Available online at www.sciencedirect.com
SCIENCE @ DIRECT®
An International Journal
computers & mathematics with applications
ELSEVIER Computers and Mathematics with Applications 50 (2005) 1669–1676
www.elsevier.com/locate/camwa

Resonant Oscillations of a Plate in an Electrically Conducting Rotating Johnson-Segalman Fluid

T. HAYAT AND S. MUMTAZ
Department of Mathematics
Quaid-i-Azam University
Islamabad 45320, Pakistan

(Received March 2004; revised and accepted June 2005)

Abstract—An analysis of hydromagnetic flow is examined in a semi-infinite expanse of electrically conducting rotating Johnson-Segalman fluid bounded by nonconducting plate in the presence of a transverse magnetic field and the governing equations are modeled first time. The structure of the velocity distribution and the associated hydromagnetic boundary layers are investigated including the case of resonant oscillations. It is shown that unlike the hydrodynamic situation for the case of resonance, the hydromagnetic steady solution satisfies the boundary condition at infinity. The inherent difficulty involved in the hydrodynamic resonance case has been resolved in the presence analysis.

Keywords—Resonance, Johnson-Segalman fluid, Velocity field, Rotating fluid, Magnetic field.

1. INTRODUCTION

In many fluids, such as food, industry, drilling operations, and bio-engineering, the fluids, either synthetic or natural, are mixtures of different stuff such as water, particle, oils, red cells, and other long chain molecules; this combination imparts strong non-Newtonian characteristics to the resulting liquids; the viscosity function varies nonlinearly with the shear rate; elasticity is felt through elongational effects and time-dependent effects. In these cases, the fluids have been treated as viscoelastic fluids. Because of the difficulty to suggest a single model which exhibits all properties of viscoelastic fluids, they cannot be described as simply as Newtonian fluids. For this reason, many models or constitutive equations have been proposed and most of them are empirical or semiempirical. One of the models to account for the rheological effects of fluid is the Johnson-Segalman model. This model offers a very interesting means for explaining "spurt", it seems more likely that the phenomenon is due to the "stick-slip" that takes place at the boundary [1]. However, the model could very well describe the "shear layers" that have been observed in experiments [2], wherein the mechanism of "stick-slip" in the interior of the domain may not be natural to the problem. More recently, Rao and Rajagopal [3] and Rao [4] have explained the phenomenon of "spurt" by discussing some flows of a Johnson-Segalman fluid.

The authors are grateful to a referee for fruitful suggestions.

0898-1221/05/$ - see front matter Typeset by AMS-TEX
doi:10.1016/j.camwa.2005.06.010

After the initiation by Lighthill [5], there has been much work on the subject of laminar boundary layers which has a regular fluctuating flow superimposed on the mean flow. Owing to mathematical difficulties, most of them include restrictions on an oscillation amplitude or a frequency in the course of their theoretical developments. One of the exact solutions of the Navier-Stokes equation in which no restriction is placed on the amplitude and frequency is obtained by Stuart [6]. In 1982, Rajagopal [7] obtained exact solutions for a class of unsteady unidirectional flows of a second-grade fluid. In continuation, Hayat *et al.* [8–10] discussed the periodic, unidirectional flows of a second-grade fluid. Very recently, Erdogan [11] analyzed the unsteady flow of a viscous fluid due to an oscillating plane wall. However, the interesting and important problem of hydromagnetic rotating flow of a Johnson-Segalman fluid on an oscillating plate has not been treated so far, which has many practical applications in geophysical and astrophysical problems. Several authors including Soundalgekar and Pop [12], Debnath [13,14], Singh [15] and Hayat *et al.* [16] have studied the theory of rotating hydromagnetic viscous fluid flows in various geometrical configurations.

