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Tensionless strings, correspondence with $SO(D,D)$ sigma-model

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Abstract

The string theory with perimeter action is tensionless by its geometrical nature and has a pure massless spectrum of higher spin gauge particles. I demonstrate that the linear transformation of the world-sheet fields defines a map to the $SO(D,D)$ sigma-model equipped by additional Abelian constraint, which breaks $SO(D,D)$ to a diagonal $SO(1,D-1)$. The effective tension is equal to the square of the dimensional coupling constant of the perimeter action. This correspondence allows to view the perimeter action as a "square root" of the Nambu–Goto area action. The aforementioned correspondence between tensionless strings and $SO(D,D)$ sigma-model allows to introduce vertex operators in full analogy with the standard string theory and to confirm the form of the vertex operators introduced earlier, the value of the intercept $a=1$ and the critical dimension $D=13$.

It is generally expected that high energy limit or, what is equivalent, the tensionless limit $\alpha' \to \infty$ of string theory should have massless spectrum $M_N^2 = (N-1)/\alpha' \to 0$ and should recover genuine symmetries of the theory [1–3]. Of course this observation ignores the importance of the high genus G diagrams, the contribution of which $A_G \simeq \exp\{-\alpha' s/(G+1)\}$ is exponentially large compared to the tree level diagram [1–3]. The ratio of the corresponding scattering amplitudes behaves as $A_{G+1}/A_G \simeq \exp\{\alpha' s/G^2\}$ and makes any perturbative statement unreliable and requires therefore a nonperturbative treatment of the problem [4–8].[1]

The tensionless model with perimeter action suggested in [17–20] does not appear as an $\alpha' \to \infty$ limit of the standard string theory, as one could probably think, but has a tensionless character by its geometrical nature [17]. Therefore it remains mainly unclear at the moment how these two models are connected. However the perimeter model shares many properties with the area strings in the sense that it has world-sheet conformal invariance and contains the correspond-

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[1] The different aspects and models of tensionless theories can be found in [9–16].

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ing Virasoro algebra, which is extended by additional Abelian generators. This makes mathematics used in the perimeter model very close to the standard string theory and allows to compute its massless spectrum, critical dimension $D_c = 13$ [18–20] and to construct appropriate vertex operators [21,22].

Comparing literally the spectrum of these two models one can see that instead of usual exponential growing of states, in the perimeter case we have only linear growing. In this respect the number of states in the perimeter model is much less compared with the standard string theory and is larger compared with the field theory models of the Yang–Mills type. From this point of view it is therefore much closer to the quantum field theory rather than to the standard string theory. At the same time its formulation and the symmetry structure is more string-theoretical. Perhaps there should be a strong nonperturbative rearrangement of the spectrum in the limit $\alpha' \to \infty$ before the spectrum of the area and the perimeter strings can become close to each other.

Our aim here is to give a partial answer to these questions. As we shall see the linear transformation of the world-sheet fields defines a map to the $SO(D,D)$ σ-model equipped by an additional Abelian constraint, which breaks $SO(D,D)$ to a diagonal $SO(1,D-1)$. The effective string tension is equal to the square of the dimensional coupling constant m of the perimeter action

$$\frac{1}{2\pi\alpha'} = \frac{m^2}{\pi}.$$

This relation allows to view the perimeter action as a "square root" of the Nambu–Goto area action $m = \sqrt{1/2\alpha'}$. The mass-shell quantization condition of the $SO(13,13)$ σ-model

$$\alpha' M_N^2 = -\alpha' K^2 = (N-1),$$

which defines the value of the first Casimir operator K^2 of the Poincaré algebra in 26 dimensions, is translated through the dictionary into the quantization condition for the square $W = w_{D-3}^2$ of the Pauli–Lubanski form w_{D-3} of the Poincaré algebra in 13 dimensions

$$W_N = \frac{(k\cdot\pi)^2}{m^2} = (N-a)^2 = (N-1)^2,$$

because, as we shall see (26), $K \circ K|_{\text{in 26 dim.}} = 2m \times (k\cdot\pi)|_{\text{in 13 dim.}}$ This demonstrates that in the tensionless string theory the intercept $a = 1$ and *therefore only $N = 1$ state realizes fixed helicity representations, $W = 0$, whereas the ground state $N = 0$ and the rest of the excited states $N \geqslant 2$ realize continuous spin representations, $W \neq 0$, of the massless little group $SO(11)$.*

