# Dual Drive Curve Tool Path Planning Method for 5-axis NC Machining of Sculptured Surfaces 

Xu Rufeng ${ }^{\mathrm{a}}$, Chen Zhitong ${ }^{\mathrm{a}, *}$, Chen Wuyi ${ }^{\mathrm{a}}$, Wu Xianzhen ${ }^{\mathrm{b}}$, Zhu Jianjun ${ }^{\text {b }}$<br>${ }^{\text {a }}$ School of Mechanical Engineering and Automation, Beijing University of Aeronautics and Astronautics, Beijing 100191, China

${ }^{\mathrm{b}}$ Changhe Aircraft Industries Group, Jingdezhen 333000, China
Received 15 September 2009; accepted 4 March 2010


#### Abstract

The problem of finished surface being not first-order continuous commonly exists in machining sculptured surfaces with a torus cutter and some other types of cutters. To solve this problem, a dual drive curve tool path planning method is proposed in this article. First, the maximum machining strip width of a whole tool path can be obtained through optimizing each tool position with multi-point machining (MPM) method. Second, two drive curves are then determined according to the obtained maximum machining strip width. Finally, the tool is positioned once more along the dual drive curve under the condition of tool path smoothness. A computer simulation and cutting experiments are carried out to testify the performance of the new method. The machined surface is measured with a coordinate measuring machine (CMM) to examine the machining quality. The results obtained show that this method can effectively eliminate sharp scallops between adjacent tool paths, keep tool paths smooth, and improve the surface machining quality as well as machining efficiency.


Keywords: dual drive curve; tool path planning; multi-point machining; 5-axis; sculptured surfaces; numerical control (NC); machining

## 1. Introduction

Machining of sculptured surfaces with a torus cutter or barrel cutter can dramatically increase the machining strip width and decrease the scallop height, which has actually been attracting much attention in recent years. H. Damsohn ${ }^{[1]}$ primarily analyzed the relationship of cutter types and tool positions within a given tolerance. H. Zhang ${ }^{[2]}$ suggested that increasing the strip width and decreasing the scallop height would be the developing trend of sculptured surface machining. Then he proposed the concept of strip-width maximization machining which is a method to obtain the maximum machining strip width through optimizing the tool position, tool path, tool geometry etc. The shortest distance curve pair was published by Y. R. Ni, et al. ${ }^{[3-4]}$ to obtain the error distribution between the

[^0]1000-9361 © 2010 Elsevier Ltd. Open access under CC BY-NC-ND license. doi: 10.1016/S1000-9361(09)60245-4
cutter and workpiece surface. J. Kruth, et al. ${ }^{[5]}$ presented a tool positioning method which varies the tool inclination during the tool path generation to achieve the best combination of scallop height, workpiece accuracy, surface accuracy, surface roughness and machining time. A. Warkentin, et al. ${ }^{[6-9]}$ pointed out that at least two contact points are available between the tool and workpiece surface and developed a multi-point machining (MPM) method, which actually broke through the limitation of tool positioning using only a single contact point and local surface curvatures. M. Engeli, et al. ${ }^{[10]}$ pointed out that the relative movement of the torus cutter and the workpiece surface is approximately equivalent to that of the torus central circle and the parallel surface, and proposed three tool positioning methods, i.e. a Hermite method, a Her-mite-Chebyshev method and a Taylor method. Besides, they fitted the error distribution curve between the tool and the design surface into Hermite polynomial, and described the "W" shape of the error distribution curve by the Hermite-Chebyshev method. R. Q. Wang, et al. ${ }^{[11-14]}$ investigated both the MPM method and Hermite method. The theory frame of multi-point tangent contact machining was thus put forward. All the methods above have effectively promoted the progress of strip-width maximization machining technology.