The main purpose of this paper is to study the hydromagnetic rotating flow at an oscillating plate. The fluid is assumed to be incompressible, non-Newtonian (Johnson-Segalman), rotating and electrically conducting and the magnetic field is applied transversely to the direction of the flow. Examples of non-Newtonian fluids which might be conductors of electricity are flow of nuclear slurries and of mercury amalgams, and lubrication with heavy oils and greases. This theoretical study of magnetohydrodynamic (MHD) flow has been a object of great interest due to its widespread applications in designing cooling systems with liquid metals, MHD generators, accelerators, pumps, and flow meters. The hydromagnetic flow is generated in the uniformly rotating fluid system by oscillations of the plate at $t = 0^+$. The governing equations of the problem are modelled and then solved subject to the relevant boundary conditions. Special attention is given to the physical nature of the solution and the structure of the boundary layers. In hydrodynamic situation, the solution remains mathematically valid and physically meaningful for all values of frequencies except the resonant frequency. Thus, it remains to answer the question of finding a meaningful solution for the case of resonant frequency. An attempt is made to answer this question by posing a hydromagnetic boundary layer problem.

2. MATHEMATICAL FORMULATION

We consider the unsteady hydromagnetic flow induced in a semi-infinite expanse of an electrically conducting Johnson-Segalman fluid bounded by an infinite plate at $z = 0$. A uniform magnetic field is applied normal to the plate. The fluid as well as the plate is in a state of solid body rotation with constant angular velocity Ω about the z-axis normal to the plate and additionally, oscillations of frequency α is superimposed on the plate at time $t > 0$.

The unsteady hydromagnetic flow in a rotating coordinate system is governed by the equation of motion, continuity equation, and the Maxwell equations in the form,

$$\rho\left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\,\mathbf{V} + 2\boldsymbol{\Omega}\times\mathbf{V} + \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})\right] = \nabla\cdot\boldsymbol{\sigma} + \mathbf{j}\times B, \tag{1}$$

$$\operatorname{div}\mathbf{V} = 0, \tag{2}$$

$$\operatorname{div}\boldsymbol{B} = 0, \qquad \operatorname{curl}\,\mathbf{B} = \mu_m \mathbf{j}\,\operatorname{curl}\,\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{3}$$

$$\mathbf{j} = \sigma_1\left(\mathbf{E} + \mathbf{V}\times\mathbf{B}\right), \tag{4}$$

where $\mathbf{V} = (u, v, w)$ is the velocity field, $\mathbf{j}$ is the electric current density, $\mathbf{B}$ is the total magnetic field so that $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$, $\mathbf{b}$ is the induced magnetic field, $\mathbf{E}$ is the total electric field, ρ is the density, σ_1 is the electric conductivity, μ_m is the magnetic permeability, and r the radial coordinate,

$$r^2 = x^2 + y^2. \tag{5}$$

The Cauchy stress tensor $\boldsymbol{\sigma}$ in a Johnson-Segalman fluid is given by [17],

$$\boldsymbol{\sigma} = -p\mathbf{I} + \mathbf{T}, \tag{6}$$

$$\mathbf{T} = 2\mu\mathbf{D} + \mathbf{S}, \tag{7}$$

$$\mathbf{S} + \lambda\left(\frac{\partial \mathbf{S}}{\partial t} + (\mathbf{V}\cdot\boldsymbol{\nabla})\,\mathbf{S} + \mathbf{S}\,(\mathbf{W} - a\mathbf{D}) + (\mathbf{W} - a\mathbf{D})^{\top}\,\mathbf{S}\right) = 2\eta\mathbf{D}, \tag{8}$$

$$\mathbf{D} = \frac{1}{2}\left(\mathbf{L} + \mathbf{L}^{\top}\right), \qquad \mathbf{W} = \frac{1}{2}\left(\mathbf{L} - \mathbf{L}^{\top}\right), \qquad \mathbf{L} = \nabla\mathbf{V}. \tag{9}$$

where p is the pressure field, λ is the relaxation time, μ and η are the viscosities, and "a" is called the slip parameter.

It should be noted that for $a = 1$, $\mu = 0$, model (6) reduces to the Maxwell model and when $\lambda = 0$, the model reduces to the Navier-Stokes fluid.

