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# Reasoning over decomposing fuzzy description logic

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## ABSTRACT

A DF-ALC (Decomposing fuzzy ALC) is proposed in this paper to satisfy the need for representing and reasoning with fuzzy ontologies in the context of semantic Web. A DF-ALC is also proposed to satisfy the need for seeing the necessity of decomposing ontology into several sub-ontologies in order to optimize the fuzzy reasoning process.

The main contribution of this work is to decompose the axioms of the ontology into sub-axioms according to a degree of certainty which is assigned to the fuzzy concepts and roles. It is also to define the syntax and semantics and to propose a local reasoning algorithm and a way of using gateways to infer between local TBox.

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## 1. Introduction

The consideration of semantics is also essential in the research for information and the evaluation of Web queries. Many works from the Semantic Web community were realized to describe the semantic of applications by building ontologies. Indeed, Semantic Web is very important for the Internet users and researchers on which they give hopes in a lot of fields such as; information search, e-business, Competitive Intelligence, etc., Its functions are to give a meaning to the data, to allow the machines to analyze and to understand the circulating information.

Have a semantic, we must first give a description to information (create meta-data), then trying to link them together through inference and deducing rules to construct ontologies. These are so central to the Semantic Web, which

on the one hand, seeks to rely on the modeling of Web resources from conceptual representations of the concerned domain, on the other hand aims to enable programs to make inferences above.

“Toleration of inconsistency can only be done by fuzzy systems. We need a semantic web which will provide guarantees and about which one can reason with logic” [1], such are the words of Tim Berners-Lee, founder and President of the World Wide Web Consortium. Where he tries to show us that all these metadata are created by humans, and so they should contain many uncertainties and inaccuracies which will affect the construction of ontologies. Because fuzzy logic was conceived to find solutions to the problems of inaccuracies and uncertainties in a flexible way, researchers have had the idea to integrate this logic in the field of the Semantic Web in

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general and to use it in the construction of ontologies by the logic of description in particular.

Description logics are a good model to describe the semantics of the data from the Web by restrictions, which are necessary to obtain reasoning algorithms that pass in the scale to detect inconsistencies or logical correlations between data or data sources, and to compute the set of answers to conjunctive queries on one hand. On the other hand they are very weak when you want to model a domain whose knowledge and information is vague and imprecise [2]. For this reason there were many proposals to extend description logics by mathematical theories which treat the uncertain and the imprecise. As a result the birth of *fuzzy description logic* appears.

To reflect our objectives, and after the exposure of our motivation, this article is organized as follows: Section 3 gives basic concepts onto the ALC DL and preliminary on the fuzzy DL and distributed DL. Section 4 presents our proposed description logic. Section 5 details the method of reasoning on a fuzzy and decomposed DL; and finally the article ends with a conclusion and perspectives.

## 2. Motivation

The fuzzy description logic does not look for the precision in the assertions; on the contrary, it looks for the answer of the vague proposals, requiring a certain uncertainty (vagueness). For example, in the classical logic, to the question: “is this person taller?” We can answer only by true, if this is the case or false if not. With fuzzy logic, we can represent cases where the person is very small, medium small, normal, not very tall, tall, etc.

However, the reasoning problems considered on ontologies have often taken a secondary position in the fuzzy knowledge bases, in most cases researchers based their efforts on uncertain knowledge representation method focusing on mathematical concepts and fuzzy sets theories, existing work, dealing with the reasoning in Fuzzy knowledge bases, merely small KB and also without giving any importance to the optimization of the reasoning algorithm.

To arrive at an effective treatment for the fuzzy KB,

we shall opt for the Organization of knowledge in categories of axioms according to certain characteristics that specify a sub domain of the ontology’s general domain. The specific categories represent subsets of axioms composed by fuzzy concepts (roles) of the ontology. This structuring will be represented by the distributed description logic, while the reasoning will be parallel on these subcategories of axioms, what will reduce the area of research on the one hand and on the other hand will reduce the relative to the reasoning time on the other hand.