Tool path planning methods of sculptured surfaces machining generally include iso-parametric methods ${ }^{[15-17]}$, iso-planar methods ${ }^{[18-20]}$, iso-scallop methods $^{[20-26]}$ and so on. Y. S. Lee ${ }^{[23]}$ proposed a nonisoparametric tool path planning method (i.e. an iso-scallop method), which uses all right side end points of the previous tool path as the left side end points of the current one. The two adjacent tool paths are hence closely joined together. Yet this method of tool path planning easily leads to serious malformation of the subsequent tool paths, which actually not only results in the high acceleration of the cutter that often brings harm to the machine tool, but also decreases the effective machining strip width of the subsequent tool paths. As a result, this method actually does not provide satisfying machining efficiency and quality, and proves to be of no practical value. M. Jin, et al. ${ }^{[27]}$ presented end-points error controlling (EPEC) method for 5-axis sculptured surfaces machining, which increases the machining strip width effectively. If the scallops at the juncture between two adjacent tool paths are not taken into consideration, this method does provide higher machining efficiency. However, the inevitably produced sharp scallops seriously lower the surface machining quality and thus result in more operations of manual grinding and polishing. M. Engeli, et al. ${ }^{[10]}$ developed a Hermite method for the cylindrical cutter machining sculptured surfaces. However, they did not provide an effective tool path planning method for the circumstance when the separation distances between the pairs of contact points on a single tool path vary largely. According to MPM method and Hermite method, if all tool positions on a single tool path are thoroughly optimized for a given tolerance, the maximum machining errors between the pairs of contact points are definitely equal. Thus, if the tool paths are planned by using traditional iso-scallop methods, serious malformation of the subsequent tool paths will happen, as illustrated in Fig.1; if the tool paths are planned in terms of the narrowest machining strip width of each tool path by using traditional iso-parametric method, the problem of exceeding the allowable tolerance between two adjacent tool paths may have been well avoided, but sharp scallops still exist at other positions, which are still required to be removed by manual polishing as shown in Fig.2. Besides, many researchers ${ }^{[28-30]}$ investigated the effect of tool path optimization on the cutting forces from the viewpoint of process mechanics. They presented tool path selection based on the minimum cutting forces. In this way, an improvement on the accuracy of machined surfaces is achieved.
All the analyses above show that the tool path planning method using contact points as the juncture of adjacent tool paths can produce smoother surface than the EPEC method can, yet it still cannot avoid the malformation of the subsequent tool paths. Based on the MPM method, this article proposes a new tool path planning method, which can effectively get rid of
sharp scallops between the adjacent tool paths, improve the surface machining quality, and reduce the time of manual polishing.


Fig. 1 Malformation of subsequent tool paths.


Fig. 2 Sharp scallops produced by iso-parametric methods.

## 2. Principle of Tool Path Planning Method

Through experimentation A. Warkentin, et al. ${ }^{[7]}$ found that in general two cutter contact (CC) points exist between the tool and the surface to be machined, and that these points are approximated symmetrical about the direction of surface minimum curvature. The distance between a pair of contact points can be maximized through optimization of tool position, when the tool geometry closely matches the part surface. For a given programming tolerance, this method can largely increase the machining strip width and thus greatly improve the machining efficiency. However, when the surface curvature varies greatly, the maximum machining strip width of each tool position can be significantly diverse. If constant maximum machining error or scallop height is set as the constraint condition
and all right side end points along one tool path will be taken as the left side end points of the next path, serious malformation of the subsequent tool paths will then be produced, as shown in Fig.1. Therefore, the smoothness of tool paths and the maximization of machining strip width must be considered together. The dual drive curve tool path planning method can meet the above requirement. Its principle is described as follows.

The first step is to find the optimal tool position and the corresponding maximum machining strip width at each CC point on the left drive curve, which is initially specified by iso-parametric or iso-planar method. At this stage, a set of the right CC points on the surface, forming the right CC path, is also obtained. The second step is to plan a new curve (i.e. the right drive curve) from the left drive curve with the narrowest strip width among the above maximum machining strip width at each CC point. Then, the tool is repositioned by a pair of CC points along the two drive curves, and smooth tool paths without gouging and excess are also generated. The right drive curve is then set as the left drive curve of the following tool path. In this way, the tool paths are arranged sequentially one after another until the workpiece surface is completely machined. Undoubtedly, the region between two adjacent tool paths is smooth and its machining errors are equal to zero. Meanwhile, sharp scallops can be avoided and at least first-order tangent contact joining between two tool paths is realized. Fig. 3 shows the tool path generation on the part surface using a dual drive curve tool path planning method. The distribution rule of the right CC point and the determination of dual drive curve will be discussed in detail as follows.


Fig. 3 Surface machined by the proposed method.

