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A Local Optimization Technique for the Solution of the Problem of Nonlinear Seismic Tomography

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Abstract—A new method for the solution of the inverse kinematic problem of seismics which is based on the local optimization approach is presented. The method has higher stability and performance in comparison to the known methods and can be well adapted to the real data.

Keywords—Inverse kinematic problem, Optimization, Volterra equation, Nonlinear tomography.

PROBLEM STATEMENT

A model of the elastic isotropic nonhomogeneous medium filling the half-space $z > 0$ is considered. The material characteristics of the medium do not depend on the horizontal coordinate y. It is assumed that the velocity $v(x, z)$ of seismic wave propagation is a sufficiently smooth function and satisfies the following preliminary condition of regularity: for any positions of the source $(x_1, 0)$ and the receiver $(x_2, 0)$ of oscillations on the linear profile of the daily surface $z = 0$ from the point 0 to the point L, there exists the unique (rectifiable) curve $\gamma(x_1, x_2)$ such that the running time of the perturbation along γ at the speed v; i.e., the value

$$\tau(x_1, x_2) = \int_{\gamma(x_1,x_2)} \frac{ds}{v(x, z)} \tag{1}$$

gives the minimum to the Fermat functional

$$\tilde{\gamma}(x_1, x_2) \longrightarrow I(\tilde{\gamma}) := \int_{\tilde{\gamma}(x_1,x_2)} \frac{ds}{v(x, z)}. \tag{2}$$

The curve γ is the seismic ray, the value $\tau(x_1, x_2)$ is the time of the first arrival, and the function $(x_1, x_2) \to \tau(x_1, x_2)$ is the 2D hodograph of the refracted waves.

The inverse kinematic problem (IKP) of seismics or the problem of nonlinear seismic tomography, as it is now called, consists in the determination of the function $v(x, z)$ in some domain Ω of the half-space $z > 0$ "adjacent to" the aperture $[0, L] \times \{0\}$, using the function $\tau(x_1, x_2)$ known in the triangle $0 \le x_1 \le x_2 \le L$ (or in the trapezium) of the variables x_1, x_2. The domain Ω is unknown, but it must be contained in some lune-shaped domain, i.e., the totality of all points which are "filled" by the rays, the ends of which are on the aperture OL. Taking into account the last remark, IKP under consideration will be called the "lune problem," in contrast to IKP

Typeset by $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-TEX for the convex domain, when there is information on the times of arrivals for any pairs of points at the boundary of the domain.

It should be pointed out that the nonlinearity of the tomographic problem formulated here is not due to the fact that the curves γ in (1) are not rectilinear, but due to the fact that the curves γ are unknown. They are the extremals of the functional (2) which satisfies the usual differential second-order equation (ray equation) of the form

$$\frac{d\left(\dot{\vec{r}}/v\right)}{ds} = \operatorname{grad}\left(\frac{1}{v}\right), \tag{3}$$

($\vec{r} \equiv \vec{r}(s)$ is the radius-vector of the ray point, s is the length of the ray arc). This equation "depends" on the function $v(x, y)$ which is to be determined.

The mathematical-physical statement of IKP was first formulated in [1]. In this paper, IKP is reduced to the initial problem for the nonclassical, first-order equation in partial derivatives relative to the function $\tau(x_1, x_2, z)$ (the minimal running time of perturbation between the points (x_1, z) and (x_2, z) at the same depth z)

$$\tau_z = \pm\sqrt{n^2(x_1, z) - (\tau_{x_1})^2} \pm \sqrt{n^2(x_2, z) - (\tau_{x_2})^2}, \tag{4}$$

where $n(x_1, z) = -\tau_{x_1}(x_1, x_1, z)$, $n(x_2, z) = \tau_{x_2}(x_2, x_2, z)$.