The aforementioned correspondence allows to introduce the vertex operators in full analogy with the standard string theory and to confirm the form of the vertex operators introduced earlier in [21,22]. The n-point scattering amplitude of *fixed helicity states* $W_1 = 0$ $(N = 1)$ in terms of 13-dimensional momenta k_i and polarizations e_i is[2]

$$\begin{aligned}\mathcal{A}(k_1,e_1;\ldots;k_n,e_n) &= \int d\pi_1\cdots d\pi_n\, e^{ie_1\pi_1+\cdots+ie_n\pi_n} \\ &\quad\times \int \prod_i^n d^2\zeta_i \left\langle U_{k_1,\pi_1}(\zeta_1)\cdots U_{k_n,\pi_n}(\zeta_n)\right\rangle, \end{aligned} \tag{1}$$

where U_{k_i,π_i} (32) are fixed helicity vertex operators $(k_i\cdot\pi_i) = 0$, $i = 1,\ldots,n$. This scattering amplitude exhibits important gauge invariance with respect to the gauge transformations [19]:

$$e_i \to e_i + k_i \Lambda_i(k_1,\ldots,k_n), \tag{2}$$

where $\Lambda_i(k_1,\ldots,k_n)$ are gauge parameters. This invariance is valid only for the states which are described by the fixed helicity vertex operator $U_{k,\pi}$ (32), for which $W_1 \sim (k\cdot\pi)^2 = 0$.

The perimeter string model was suggested in [17] and describes random surfaces embedded in D-dimensional space–time with the following action

$$S = mL = \frac{m}{\pi}\int d^2\zeta\,\sqrt{h}\sqrt{\left(\Delta(h)X_\mu\right)^2},$$

where $h_{\alpha\beta}$ is the world-sheet metric, $\Delta(h) = 1/\sqrt{h}\partial_\alpha\sqrt{h}h^{\alpha\beta}\partial_\beta$ is Laplace operator and m has dimension of mass. There is no Nambu–Goto area term in this action. The action has dimension of length L and the dimensional coupling constant is m. Multiplying and dividing the Lagrangian by the square root $\sqrt{(\Delta(h)X_\mu)^2}$ one can represent it in the σ-model

[2] W_N defines fixed helicity states, when $W_1 = 0$ $(N = 1)$, and continuous spin representations (CSR), when $W_N \neq 0$ $(N \neq 1)$ [18,19,27–29].

form [18]:

$$S = -\frac{1}{\pi}\int d^2\zeta\ \eta_{\mu\nu}\sqrt{h}h^{\alpha\beta}\partial_\alpha\Pi^\mu\partial_\beta X^\nu, \tag{3}$$

where the operator Π^μ is

$$\Pi^\mu = m\frac{\Delta(h)X^\mu}{\sqrt{(\Delta(h)X^\mu)^2}}.$$

We shall consider the model B, in which two field variables X^μ and $h_{\alpha\beta}$ are independent. The classical equation is

$$\text{(I)}\quad \Delta(h)\Pi^\mu = 0 \tag{4}$$

and world-sheet energy–momentum tensor

$$\text{(II)}\quad T_{\alpha\beta} = \partial_{\{\alpha}\Pi^\mu\partial_{\beta\}}X^\mu - h_{\alpha\beta}h^{cd}\partial_c\Pi^\mu\partial_d X^\mu = 0. \tag{5}$$