### 2.1. Distribution rule of right CC point

As shown in Fig.4(a), a torus cutter $T$ machines the part surface $S$ : $\boldsymbol{r}(u, v)(u \in[0,1], v \in[0,1]), h$ is a given tolerance, $\delta$ a maximum machining error, $S_{i}$ the $i$ th tool path $\left(i=1,2, \cdots, N_{\text {path }}\right.$, where $N_{\text {path }}$ is the number of the
total tool path), $\mathrm{tp}_{i}$ the left drive curve of tool path $S_{i}$, $\operatorname{tp}_{i+1}$ the right drive curve of tool path $S_{i} . c_{i, j-1}$ and $c_{i, j}$ are the left CC points on $\mathrm{tp}_{i}, c_{i+1, j-1}$ and $c_{i+1, j}$ are the right CC points calculated by using the MPM method when $\delta=h, c_{i+1, j}^{\prime}$ and $c_{i+1, j}^{\prime \prime}$ are also the right CC points calculated by using the MPM method when $\delta \neq h, K, K^{\prime}$ and $K^{\prime \prime}$ denote the simplified form of the torus cutter $T$. Analysis from a computational example shows that the right CC points can be located on a continuous curve $c_{i+1, j}^{\prime} c_{i+1, j} c_{i+1, j}^{\prime \prime}$, and the distance between a pair of CC points reduces as the maximum machining error $\delta$ decreases (Fig.4(b)). M. Engeli, et al. ${ }^{[10]}$ and R. Q. Wang, et al. ${ }^{[11,14]}$ sought for the optimal tool position and the maximum machining strip width at each CC point by using the same principle.


Fig. 4 Relationship between the right CC point and maximum machining error.

### 2.2. Determination of dual drive curve

For a given part surface $S$, the feed direction and pattern of tool paths need to be specified according to the surface property. Suppose the left drive curve $\mathrm{tp}_{i}$ is discretized into 5 sampling CC points $c_{i, j}(j=1,2, \cdots, 5)$,
as shown in Fig.5. For every CC point on $\operatorname{tp}_{i}$, one can obtain the corresponding right CC point $\mathrm{cr}_{i+1, j}$, which is calculated by using MPM method in terms of the given tolerance. The curve linking these contact points $\mathrm{cr}_{i+1, j}$ is called the right CC path $\operatorname{tp}_{\mathrm{r} i}$. Meanwhile, the corresponding machining strip width $w_{i, j}$ at the CC point $c_{i, j}$ can also be obtained. Among these machining strip widths $w_{i, j}$, the optimal machining strip width, $W\left(S_{i}\right)=$ $\min \left\{w_{i, j}\right\}=w_{i, 3}$, can thus be obtained as the path interval of $S_{i}$ (Fig.5). Furthermore, the right drive curve $\mathrm{tp}_{i+1}$ (i.e. the left drive curve of the next tool path $S_{i+1}$ ) can be derived from $\operatorname{tp}_{i}$ and $W\left(S_{i}\right)$. Accordingly, two drive curves $\operatorname{tp}_{i}$ and $\mathrm{tp}_{i+1}$ are determined on the part surface $S$.


Fig. 5 Scheme of determination of dual drive curve.

## 3. Tool Path Planning Method

### 3.1. Mathematical model of tool path optimization

Suppose $r(u, v)(u \in[0,1], v \in[0,1])$ is the equation of design surface $S$, which is machined on a 5 -axis machine tool, and tool geometry is also given. If the total length of tool path is shorter, machining efficiency will be higher. The total length of tool path is thus a target function of tool path optimization. It must satisfy the following four constraint conditions. First, the summation of all machining strip widths of tool path is just equal to the surface machining limitation. Second, the intersection of effective machined surfaces of any two adjacent tool paths is just a drive curve between them, i.e. scallop height between any two adjacent tool paths is zero. Third, the machined surface errors are less than or equal to the given tolerance. Fourth, keep each tool path smooth. Hence, a mathematical model of tool path optimization can be expressed as

$$
\min \sum_{i=1}^{N_{\text {palh }}} f\left(S_{i}\right)
$$

$$
\text { s.t. }\left\{\begin{array}{l}
\sum_{i=1}^{N_{\text {path }}} W\left(S_{i}\right)=1 \\
\Omega\left(S_{i}\right) \cap \Omega\left(S_{i+1}\right)=\left\{\operatorname{tp}_{i+1}\right\}  \tag{1}\\
\operatorname{dist}\left(P_{\Omega}, S\right) \leq h, P_{\Omega} \in \Omega\left(S_{i}\right) \\
S_{i} \in D_{0}
\end{array}\right.
$$

where $f\left(S_{i}\right)$ is the length of tool path $S_{i}, \Omega\left(S_{i}\right)$ the effective machined surface of $S_{i}, \operatorname{tp}_{i+1}$ the right drive curve of $S_{i}$, i.e. the left drive curve of $S_{i+1}, P_{\Omega}$ any point on the machined surface $\Omega\left(S_{i}\right)$, dist ( $P_{\Omega}, S$ ) the minimum distance between point $P_{\Omega}$ and surface $S, D_{0}$ the constraint condition of smoothness of tool path $S_{i}$.