In our contribution also, the axioms may be composed of concepts (roles) that belong to two different categories; the fuzziness will be represented by an annotation related to each concept and each role, and will be treated by the notions of fuzzy sets proposed by Zadeh. We shall note then two types of axioms, axioms intra-categories and inter-categories using the concept of bridge.

## 3. Preliminary

We start with a brief introduction to classical description logic, fuzzy description logic and distributed description logic, which will be useful for defining decomposing fuzzy description logic.

### 3.1. Description logic

Description logics [3–7] forms a family of knowledge representation languages which can be used to represent knowledge in an application domain in a structured and formal way. A fundamental characteristic of these languages is that they have a formal semantics. Description logics are used for numerous applications.

They have a common basis AL enriched with different extensions: the description logic ALC, object of the present work, adds the negation to AL and thus makes a modal propositional logic extension. Other extensions add the transitive closure of roles, restrictions number on roles and the concept of sub-role, etc.

Description logics use the notion of concept, role and individual. Concepts correspond to classes of individuals and roles are relations between these individuals. Both a concept and a role have a structured description defined from a set of constructors.

In description logics there are two levels of processing:

- Terminological level Tbox: the generic level (global) true in all models and for any individual.
- Assertion Level Abox: Provides instances of concepts and roles.

#### 3.1.1. Syntax

NC is a set of concept names and NR is a set of role names. The ALC-concept is constructed by induction using the following grammar:

$C, D ::=$

- A: Atomic Concept
- T: Universal concept Top
- $\perp$ : Bottom concept
- $\neg$ : Atomic negation
- $C \sqcap D$ : Conjunction concepts
- $C \sqcup D$ : Disjunction de concepts
- $\forall r.C$ : Value restriction
- $\exists r$ : Limited exist restriction

where  $A \in NC$  and  $R \in NR$ .

#### 3.1.2. Semantic

A semantic is an associated description of the concepts and the roles: the concepts are interpreted like subsets of a domain  $\Delta^I$  and the roles like subsets of a product  $\Delta^I \times \Delta^I$ .

An interpretation  $I$  is essentially a couple  $(\Delta_I, \cdot^I)$  where  $\Delta_I$  is called an interpretation domain and  $(\cdot^I)$  an interpretation function that assigns to a concept  $C$ , a subset  $C^I$  of  $\Delta_I$  and a role  $r$  a subset  $r^I$  of  $\Delta_I \times \Delta_I$ . In mathematical notation, it is defined as follows:

- $T^I = \Delta_I$
- $\perp^I = \emptyset$
- $\neg C = \Delta_I - C^I$
- $(C \sqcap D)^I = C^I \cap D^I$
- $(\forall r.C)^I = \{x \in \Delta_I / \forall y : (x, y) \in r^I \rightarrow y \in C^I\}$
- $(\exists r.C)^I = \{x \in \Delta_I / \exists y : (x, y) \in r^I \wedge y \in C^I\}$ .

### 3.2. Fuzzy description logic

Fuzzy logic appeared in 1965 in Berkeley in the laboratory of Lotfi Zadeh [8] with the theory of fuzzy sets. It is a classic extension of the set theory for the consideration of sets defined in an imprecise way. Contrary to the classic logic, fuzzy logic allows a declaration to be in another state than true or false, the declaration could be true or false for a certain degree, which is taken starting from a space of truth (e.g. Mohamed is taller). We are unable to establish whether the statement is true or false completely because of the implication of the vague concept, As "Taller", which does not have a precise definition.

**Definition.** Let  $X$  be a set. A fuzzy set  $A$  of  $A$  is defined by a membership function  $F_A$  on  $X$  with values in the interval  $[0, 1]$ .

In fuzzy logic, there are generally three different approaches: Lukasiewicz, Gödel, and product logic [9,10]. Popular logic of Zadeh is a sub logical of Lukasiewicz. These logics offer different operators for conjunction, disjunction, negation and implication. They are shown in Table 1.

Fuzzy description Logics (Fuzzy DLs) are extensions of classical description logics, they have been proposed as a language that can represent and reason about uncertain and imprecise knowledge [11-13]. These extensions have gained a considerable attention in recent years, partly because they are essential for applications that are inherently imprecise as multimedia analysis, geospatial applications and much more, whereas on the other hand they can be applied to the semantic Web applications such as the representation of ontologies those model domains whose knowledge is imprecise.

