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A Basic Reference State Suitable for Anomaly-Coupled Ocean-Atmosphere Climate Models

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Abstract—A large class of ocean-atmosphere models exists in which the ocean state is coupled to the model of the atmosphere only through the anomalies of the ocean state. The sea surface temperatures are defined with respect to a mean reference state, i.e., they are the difference between the ocean state and a reference state. Due to coupled model drift, the choice of reference state is important and it can have a large impact on the variability in the model. The reference state can be calculated as an average throughout the coupled simulation and various methods of doing this (moving average, exponentially weighted moving average and accumulated mean) are compared in this note. The accumulated mean method appears to be the sole method of the three which gives both unbiased anomalies and a convergent reference state. It is recommended for use in anomaly-coupled models for improving variability and predictability. © 1998 Elsevier Science Ltd. All rights reserved.

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1. MOTIVATION

A large class of ocean-atmosphere models exists in which the ocean state is coupled to the model of the atmosphere only through the anomalies of the ocean state. The anomalies are defined with respect to a mean reference state, i.e., they are the difference between the ocean state and the reference state. In general, the reasons for this are

- to minimize the coupled model drift which can lead to poor predictability [1], or
- to accommodate an atmosphere model based on a linearisation about the mean state [2].

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In such nonlinear coupled models, the results can depend strongly on the choice of the reference state. This has been shown by Macías and Stephenson [3] for a hybrid coupled model of the Pacific. There is no *a priori* reason that the mean state for the coupled model must also be the reference state and, indeed, the two states differ in general. However, this gives rise to anomalies which do not vary about zero but which are permanently either positively or negatively biased. Realistic variability and good predictability require that one choose the reference state equal to the coupled model's mean state but this leads to a question because the coupled model's mean is not known in advance.

One can imagine an iterative method whereby a series of coupled model runs is performed, each run using the mean state of all the previous runs as its reference state. This seems a reasonable approach, though due to hysteresis, there is no proof that this method will converge. The limiting case of this method wherein the length of each run goes to zero gives rise to the continuous ACcumulated MEan (ACME) adaptive method for which one accumulates the mean state throughout the run and uses it as an evolving reference state. The ACME method was developed by Macías and Stephenson [3] for use in an intermediate coupled model of the tropical Pacific; it was shown to give improved variability. The use of this method in predictability studies of anomaly-coupled models would be of great interest. It could help reduce the erroneous drift which can occur once the model is coupled. The aim of this short note is to present the method and to compare it with other possible methods.

2. NOTATION

Consider the Sea Surface Temperature (SST) at some grid point on a certain day in year n of the coupled run (as an example) and denote this as T_n. The reference temperature can be allowed to evolve throughout the coupled run and is denoted by $\widetilde{T}_n$. Hence, the SST anomaly in year n can be written as $\Delta_n = T_n - \widetilde{T}_n$ and it is this anomaly which is passed to the atmosphere model for coupling. In order to quantify the convergence of the reference temperature, it is also useful to introduce the definition of a convergence error, ϵ_n, where $\epsilon_n = \widetilde{T}_n - \widetilde{T}_{n-1}$. Note that in all these definitions, there is no explicit mention of the calendar date. Thus, the methods presented below are valid no matter what the frequency of the coupling of atmosphere and ocean models, be it days, weeks, months, etc.

It is desirable to define the reference temperature so that the following properties are satisfied.

1. **Unbiased anomalies.** After a sufficiently long time, the reference temperature should represent the mean of the coupled model temperatures so that the anomalies are unbiased and fluctuate about zero. The long-time mean of the anomalies should be zero, $E(\Delta) = 0$.
2. **Convergence.** After many years of a coupled run, the reference temperature should converge uniformly to a climatological value independent of the year. That is to say,

$$\lim_{n \to \infty} \| \epsilon_n \| \to 0. \tag{1}$$

   This implies that the long-time variance of the convergence error equals zero, $\mathrm{Var}(\epsilon) = E(\epsilon^2) = 0$, where the identity $E(\epsilon) = 0$ has been used.

A method sufficient to guarantee these properties will be presented in the next section.

3. DEFINING THE REFERENCE

In this section, various methods for defining an average reference temperature throughout the coupled simulation will be described and their strengths and weaknesses discussed.