The tool path length along parameter $u$ or $v$ direction for given design surface changes slightly for most sculptured surfaces, the total length of tool path is thus dependent on the number of tool paths. In other words, if the number of tool paths is smaller, the total tool path length becomes shorter. Likewise, if the machining strip width of each tool path is wider, the number of tool paths also becomes smaller. Consequently, the mathematical model of tool path optimization in Eq.(1) can be transformed into how to solve the optimal machining strip width of tool path $S_{i}$ and how to generate optimal tool positions along tool path $S_{i}$.
(1) Searching for the optimal machining strip width of a tool path

Section 2.2 presents an approach of dual drive curve to calculate the optimal machining strip width of tool path $S_{i}$. Through discretizing the left drive curve into some sampling CC points, the tool positions without gouging and excess and the corresponding maximum machining strip widths at these CC points can be obtained. The minimum strip width among these machining strip widths is referred as the optimal machining strip width of the current tool path. Therefore, a mathematical model of calculating the optimal machining strip width of tool path is expressed as

$$
\begin{gather*}
W\left(S_{i}\right)=\min \left\{w_{i, j}^{*}=\max w_{i, j}(\rho, \phi) \mid j=1,2, \cdots, M_{\mathrm{sam}}\right\} \\
\text { s.t. }(\rho, \phi) \in D_{1} \cap D_{2} \tag{2}
\end{gather*}
$$

where $w_{i, j}(\rho, \phi)$ the machining strip width at CC point $c_{i, j}$ on the drive curve $\operatorname{tp}_{i}, M_{\text {sam }}$ the number of sampling CC points on $\mathrm{tp}_{i}, \rho$ the distance between two CC points in parametric fields, $\phi$ an angle between the line joining two CC points and $u$ or $v$ direction in parametric fields. Besides, $\rho$ and $\phi$ must also meet the constraint conditions of $D_{1}$ and $D_{2}$.
$D_{1}$ and $D_{2}$ may be defined respectively by

$$
\begin{align*}
& D_{1}=\left\{(\rho, \phi) \mid \min E_{\mathrm{f}}(\rho, \phi) \geq 0, \max E_{\mathrm{f}}(\rho, \phi) \leq h\right\}  \tag{3}\\
& D_{2}=\left\{(\rho, \phi) \mid \min E_{\mathrm{f}}(\rho, \phi) \geq 0, \min E_{\mathrm{b}}(\rho, \phi) \geq 0\right\} \tag{4}
\end{align*}
$$

where $E_{\mathrm{f}}(\rho, \phi)$ is a set of machining errors between the front cutting portion of the cutter and the design sur-
face, $E_{\mathrm{b}}(\rho, \phi)$ a set of machining errors between the back cutting portion of the cutter and the design surface.
(2) Calculating the optimal tool position

The right drive curve $\mathrm{tp}_{i+1}$ can be determined based on the above optimal machining width $W\left(S_{i}\right)$. Then, the tool can move along the dual drive curve, and be repositioned. Meanwhile, the tool must tangentially contact the design surface on the dual drive curve without gouging the surface. It can be concluded that the tilt angle at each CC point has an important impact on the tool path quality from the actual computation of tool path generation. As the value of parameter $u$ or $v$ on the left drive curve changes, the tilt angle must vary continuously and smoothly. Only in this way, the generated tool path becomes smooth. A cubic spline function, $\phi=S(t)$, is constructed by using the sampling CC points $c_{i, j}$ on the left drive curve of tool path $S_{i}$ as an interpolation point and its corresponding tilt angle $\phi_{i, j}$ as a function value, where $t$ is the value of parameter $u$ or $v$. Assuming the tool moves along the parameter $v$ direction, there is a corresponding tilt angle $\phi_{i, k}$ for $\forall c_{i, k}\left(u_{i}, v_{i, k}\right)\left(k=1,2, \cdots, M_{\mathrm{cc}}\right.$, where $M_{\mathrm{cc}}$ is the number of CC points on $\mathrm{tp}_{i}$ according to step length tolerance) on the drive curve $\mathrm{tp}_{i}$. Hence, the right CC point $c_{i+1, k}\left(u_{i+1}, v_{i+1, k}\right)$ on $\mathrm{tp}_{i+1}$ can be determined in terms of the above angle $\phi_{i, k}$ and $W\left(S_{i}\right)$.

$$
\left.\begin{array}{l}
u_{i+1}=u_{i}+W\left(S_{i}\right)  \tag{5}\\
v_{i+1, k}=v_{i, k}+W\left(S_{i}\right) \tan \phi_{i, k}
\end{array}\right\}
$$

Finally, the optimal tool position can be calculated by using the MPM method for a pair of $c_{i, k}$ and $c_{i+1, k}$ on two drive curves.