The operator Π is a *space-like vector*,

$$\text{(III)}\quad \Theta \equiv \Pi^\mu\Pi^\mu - m^2 = 0. \tag{6}$$

The energy–momentum tensor is conserved $\nabla^a T_{ab} = 0$ and is traceless $h^{ab}T_{ab} = 0$, thus we have two-dimensional world-sheet conformal field theory with the central charge $c = 2D$ [18]. We have equation of motion (4) together with the primary constraints (5) and (6) and secondary constraints $\Theta^{1,0} = \Pi\partial_+\Pi$, $\Theta^{0,1} = \Pi\partial_-\Pi$, $\Theta^{1,1} = \partial_+\Pi\partial_-\Pi$ of conformal weights $(1,0)$, $(0,1)$ and $(1,1)$ [18]. The equivalent form of the action (3) is [23]

$$\acute{S} = -\frac{1}{\pi}\int d^2\zeta\ \sqrt{h}h^{\alpha\beta} \times \left\{\eta_{\mu\nu}\partial_\alpha\Pi^\mu\partial_\beta X^\nu + \omega_{\alpha\beta}\left(\Pi^2 - m^2\right)\right\}, \tag{7}$$

where the Π^μ field is now an independent variable and the $\omega_{\alpha\beta}$ are Lagrange multipliers. The system of equations which follows from $\acute{S}$

$$\Delta(h)\Pi^\mu = 0, \qquad \Delta(h)X^\mu - 2h^{\alpha\beta}\omega_{\alpha\beta}\Pi^\mu = 0,$$
$$\Pi^\mu\Pi_\mu = m^2 \tag{8}$$

is equivalent to the original equations (4) and (6) and the corresponding new energy–momentum tensor $\acute{T}_{\alpha\beta}$ acquires an additional term which depends only on the field Π

$$\acute{T}_{\alpha\beta} = T_{\alpha\beta} + \left(\omega_{\alpha\beta} - \frac{1}{2}h_{\alpha\beta}h^{\gamma\delta}\omega_{\gamma\delta}\right)\left(\Pi^2 - 1\right), \tag{9}$$

where $h^{ab}\acute{T}_{ab} = 0$. The central charge $c = 2D$ of the Virasoro algebra remains untouched and demonstrates the absence of additional contributions to the central charge due to the primary constraint (6) (see also [24] for alternative calculation).

Correspondence with the $SO(D,D)$ σ-model. Let us introduce new variables as follows:

$$\frac{1}{m^2}\Pi^\mu = \frac{1}{\sqrt{2}}\left(\Phi_1^\mu + \Phi_2^\mu\right),$$
$$X^\mu = \frac{1}{\sqrt{2}}\left(\Phi_1^\mu - \Phi_2^\mu\right). \tag{10}$$

Then the action (3) will take the form

$$S = -\frac{m^2}{2\pi}\int d^2\zeta\ \eta_{\mu\nu}\sqrt{h}h^{\alpha\beta} \times \left(\partial_\alpha\Phi_1^\mu\partial_\beta\Phi_1^\nu - \partial_\alpha\Phi_2^\mu\partial_\beta\Phi_2^\nu\right). \tag{11}$$

If one considers the $2D$-dimensional target space with the combined coordinates

$$\Phi^M = \left(\Phi_1^{\mu_1}, \Phi_2^{\mu_2}\right), \quad M = 1,\ldots,2D,$$

and fully symmetric Lorentzian signature space–time metric with D pluses and D minuses

$$\eta^{MN} = \begin{pmatrix} \eta^{\mu_1\nu_1} & \\ & -\eta^{\mu_2\nu_2} \end{pmatrix} = \begin{pmatrix} -,+,\ldots,+ & \\ & +,-,\ldots,- \end{pmatrix}, \tag{12}$$

then the action (11) will have formal interpretation in terms of σ-model being defined on a $2D$-dimensional target space with the symmetry group $SO(D,D)$

$$S = -\frac{m^2}{2\pi}\int d^2\zeta\ \eta_{MN}\sqrt{h}h^{\alpha\beta}\partial_\alpha\Phi^M\partial_\beta\Phi^N. \tag{13}$$

From this expression of the action we can deduce that the effective string tension T_{eff} is equal to the square of the mass m

$$\frac{1}{2\pi\alpha'} = \frac{m^2}{\pi}. \tag{14}$$

The last relations allow to view the tensionless string theory, which is defined by the perimeter action (3), as a "square root of the Nambu–Goto area action" $m = \sqrt{1/2\alpha'}$. This interpretation has deep geometrical origin because in some sense the perimeter L, which was defined for the two-dimensional surfaces in (3), can be considered as a square root of the surface area.