METHODS OF SOLUTION

The solutions to the IKP of the lune-shaped domain have been proposed by many investigators, beginning from the work mentioned above. The algorithms proposed can be divided into three big classes. The algorithms of the first class are based on evolution equation (4) for the layer-by-layer recalculation of the times $\tau(x_1, x_2, z)$ from the depth $z = z_k$ to the depth $z = z_{k+1} = z_k + h_z$ using either the finite difference approximation or the method of characteristics. After each recalculation, the velocity v at the new depth is calculated using the values τ for the neighbouring points, when the rays are practically straight. In the algorithms of this type, we have to deal with substantial computational instability. It consists of the following: the velocity at the depth, at the point (x, z), is computed as the ratio $\tau(x_1, x_2, z)/(x_2 - x_1)$ at the points x_1, x_2 which are close to x (usually, $x = (x_1 + x_2)/2$). In this case, the value $\tau(x_1, x_2, z)$ is obtained from some $\tau(\tilde{x_1}, \tilde{x_2}, 0)$—the time along the big ray passing through the points (x_1, z), (x_2, z)—by means of multiple subtraction of small values (in the process of layer-by-layer recalculation). As a result of this procedure, the value $\tau(x_1, x_2, z)$ may consist only of the round-off errors.

The other class of algorithms is based on the method of successive approximations of Newton-Kantorovich. The problem of determination of the term Δv_n, as addition to the previous approximation v_n, $n = 0, 1, 2, \ldots$, at each step is the problem of integral geometry using the known family of curves. The problem is solved by some of the optimization methods. The effectiveness of this class of algorithms essentially depends on the choice of the zero approximation.

Our method refers to the third class which is intermediate between the indicated classes. It includes both the recalculation of the function $\tau(x_1, x_2, z)$ and the optimization. However, the recalculation is made for the depth $H \gg h_Z$ (the discretization step of equation (4) over the variable z providing the approximation of this equation). The velocity in the layer is determined using the optimization method for the subdomains with the horizontal size $\Delta \ll L$. The local approximations obtained are pasted together in a function which is assumed to be the sought-for velocity in the layer.

If we restrict ourselves to the zero approximations in the form of the linear velocities $v_i(x, z) = a_i + b_i z + c_i x$ in the i^{th} subdomain, $i = 1, 2, \ldots, N$, as local approximations (here a_i, b_i, c_i are determined by the optimization method), we shall obtain a preliminary, very rough idea of the algorithm proposed. Omitting the problem of recalculation in this paper, we shall dwell in more detail on the algorithm for the velocity determination in a layer.

CONSTRUCTION OF VELOCITY IN VERTICAL CROSS SECTIONS

The algorithm for the velocity determination in a layer uses the local approximations by linear velocities in a much larger number of subdomains than could be inferred from the preliminary description. Namely, for each vertical cross-section $x = \text{const} = x_k$, $k = k_0, k_0 + 1, \dots, N - k_0$, ($x_k$ is a grid node along x), a family of linear approximations $v_l(x, z) = a_l + b_l z + c_l x$ is constructed which depends on the positive parameter l, where l is the half-width of the linear aperture with the centre at the point $(x_k, 0)$, and the coefficients a_l, b_l, c_l are determined using the optimization over the rays (with some weights), the ends of which are on this aperture. The domain of definition of the velocity $v_l(x, z)$ is a circular lune $\Omega_l(x_k)$. The further description is based on two ideas.

The first idea is in the following. On the basis of the linear approximations $v_l(x, z)$, we get an idea of the integral means of the function $n(x, z) = 1/v(x, z)$ or the function $v(x, z)$ itself for the domains $\Omega_l(x_k)$. That is, the asymptotic equalities at $l \to 0$ hold:

$$\iint\limits_{\Omega_l(x_k)} \frac{dx\,dz}{v(x,z)} = \iint\limits_{\Omega_l} \frac{dx\,dz}{v_l(x,z)} + o(1) := F_l + o(1), \tag{5}$$

$$\iint\limits_{\Omega_l(x_k)} v(x,z)\,dx\,dz = \iint\limits_{\Omega_l} v_l(x,z)\,dx\,dz + o(1) := \Phi_l + o(1). \tag{6}$$

The second idea uses the hypothesis of compensation of the horizontal gradient of the function $n(x, z) = 1/v(x, z)$ or the function $v(x, z)$. It is in the assumption of "good" representability of these functions near the cross section $x = x_k$ in the form

$$\frac{1}{v(x,z)} = n(x,z) \approx n(z) + a(x - x_k), \qquad v(x,z) \approx v(z) + a'(x - x_k),$$

where a, a' are some constants.