### 3.2. Detailed process of tool path planning

First of all, the optimal feed direction can be specified by analyzing the property of the part surface $S$ : $\boldsymbol{r}(u, v)(u \in[0,1], v \in[0,1])$. As shown in Fig.6, the process of generating tool paths is described in the parametric domains. Suppose the tool machines the surface $S$ along the direction of parameter $v$, and the machining parametric region needs to satisfy $u \in\left[u_{\text {min }}\right.$, $\left.u_{\text {max }}\right]$ and $v \in\left[v_{\text {min }}, v_{\text {max }}\right]$, where $u_{\text {min }}$ and $u_{\text {max }}$ are the left and right boundaries of machining region, $v_{\text {min }}$ and $v_{\text {max }}$ the front and back boundaries of machining region. In Fig.6, $\rho$ is the parametric distance between two CC points, i.e. the radius of the parametric circle $\pi$. Fig. 7 shows the flowchart of tool path planning, and the detailed process of tool path planning is described as follows.

Step 1 Set the machining range of the part surface $S, u \in\left[u_{\min }, u_{\max }\right]$ and $v \in\left[v_{\min }, v_{\max }\right]$, and a given programming tolerance $h$.

Step 2 Assume the parametric equation of the left drive curve $\mathrm{tp}_{i}$ is $u=u_{i}$ (see Fig.6).


Fig. 6 Tool path distribution in parametric domains.


Fig. 7 Flowchart of tool path planning.

Step 3 The left drive curve $\mathrm{tp}_{i}$ is evenly discretized into $M_{\text {sam }}$ sampling CC points $c_{i, j}\left(j=1,2, \cdots, M_{\text {sam }}\right)$. For $\forall c_{i, j}$ on $\operatorname{tp}_{i}$, we can obtain its parametric coordinates $\left(u_{i}, v_{j}\right)$, where $v_{j}=v_{\min }+(j-1)\left(v_{\max }-v_{\min }\right) /\left(M_{\mathrm{sam}}-1\right)$.

Step 4 For $\forall c_{i, j}$ on the curve $\operatorname{tp}_{i}$, one can obtain the second CC point $\mathrm{cr}_{i+1, j}$ which are calculated by using the MPM method when the maximum machining error $\delta$ is equal to a given tolerance $h$. Likewise, the maximum machining strip width $w_{i, j}^{*}\left(\rho_{i, j}^{*}, \phi_{i, j}^{*}\right)=$ max $w(\rho, \phi)$ at the sampling points $c_{i, j}$ and the corresponding tilt angle $\phi_{i, j}^{*}$ are also calculated.

Step 5 If $j<M_{\text {sam }}, j=j+1$, go to Step 4; else go to Step 6.
Step 6 The optimal machining strip width $W\left(S_{i}\right)$ of tool path $S_{i}$ can be calculated according to Eq.(2). Then, the right drive curve $\mathrm{tp}_{i+1}$ of $S_{i}$ (i.e. the left drive curve of $S_{i+1}$ ) can be calculated from the equation $u=u_{i+1}=u_{i}+W\left(S_{i}\right)$. If $u_{i+1}>u_{\text {max }}$, then $u_{i+1}=u_{\text {max }}$.

Step 7 A cubic spline function, $\phi=S(t)$, is constructed by using the sampling CC points $c_{i, j}$ on the curve $\mathrm{tp}_{i}$ of $S_{i}$ as an interpolation point and the corresponding tilt angle $\phi_{i, j}^{*}$ as a function value, where $t$ is the value of parameter $v$ of the curve $\mathrm{tp}_{i}$.

Step 8 According to a given step length tolerance $h$, the curve $\mathrm{tp}_{i}$ is evenly discretized again into $M_{\mathrm{cc}}$ CC points $c_{i, k}\left(k=1,2, \cdots, M_{\text {cc }}\right)$.

Step 9 For $\forall c_{i, k}$ on the curve $\operatorname{tp}_{i}$, we can obtain its parametric coordinates $\left(u_{i}, v_{i, k}\right)$, where $v_{i, k}=v_{\text {min }}+$ $(k-1)\left(v_{\max }-v_{\min }\right) /\left(M_{\mathrm{cc}}-1\right)$, and its corresponding tilt angle $\phi_{i, k}=S\left(v_{i, k}\right)$. According to Eq.(3), we can then obtain parametric coordinate $\left(u_{i+1}, v_{i+1, k}\right)$ of the right CC point $c_{i+1, k}$, where $u_{i+1}=u_{i}+W\left(S_{i}\right), v_{i+1, k}=v_{i, k}+$ $W\left(S_{i}\right) \tan \phi_{i, k}$.

