Resolving the black-hole information paradox by treating time on an equal footing with space

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ABSTRACT

Pure states in quantum field theory can be represented by many-fingered block-time wave functions, which treat time on an equal footing with space and make the notions of "time evolution" and "state at a given time" fundamentally irrelevant. Instead of information destruction resulting from an attempt to use a "state at a given time" to describe semi-classical black-hole evaporation, the full many-fingered block-time wave function of the universe conserves information by describing the correlations of outgoing Hawking particles in the future with ingoing Hawking particles in the past.

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1. Introduction

The semi-classical description of black-hole evaporation [1] predicts that the final state after the complete evaporation cannot be represented by a pure state [2]. A transition from a pure to a non-pure (i.e., mixed) state contradicts unitarity of quantum mechanics and leads also to other pathologies [3]. Many approaches to restore a pure-state description at late times have been attempted, but none of them seems to be completely satisfying (for reviews see, e.g., [4]).

To overcome this problem, we start with the observation that all these previous approaches (with a notable exception in [5]) share one common assumption: that the quantum state (either pure or mixed) should be a function of time, or more generally, a functional of the spacelike hypersurface. Indeed, such an assumption is deeply rooted in our intuitive understanding of the concept of time, according to which universe evolves with time. Yet, such a view of time does not seem to be compatible with the classical theory of relativity (both special and general). The picture of a "time-evolving" universe seems particularly unappealing when the universe violates the condition of global hyperbolicity, which, indeed, is the case with completely evaporating black holes (see Fig. 1). Instead, one of the main messages of the theory of relativity is that time should be treated on an equal footing with space. In particular, it seems natural to adopt the *block time* (also known under the name block universe; see, e.g., [6] and references therein) picture of the universe, according to which the universe does *not evolve* with time, but is a "static" 4-dimensional object in which "past", "presence", and "future" equally exist. For exam-

ple, such a view automatically resolves causal paradoxes associated with closed causal curves [7].

The basic intuitive idea how the block-time picture of the universe resolves the black-hole information paradox can be seen from Fig. 1 (see also [5]). From the standard point of view, only the outgoing particle exists in the far future, while the ingoing particle is destroyed. Consequently, information encoded in the correlations between outgoing and ingoing particles is destroyed. On the other hand, from the block-time point of view the past also exists, so the information is not destroyed because the outgoing particle in the far future is correlated with the ingoing particle in the past. The aim of this Letter is to put this intuitive idea into a more precise framework. In the next section we briefly review the main ideas of the general formalism of treating time in quantum theory on an equal footing with space, while the implications on Hawking evaporation are discussed in Section 3.

2. Treating time in QM on an equal footing with space

The first step towards treating time on an equal footing with space in quantum mechanics (QM) is to extend the probabilistic interpretation of a 1-particle wave function $\psi(\mathbf{x}, t) \equiv \psi(x)$ [8,9]. Instead of the usual infinitesimal probability of finding particle at the space position $\mathbf{x}$

$$dP_{(3)} = |\psi(\mathbf{x}, t)|^2 d^3x, \tag{1}$$

one has the infinitesimal probability of finding particle at the spacetime position x

$$dP = |\psi(x)|^2 d^4x. \tag{2}$$

The usual probability (1) is then recovered from (2) as a special case, corresponding to the *conditional* probability that the particle
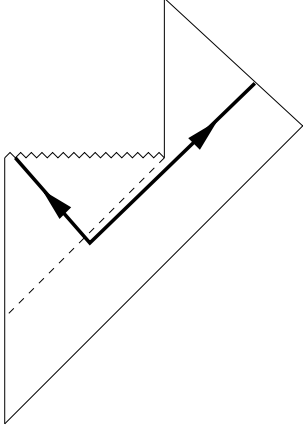
Fig. 1. Penrose diagram of a completely evaporating black hole. The lines with arrows represent a Hawking pair of particles.

will be found at $\mathbf{x}$ if it is already known that it is detected at time t. More precisely, since ψ in (1) and (2) do not have the same normalizations, the variant of (1) that emerges from (2) should be written as

$$dP_{(3)} = \frac{|\psi(\mathbf{x}, t)|^2 \, d^3 x}{N_t}, \tag{3}$$

where

$$N_t = \int |\psi(\mathbf{x}, t)|^2 \, d^3 x \tag{4}$$

is the normalization factor. As discussed in [9], such a generalized probabilistic interpretation allows to define the time operator in QM, solves the problem of probabilistic interpretation of solutions to the Klein–Gordon equation, and provides a better explanation of the standard rule that transition amplitudes should be interpreted in terms of transition probabilities *per unit time*.