3.1. Moving Average (MA)

Perhaps the simplest method that can be envisaged is a climatological *moving average*[1] over the previous few years of the coupled simulation. Mathematically this can be expressed as

[1] Also commonly known as a running mean.

$$\widetilde{T}_n = \frac{1}{m}\sum_{k=0}^{m-1} T_{n-k}, \tag{2}$$

where m equals the number of years averaged over. It can be seen from this that the time-mean of the reference temperature is equal to the time-mean of the temperature and that the anomalies are unbiased. The expression can be rewritten in an iterative form as

$$\widetilde{T}_n = \widetilde{T}_{n-1} + \frac{1}{m}(T_n - T_{n-m}), \tag{3}$$

and hence, it can be shown that $\text{Var}(\epsilon) = \text{Var}(T_n - T_{n-m})/m$ which can be expressed as $\text{Var}(\epsilon) = 2\,\text{Var}(T)(1 - r_m)/m$ where r_m is the m-year lag autocorrelation coefficient of T and generally is less than unity. From this it can be seen that the long-time variance of the error is in general not zero, and hence, that the reference temperature does not converge. As a consequence of the sharp edges on the moving-averages filter, the reference temperature oscillates in time. This effect is well known; even the moving average of a stochastic series can give rise to an oscillatory series, a phenomenon known as the Slutsky-Yule effect [4]. Such spurious oscillations on the reference temperature are dangerous; they can give rise to artificial variability via nonlinearities in the coupled model.

3.2. Exponentially Weighted Moving Average (EWMA)

An alternative method proposed by Holt [5] and frequently used in econometric and financial predictions, is the Exponentially Weighted Moving Average defined as

$$\widetilde{T}_n = \alpha\widetilde{T}_{n-1} + (1 - \alpha)T_n, \tag{4}$$

where α is an arbitrary constant. In econometric predictions, a typical value of the constant is $\alpha = 0.7$ (see for example [6]) his method weights recent values more highly than previous values in contrast to the previous moving average method. The time-mean of this equation is $(1 - \alpha)E(\Delta) = 0$, and hence, the anomalies are guaranteed to be unbiased if α is not unity. One can also easily obtain the identity

$$\alpha\epsilon_n = (1 - \alpha)\Delta_n, \tag{5}$$

which relates the convergence errors to the anomalies. From this identity, it can be seen that the errors can only converge to zero if α is unity or if the anomalies converge to zero. Since neither condition holds, it is evident that this method also does not converge. An adaptive method whereby α is allowed to converge to unity in the long-time limit would alleviate this problem, but at the price of having biased anomalies. As an alternative, the following method will be shown to have an α which converges to unity giving a convergent reference temperature as well as anomalies that are unbiased.

3.3. Accumulated Mean (ACME)

Consider a reference temperature defined as the mean accumulated since m years before the start of the coupled simulation

$$\widetilde{T}_n = \frac{1}{n + m + 1}\left(\sum_{k=0}^{n} T_k + mT_c\right), \tag{6}$$

where it is assumed that the temperature is equal to the reference climatology T_c before the start of the simulation and that T_0 is the temperature from the initial year of the coupled run. This can be recast as

$$\widetilde{T}_n = \alpha_n\widetilde{T}_{n-1} + (1 - \alpha_n)T_n, \tag{7}$$

where a time evolving α_n is defined as

$$\alpha_n = \frac{n+m}{n+m+1}. \tag{8}$$

This value monotonically increases from $m/(m+1)$ to unity throughout the simulation, and hence, represents an elaboration of the EWMA method. After numerical testing the choice of $m = 2$ appears to be desirable giving an initial $\alpha = 2/3$ which is close to $\alpha = 0.7$. The identity of equation (5) still holds albeit with a time-varying α. Because α converges to unity in the long time limit, this method is guaranteed to converge, providing only that the variance of the temperature anomalies remains finite. Furthermore, the reference temperature converges to the mean temperature in the long-time limit and hence the anomalies are unbiased. This ACME method appears desirable to use in defining a mean reference temperature for anomaly-coupled models.

3.4. Related Methods

In [7], a related method of using long-term annual means has been successfully applied in calculating the flux correction in the spin-up phase of a fully coupled global climate model simulation. A double time integration was needed in order to damp out spurious oscillations coming from the first bias correction.

4. CONCLUDING REMARKS

Various methods have been presented for defining a reference state. All have the property that the mean of the reference state is the mean state. Hence, all the anomalies have zero mean and are, therefore, unbiased. Of the methods examined, only the ACME method has the property that the reference state converges to the mean state in the long time limit. It is, therefore, preferable for use in climate models. The ACME method affords a way of smoothly interpolating in time between the reference climatology and the mean state of the coupled model. The choice of $m = 2$ appears to be reasonable and corresponds in the initial year to the typical EWMA model used in financial predictions. Using the ACME method to define the reference state obviates the difficult choice of reference state and provides a natural basic state about which to define anomalies in anomaly coupled climate models. The method gives unbiased anomalies, helps to improve variability and, in many cases, will improve even short term predictions. It is currently being employed today (see [8]), and has performed favorably.

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