 11, 12.5, 13.75, 16.75, 18.25, 19, 20, 20.25, 21.5, 23.25, 25, 27, 28.5, 30, 31, 31.25, 32.25, 32.5, 33, 33.5, 35, 36.75, 37, 37, 37.75, 38, 38, 39.5, 45.25, 47.5, 48.25, 48.5, 53.25, 53.75.

Now taking into account the whole set of survival data (both censored and uncensored), and computing the statistic. It was found that $\hat{\Delta}^c_F = 0.003788$ which is less than the critical value of the Table 3.1. Then we accept the null hypothesis of exponentiality property.

Example 4.4. On the basis of right-censored data for lung cancer patients from Pena (2002). These data consists of 86 survival times (in month) with 22 right censored. The whole life times (non-censored data) are:

0.99, 1.28, 1.77, 1.97, 2.17, 2.63, 2.66, 2.76, 2.79, 2.86, 2.99, 3.06, 3.15, 3.45, 3.71, 3.75, 3.81, 4.11, 4.27, 4.34, 4.40, 4.63, 4.73, 4.93, 4.93, 5.03, 5.16, 5.17, 5.49, 5.68, 5.72, 5.85, 5.98, 8.15, 8.62, 8.48, 8.61, 9.46, 9.53, 10.05, 10.15, 10.94, 10.94, 11.24, 11.63, 12.26, 12.65, 12.78, 13.18, 13.47, 13.96, 14.88, 15.05, 15.31, 16.13, 16.46, 17.45, 17.61, 18.20, 18.37, 19.06, 20.70, 22.54, 23.36.

The ordered censored observations are:

11.04, 13.53, 14.23, 14.65, 14.91, 15.47, 15.47, 17.05, 17.28, 17.88, 17.97, 18.83, 19.55, 19.55, 19.75, 19.78, 19.95, 20.04, 20.24, 20.73, 21.55, 21.98.

Now taking into account the whole set of survival data (both censored and uncensored), and computing the statistic. It was found that $\hat{\Delta}_F = 3.3764$ which is greater than the critical value of the Table 3.1. Then we reject the null hypothesis of exponentiality and accept H_1 which states that the data set has NBU(2) property.

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