The next step is to generalize this to the case of many-particle wave functions. To treat time on an equal footing with space, one needs to introduce a many-fingered time wave function [10]. A state describing n particles is described by the many-fingered time wave function $\psi(\mathbf{x}_1, t_1, \ldots, \mathbf{x}_n, t_n) \equiv \psi(x_1, \ldots, x_n)$. Consequently, (2) generalizes to [9]

$$dP = |\psi(x_1, \ldots, x_n)|^2 \, d^4 x_1 \cdots d^4 x_n. \tag{5}$$

In particular, if the first particle is detected at t_1, second particle at t_2, etc., then Eq. (3) generalizes to

$$dP_{(3n)} = \frac{|\psi(\mathbf{x}_1, t_1, \ldots, \mathbf{x}_n, t_n)|^2 \, d^3 x_1 \cdots d^3 x_n}{N_{t_1, \ldots, t_n}}, \tag{6}$$

where

$$N_{t_1, \ldots, t_n} = \int |\psi(\mathbf{x}_1, t_1, \ldots, \mathbf{x}_n, t_n)|^2 \, d^3 x_1 \cdots d^3 x_n. \tag{7}$$

Indeed, (6) coincides with the usual probabilistic interpretation of the many-fingered time wave function [10]. The more familiar single-time wave function is a special case corresponding to the time-coincidence limit

$$\psi(\mathbf{x}_1, \ldots, \mathbf{x}_n; t) = \psi(\mathbf{x}_1, t_1, \ldots, \mathbf{x}_n, t_n)\big|_{t_1 = \cdots = t_n \equiv t}. \tag{8}$$

In this case (6) reduces to the familiar single-time probabilistic interpretation

$$dP_{(3n)} = \frac{|\psi(\mathbf{x}_1, \ldots, \mathbf{x}_n; t)|^2 \, d^3 x_1 \cdots d^3 x_n}{N_t}, \tag{9}$$

where N_t is given by (7) at $t_1 = \cdots = t_n \equiv t$.

A more difficult step is to generalize this to quantum field theory (QFT), where the number of particles may be uncertain and may change. The appropriate formalism has recently been developed in [11]. Instead of repeating the whole analysis, let us briefly review the final results. In general, a QFT state is described by a wave function $\Psi(x_1, x_2, \ldots)$ that depends on an infinite number of spacetime positions x_A, $A = 1, 2, \ldots, \infty$. Introducing the notation

$$\vec{x} = \{x_1, x_2, \ldots\}, \tag{10}$$

a QFT state $|\Psi\rangle$ can be represented by the wave function

$$\Psi(\vec{x}) = \langle \vec{x} | \Psi \rangle \tag{11}$$

satisfying the normalization condition

$$\int \mathcal{D}\vec{x} \, |\Psi(\vec{x})|^2 = 1, \tag{12}$$

where

$$\mathcal{D}\vec{x} = \prod_{A=1}^{\infty} d^4 x_A. \tag{13}$$

Each state can be expanded as

$$\Psi(\vec{x}) = \sum_{n=0}^{\infty} \tilde{\Psi}_n(\vec{x}), \tag{14}$$

where $\tilde{\Psi}_n(\vec{x})$ really depends only on n coordinates x_A and represents an n-particle wave function. The tilde on $\tilde{\Psi}_n$ denotes that this wave function is not normalized. For free fields, i.e., when the number of particles does not change, the expansion (14) can be written in the form

$$\Psi(\vec{x}) = \sum_{n=0}^{\infty} c_n \Psi_n(\vec{x}), \tag{15}$$

where $\Psi_n(\vec{x})$ are normalized n-particle wave functions

$$\int \mathcal{D}\vec{x} \, |\Psi_n(\vec{x})|^2 = 1, \tag{16}$$

and c_n are coefficients satisfying the normalization condition

$$\sum_{n=0}^{\infty} |c_n|^2 = 1. \tag{17}$$

In particular, the vacuum (i.e., the state without particles) is represented by a constant wave function