Several fuzzy extensions for description logics were proposed like formalisms able to collect and reason on imprecise and uncertain knowledge [14,15].

To support services inference fuzzy DL, several different reasoning algorithms have been proposed, such as tableaux algorithms [16,17] as well techniques that reduce the fuzzy DL reasoning to a classical DL reasoning [18,15]. Some other works were treats the classification of fuzzy DLs according to the decidability of their consistency problem [19,11,12,20]. also we find effort to study the optimizing techniques to reason with fuzzy DLs [21-23], but neither of them considers reasoning with decomposing KB.

### 3.3. Distributed description logics

Borgida and Serafini proposed a Distributed Description Logics (DDL) [24], which generalizes the description logic with a semantic space model to represent knowledge bases (ontologies) in distributed environments, and ensure reasoning this KB.

Using the same principle as Distributed First Order Logics (DFOL), distributed description logics (DDL) allows connecting and reasoning with multiple ontologies on the Semantic Web. Gateways in distributed description logics are restricted to relations between concepts, roles and individuals of different sub ontologies. Their semantics allow foremost deduct only subsumption relations ontology provides.

### Syntax

Network ontologies in DDL are composed of various KB in description logics, whose syntax has been presented above. Moreover, ontologies are interconnected through gateways. These are represented in the abstract syntax as follows:

**Definition.** Let  $O_i$  and  $O_j$  two ontologies. A gateway from  $O_i$  to  $O_j$  ( $i \neq j$ ), is an expression of the following forms:

- $i : X \sqsubseteq j : Y$  (into-bridge rule);
- $i : X \sqsupseteq j : Y$  (onto-bridge rule);
- $i : a \rightarrow j : b$  (individual correspondence).

where  $i : X$  and  $j : Y$  are either concepts or roles  $O_i$  and  $O_j$  respectively and  $i : a$  is an individual to  $O_i$  and  $j : b$  is an individual  $O_j$ .

### Semantic

In a network of ontologies in DDL we assigned an interpretation in description logics to each ontology. Different description logics may exist in each network node. To connect knowledge of two ontologies, DDL uses the relationships of domains.

**Definition.** Let  $\Delta_i$  and  $\Delta_j$  two interpretation domains. A relationship homogeneous domains  $r_{ij}$  from  $i$  to  $j$  is sub-set of  $\Delta_i X \Delta_j$ . For all  $d \in \Delta_i$ , we use  $r_{ij}(d)$  to denote the set  $\{d' \in \Delta_j \mid \langle d, d' \rangle \in r_{ij}\}$ , For all  $D \in \Delta_i$ , we use  $r_{ij}(D)$  to denote the set  $\bigcup_{d \in D} r_{ij}(d)$  and for all  $R \subseteq \Delta_i X \Delta_j r_{ij}(R)$  denote  $\bigcup_{(d,e) \in R} r_{ij}(d) X r_{ij}(e)$ .

A domains relationship  $r_{ij}$  represents one possible way of matching elements  $\Delta_i$  with elements of  $\Delta_j$ , according to the view  $j$ .

**Definition.** A domain relation  $r_{ij}$  satisfies a homogeneous gateway towards two local interpretations  $I_i$  and  $I_j$  (noted  $\langle I_i; r_{ij}; I_j \rangle = rp$ ) if and only if:

- $\langle I_i; r_{ij}; I_j \rangle \models i : X \sqsubseteq j : Y$   
if and only if  $r_{ij}(X^I_i) \subseteq Y^I_j$ ,
- $\langle I_i; r_{ij}; I_j \rangle \models i : X \sqsupseteq j : Y$

if and only if  $r_{ij}(X^I_i) \supseteq Y^I_j$ ,

- $\langle I_i; r_{ij}; I_j \rangle \models i : a \rightarrow j : b$  then  $\langle a^I_i, b^I_j \rangle \in r_{ij}$ .

A heterogeneous relationship indicates an association between an element of a concept and the reification of a relationship, so associates an object of the domain to a pair of objects.