We define the stress tensor and velocity as

$$\mathbf{S}\,(z,t) = \begin{pmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{pmatrix}, \qquad \mathbf{V}\,(z,t) = \begin{pmatrix} u \\ v \\ 0 \end{pmatrix}. \tag{10}$$

We assume that the induced magnetic field produced by the motion of an electrically conducting fluid is negligible. The assumption is justified since the magnetic Reynolds number is small, which is generally the case in normal aerodynamic applications. Since no external electric field is applied and the effect of polarization of the ionised fluid is negligible, we also can assume that the electric field $\mathbf{E} = 0$. Under these assumptions and using (1), (3)–(6), and (10), we have

$$\rho\left[\frac{\partial u}{\partial t} - 2\Omega v\right] = -\frac{\partial \hat{p}}{\partial x} + \frac{\partial S_{xz}}{\partial z} + \mu\frac{\partial^2 u}{\partial z^2} - \sigma B_0^2 u, \tag{11}$$

$$\rho\left[\frac{\partial v}{\partial t} + 2\Omega u\right] = -\frac{\partial \hat{p}}{\partial y} + \frac{\partial S_{yz}}{\partial z} + \mu\frac{\partial^2 v}{\partial z^2} - \sigma B_0^2 v, \tag{12}$$

$$0 = -\frac{\partial \hat{p}}{\partial z} + \frac{\partial S_{zz}}{\partial z}, \tag{13}$$

with

$$\hat{p} = p - \frac{\rho}{2}r^2\Omega^2, \tag{14}$$

$$S_{xx} + \lambda\left[\frac{\partial S_{xx}}{\partial t} - (1+a)\,\frac{\partial u}{\partial z}S_{xz}\right] = 0, \tag{15}$$

$$S_{xy} + \lambda\left[\frac{\partial S_{xy}}{\partial t} - \frac{(1+a)}{2}\left\{\frac{\partial u}{\partial z}S_{yz} + \frac{\partial v}{\partial z}S_{xz}\right\}\right] = 0, \tag{16}$$

$$S_{xz} + \lambda\left[\frac{\partial S_{xz}}{\partial t} + \frac{(1-a)}{2}\left\{\frac{\partial u}{\partial z}S_{xx} + \frac{\partial v}{\partial z}S_{xy}\right\} - \frac{(1+a)}{2}\frac{\partial u}{\partial z}S_{zz}\right] = \eta\frac{\partial u}{\partial z}, \tag{17}$$

$$S_{yy} + \lambda\left[\frac{\partial S_{yy}}{\partial t} - (1+a)\,\frac{\partial v}{\partial z}S_{yz}\right] = 0, \tag{18}$$

$$S_{yz} + \lambda\left[\frac{\partial S_{yz}}{\partial t} + \frac{(1-a)}{2}\left\{\frac{\partial u}{\partial z}S_{xy} + \frac{\partial v}{\partial z}S_{yy}\right\} - \frac{(1+a)}{2}\frac{\partial v}{\partial z}S_{zz}\right] = \eta\frac{\partial v}{\partial z}, \tag{19}$$

$$S_{zz} + \lambda\left[\frac{\partial S_{zz}}{\partial t} + (1-a)\left\{\frac{\partial u}{\partial z}S_{xz} + \frac{\partial v}{\partial z}S_{yz}\right\}\right] = 0, \tag{20}$$

and equation of continuity is identically satisfied.

The boundary conditions are taken as

$$u + iv = U_0\left[ce^{i\alpha t} + de^{-i\alpha t}\right], \qquad \text{on } z = 0, \quad t > 0, \tag{21}$$

$$u, v \to 0, \qquad \text{as } z \to \infty, \quad t > 0, \tag{22}$$

where c and d are complex.

3. DIMENSIONLESS ANALYSIS

Introducing

$$
\begin{gathered}
x^* = \frac{U_0}{\nu}x, \qquad y^* = \frac{U_0}{\nu}y, \qquad z^* = \frac{U_0}{\nu}z, \qquad u^* = \frac{u}{U_0}, \qquad v^* = \frac{v}{U_0}, \\
S^* = \frac{\nu S}{(\mu+\eta)\,U_0^2}, \qquad t^* = \frac{U_0^2}{\nu}t, \qquad p^* = \frac{p}{\rho U_0^2}, \qquad \Omega^* = \frac{\nu\Omega}{U_0^2},
\end{gathered} \tag{23}
$$

where $\nu(=\eta/\rho)$ is the kinematic viscosity.