$$\Psi_0(\vec{x}) = \frac{1}{\sqrt{\mathcal{V}}}, \tag{18}$$

where $\mathcal{V}$ is the volume of the configuration space

$$\mathcal{V} = \int \mathcal{D}\vec{x}. \tag{19}$$

The probabilistic interpretation of (14) is given by a natural generalization of (5)

$$\mathcal{D}P = |\Psi(\vec{x})|^2 \mathcal{D}\vec{x}. \tag{20}$$

By using the techniques developed in [11], the wave functions $\tilde{\Psi}_n(\vec{x})$ can in principle be calculated for any interacting QFT. These wave functions contain a complete information about probabilities of particle creation and destruction. Let us briefly discuss how this probabilistic interpretation works. Let $x_{n,1}, \ldots, x_{n,n}$ denote n coordinates x_A on which $\tilde{\Psi}_n(\vec{x}) \equiv \tilde{\Psi}_n(x_{n,1}, \ldots, x_{n,n})$ really depends. (With respect to other coordinates x_A, $\tilde{\Psi}_n(\vec{x})$ is a constant.) If $\tilde{\Psi}_n(x_{n,1}, \ldots, x_{n,n})$ vanishes for $x_{n,a_n}^0 = t$, then the probability that

the system will be found in the n-particle state at time t vanishes. If $\tilde{\Psi}_n(x_{n,1}, \ldots, x_{n,n})$ does not vanish for $x_{n,a_n}^0 = t' \neq t$, then there is a finite probability that the system will be found in the n-particle state at time t'. This corresponds to a probabilistic description of particle creation or destruction when $t' > t$ or $t > t'$, respectively. As shown in [11], for coincidence times the probabilities obtained this way coincide with those obtained by more conventional single-time methods in QFT. Thus, the many-fingered time formalism is not really a modification, but only an extension of the conventional QFT formalism.

3. Implications on unitarity of Hawking evaporation

Now let us discuss how such a general formulation of QFT enriches our understanding of Hawking evaporation. Unfortunately, the explicit calculation of $\Psi(\vec{x})$ describing the Hawking evaporation is prohibitively difficult. Nevertheless, some qualitative features of $\Psi(\vec{x})$ can easily be inferred from the standard results [1]. It turns out that these qualitative features are sufficient to understand how the description of Hawking evaporation by $\Psi(\vec{x})$ resolves the information paradox.

For simplicity, we assume that the set of all particles can be divided into ingoing particles that never escape from the horizon and outgoing particles that go to the future infinity (Fig. 1 shows a pair of such particles). Therefore, all these particles are described by a wave function of the form

$$\Psi(\vec{x}) = \Psi(\vec{x}_{\text{in}}, \vec{x}_{\text{out}}). \tag{21}$$

Since the Hawking particles are created in pairs, this wave function can be expanded as

$$\Psi(\vec{x}_{\text{in}}, \vec{x}_{\text{out}}) = \sum_{n=0}^{\infty} \tilde{\Psi}_{2n}(\vec{x}_{\text{in}}, \vec{x}_{\text{out}}), \tag{22}$$

where $\tilde{\Psi}_{2n}(\vec{x}_{\text{in}}, \vec{x}_{\text{out}})$ really depends on n "ingoing coordinates" $x_{\text{in}\,A}$ and n "outgoing coordinates" $x_{\text{out}\,A}$. (In fact, the wave functions $\tilde{\Psi}_{2n}$ depend also on $\vec{x}_{\text{back}}$ describing the background particles of initial black-hole matter, but for the sake of notational simplicity the dependence on $\vec{x}_{\text{back}}$ is suppressed.) The fact that the state is initially in the vacuum means that all $\tilde{\Psi}_{2n}(\vec{x}_{\text{in}}, \vec{x}_{\text{out}})$ with $n \geqslant 1$ vanish for small values of $x_{\text{in}\,A}^0$ and $x_{\text{out}\,A}^0$.