Step 10 For a pair of $c_{i, k}$ and $c_{i+1, k}$ on two drive curves, the optimal tool position, i.e. the tool axis vec$\operatorname{tor} \boldsymbol{T}_{i, k}^{\text {axis }}$ and the tool position vector $\boldsymbol{T}_{i, k}^{\mathrm{pos}}$, can be calculated by using the MPM method.

Step 11 If $k<M_{\mathrm{cc}}, k=k+1$, go to Step 9; else go to Step 12.

Step 12 If $i<N_{\text {path }}$, go to Step 13; else the procedure ends.

Step 13 If $u_{i+1}<u_{\text {max }}$, i.e. the right drive curve $\mathrm{tp}_{i+1}$ of $S_{i}$ does not exceed the right boundary $u_{\text {max }}$ of machining region, $i=i+1$, go to Step 2; else the part surface has been machined completely and the procedure ends.

## 4. Machining Test

To verify the new tool path planning method discussed above, we selected an open form surface ${ }^{[9]}$ (see Fig.8) which is commonly found in the die industry as an example surface. The surface is defined as follows:

$$
\boldsymbol{r}(u, v)=\left[\begin{array}{c}
-94.4+88.9 v+5.6 v^{2}  \tag{6}\\
-131.3 u+28.1 u^{2} \\
a_{1}+a_{2}
\end{array}\right]
$$

where $a_{1}=5.9\left(u^{2} v^{2}+u^{2} v\right)-3.9 v^{2} u+76.2 u^{2} ; a_{2}=6.7 v^{2}-$ $27.3 u v-50.8 u+25.0 v+12.1$.


Fig. 8 Example surface.
A torus tool with a torus radius of 5 mm and an insert radius of 3 mm was used for simulations and cutting tests. The given programming tolerance $h$ was set as 0.01 mm . Because the minimum principal direction of the part surface is approximately along the direction of parameter $v$, the tool machines the part surface along the direction of parameter $v$. The surface machining range satisfies the condition of $u \in[0,1]$ and $v \in[0,1]$ in parametric domains. Assume each left drive is evenly discretized into 10 sampling CC points (it is mainly dependent on the surface property), and each left drive is evenly discretized into 150 CC points according to a step length tolerance. The calculating procedure of tool path optimization is programmed according to the developed method in Section 3, based on UG/OPEN API and C programming language. Tool paths can be generated by using the developed method in the software UG environment (see Fig.9).


Fig. 9 Tool paths generated by using the developed method.

### 4.1. Simulation results

The surface machining process was simulated in the software VERICUT with the cutter location resource file (CLSF) from UG. Results of computer simulation are shown in Fig.10. The thick solid lines denote the actual cutting tool paths, and the dashed ones denote the traverse tool paths. The dark regions on the machined surface denote actual areas with error varying from 0.005 mm to 0.010 mm , while the white regions denote areas with error less than 0.005 mm or even with no excess. There is also no gouging on the whole machined surface. As shown in Fig.10, there is no excess between two adjacent tool paths, i.e. machined surface errors at the juncture between each two adjacent tool paths are all approximately zero.


Fig. 10 Simulation results in software VERICUT.
Therefore, areas between two adjacent tool paths are connected smoothly and without sharp scallops. Besides, machined surface errors only exist near the thick solid lines (i.e. the cutting tool paths). In other words, excess only exists in the middle of each tool path. The error distribution curve between the tool and the part surface is first-order continuous, so sharp scallops do not exist.

### 4.2. Experimental results

A machining test was conducted on a FIDIA's G996RT 5-axis machining center with a tilt-rotary table by using the proposed tool path planning method. The cutting conditions for the 5 -axis machining were as follows: the spindle speed was $16000 \mathrm{r} / \mathrm{min}$, the feed rate $5000 \mathrm{~mm} / \mathrm{min}$, and depth of cut 0.5 mm , respectively. The total time of surface machining was about 1 min . Fig. 11 presents a photo of the machined surface. A CMM (Hexagonmetrology Global Advantage 153010) was used to measure three selected sections of the machined surface with measuring step length of