Equations (11)–(13), (15)–(20) and boundary conditions (21) and (22) now take the following form,

$$\frac{\partial u^*}{\partial t^*} - 2\Omega^* v^* = -\frac{\partial \hat{p}^*}{\partial x^*} + \frac{\mu}{\eta}\frac{\partial^2 u^*}{\partial z^{*2}} + \phi\frac{\partial}{\partial z^*}S^*_{xz} - Nu^*, \tag{24}$$

$$\frac{\partial v^*}{\partial t^*} + 2\Omega^* u^* = -\frac{\partial \hat{p}^*}{\partial y^*} + \frac{\mu}{\eta}\frac{\partial^2 v^*}{\partial z^{*2}} + \phi\frac{\partial}{\partial z^*}S^*_{yz} - Nv^*, \tag{25}$$

$$0 = -\frac{\partial \hat{p}^*}{\partial z^*} + \phi\frac{\partial}{\partial z^*}S^*_{zz}, \tag{26}$$

$$\frac{\partial^2 F^*}{\partial t^*\partial z^*} + 2i\Omega^*\frac{\partial F^*}{\partial z^*} = \frac{\mu}{\eta}\frac{\partial^3 F^*}{\partial z^{*3}} + \phi\frac{\partial^2}{\partial z^{*2}}\left(S^*_{xz} + iS^*_{yz}\right) - NF^*, \tag{27}$$

$$S^*_{xx} + W_e\left[\frac{\partial}{\partial t^*}S^*_{xx} - (1+a)\,S^*_{xz}\frac{\partial u^*}{\partial z^*}\right] = 0, \tag{28}$$

$$S^*_{xy} + W_e\left[\frac{\partial}{\partial t^*}S^*_{xy} - \left(\frac{1+a}{2}\right)\left\{\frac{\partial u^*}{\partial z^*}S^*_{yz} + \frac{\partial v^*}{\partial z^*}S^*_{xz}\right\}\right] = 0, \tag{29}$$

$$S^*_{xz} + W_e\left[\frac{\partial}{\partial t^*}S^*_{xz} + \left(\frac{1-a}{2}\right)\left\{\frac{\partial u^*}{\partial z^*}S^*_{xx} + \frac{\partial v^*}{\partial z^*}S^*_{xy}\right\} - \left(\frac{1+a}{2}\right)\frac{\partial u^*}{\partial z^*}S^*_{zz}\right] = \frac{1}{\phi}\frac{\partial u^*}{\partial z^*}, \tag{30}$$

$$S^*_{yy} + W_e\left[\frac{\partial}{\partial t^*}S^*_{yy} - (1+a)\,\frac{\partial v^*}{\partial z^*}S^*_{yz}\right] = 0, \tag{31}$$

$$S^*_{yz} + W_e\left[\frac{\partial}{\partial t^*}S^*_{yz} + \left(\frac{1-a}{2}\right)\left\{\frac{\partial u^*}{\partial z^*}S^*_{xy} + \frac{\partial v^*}{\partial z^*}S^*_{yy}\right\} - \left(\frac{1+a}{2}\right)\frac{\partial v^*}{\partial z^*}S^*_{zz}\right] = \frac{1}{\phi}\frac{\partial v^*}{\partial z^*}, \tag{32}$$

$$S^*_{zz} + W_e\left[\frac{\partial}{\partial t^*}S^*_{zz} + (1-a)\left\{\frac{\partial u^*}{\partial z^*}S^*_{xz} + \frac{\partial v^*}{\partial z^*}S^*_{yz}\right\}\right] = 0, \tag{33}$$

$$u^* + iv^* = ce^{i\omega t^*} + de^{-i\omega t^*}, \qquad \text{on } z^* = 0, \quad t^* > 0, \tag{34}$$

$$u^*,\ v^* \to 0, \qquad \text{as } z^* \to \infty, \quad t^* > 0, \tag{35}$$

where

$$\omega = \frac{\nu\alpha}{U_0^2}, \qquad W_e = \frac{\lambda U_0^2}{\nu}, \qquad \phi = \frac{\mu+\eta}{\eta}, \qquad N = \frac{\sigma B_0^2\nu}{\rho U_0^2}, \tag{36}$$

$$F^* = u^* + iv^*, \tag{37}$$

in which W_e is the Weissenberg number (ratio of the relaxation time of the fluid to a characteristic time with the flow), ϕ is the ratio of the viscosities, and N is the Hartmann number. The compatibility equation (27) is obtained by eliminating $\hat{p}^*$ from equations (24)–(26).