Substituting the functions $n(x, z)$ and $v(x, z)$ in equalities (5), (6) for the right-hand sides of the last two formulas, reducing the integrals obtained to the single integrals, and rejecting the infinitely small $o(1)$ on the right, we obtain 1D integral equations of the Volterra type of the first kind for the determination of approximate values of the sought-for functions $n(x, z)$ or $v(x, z)$ on the given vertical cross section $x = x_k$. These equations have the form

$$\int_0^{h(l)} n(z) K(z,l)\,dz = F_l, \qquad 0 < l \le m_k h_x, \tag{7}$$

$$\int_0^{h(l)} v(z) K(z,l)\,dz = \Phi_l, \qquad 0 < l \le m_k h_x, \tag{8}$$

where $K(z, l) = \sqrt{h - z}\sqrt{2r - (h - z)}$, $h \equiv h(l)$ is the depth, $r \equiv r(l)$ is the radius of the lune arc $\Omega_l(x_k)$, h_x is the grid step of the sources-receivers on the daily surface, the values m_k are natural, giving the maximal value of the aperture associated with the cross section $x = x_k$.

Equations (7),(8) are solved by the method of Bogolyubov-Krylov [2]. As a result, we obtain the approximate values $n(z)$ or $v(z)$ at the points $z_i \equiv z_i(x_k) = h(l_i)$, where $l_i = ih_x$, $i = 1, 2, \dots, m_k$.

The "lower" part of the boundary of the domain Ω (named the "lower bound of the velocity recovery zone" and shown below in digital examples) is constructed using the points $(x_k, z_{m_k}(x_k))$, $k = 1, 2, \dots, N - 1$ of the plane (x, z). In this domain, the velocity is considered to be reconstructed on the basis of the given algorithm. Its values are computed as the bicubic spline, which is constructed using the data recalculated into the uniform grid from the grid $(x_k, z_i(x_k))$, $k = 1, 2, \dots, N-1$, $i = 0, \dots, m_k$ with the help of the special algorithm of smoothing recovery [3].

It should be noted that the layer or, more precisely, the rectangle H as discussed above, must be inserted into the domain Ω. It is clear that in this case the length of the rectangle will be smaller than the previous length which is equal to L.

STRONG HYPOTHESIS OF COMPENSATION

The idea of local one-dimensionality can be considered already at the stage of the minimization of the time functional in the class of linear velocities, if we use the arrival times only for the symmetric arrangements of receivers-sources (relative to the cross section $x = x_k$). Here, due to the weak sensitivity of the functional to the variation of the coefficient for the horizontal variable, it is reasonable not to determine this coefficient, but to set it equal to zero.

In order to estimate this sensitivity, we shall analyze, at $l \to 0$, the relative contribution into the times of arrivals associated with the presence of the horizontal gradient of the linear velocity $v(x, z) = a + bz + c(x - x_k)$, $c \neq 0$. Thus, we have

$$\tau(x_k - l, x_k + l) = \int\limits_{\gamma(x_k - l, x_k + l)} \frac{ds}{a + bz + c(x - x_k)} = \int\limits_{\gamma_-} \cdots + \int\limits_{\gamma_+} \cdots,$$

where γ_+ and γ_- are the left and right parts of the arc $\gamma(x_k - l, x_k + l)$ relative to the straight line $x = x_k$. Reducing the integral over γ_- to the integral over γ_+ with the help of simple transformations, we obtain

$$\tau(x_k - l, x_k + l) = 2 \int\limits_{\gamma_+} \frac{ds}{v(z)} + 2 \int\limits_{\gamma_+} \frac{c^2 (x - x_k)^2}{v^2(z) - c^2 (x - x_k)^2} \frac{ds}{v(z)},$$

where $v(z) = a + bx$. Applying the weight theorem of the mean in the second integral, we can write

$$\tau(x_k - l, x_k + l) = (1 + \alpha(\xi)) \int\limits_{\gamma} \frac{ds}{v(z)},$$

where

$$\alpha(\xi) = \frac{c^2 \xi^2}{v^2(\zeta) - c^2 \xi^2}, \qquad (\xi + x_k, \zeta) \in \gamma_+, \quad 0 < \xi < l.$$