## 0.5 mm .

Fig. 12 shows the surface deviation $E$ along the normal vectors between the actual machined surface and design surface in the three sections. The three sections are selected in the $O Y Z$ plane at $X=-5 \mathrm{~mm}$, $X=-30 \mathrm{~mm}$, and $X=-60 \mathrm{~mm}$, respectively. In Fig.12, "*" denotes the error between the actual machined surface and design surface, and the three traces joining the mark "*" approximately reflect the error distribution in the resulting profile. From Fig.12, it can be observed that the machining errors vary from -0.05 mm to 0.04 mm as the $Y$ coordinates increase. This type of error is called form error caused by fixture, tool deflections, the CMM measurements, the inconsistency of the workpiece coordinate system and the measured coordinate system as well as the machine coordinate system, etc. If the form error is disregarded, the scallop heights are approximately within 0.01 mm . Regions of large scallops mainly distribute in the range from -65 mm to -95 mm , which corresponds with the simulation results (see Fig.10). At the junctures between adjacent tool paths, which are shown as the valleys in Fig.12, no sharp scallops exist, and the traces are also continuous and smooth; while in the regions between two adjacent contact points, which are shown as the peaks in the figure, large rounded scallops exist, but these scallops are generally smooth connections


Fig. 11 Photo of machined surface.

(a) $X=-5 \mathrm{~mm}$

(b) $X=-30 \mathrm{~mm}$


Fig. 12 Measured results of the machined surface.
without sharp ones. From the good consistency of measured and simulated results, we can come to the conclusion that the finish surface is first-order continuous, and the proposed tool path planning method can eliminate sharp scallops, keep tool paths smooth, and improve the surface machining quality as well as machining efficiency.

## 5. Conclusions

(1) The problem of finished surface being not first-order continuous commonly exist in machining sculptured surfaces with a torus cutter and some other types of cutters. To solve this problem, a dual drive curve tool path planning method is proposed, and a mathematical model of tool path optimization is also established.
(2) The methods of determining dual drive curve and calculating the tilt angle using the cubic spline interpolating approach are presented in the article. The tool can be repositioned along two drive curves. Tool paths without gouging and excess are thus generated.
(3) The proposed method of tool path optimization is verified through computer simulation and machining test. The results obtained show that scallop heights between adjacent tool paths are zero, and the measured section curve is first-order continuous. This method can thus effectively eliminate sharp scallops, keep tool paths smooth, and improve the surface machining quality as well as machining efficiency.

## Acknowledgements

The authors would like to thank Xiong Xiyao, Lin Dong, Xu Yiping, Jiang Like, Tao Jianfeng, et al. for their help, and Changhe Aircraft Industries Group for the use of equipment.