4. ANALYTICAL SOLUTION

It is known that the analysis of the flow of the non-Newtonian fluids is more challenging mathematically and computationally. In closed form, the analytical solutions governing the flow of non-Newtonian fluids are not possible. In general, the order of the differential equation(s) characterizing the flow of these fluids is more than the number of the available boundary conditions. Thus, the adherence boundary condition is insufficient for determinacy. The difficulty is further accentuated by the fact that non-Newtonian parameters of the fluid usually occurs in the coefficient of the highest derivative. In the past, the usual attempt to solve this difficulty centered around seeking a perturbation solution assuming the non-Newtonian fluid parameter to be small; the classical paper being by Beard and Walters [18]. One may also refer, for example, to [19–23] for other problems in various geometries. Teipel [24] and Ariel [25] obtained an exact numerical solution of the problem in reference [18]. Their investigations revealed that perturbation solution is only valid for small values of non-Newtonian parameter less than the some "critical value". Therefore, the solution by perturbation technique gives only a trend as the fluid shows departure from the Newtonian nature or the fluid is slightly non-Newtonian. Since we have an interest in finding the perturbation solution so the present analysis is limited only to indicate the trend of the flow for the non-Newtonian fluid and is not a solution for the non-Newtonian fluid *per se*.

We note that equations (24)–(33) are nonlinear partial differential equations. Therefore, to carry out the perturbation, we shall assume all the functions can be expanded as

$$\Psi = \Psi_0 + W_e \Psi_1 + \dots, \tag{38}$$

where Ψ in turn stands for the functions u^*, v^*, $\hat{p}^*$, F^*, and S^*. Substituting form (38) for the functions u^*, v^*, $\hat{p}^*$, F^*, and S^* into (24) to (35), (37), and equating the terms at O(1), O(W_e), and O(W_e^2), we obtain, respectively, the following.

ORDER 1.

$$\frac{\partial u_0^*}{\partial t^*} - 2\Omega^* v_0^* = -\frac{\partial \hat{p}_0^*}{\partial x^*} + \phi \frac{\partial^2 u_0^*}{\partial z^{*2}} - N u_0^*, \tag{39}$$

$$\frac{\partial v_0^*}{\partial t^*} + 2\Omega^* u_0^* = -\frac{\partial \hat{p}_0^*}{\partial y^*} + \phi \frac{\partial^2 v_0^*}{\partial z^{*2}} - N v_0^*, \tag{40}$$

$$0 = -\frac{\partial \hat{p}_0^*}{\partial z^*}, \tag{41}$$

$$\frac{\partial^2 F_0^*}{\partial t^* \partial z^*} + (N + 2i\Omega^*) \frac{\partial F_0^*}{\partial z^*} = \phi \frac{\partial^3 F_0^*}{\partial z^{*3}}, \tag{42}$$

$$S_{0xx}^* = S_{0xy}^* = S_{0yy}^* = S_{0zz}^* = 0, \tag{43}$$

$$S_{0xz}^* = \frac{1}{\phi} \frac{\partial u_0^*}{\partial z^*}, \tag{44}$$

$$S_{0yz}^* = \frac{1}{\phi} \frac{\partial v_0^*}{\partial z^*}, \tag{45}$$

$$F_0^* = u_0^* + i v_0^*, \tag{46}$$

$$F_0^* = c e^{i\omega t^*} + d e^{-i\omega t^*}, \qquad \text{on } z^* = 0, \quad t^* > 0, \tag{47}$$

$$F_0^* \to 0, \quad \text{as } z^* \to \infty, \qquad t^* > 0. \tag{48}$$

ORDER W_e.

$$\frac{\partial u_1^*}{\partial t^*} - 2\Omega^* v_1^* = -\frac{\partial \hat{p}_1^*}{\partial x^*} + \phi \frac{\partial^2 u_1^*}{\partial z^{*2}} - \frac{\partial^3 u_0^*}{\partial t^* \partial z^{*2}} - N u_1^*, \tag{49}$$

$$\frac{\partial v_1^*}{\partial t^*} + 2\Omega^* u_1^* = -\frac{\partial \hat{p}_1^*}{\partial y^*} + \phi \frac{\partial^2 v_1^*}{\partial z^{*2}} - \frac{\partial^3 v_0^*}{\partial t^* \partial z^{*2}} - N v_1^*, \tag{50}$$