So, the value $\alpha(\xi)$ is the sought-for relative contribution of the horizontal gradient into the times of arrivals for the symmetric arrangements with the aperture equal to $2l$. Writing $\alpha(\xi)$ in the form $\alpha(\xi) = \chi(\xi)/(1 - \chi(\xi))$, $\chi(\xi) = c^2 \xi^2 / v^2(\zeta)$ and taking into account that, at $0 < \xi < l$, $0 < \chi(\xi) < c^2 l^2 / a^2 := \chi$, we find that $0 < \alpha < \chi/(1 + \chi)$. Let us represent χ in the form

$$\chi = \left(\frac{c}{b}\right)^2 \left(\frac{bl}{a}\right)^2 \equiv \left(\frac{a}{b}\right)^2 \operatorname{tg}^2 \varphi.$$

We see that χ can be small for two reasons: due to small angular apertures (φ is small) and small ratios of the horizontal gradient to the vertical gradient (a/b is small).

And, to summarize, the strong hypothesis of compensation is in the fact that the local approximations are sought in the form of linear 1D velocities, and the minimization of the time functional is realized using all possible symmetric arrangements of sources-receivers relative to the cross section $x = x_k = \text{Const}$.

THE LOCALLY 1D METHOD

The reasoning of the previous section and the numerical results obtained on the basis of these considerations suggest that it is reasonable to abandon local optimizations in the class of 1D linear velocities and use the arrival times of the symmetrical arrangements as the hodograph for the reconstruction of the 1D velocity in the cross section $x = x_k$ by the Herglotz-Wiechert formula [4]. It is evident that the data formed must be incorporated into the class of solvability of the 1D inverse kinematic problem.

In the realization of this approach, we have obtained the most efficient algorithm which is stable in its application to the real data, and developed the software for the use of the method in the calculation of static corrections in the conditions of considerable variability of the earth's structure in the horizontal direction.