**Definition (Relations of Heterogeneous Domain).** Let  $I_i$  and  $I_j$  two interpretations. A relationship of domain concept-role  $cr_{ij}$  from  $i$  to  $j$  is a subset of  $\Delta_i X \Sigma_j$ . A relationship of domain role-concept  $rc_{ij}$  from  $i$  to  $j$  is a subset of  $\Sigma_i X \Delta_j$ .

The  $rc_{ij}$  relation represents a possible way to reify relations between objects, whereas the  $cr_{ij}$  relation represents the inverse process. In our work we introduce the notion of fuzzy based on the formalism of Fuzzy DLs

The classic DLs are interpreted by conventional classic set-concepts: set, binary relation, membership, etc. Fuzzy DL extensions are expressed semantically through fuzzy sets theory: While in the classical set theory, an element belongs

**Table 1 – Fuzzy logic approaches.**

Fonction	$x \wedge y$	$x \vee y$	$x \rightarrow y$	$\neg x$
Lukasiewicz	$\max(x + y - 1, 0)$	$\min(x + y, 1)$	$\min(x + y, 1)$	$1 - x$
Gödel	$\min(x, y)$	$\max(x, y)$	$\begin{cases} 1 & \text{if } x \leq y \\ y & \text{sinon} \end{cases}$	$\begin{cases} 1 & \text{if } x = y \\ 0 & \text{sinon} \end{cases}$
Produit	$\max(x + y - 1, 0)$	$\min(x + y, 1)$	$\begin{cases} 1 & \text{if } x \leq y \\ x/y & \text{sinon} \end{cases}$	$\begin{cases} 1 & \text{if } x = y \\ 0 & \text{sinon} \end{cases}$
Zadeh	$\min(x, y)$	$\max(x, y)$	$\max(1 - x, y)$	$1 - x$

or does not belong to a set, unlike in the fuzzy subsets theory, an element belongs to the set with a certain degree. More formally, let  $X$  be a set of elements, a fuzzy subset  $A$  of  $X$  is defined by a membership function  $\mu_A(x)$ , This function affects all  $x \in X$  to an including value between 0 and 1 representing the degree to which this element belongs  $X$ .

Fuzzy LDs differ among themselves mainly by the means by which they introduce fuzziness, that is to say, the syntactical elements (constructors, axioms, assertions) for which the classical interpretation is insufficient.

## 4. DF-ALC

### 4.1. Decomposition

The explosion of the number of accessible information sources via Web multiplies the needs for technics allowing the reasoning in these sources. The decomposition of ontologies is a very important research topic in fuzzy knowledge bases since it allows a multi knowledge representation based on degrees of certainty on one hand, the thing that allows us to optimize the reasoning on the other hand.

The reasoning in a wide vague ontology may be reduced to some reasoning procedures in the sub-axioms of the global ontology by basing on the inference services in DLs; which are solved by the tableau algorithm which is based on the satisfiability and subsumptions cheking.

Our approach will be applied on the description logic ALC.

A decomposition of fuzzy ALC TBox is presented by the following rules:

- An axiom is in one of the forms:

$$i : C_{x1} \sqsubseteq D_{x2}$$

$$i : C_{x1} \equiv D_{x2}$$

$$i : R_{x1} \sqsubseteq S_{x2}$$

where  $C$  and  $D$  are concept expressions,  $R$  and  $S$  are role names,  $x1, x2 \in [0, 1]$ .

- An expression of concept is in one of the forms:

$$i : CN|T|\perp|i : \neg C|i : C \cup D|i : C \cap D|i :$$

$$\forall R.C|i : \exists R.C.$$

- A rule of the bridge is in the form:

$$i : C_{x1} \equiv j : C_{x2}/x1, x2 \in [0, 1] \wedge x2 \geq x1.$$

$C$  an expression concept.

In the following sections, we present two approaches of reasoning on decomposing ontologies, we give more details about reasoning techniques (paralleled and distributed). The tableau algorithm will be applied in the local ontologies

before merged into the reasoning parallel case or propagated in the reasoning distributed case.