$$\frac{\partial \hat{p}_1^*}{\partial z^*} = -(1-a)\frac{\partial}{\partial z^*}\left[\left(\frac{\partial u_0^*}{\partial z^*}\right)^2 + \left(\frac{\partial v_0^*}{\partial z^*}\right)^2\right], \tag{51}$$

$$\frac{\partial^2 F_1^*}{\partial t^* \partial z^*} + (N + 2i\Omega^*)\frac{\partial F_1^*}{\partial z^*} = \phi\frac{\partial^3 F_1^*}{\partial z^{*3}} - \frac{\partial^4 F_0^*}{\partial t^* \partial z^{*3}}, \tag{52}$$

$$S_{1xx}^* = \left(\frac{1+a}{\phi}\right)\left(\frac{\partial u_0^*}{\partial z^*}\right)^2, \tag{53}$$

$$S_{1xy}^* = \left(\frac{1+a}{\phi}\right)\frac{\partial u_0^*}{\partial z^*}\frac{\partial v_0^*}{\partial z^*}, \tag{54}$$

$$S_{1xz}^* = \frac{1}{\phi}\left[\frac{\partial u_1^*}{\partial z^*} - \frac{\partial^2 u_0^*}{\partial t^* \partial z^*}\right], \tag{55}$$

$$S_{1yy}^* = \left(\frac{1+a}{\phi}\right)\left(\frac{\partial v_0^*}{\partial z^*}\right)^2, \tag{56}$$

$$S_{1yz}^* = \frac{1}{\phi}\left[\frac{\partial v_1^*}{\partial z^*} - \frac{\partial^2 v_0^*}{\partial t^* \partial z^*}\right], \tag{57}$$

$$S_{1zz}^* = -\left(\frac{1-a}{\phi}\right)\left[\left(\frac{\partial u_0^*}{\partial z^*}\right)^2 + \left(\frac{\partial v_0^*}{\partial z^*}\right)^2\right], \tag{58}$$

$$F_1^* = u_1^* + iv_1^*, \tag{59}$$

$$F_1^* = 0, \qquad \text{on } z^* = 0, \quad t^* > 0, \tag{60}$$

$$F_1^* \to 0, \qquad \text{as } z^* \to \infty, \quad t^* > 0. \tag{61}$$

ORDER 1 SOLUTION.

We seek a periodic solutions of the form,

$$F_0^* = F_{01}^* e^{i\omega t^*} + F_{02}^* e^{-i\omega t^*}. \tag{62}$$

This together with (42), (47) and (48) gives a solution of the following form,

$$F_0^* = c e^{(i\omega t^* - \lambda_{01} z^*)} + d e^{-(i\omega t^* + \lambda_{02} z^*)}, \tag{63}$$

with

$$\lambda_{01}, \lambda_{02} = \left\{\frac{N + i(2\Omega^* \pm \omega)}{\phi}\right\}^{1/2}, \tag{64}$$

or

$$\lambda_{01} = \left(\frac{2\Omega^* + \omega}{2\phi}\right)^{1/2}(\alpha_{01} + i\beta_{01}), \tag{65}$$

$$\lambda_{02} = \left(\frac{2\Omega^* - \omega}{2\phi}\right)^{1/2}(\alpha_{02} + i\beta_{02}), \tag{66}$$

$$\alpha_{01} = \left[\sqrt{\gamma_{01}^2 + 1} + \gamma_{01}\right]^{1/2}, \qquad \gamma_{01} = \frac{N}{2\Omega^* + \omega},$$

$$\beta_{01} = \left[\sqrt{\gamma_{01}^2 + 1} - \gamma_{01}\right]^{1/2} = \frac{1}{\alpha_{01}},$$

$$\alpha_{02} = \left[\sqrt{\gamma_{02}^2 + 1} + \gamma_{02}\right]^{1/2}, \qquad \gamma_{02} = \frac{N}{2\Omega^* - \omega},$$

$$\beta_{02} = \left[\sqrt{\gamma_{02}^2 + 1} - \gamma_{02}\right]^{1/2} = \frac{1}{\alpha_{02}}.$$

ORDER W_e SOLUTION.