## References

[1] Damsohn H. Fünfachsiges NC-fräsen, ein beitrag zur technologie teileprogrammierung und postprozessorverarbeitung. PhD thesis, University of Stuttgart, 1976. [in German]
[2] Zhang H. The developing trend of sculptured surface machining in increasing the strip width and decrease the scallop height. Surface Machining and Checking. Beijing: Beijing Aeronautics Institute, 1987. [in Chinese]
[3] Ni Y R, Ma D Z, Zhang H, et al. Optimal orientation control for torus tool 5-axis sculptured surface NC machining. Chinese Journal of Mechanical Engineering 2001; 37(2): 87-91. [in Chinese]
[4] Ni Y R. Optimal torus-shaped end-mill orientation control for 5 -axis sculptured surface NC machining and graphics simulation. MS thesis, Beijing University of Aeronautics and Astronautics, 1999. [in Chinese]
[5] Kruth J, Klewais P. Optimization and dynamic adaptation of the cutter inclination during five-axis milling of sculptured surfaces. CIRP Annals - Manufacturing Technology 1994; 43(1): 443-448.
[6] Warkentin A, Bedi S, Ismail F. Five-axis milling of spherical surfaces. International Journal of Machine Tools and Manufacture 1996; 36(2): 229-243.
[7] Warkentin A, Ismail F, Bedi S. Intersection approach to multi-point machining of sculptured surfaces. Computer Aided Geometric Design 1998; 15(6): 567-584.
[8] Warkentin A, Ismail F, Bedi S. Multi-point tool positioning strategy for 5 -axis machining of sculptured surfaces. Computer Aided Geometric Design 2000; 17(1): 83-100.
[9] Warkentin A, Ismail F, Bedi S. Comparison between multi-point and other 5-axis tool positioning strategies. International Journal of Machine Tools and Manufacture 2000; 40(2): 185-208.
[10] Engeli M, Waldvogel J, Schnider T. Method for processing work pieces by removing material. US Patent: No.6,485,236 B1, 2002.
[11] Wang R Q, Chen W Y. Secant algorithm in multi-point contact machining. Chinese Journal of Mechanical Engineering 2006; 42(7): 203-206. [in Chinese]
[12] Wang R Q, Chen W Y, Jin M. Local interference identification of multi-point machining. Journal of Beijing University of Aeronautics and Astronautics 2006; 32(5): 580-584. [in Chinese]
[13] Wang R Q, Bian Y M, Chen W Y. Hermite method convergence analyses and improvement. Journal of Beijing University of Aeronautics and Astronautics 2006; 32(2): 239-243. [in Chinese]
[14] Wang R Q. Theory analyses on wider-stripe tool path generation methods and their applications on blade machining. PhD thesis, Beijing University of Aeronautics and Astronautics, 2007. [in Chinese]
[15] Broomhead P, Edkins M. Generating NC data at the machine tool for the manufacture of free-form surfaces. International Journal of Production Research 1986; 24(1): 1-14.
[16] Loney G, Ozsoy T. NC machining of free-form surfaces. Computer-Aided Design 1987; 19(2): 85-90.
[17] Elber G, Cohen E. Toolpath generation for freeform surface models. Computer-Aided Design 1994; 26(6): 490-496.
[18] Bobrow J. NC machine tool path generation from CSG part representations. Computer-Aided Design 1985; 17(2): 69-76.
[19] Yong S S, Lee K. NC milling tool path generation for arbitrary pockets defined by sculptured surfaces. Com-puter-Aided Design 1990; 22(5): 273-284.
[20] Huang Y, Oliver J. Non-constant parameter NC tool path generation of sculptured surfaces. International Journal of Advanced Manufacturing Technology 1994; 9(5): 281-290.
[21] Suresh K, Yang D C H. Constant scallop-height machining of free-form surfaces. Journal of Engineering for Industry 1994; 116(2): 253-259.
[22] Sarma R, Dutta D. Geometry and generation of NC tool paths. Journal of Mechanical Design, Transactions of the ASME 1997; 119(2): 253-258.
[23] Lee Y S. Non-isoparametric tool path planning by machining strip evaluation for 5 -axis sculptured surface machining. Computer-Aided Design 1998; 30(7): 559-570.
[24] Feng H, Li H. Constant scallop-height tool path generation for three-axis sculptured surface machining. Computer-Aided Design 2002; 34(9): 647-654.
[25] Tournier C, Duc E. A surface based approach for con-
stant scallop height tool-path generation. The International Journal of Advanced Manufacturing Technology 2002; 19(5): 318-324.
[26] Tournier C, Duc E. Iso-scallop tool path generation in 5 -axis milling. The International Journal of Advanced Manufacturing Technology 2005; 25(9): 867-875.
[27] Jin M, Zhang L, Chen Z T. End-points error controlling method for torus tool position optimization in five-axis NC machining. Journal of Beijing University of Aeronautics and Astronautics 2006; 32(9): 1125-1128. [in Chinese]
[28] López de Lacalle L N, Lamikiz A, Muñoa J, et al. The CAM as the centre of gravity of the five-axis high speed milling of complex parts. International Journal of Production Research 2005; 43(10): 1983-1999.
[29] López de Lacalle L N, Lamikiz A, Sáchez J A, et al. Toolpath selection based on the minimum deflection cutting forces in the programming of complex surfaces milling. International Journal of Machine Tools and Manufacture 2007; 47(2): 388-400.
[30] Lazoglu I, Manav C, Murtezaoglu Y. Tool path optimization for free form surface machining. CIRP An-nals-Manufacturing Technology 2009; 58(1): 101104.

## Biographies:

Xu Rufeng Born in 1980, he received B.S. and M.S. degrees from Shandong University of Technology in 2003 and 2006 respectively, and is currently a doctoral candidate at Beijing University of Aeronautics and Astronautics. His main research interest includes 5 -axis NC machining of sculptured surfaces and CAM software development. E-mail: xurufeng2003@126.com

Chen Zhitong Born in 1967, he received M.S. and Ph.D. degrees from Beijing University of Aeronautics and Astronautics in 1992 and 2001, respectively. He is a professor at School of Mechanical Engineering and Automation, Beijing University of Aeronautics and Astronautics. His research interest includes 5 -axis NC machining of sculptured surfaces and CAM software development.
E-mail: ztchen@buaa.edu.cn


[^0]:    *Corresponding author. Tel.: +86-10-82316091.
    E-mail address: ztchen@buaa.edu.cn
    Foundation items: National Natural Science Foundation of China (50875012); National High-tech Research and Development Program (2008AA04Z124); National Science and Technology Major Project (2009ZX04001-141); Joint Construction Project of Beijing Municipal Commission of Education

