Physics Letters B 669 (2008) 357–358

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Schnabl's solution and boundary states in open string field theory

Teruhiko Kawano [a], Isao Kishimoto [b,*], Tomohiko Takahashi [c]

[a] Department of Physics, University of Tokyo, Hongo, Tokyo 113-0033, Japan
[b] Theoretical Physics Laboratory, RIKEN, Wako 351-0198, Japan
[c] Department of Physics, Nara Women's University, Nara 630-8506, Japan

ARTICLE INFO

Article history:
Received 14 May 2008
Received in revised form 16 September 2008
Accepted 3 October 2008
Available online 8 October 2008
Editor: T. Yanagida

ABSTRACT

We discuss that Schnabl's solution is an off-shell extension of the boundary state describing a D-brane in the closed string sector. It gives the physical meaning of the gauge-invariant overlaps for the solution in our previous paper and supports Ellwood's recent proposal in the operator formalism.

In our previous paper [1], it was discussed that a class of gauge-invariant observables gives the same values for the analytic solution [2] given by Schnabl as the ones for the numerical solution [3–5] in the level truncation in open string field theory [6]. It gives another interesting evidence that the numerical solution is gauge equivalent to the analytic solution.

The gauge-invariant observables $\mathcal{O}_V(\Psi)$ for an open string field Ψ are called gauge-invariant overlaps in [1] and are given by

$$\mathcal{O}_V(\Psi) = \big\langle V(i) f_{\mathcal{I}}[\Psi]\big\rangle = \langle \mathcal{I}|V(i)|\Psi\rangle, \quad f_{\mathcal{I}}(z) \equiv \frac{2z}{1-z^2},$$

where the CFT correlator is defined on an upper half plane. The on-shell closed string vertex operator $V(i)$ is inserted at the midpoint of the open string, and the conformal mapping $f_{\mathcal{I}}(z)$ plays the role of the identity state $\langle\mathcal{I}|$, identifying the left half of the string with its right half. They were originally discussed in [7] to give the interaction of an on-shell closed string with open strings in the open string field theory.

For the analytic solution

$$|\Psi_{\lambda=1}\rangle = |\psi_0\rangle + \sum_{n=0}^{\infty}\big(|\psi_{n+1}\rangle - |\psi_n\rangle - \partial_r|\psi_r\rangle|_{r=n}\big) \equiv |\psi_0\rangle + |\chi\rangle,$$

since it was shown [1,8] that $\mathcal{O}_V(\psi_r)$ does not depend on r, one can see that

$$\mathcal{O}_V(\Psi_{\lambda=1}) = \langle\mathcal{I}|V(i)|\Psi_{\lambda=1}\rangle = \langle\mathcal{I}|V(i)|\psi_0\rangle.$$

Therefore, it suggests that only the first term $|\psi_0\rangle$ contributes to the gauge-invariant overlaps $\mathcal{O}_V(\Psi_{\lambda=1})$.

In the paper [1], the gauge-invariant overlaps were used to examine whether the numerical solution is gauge equivalent to the analytic solution, but their physical meaning was not clear. Recently, Ellwood has made an interesting proposal [8] that the gauge-invariant overlaps are related to the closed string tadpoles as

$$\mathcal{O}_V(\Psi) = \mathcal{A}_\Psi(V) - \mathcal{A}_0(V),$$

where $\mathcal{A}_\Psi(V)$ is the disk amplitude for a closed string vertex operator V with the boundary condition of the CFT given by the open string field solution Ψ, and $\mathcal{A}_0(V)$ is the usual disk amplitude in the perturbative vacuum.

In fact, for the analytic solution, he has shown that this is the case. Since, after tachyon condensation, there are no D-branes, no closed tadpoles are available, and thus $\mathcal{A}_\Psi(V)=0$. Therefore, the gauge-invariant overlap $\mathcal{O}_V(\Psi)$ for the solution gives the usual disk amplitude of the opposite sign with one closed string emitted. It suggests that the analytic solution given by Schnabl is closely related to the boundary state describing a D-brane in the closed string perturbation theory, as expected.

In this Letter, we will explicitly demonstrate this in the operator formalism of open string field theory by using the Shapiro–Thorn vertex $\langle\hat\gamma(1_c,2)|$ [9], which maps an open string state to the corresponding closed string state. (For more detail, see Appendix B in [1].) By using the open-closed string vertex $\langle\hat\gamma(1_c,2)|$, as discussed in detail in [1], one may rewrite a gauge-invariant overlap as

$$\mathcal{O}_V(\psi) = \langle\hat\gamma(1_c,2)|\phi_c\rangle_{1_c}|\psi\rangle_2,$$

where $|\phi_c\rangle_{1_c}$ is an on-shell state given by the vertex operator V of the closed string 1_c, and $|\psi\rangle_2$ is a state of the open string 2.

For the analytic solution, one has

$$\begin{aligned}\mathcal{O}_V(\Psi_{\lambda=1}) &= \langle\hat\gamma(1_c,2)|\phi_c\rangle_{1_c}|\Psi_{\lambda=1}\rangle_2\\ &= \langle\hat\gamma(1_c,2)|\phi_c\rangle_{1_c}|\psi_0\rangle_2 + \langle\hat\gamma(1_c,2)|\phi_c\rangle_{1_c}|\chi\rangle_2,\end{aligned}$$

* Corresponding author.
E-mail address: ikishimo@yukawa.kyoto-u.ac.jp (I. Kishimoto).