Let us take

$$F_1^* = F_{11}^* e^{i\omega t^*} + F_{12}^* e^{-i\omega t^*}. \tag{67}$$

Substituting (67) into (52), we have two ordinary differential equations for F_{11}^* and F_{12}^* and solutions of these equations satisfying the corresponding boundary conditions derived from (60) and (61) are obtained and the expression for F_1^* is given by

$$F_1^* = -\frac{i\omega z^*}{2\phi}\left[c\lambda_{01} e^{(i\omega t^* - \lambda_{01} z^*)} - d\lambda_{02} e^{-(i\omega t^* + \lambda_{02} z^*)}\right] \tag{68}$$

and finally, the expression for F^* up to $O(W_e)$ is of the following form,

$$F^*(z^*, t^*) = c\left[1 - \frac{i\omega W_e z^* \lambda_{01}}{2\phi}\right] e^{(i\omega t^* - \lambda_{01} z^*)} + d\left[1 + \frac{i\omega W_e z^* \lambda_{02}}{2\phi}\right] e^{-(i\omega t^* + \lambda_{02} z^*)}. \tag{69}$$

5. CONCLUDING REMARKS

We present the oscillatory solution for the rotating flow of a Johnson-Segalman fluid bounded by a rigid plate. The time-dependent governing equations of this boundary value problem are expressed in terms of nondimensional parameters. The Weissenberg number W_e and Hartmann number N appear as the significant physical scales. Perturbation solutions up to $O(W_e)$ for the flow have been constructed. The following results are found from (69).

- It is noted that structure of the associated magnetohydrodynamic boundary layers on the plate are qualitatively similar to those of the classical hydrodynamic Stokes and Ekman layers.
- The thicknesses of the boundary layers are

$$\frac{1}{\alpha_{01}}\left[2\phi/\left(2\Omega^* + \omega\right)\right]^{1/2} \qquad \text{and} \qquad \frac{1}{\alpha_{02}}\left[2\phi/\left(2\Omega^* - \omega\right)\right]^{1/2}.$$

  Clearly, these thicknesses are in the combination of hydrodynamic and hydromagnetic boundary layers and are smaller than the classical Stokes and Ekman layers.
- The thicknesses of the boundary layers in the hydrodynamic case are $\left[2\phi/\left(2\Omega^* + \omega\right)\right]^{1/2}$ and $\left[2\phi/\left(2\Omega^* - \omega\right)\right]^{1/2}$ and these correspond to the case of [26] when $\mu = 0$.
- The most important feature of (69) is that unlike the hydrodynamic situation for the resonant case, (69) satisfies the boundary condition at infinity for all values of frequencies including the resonant frequency. Consequently, the associated boundary layers remain bounded for the resonant case. The physical implication of this conclusion is that for the case of resonance, the unbounded spreading of the oscillations away from the plate is controlled by the external magnetic field. Consequently, the hydromagnetic oscillations are confined to the ultimate boundary layers. This observation holds for both Johnson-Segalman and Newtonian fluids.
- The results for rotating and conducting Maxwell fluid can be recovered as a special case from the present analysis by taking $\mu = 0$ and $a = 1$. Moreover, if $\lambda = \mu = 0$ and $a = 1$, we are left with the results governing the flow of rotating and conducting Newtonian fluid.
- Finally, the consideration of the vorticity vector,

$$\text{curl } \mathbf{V} = \left(-\frac{\partial v}{\partial z}, \frac{\partial u}{\partial z}, 0\right),$$

  leads us to the conclusion that the rotation generates vorticity.
- It is remarked that the present study only predicts the trends of the flow for a Johnson-Segalman fluid and that for a full revelation of all the characteristics of the flow an alternate study, possibly a numerical one is required.