This intuitive interpretation can be made more precise if one recalls Zenodor–Minkowski isoperimetric inequality [25,26], which tells that $L^2 \geqslant 4\pi S$, with the equality taking place only for a sphere.

The crucial constraint (6) will take the form $m^2 \times (\Phi_1 + \Phi_2)^2 = 2$ and it breaks $SO(D, D)$ group of fully symmetric space–time $M^{D,D}$ down to the diagonal group $SO(1, D-1)$ of the standard space–time $M^{1,D-1}$ with one time coordinate

$$SO(D, D) \to \operatorname{diag} SO(1, D-1),$$

as one can see from the component form of the above constraint[3]

$$-\left(\Phi_1^0 + \Phi_2^0\right)^2 + (\vec{\Phi}_1 + \vec{\Phi}_2)^2 = -\Phi_+^2 + \vec{\Phi}_+^2 = 2/m^2. \tag{15}$$

The X_μ and Π_μ fields (10) are actually light-cone coordinates on $M^{D,D}$ and one can heuristically say that our strings are massless because they propagate on the light cone of $M^{D,D}$. The Abelian constraint (15) can also be considered as a "compactification" to a hyperboloid manifold H^D.

The energy–momentum tensor (5) will take the form

$$T_{\alpha\beta} = \partial_\alpha \Phi_1 \partial_\beta \Phi_1 - \partial_\alpha \Phi_2 \partial_\beta \Phi_2 - \frac{1}{2} h_{\alpha\beta} h^{\gamma\delta}(\partial_\gamma \Phi_1 \partial_\delta \Phi_1 - \partial_\gamma \Phi_2 \partial_\delta \Phi_2). \tag{16}$$

It is therefore clear that we shall have $2D_c = 26$ and shall recover the previous result [18]

$$D_c = 13. \tag{17}$$

Operator algebra and vertexes. For the open strings the solution of this two-dimensional world-sheet CFT is [18]:

$$X^\mu = x^\mu + \frac{1}{m}\hat{\pi}^\mu \tau + i \sum_{n\neq 0} \frac{1}{n} \beta_n^\mu e^{-in\tau} \cos n\sigma,$$
$$\Pi^\mu = m e^\mu + \hat{k}^\mu \tau + i \sum_{n\neq 0} \frac{1}{n} \alpha_n^\mu e^{-in\tau} \cos n\sigma, \tag{18}$$

where $\hat{k}^\mu = -i\partial/\partial x_\mu$ and $\hat{\pi}^\mu = -i\partial/\partial e_\mu$ are momentum operators and α_n, β_n are oscillators with the following commutator relations

$$\left[x^\mu, \hat{k}^\nu\right] = i\eta^{\mu\nu}, \qquad \left[e^\mu, \hat{\pi}^\nu\right] = i\eta^{\mu\nu},$$
$$\left[\alpha_n^\mu, \beta_l^\nu\right] = n\eta^{\mu\nu}\delta_{n+l,0} \tag{19}$$

and $[\alpha, \alpha] = [\beta, \beta] = 0$ (the indexes are not shown). It is also convenient to introduce the zero momentum operators $\alpha_0^\mu = \hat{k}^\mu$, $\beta_0^\mu = \hat{\pi}^\mu$. The appearance of the additional zero mode means that the wave function is a function of the coordinate variables x^μ and e^μ:

$$\Psi_{\text{Phys}} = \Psi(x, e).$$

The coordinate variable x^μ belongs to a Minkowski space $x^\mu \in M^{13}$ and e^μ belongs to a hyperboloid $e^\mu \in H^{13}$ which is defined by the relation $e^2 = -e_0^2 + \vec{e}^{\,2} = 1$ (6), (15)

$$M^{26} \to M^{13} \otimes H^{13}. \tag{20}$$

It was suggested therefore in [18] that e^μ should be interpreted as a polarization vector, because from the constraint (6), (15) it follows that [18]