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2008.10.002

and as mentioned above, the second term on the right-hand side is zero for the on-shell closed string state $|\phi_c\rangle_{1_c}$. As for the first term on the right-hand side, one can see that the open string tachyon state $|\psi_0\rangle = (2/\pi)c_1|0\rangle$ is transformed via the vertex $\langle\hat{\gamma}(1_c,2)|$ into the boundary state $\langle B|$. In fact, one can obtain the relation

$$\langle\hat{\gamma}(1_c,2)|\psi_0\rangle_2\mathcal{P}_{1_c} = \frac{1}{2\pi}\langle B|c_0^-$$

with the level matching projection $\mathcal{P}_{1_c}$ for the closed string 1_c, where the boundary state is the usual one

$$\langle B| = \langle 0|c_{-1}\bar{c}_{-1}c_0^+ e^{-\sum_{n=1}^{\infty}[\frac{1}{n}\alpha_n\cdot\bar{\alpha}_n + c_n\bar{b}_n + \bar{c}_n b_n]}$$

in the closed string perturbation theory. One thus finds that

$$\mathcal{O}_V(\Psi_{\lambda=1}) = \frac{1}{2\pi}\langle B|c_0^-|\phi_c\rangle,$$

which is in precise agreement with Ellwood's result.

Since the second term $\langle\hat{\gamma}(1_c,2)|\phi_c\rangle_{1_c}|\chi\rangle_2$ does not necessarily vanish for off-shell closed string states, one may conclude that the transform of the analytic solution $|\Psi_{\lambda=1}\rangle$ via the Shapiro–Thorn vertex is an off-shell extension of the boundary state $|B\rangle$. Although it seems more elaborate to calculate $\langle\hat{\gamma}(1_c,2)|\chi\rangle_2$, it would be interesting to find the relation of the off-shell boundary state with the equation of motion in closed string field theory [10–12].

Furthermore, given all the interactions between open strings and closed strings in the open-closed string field theory [13], one may raise a question whether the Schnabl solution is consistent with the equations of motion of the open-closed string field theory, even in the vanishing string coupling constant limit, but it is beyond the scope of this Letter. However, if it is consistent, the relation of the transform of the analytic solution $|\Psi_{\lambda=1}\rangle$ via the Shapiro–Thorn vertex with the boundary state $|B\rangle$ could be clearer along with the interactions of the theory [13]. We think that our observation in this Letter may serve as an encouraging step in the investigation.[1]

Acknowledgements

We are grateful to Taichiro Kugo for relevantly useful discussions. The work of T.K. was supported in part by a Grant-in-Aid (#19540268) from the MEXT of Japan. The work of I.K. was supported in part by the Special Postdoctoral Researchers Program at RIKEN and in part by a Grant-in-Aid (#19740155) from the MEXT of Japan. The work of T.T. was supported in part by a Grant-in-Aid (#18740152) from the MEXT of Japan.

References

[1] T. Kawano, I. Kishimoto, T. Takahashi, Nucl. Phys. B 803 (2008) 135, arXiv: 0804.1541.
[2] M. Schnabl, Adv. Theor. Math. Phys. 10 (2006) 433, hep-th/0511286.
[3] A. Sen, B. Zwiebach, JHEP 0003 (2000) 002, hep-th/9912249.
[4] N. Moeller, W. Taylor, Nucl. Phys. B 583 (2000) 105, hep-th/0002237.
[5] D. Gaiotto, L. Rastelli, JHEP 0308 (2003) 048, hep-th/0211012.
[6] E. Witten, Nucl. Phys. B 268 (1986) 253.
[7] B. Zwiebach, Mod. Phys. Lett. A 7 (1992) 1079, hep-th/9202015.
[8] I. Ellwood, JHEP 0808 (2008) 063, arXiv: 0804.1131.
[9] J.A. Shapiro, C.B. Thorn, Phys. Lett. B 194 (1987) 43.
[10] M. Saadi, B. Zwiebach, Ann. Phys. 192 (1989) 213.
[11] T. Kugo, H. Kunitomo, K. Suehiro, Phys. Lett. B 226 (1989) 48;
T. Kugo, K. Suehiro, Nucl. Phys. B 337 (1990) 434.
[12] B. Zwiebach, Nucl. Phys. B 390 (1993) 33, hep-th/9206084.
[13] B. Zwiebach, Ann. Phys. 267 (1998) 193, hep-th/9705241.
[14] M. Schnabl, Phys. Lett. B 654 (2007) 194, hep-th/0701248;
M. Kiermaier, Y. Okawa, L. Rastelli, B. Zwiebach, JHEP 0801 (2008) 028, hep-th/0701249.
[15] S. Hellerman, M. Schnabl, arXiv: 0803.1184.
[16] I. Kishimoto, arXiv: 0808.0355.

[1] More recently, the idea of this Letter is extended to the marginal solutions [14] and the rolling tachyon solution [15] by one of the authors [16]. It may suggest that our observation of this Letter could be more generic.