### 4.2. DF-ALC TBox and ABox

Most description logics consist of a TBox which represent the domain, and an ABox which declare particular individuals in this domain. Our job is also to introduce the notion of fuzziness in these two components, adding a degree of certainty to terminological axioms (TBox) and also a degree of membership to fuzzy individuals (Fuzzy-ABox). Moreover, the construction of description logic should consider the following points:

- A Tbox is composed by concepts and atomic roles.
- The concepts are divided into two types:
  - The concepts names  $NC$  that appear in the left side of the axiom.
  - The basic concepts  $BC$  that appear in the right.
- The concept names can appear only once on the left side of an axiom of the same sub-ontology.
- The atomic concepts that define the axioms may be imprecise.
- The axioms defined in the description logic ALC is not necessarily just in DF-ALC.

DF-ALC can represent imprecise atomic concepts by fuzzy property that takes a value in the interval  $[0, 1]$ , this property does not require a big change during the extension of the syntax of the classic description logic.

The theory of fuzzy subsets proposed by Zadeh is used in our approach for calculating the concept's certainty degree for a conjunction, disjunction or negation operation.

More fuzzy TBox proposed in DF-ALC, vagueness also appears in ABox with a degree of membership of the individual such as the concept ( $Ex : C_I(x) = a/a \in [0, 1]$ ) so that the assertions in ABox can be represented as follows:  $C_a(x)_b$  such as:

- The concept “ $C$ ” should be satisfied in TBox.
- The instance “ $x$ ” belongs to the concept “ $C$ ” in ABox.

### 4.3. DF-ALC syntax and semantic

We will now go into the details of the formal definition of description logic DF-ALC: its syntax and semantics. To facilitate the reading we put respectively  $A, C$  and  $R$ ; the whole atomic concept, complex concept and Role (see Table 2).

A fuzzy interpretation is a pair  $I = (\Delta^I, \cdot^I)$ , where  $\Delta^I$  called domain, while  $\cdot^I$  an interpretation function that associates a fuzzy concept/role to a subset  $A^I$  of  $\Delta^I$  with a membership

**Table 2 – Syntax and semantic of DF-ALC constructors.**

Constructor	Syntax	Semantics
Top	$T$	$\Delta^I$
Bottom	$\perp$	$\emptyset$
Atomic concept	$A_a$	$A_a^I \subseteq \Delta^I$
Atomic role	$R_a$	$R_a^I \subseteq \Delta^I \times \Delta^I$
Conjunction	$C_a \cap D_b$	$(C \cap D)^I_{[\text{Min}(a,b)]}$
Disjunction	$C_a \cup D_c$	$(C \cup D)^I_{[\text{Max}(a,b)]}$
Negation	$\neg C_a$	$C_{[1-a]}^I$
Universal quantification	$\forall R_a.C_b$	$\text{Inf}\{\max\{1 - R_a, C_b\}\}$
Existential quantification	$\exists R_a.C_b$	$\text{Sup}\{\min\{R_a, C_b\}\}$

degree  $C : \Delta^I \rightarrow [0, 1]/R : \Delta^I \times \Delta^I \rightarrow [0, 1]$ . The interpretation function in DF-ALC must satisfy these following equations for all  $d \in \Delta^I$

$$T^I(d) = 1;$$

$$\perp^I(d) = 0;$$

$$C^I(d) = \mu(C(d));$$

$$(C \cap D)^I(d) = \min(\mu(C(d)), \mu(D(d)));$$

$$(C \cup D)^I(d) = \max(\mu(C(d)), \mu(D(d)));$$

$$\neg C^I(d) = 1 - \mu(C(d));$$

$$\forall R.C(d) = \text{Inf}_{d' \in \Delta} \{\max\{1 - \mu(R(d, d')), C(d')\}\};$$

$$\exists R.C(d) = \text{Sup}_{d' \in \Delta} \{\min\{\mu(R(d, d')), C(d')\}\}.$$

A collection of description logics, where  $k$  is a non-empty set of indexes for each  $k \in K$ , a Tbox  $\Gamma^k$  is presented in a concrete  $DL_k$ . In order to distinguish from descriptions in each Tbox  $\Gamma^k$ , we put in the top of descriptions, the index of their TBox. For example  $k$ , it denotes a concept of  $\{C\}$  of  $\{DL_k\}/k \in K$  with a degree of certainty “ $a$ ”. Semantic morphisms between TBox are presented using the bridges rules.