$$k^2 = 0, \qquad e \cdot k = 0, \qquad e^2 = 1.$$

It is important to get a better idea about the algebra (19). The transformation (10) naturally leads to the oscillators

$$A_n^\mu = \frac{1}{\sqrt{2}}\left(\alpha_n^\mu + \beta_n^\mu\right), \qquad B_n^\mu = \frac{1}{\sqrt{2}}\left(\alpha_n^\mu - \beta_n^\mu\right)$$

and brings the algebra (19) to the form

$$\left[A_n^\mu; A_m^\nu\right] = +\eta^{\mu\nu} n\delta_{n+m},$$
$$\left[B_n^\mu; B_m^\nu\right] = -\eta^{\mu\nu} n\delta_{n+m},$$
$$\left[A_n^\mu; B_m^\nu\right] = 0. \tag{21}$$

This is a standard algebra of the oscillators with the following signature

$$\eta^{\mu\nu} = (-, +, \ldots, +) \in SO(1, D-1),$$
$$-\eta^{\mu\nu} = (+, -, \ldots, -) \in SO(D-1, 1).$$

In terms of the above oscillators the "target space" coordinates (10) $\Phi^M = (\Phi_1^{\mu_1}, \Phi_2^{\mu_2})$ have the form:

$$\sqrt{2}\Phi_1^\mu = x^\mu + \frac{1}{m}e^\mu + \left(\frac{1}{m^2}\hat{k}^\mu + \frac{1}{m}\hat{\pi}^\mu\right)\tau + \frac{i}{m}\sum_{n\neq 0}\frac{1}{n} A_n^\mu e^{-in\tau} \cos n\sigma,$$

[3] The $SO(D, D)$ signature allows the D light-cone coordinates $\Phi_\pm = \Phi_1^0 \pm \Phi_2^0$, $\vec{\Phi}_\pm = \vec{\Phi}_1 \pm \vec{\Phi}_2$.

$$\sqrt{2}\Phi_2^\mu = -x^\mu + \frac{1}{m}e^\mu + \left(\frac{1}{m^2}\hat{k}^\mu - \frac{1}{m}\hat{\pi}^\mu\right)\tau + \frac{i}{m}\sum_{n\neq 0}\frac{1}{n}B_n^\mu e^{-in\tau}\cos n\sigma. \tag{22}$$

The above $SO(D,D)$ σ-model interpretation of the tensionless string theory allows to introduce vertex operators in the full analogy with the standard string theory case. Indeed the vertex operator for the ground state has the form:

$$V_K = :e^{iK\circ\Phi}:$$

where $K^M = (k_1^{\mu_1}, k_2^{\mu_2})$, $\Phi^M = (\Phi_1^{\mu_1}, \Phi_2^{\mu_2})$, $K\circ\Phi = \eta_{MN}K^M\Phi^N$, and has conformal dimension equal to the square of the momentum K^M

$$\Delta = \alpha' K^2 = \frac{K\circ K}{2m^2}. \tag{23}$$

Therefore substituting the expressions for the field $\Phi^M = (\Phi_1^{\mu_1}, \Phi_2^{\mu_2})$ in terms of the original world-sheet fields X^μ and Π^μ (10) we shall get

$$V_K = :e^{ik_1\Phi_1 - ik_2\Phi_2}: = :e^{ikX + i\pi\Pi/m}:, \tag{24}$$

where the momenta k and π are:

$$k = \frac{1}{\sqrt{2}}(k_1 + k_2), \qquad \pi = \frac{1}{\sqrt{2}m}(k_1 - k_2). \tag{25}$$

This is an interesting relation because it demonstrates how the $2D$-dimensional momentum K^M of the $SO(D,D)$ σ-model splits into two parts which form the physical momentum variable k_μ of the tensionless strings propagating in a 13-dimensional Minkowski space–time $x^\mu \in M^{1,D-1}$ and the momentum π^μ, which is conjugate to the polarization vector $e^\mu \in H^{1,D-1}$

$$M_{SO(13,13)} \to M_{SO(1,12)} \otimes H_{SO(1,12)}.$$

We can now translate the conformal dimension Δ of the ground state vertex operator V_K into the language of our momenta k and π

$$\Delta = \frac{K\circ K}{2m^2} = \frac{k_1^2 - k_2^2}{2m^2} = \frac{(k+m\pi)^2}{4m^2} - \frac{(k-m\pi)^2}{4m^2} = \frac{(k\cdot\pi)}{m}. \tag{26}$$

This clearly confirms the form of the vertex operator and its conformal dimension obtained earlier in [21].