The wave function (22) is a pure state. It describes the whole system of ingoing and outgoing particles for all possible values of times of each particle. The correlations between all these particles can also be described by the density matrix

$$\rho(\vec{x}_{\text{in}}, \vec{x}_{\text{out}} | \vec{x}'_{\text{in}}, \vec{x}'_{\text{out}}) = \Psi(\vec{x}_{\text{in}}, \vec{x}_{\text{out}}) \Psi^*(\vec{x}'_{\text{in}}, \vec{x}'_{\text{out}}), \tag{23}$$

which is nothing but a density-matrix representation of the pure state (22). However, an outside observer cannot detect the inside particles. Consequently, his knowledge is described by a mixed state obtained by tracing out over unobservable ingoing particles

$$\rho_{\text{out}}(\vec{x}_{\text{out}} | \vec{x}'_{\text{out}}) = \int \mathcal{D}\vec{x}_{\text{in}}\, \rho(\vec{x}_{\text{in}}, \vec{x}_{\text{out}} | \vec{x}_{\text{in}}, \vec{x}'_{\text{out}}). \tag{24}$$

(Of course, since now we work in a curved background, the measure (13) is now modified by the replacement $d^4x_A \to \sqrt{|g(x_A)|}\, d^4x_A$.) Nevertheless, the whole system is still described by the pure state (23).

Now we are ready to discuss how our approach resolves the information paradox. For convenience, we choose the global time coordinate such that equal-time hypersurfaces correspond to (undrawn) horizontal lines in Fig. 1. Let us assume that the complete evaporation ends at time T, after which neither a black hole nor a remnant is present. From the standard semi-classical analysis [1], we know that ingoing particles have zero probability of being found at times larger than T. They are destroyed at the singularity that does not exist for times after the complete evaporation, as illustrated by Fig. 1. Nevertheless, the pure state (22) is well defined for all values of $x_{\text{out}\,A}^0 > T$. But what happens if we put $x_{\text{in}\,A}^0 > T$? For such values of $x_{\text{in}\,A}^0$ the wave function (22) is still well defined, but the value of Ψ turns out to be equal to zero, because the probability of finding the ingoing particles at $x_{\text{in}\,A}^0 > T$ is zero. A wave function with the value zero does not encode much information, which corresponds to an apparent loss of information at times larger than T. Still, a wave function with zero value is still a wave function, so the state is still pure. In fact, since only outgoing particles are present for times larger than T, there is no much point in considering the case $x_{\text{in}\,A}^0 > T$. To obtain a nontrivial information from (22) at times larger than T, one should only put $x_{\text{out}\,A}^0 > T$, while times of ingoing particles should be restricted to $x_{\text{in}\,A}^0 < T$. In that case, the pure state (22) describes how the outgoing particles at times after the complete evaporation are correlated with the ingoing particles before the complete evaporation. Such nonlocal correlations cannot be measured by local observers that cannot travel faster than light, so information seems lost from the point of view of local observers. Nevertheless, these correlations are still encoded in the total wave function of the universe, so the principles of QM are not violated – the wave function of the universe is still pure.

Thus, we see how treating time on an equal footing with space provides a new, purely kinematic solution to the black-hole information paradox, without need to understand the details of dynamics. Essentially, the block-time picture of the universe makes any time-dependent problem in 3 spacial dimensions analogous to a time-independent problem in 4 spacial dimensions. Consequently, there can be no fundamental problem with non-unitary evolution of the quantum state simply because the concept of evolution itself does not have any fundamental meaning. Instead, all we have are correlations among particles at different spacetime positions. Thus, even if the original Hawking calculation [1] is essentially correct (in the sense that the black hole eventually evaporates completely and that the outgoing radiation cannot be described by a pure state), the information is still there, encoded in the correlations between outgoing particles in the future and ingoing particles in the past. From this point of view, the original Hawking calculation may be essentially correct, but it is not complete because it only describes correlations among particles at the same spacelike hypersurface.[1] To conclude, we believe that our results represent a new step towards reconciliation of quantum mechanics (QM) with general relativity (GR). GR suggests that time should be treated on an equal footing with space, while QM demands unitarity. We have shown that the former (i.e., treating time on an equal footing with space) automatically restores the latter (the unitarity).

Acknowledgement

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