## 5. Reasoning DF-ALC

DL systems provide to the users various inference capabilities. The reasoning allows inferring implicit knowledge from explicit knowledge stored in the knowledge base. The basic inference on concept expressions in DL is the subsumption between two concepts which determines the sub-concept/super-concept relations. The basic inference on individuals is to determine whether an individual is an instance of a certain concept. The research of the reasoning in the DF LD-LAC is a new challenge for large fuzzy knowledge bases.

In this part of the article we will try to present some techniques of reasoning for fuzzy decomposing Knowledge Base.

We base on the tableau Algorithm to reduce the problem of subsumption to satisfiability problem. In fact, we remember that  $C \sqsubseteq D$  if satisfy if and only if  $C \cap \neg D$  is not satisfiability. The fuzzy tableau algorithm treats each sub-axiom (each sub ontology) in a separate way, it begins every time with an ABox  $\text{In}\{C_a(x)_b\}$  to check the insatisfiability of concept  $C_a$ .

### 5.1. Reasoning with local Tbox

The reasoning method proposed in our work based on the same process used in previous works that address the problem of fuzziness in description logic. An algorithm is

applied to each local TBox, so for check the satisfiability of a concept  $C$  in TBox  $T = \{T_1, T_2 \dots T_n\}$  we must check the satisfiability of this concept in each local TBox  $T_i$ .

For a local TBox the inference method is as follow: The initial local table  $T_i$  is constructed as follows. For every individual of assertional level  $C_a(x)_b$ , we added to the table  $T_i = \{C_a(x)_b\}$ .

The table tree construction begin with the initial table and will be replacing by the formula itself (Axiom definition). Successors operations are built for a table  $T$  using the rules listed below for objective to arrive to atomics concepts in the last leaves of the tree of the tabl:

The leaves of this tree are:

- Conflicting tables: they contain a pair of formulas  $p$  and  $\neg p$ .
- Complete tables: they are not contradictory and no rule is applicable to them.

Règles	Conditions	Résultat
$\cap$ -rule	$-(C_a \cap D_b)(x)_c \in T$	$T' = T \cup \{C_a(x)_c, C_b(x)_c\}$
$\cup$ -rule	$-(C_a \cup D_b)(x)_c \in T$	$T' = T \cup \{C_a(x)_c\}$ ou $T' = T \cup \{C_b(x)_c\}$
$\exists$ -rule	$-(\exists R_b.C_a)(x)_c \in T$	$T' = T \sqcup \{R_b(x, y)_c, C_a(y)_c\}$
$\forall$ -rule	$-(\forall R_b.C_a)(x)_c, R_b(x, y)_c \in T$ $-C_a(y)_c \notin T$	$T' = T \sqcup \{C_a(y)_c\}$

The construction of a local algorithm table stops either when meeting a complete table, in this case the formula is satisfiable, otherwise if we cant apply any rules and we find contradictory in the last leaves of the tree (nodes), in this case we say that the formula is not satisfiable.

The possible case to specify contradictory table are listed follow:

- The table contains a formula  $\neg(x)$ ;
- The table contains a pair of formulas  $C(x)$  and  $\neg C(x)$ .  $C$  is necessarily a primitive concept.
- A fuzzy conflict as  $C_0(x) = a$ .

Let  $T$  be a complete table,  $M[T]$  the model built, we have:

- $\Delta_{M[T]}$  is the set of elements appearing in  $T$ .
- Let  $x$  be an element  $x^{M[T]} = x$ .
- Let  $A$  be a primitive concept,  $x \in A^{M[T]}$  iff  $A(x) \in T$ .
- Let  $r$  be a primitive role,  $x \in (x, y) \in r^{M[T]}$  ssi  $a(x, y) \in T$ .