Indeed the general form of the vertex operators suggested in [21] is given by the formula

$$U_{k,\pi}^{\mu_1\tilde{\mu}_1\ldots,\ldots\mu_j\tilde{\mu}_j}(\zeta) = :\partial_\zeta^{n_1}X^{\mu_1}\partial_{\bar{\zeta}}^{\tilde{n}_1}X^{\tilde{\mu}_1}\ldots\ldots\partial_\zeta^{n_j}\Pi^{\mu_j}\partial_{\bar{\zeta}}^{\tilde{n}_j}\Pi^{\tilde{\mu}_j} \times e^{ik\cdot X(\zeta)+i\pi\cdot\Pi(\zeta)}:, \tag{27}$$

the conformal spin should be equal to zero, therefore $n_1 + \cdots + n_j = \tilde{n}_1 + \cdots + \tilde{n}_j = N$. Using the world-sheet energy–momentum operator [18] $T(\zeta) = -:\partial_\zeta X\cdot\partial_\zeta\Pi:$ one can compute the anomalous dimension of the open strings vertex operators [21,22]:

$$\Delta = \frac{(k\cdot\pi)}{m} + N. \tag{28}$$

It must be equal to 1 in order to describe emission of physical states, therefore in tensionless string theory the value of the intercept is equal to one, $a = 1$, because $(L_0 - a)\psi = (\frac{(k\cdot\pi)}{m} + N - a)\psi = 0$ [19]. The corresponding poles

$$\frac{(k\cdot\pi)}{m} = 1 - N \tag{29}$$

are translated to the mass-shell condition on the σ-model side: $\alpha' K^2 = 1 - N$.

Let us discuss what these relations mean. The physical meaning of the invariant $k\cdot\pi$ is given by $W = w_{D-3}^2$-square of the Pauli–Lubanski form $w_{D-3}^{\mu_1,\ldots,\mu_{D-3}} \sim \varepsilon^{\mu_1,\ldots,\mu_{D-3},\nu\lambda\rho}k_\nu M_{\lambda\rho}$ of the Poincaré algebra on M^{13}, that is [18,19]

$$W = \frac{(k\cdot\pi)^2}{m^2}. \tag{30}$$

From (29), (30) we conclude that on the level N the value of the square of the Pauli–Lubanski form is equal to

$$W_N = (1-N)^2. \tag{31}$$

As it is well known it defines fixed helicity states, when $W = 0$ and continuous spin representations, when $W \neq 0$ [18,19,27–29]. Therefore *only* $N = 1$ *state realizes the fixed helicity representations, whereas the ground state* $N = 0$ *and the rest of the excited states* $N \geqslant 2$ *realize continuous spin representations of the massless little group* $SO(11)$. The corresponding vertex operator $U_{k,\pi}$ $(N = 1)$ in open strings case is

$$U_{k,\pi} = \zeta\circ\dot{\Phi}e^{iK\circ\Phi} = (\xi\cdot\dot{\Pi} + \omega\cdot\dot{X})e^{ik\cdot X + i\pi\cdot\Pi}, \tag{32}$$

where in 26 dimensions $K \circ \zeta = 0$, $K \circ K = 0$ or, being translated through our dictionary (26) into 13 dimensions, it will take the form: $k \cdot \xi(k,\pi) + \pi \cdot \omega(k,\pi) = 0$, $k \cdot \pi = 0$. The $U_{k,\pi}$ operators are of the essential importance, because for them $W \sim (k \cdot \pi)^2 = 0$, and they create fixed helicity massless gauge particles [30–41].

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