REFERENCES

1. I.J. Rao and K.R. Rajagopal, The effect of slip boundary condition on the flow of fluids in a channel, *Acta Mech.* **135**, 113, (1999).
2. A. Sirivat, A.Z. Szeri and K.R. Rajagopal, An experimental investigation of the flow of non-Newtonian fluids between rotating disks, *J. Fluid Mech.* **186**, 243, (1988).
3. I.J. Rao and K.R. Rajagopal, Some simple flows of a Johnson-Segalman fluid, *Acta Mech.* **132**, 209, (1999).
4. I.J. Rao, Flow of a Johnson-Segalman fluid between rotating coaxial cylinders with and without suction, *Int. J. Non-Linear Mech.* **34**, 63, (1999).
5. M.J. Lighthill, The response of laminar skin friction and heat transfer to fluctuations in the stream velocity, *Proc. R. Soc. Lond.* **224**, 1, (1954).
6. J.T. Stuart, A solution of the Navier-Stokes and energy equations illustrating the response of skin friction and temperature of an infinite plate thermometer to fluctuations in the stream velocity, *Proc. R. Soc. Lond.* **A231**, 161, (1955).
7. K.R. Rajagopal, A note on unsteady unidirectional flows of a non-Newtonian fluid, *Int. J. Non-Linear Mech.* **17**, 369, (1982).
8. T. Hayat, S. Asghar and A.M. Siddiqui, Periodic flows of a non-Newtonian fluid, *Acta Mech.* **131**, 169, (1998).
9. T. Hayat, S. Asghar and A.M. Siddiqui, On the moment of a plane disk in a non-Newtonian fluid, *Acta Mech.* **136**, 125, (1999).
10. A.M. Siddiqui, T. Hayat and S. Asghar, Periodic flows of a non-Newtonian fluid between two parallel plates, *Int. J. Non-Linear Mech.* **34**, 895, (1999).
11. M.E. Erdogan, A note on an unsteady flow of a viscous fluid due to an oscillating plane wall, *Int. J. Non-Linear Mech.* **35**, 1, (2000).
12. V.M. Soundalgekar and I. Pop, On hydromagnetic rotating flow in a rotating fluid past an infinite porous wall, *Z. Angew. Math. Mech. (ZAMM)* **58**, 718, (1973).
13. L. Debnath, On Ekman and Hartmann boundary layers in a rotating fluid, *Acta Mech.* **18**, 333, (1973).
14. L. Debnath, Exact solutions of the unsteady hydrodynamic and hydromagnetic boundary layer equations in a rotating fluid system, *Z. Angew. Math. Mech. (ZAMM)* **55**, 431, (1975).
15. K.D. Singh, An oscillating hydromagnetic Couette flow in a rotating system, *Z. Angew. Math. Mech. (ZAMM)* **80**, 429, (2000).
16. T. Hayat, K. Hutter, S. Asghar and A.M. Siddiqui, MHD flows of an Oldroyd-B fluid, *Mathl. Comput. Modelling* **36** (9/10), 987, (2002).
17. M.W. Johnson Jr. and D. Segalman, A model for viscoelastic fluid behaviour which allows for non-affine deformation, *J. Non-Newtonian Fluid Mech.* **2**, 255, (1977).
18. D.W. Beard and K. Walters, Elastico-viscous boundary layer flows 1. Two dimensional flow near a stagnation point, *Proc. Camb. Phil. Soc.* **64**, 667, (1964).
19. G.M. Shrestha, Laminar elastico-viscous flow through channels with porous walls with different permeability, *Appl. Sci. Res.* **20**, 289, (1969).
20. S.P. Mishra and U. Mohapatra, Elastico-viscous flow between a rotating and a stationary disk with uniform suction at the stationary disk, *J. Appl. Phys.* **48**, 1515, (1977).
21. K.R. Rajagopal, T.Y. Na and A.S. Gupta, Flow of a viscoelastic fluid over a stretching sheet, *Rheol. Acta* **23**, 213, (1984).
22. P.D. Verma, P.R. Sharma and P.D. Ariel, Applying quasilinearization to the problem of steady laminar flow of a second grade fluid between two rotating porous disks, *J. Tribol. Trans. ASME* **106**, 448, (1984).
23. M.E. Erdogan, On the flow of a non-Newtonian fluid past a porous flat plate, *Z. Angew. Math. Mech.* **55**, 79, (1975).
24. I. Teipel, Die raumliche staupunktstromung fur ein viscoelastiches fluid, *Rheol. Acta* **25**, 75, (1986).
25. P.D. Ariel, A hybrid method for computing the flow of viscoelastic fluids, *Int. J. Numer. Meth. Fluids* **14**, 757, (1992).
26. C. Thornley, On Stokes and Rayleigh layers in a rotating system, *Quart. J. Mech. and Appl. Math.* **21**, 451, (1968).