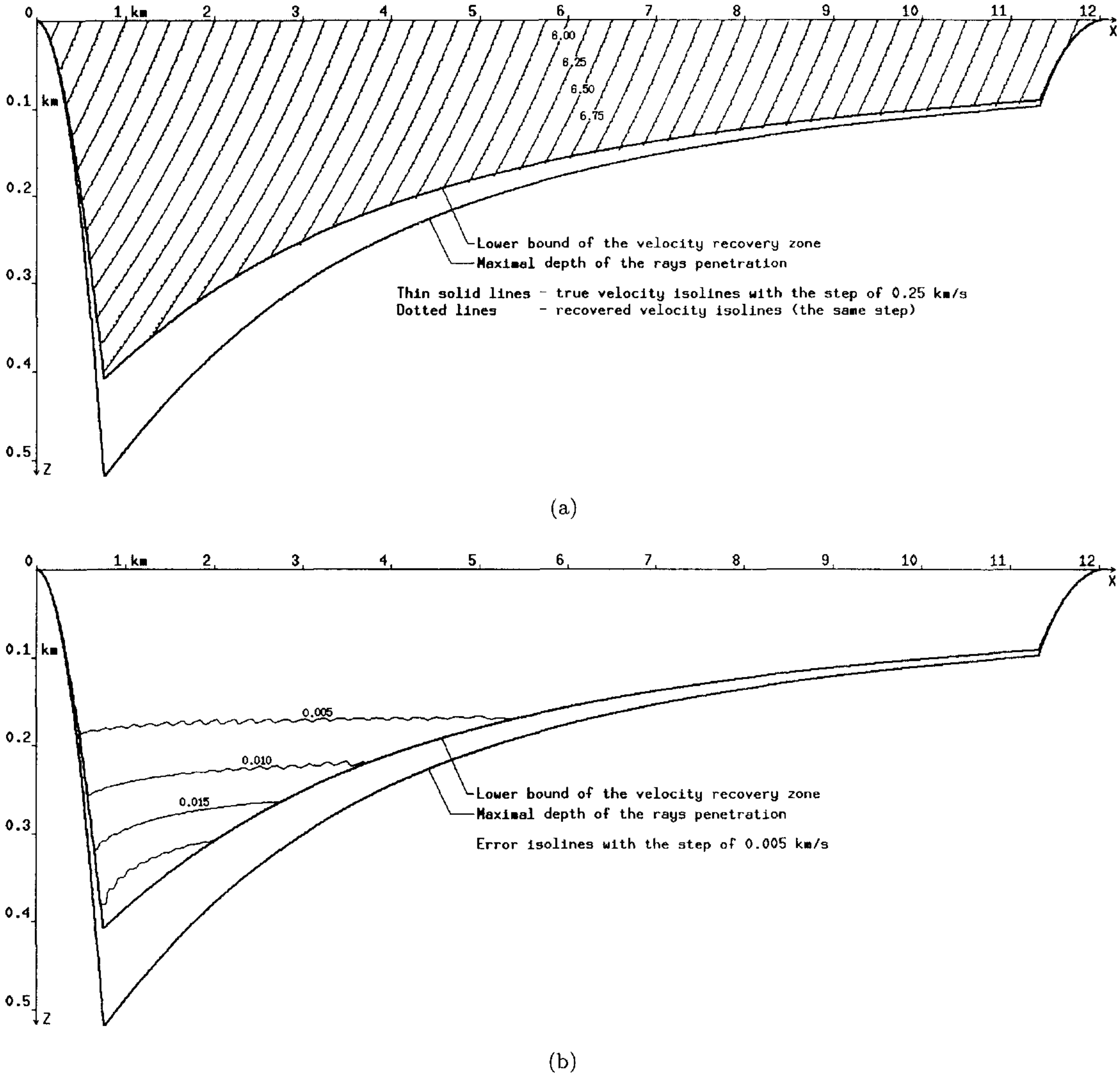


Figure 1.

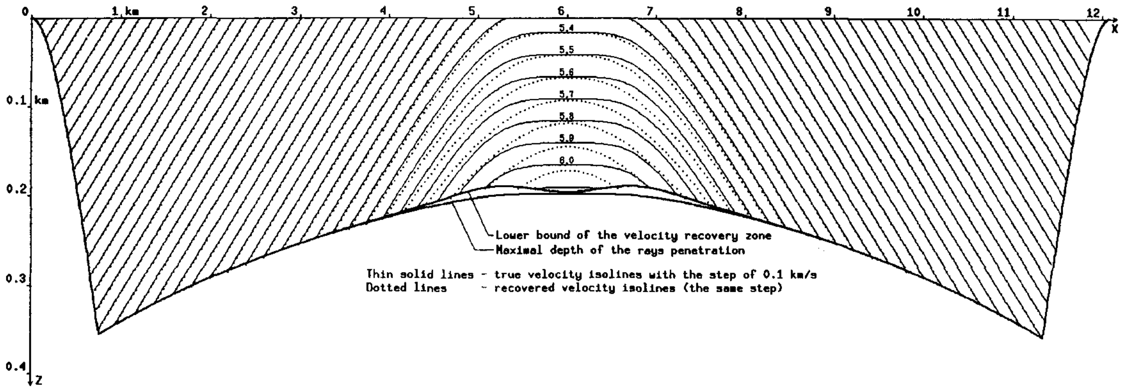
SOME NUMERICAL RESULTS

Numerical experiments for the estimation of the quality of reconstruction of the theoretical velocity distributions of different types by means of all the algorithms proposed were carried out.

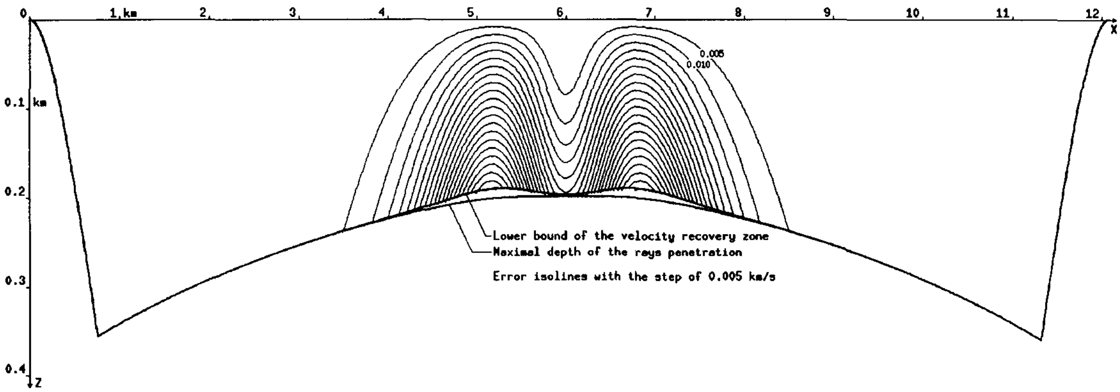
The geometrical parameters of the model were chosen as close as possible to the experimental scheme of observations using the common depth point (CDP) method for the determination of static corrections. In our case, the length of the horizontal profile of observations is 12 km, the largest distance in the arrangements of the sources-receivers is 1.5 km, the distance between the receivers is 12.2 m and the maximal number of receivers for one source is 124.