The termination of this method is not guaranteed if we don't implement a strategy of application of the rules. For that we introduce some notations. Let us Call *father* of a variable  $x$ , an element such that  $a_i(y, x)$  for  $a_i$  atomic role. In the creation a variable has only one father and review of the rules we check that cannot acquire a second father. By examination of the rules, the size of the largest formula of the concept of a variable is always strictly less than the size of the largest formula of its father concept. Similarly the size of the largest formula of an individual concept is always less than or equal to the size of the largest formula of concept of initial table.

Finally we note that any formula of concept that appears is a sub formula of a formula of the initial table.

So our strategy for sub ontology is as follows:

- 1- Apply a rule to a variable only if no rule is applicable to these ancestors.
- 2- For an element, apply primarily local rules ( $R_{\wedge}$  ou  $R_{\vee}$ ).
- 3- For an element, apply the rule generator ( $R_{\exists}$ ) if local rules are not applicable.
- 4- For an element, apply the rule of propagation ( $R_{\forall}$ ) if the local and generator rules are not applicable.

Point 1 ensures that when applying a rule to a variable, that variable no longer disappear from the table. Indeed, for a review of the rules we check that none of them never applicable to individuals and to their ancestors. Consequently during the evolution of the table only the variables which are sheets of a tree and on which we applied, any rule can disappear.

We are interested only in the definitive elements of the table because the others are not subject to rule application.

## 5.2. Distributed reasoning

In this section, we introduce a reasoning algorithm based on the main idea of the procedure for distributed reasoning Luciano Serafini, Andrei Taminin and Lephram [25–27] that takes a complex concept  $C$  as input and returns the result of the test of (in) satisfiability. We denote a TBox-decomposing  $T = \{\{Ti\}, B\}$ . A distributed TBox is composed of sources and targets TBox. However, this can be determined when you run the reasoning, ie, if a query is posed on the decomposing TBox  $Ti$  so  $Ti$  is then assigned to the source TBox and others TBox become targets.

The main idea is initially to find a source local TBox compatible with the request, and try to build a complete tree by following the steps of the algorithm table defined in Section 5.1. Once arrived at the end of execution of these steps, by traversing the generated nodes and by searching the open branches of  $Ts$ , we must check the existence of the identical rules of bridges between  $Ts$  and other TBox targets. Let us note that the rules of the bridge take only one direction.

If a complete bridge rule is found, we must assign the elements of the open branch  $Ts$  to target TBox, and we must again apply the tableaux algorithm and so on.

This means that we can initially treat a request posed on certain TBox. If we start on  $T1$ , then  $T1$  is seen like TBox source and  $T2, T3, \dots, Tn$  the targets TBox. So to quickly detect local contradictions, we must initially run on the TBox above cited. We obtain the following two cases:

1. Let  $T_1(x)$  or  $T_2(x)$  or  $\dots$  or  $T_n(x)$  is unsatisfiable (i.e. all leaf nodes  $T_1(x)$  or  $T_2(x)$  or  $\dots$  or  $T_n(x)$  are contradictions) then we conclude that  $x$  is unsatisfiable with by report to general  $T$ .
2. Either all  $T_i$  are satisfiable (ie, there is at least non-contradictory sheet in  $T_i$ ), then we apply the tableaux algorithm on  $T_2, T_3, \dots, Tn$  for open nodes  $T_1$  using the identical bridges rules.

To apply an identical bridge rule, the degree of certainty in the TBox axiom target is greater than the degree of certainty of the axiom of TBox source.

## 6. Conclusion

The integration of the vagueness notion in the description logic requires the extraction of sub axioms in the general axiom in order to have an exact degree of certainty of concept defined from other atomic concepts; This extraction drives to an increase the size of knowledge base, which leads to an increase of the run time and to a wasting of storage during the reasoning, for it we thought of proposing a decomposition of axioms to ensure the presentation and a successful reasoning.

Our proposal approach can be used to support the improvement of reasoning that supports the description logic ALC. This reasoning will handle two things at once, vagueness of the concept and the role on one side in addition to inter and intra TBox inferences on the other side. This reasoning can also optimize the reasoning of the classical description logic ALC if we know that the degree of certainty will be forever equal to 1.

The prospects of this work are double: The first is to finalize the development of a reasoning that supports this kind of description logic and the second is to try to project this work on other DLs in the most effective expressivity.

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