The vertical velocity gradient was chosen so that the maximal depth of the ray penetration at the basis of 1.5 km could be 200–500 m. The calculations showed that, when the modulus of the ratio of the horizontal gradient to the vertical gradient does not exceed 0.1, all the algorithms have, up to a depth of 200 m, approximately the same relative error of reconstruction which is close to 1%.

For reasons of space, we shall present only two examples of reconstruction of the theoretical velocity distribution. The minimization method in the class of 2D linear velocities (the 2D algorithm) with the integral equations for v along the vertical cross sections was taken as the reconstruction algorithm. This algorithm is exact on the linear velocity.



(a)



(b)

Figure 2.

Figures 1a and 1b illustrate the results of reconstruction for the "slightly nonlinear" velocity with a large horizontal gradient

$$v(x,z) = 6 + 4z + 4z^2 + x, \qquad |x-6| \le 6, \quad 0 \le z \le 0.550 \text{ (km/sec)}.$$

The inclination angle of isolines is about 14^0. The use of locally one-dimensional versions of the algorithms gives a substantially larger error.

A more complicated example is illustrated in Figures 2a and 2b for the velocity expression

$$v(x,z) = 6 + 4\left(z - \frac{1}{6}\left((x-6)^4 + 1\right)^{1/4}\right), \qquad |x-6| \le 6, \quad 0 \le z \le 0.400.$$

The velocity model has two zones with practically rectilinear isolines (see Figure 2a) with the modulus of the angular coefficient close to 1/6. The fact that the 2D-algorithm is exact on the linear velocity is clearly seen in Figure 2b, where the isolines of the error are practically absent from the zones with the constant gradient. In these areas, the modulus of the error does not exceed 0.005 km/sec.

Figure 3 illustrates the reconstruction of the velocity structure in the refractor using the real system of hodographs which were made available to us by the Green Mountain Geophysics Firm. The hodographs were obtained in order to introduce the static corrections behind the zone of small velocities in the implementation of the CDP method in the Las Animas Arch region (Colorado).

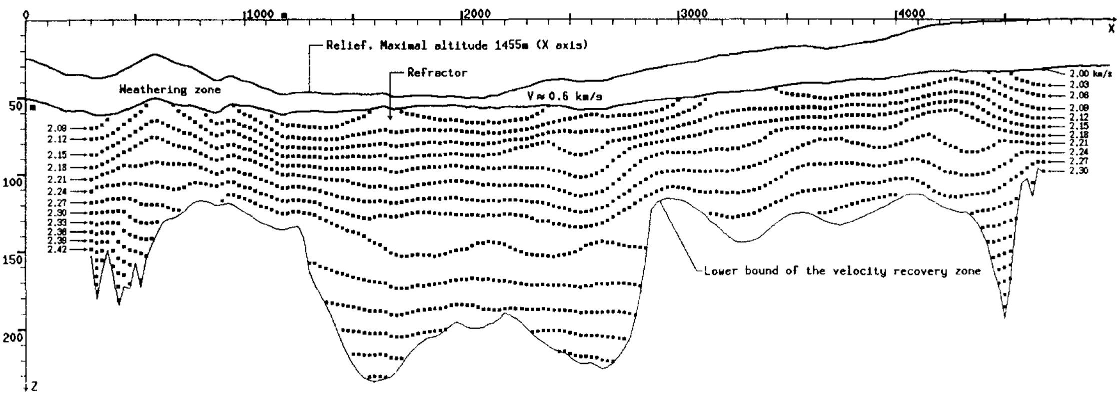


Figure 3.

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