Buckling delamination of a rectangular plate containing a rectangular crack and made from elastic and viscoelastic composite materials

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A R T I C L E   I N F O

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A B S T R A C T

This paper studies a three-dimensional buckling delamination problem for a rectangular plate made from elastic and viscoelastic composite material. It is assumed that the plate contains a rectangular band-crack (Case 1) and a rectangular edge-crack (Case 2) and that the edge-surfaces of these cracks have an initial infinitesimal imperfection. The evolution of this initial imperfection with an external compressive loading, acting along the crack (for an elastic composite) or with duration of time (for a viscoelastic composite under fixed external loading) is investigated within the framework of three-dimensional geometrically non-linear field equations of the theory of the viscoelasticity for anisotropic bodies. To determine the values of the critical force or critical time as well as the buckling delamination mode, the initial imperfection criterion is used. The corresponding boundary-value problems are solved by employing boundary form perturbation techniques, Laplace transform and FEM (Finite Element Method). The influence of the materials and/or the geometrical parameters of the plate on the critical values are discussed. In particular, it is established that for the considered change range of the problem parameters, the buckling form depends only on the initial infinitesimal imperfection mode of the crack edges.

1. Introduction

One of the most common failure mechanisms in laminated composite materials is a local buckling around the delaminated zone, i.e. of the zone in which two adjacent layers are partially debonded at their interface. Note that this zone may be formed as a consequence of various impact events, poor fabrication processes and fatigue. As is well-known, the compressive strength of the structures made from laminated composite materials may be reduced several times because of the delamination damage, which is modeled as a crack in related research. The region bounded by the crack and the laminate free surface is liable to buckle locally under compressive loads, thereby creating conditions conducive to delamination growth and consequent global failure of the structure. One of the pioneering investigations related to the buckling-failure problem was done by Kachanov (1976) and numerous studies have so far been carried out in this field. A review of these investigations is given by Kardomateas et al. (1995), Nilsson et al. (1993), Chai et al. (1981), Wang et al. (1995) and others in which it is presumed that there is a crack which is parallel to the free plane and to the direction of the compressed external forces. In this case, the beginning of the delamination growing process is modeled as a buckling of the part of the material which occupies the region between the crack and the free plane, and solutions are found in the framework of the approximate stability loss theories of plates or beams. It is evident that the results of these investigations do not apply in the cases where the thickness of this part is equal to or greater than the length of the crack. Moreover, there is a series of works, such as Hwang and Mao (1999), Short et al. (2001) and Arman et al. (2006), investigating the effects of the geometry of the delaminated portion on the buckling force of the laminate. At the same time, in these works the investigations were carried out experimentally and numerically with the use of FEM modeling. For instance, in the paper by Hwang and Mao (1999), the influence of the delaminated zone (crack) on the global buckling critical forces of the plate-strip made from glass-fiber layers was studied. In the paper by Short et al. (2001), the effect of delamination geometry (i.e. the sizes of the rectangular interface crack) on the global buckling critical forces of the layered rectangular plate was investigated experimentally and numerically by employing 3D modeling using the packet program ABAQUS V.5.8 FEM code. The
same problem for a rectangular plate containing a circular cylindrical hole through the thickness of the plate was investigated by Arman et al. (2006). It was assumed that the circular delamination (crack) was around the circular hole and to obtain numerical results, 3D FEM modeling of the plate was used.

The buckling driven delaminations of compressed films and coatings on substrates were studied experimentally by Evans and Hutchinson (1995), Gioia and Ortiz (1997), Hutchinson and Suo (1992), Hutchinson et al. (1992), Nilsson and Giannakopoulos (1995), Thouless et al. (1994), Wang and Evans (1998) and Moon et al. (2002). In these and other investigations of these authors (for examples, see Moon et al., 2004; Hutchinson et al., 2000), approximate mathematical modeling was also used to describe the experimentally studied problems. It should be noted that in all the foregoing investigations the fundamental buckling theories based on the three-dimensional non-linear equations of the deformable body mechanics were not proposed or used. Such theories, for example, the Three-Dimensional Linearized Theory of Stability (TDLTS) of the deformable solid body mechanics for local buckling problems for bodies containing cracks were proposed and employed in the papers by Guz and Nazarenko (1985a,b) and others. A detailed description of the field equations and relations of the TDLTS are given in many references, for instance in the monograph by Guz (1999). Recent reviews of related investigations were done in papers by Guz and Nazarenko (1989a,b) and Guz et al. (2004). A detailed description of some early results was given in the monograph by Guz (2008a,b). The present level of these investigations is detailed in a paper by Bogdanov et al. (2009).

It should be noted that the TDLTS has also been applied successfully for investigation of the micro-mechanical internal and near surface stability loss problems of composites: see references Guz and Dekret (2009a,b), Guz and Dekret (2008) and Dekret (2008a,b) and the references listed in those papers.

In all the investigations reviewed above, it was assumed that the materials of the composites are time-independent. The development of the TDLTS based on the initial imperfection stability loss criterion by Hoff (1954) for time dependent materials was proposed and employed in the works by Akbarov (1998, 2007), Akbarov et al. (1997), Akbarov and Yahnioğlu (2001), Akbarov and Mamedov (2009) and others. A description of some related results was also given in the monograph by Akbarov and Guz (2000). The development and application of the above-noted version of the TDLTS on the study of the buckling delamination problems of the elements of construction (such as plate-strips and circular plates) made from viscoelastic materials was done in papers by Akbarov and Rzayev (2002a,b, 2003), Rzayev and Akbarov (2002) and others. A review of these studies was given in the paper by Akbarov (2007).

In these studies, two-dimensional problems were analyzed for a plate-strip containing a crack, the edges of which were parallel to the plane of the plate and a circular plate containing a penny-shaped crack, the edge-faces of which were also parallel to the plate’s upper and lower faces. These investigations were also carried out by utilizing 2D FEM modeling. In the present paper, however, an attempt is made to develop the approach proposed in the works by Akbarov and Rzayev (2002a,b) and Rzayev and Akbarov (2002) for three-dimensional buckling delamination problems, namely, for a rectangular plate containing a rectangular crack. Two cases are considered: Case 1: it is assumed that the rectangular plate contains an embedded rectangular band crack and Case 2: it is assumed that the rectangular plate contains a rectangular edge-crack. For the concrete numerical investigation done by utilizing 3D FEM modeling, the material of the plates is modeled as a viscoelastic transversal isotropic one whereby the direction of the isotropy axis coincides with the direction of the normal vector of the face planes of the plates.

### 2. Formulation of the problem

Consider a thick rectangular plate which contains a rectangular crack. The Cartesian coordinate system $OX_1OX_3$ is associated with the plate so as to give Lagrange coordinates of the points of the plate in the natural state. Assume that the plate occupies the region

$$\Omega = \{0 < x_1 < \ell_1; \ 0 < x_2 < h; \ 0 < x_3 < \ell_3\}.$$  \hspace{1cm} (1)

We will distinguish two cases with respect to the form and location of the crack in the plate. In Case 1 we assume that the rectangular band-crack at

$$\Omega_1 = \{\ell_1 - \ell_0 < x_1 < (\ell_1 + \ell_0)/2; \ x_2 = \bar{h}_1; \ 0 < x_3 < \ell_3\},$$  \hspace{1cm} (2)

but in Case 2 we assume that the plate contains the rectangular edge-crack at

$$\Omega_2 = \{(\ell_1 - \ell_0)/2 < x_1 < (\ell_1 + \ell_0)/2; \ x_2 = \bar{h}_1; \ \ell_1 - \ell_3 < x_3 < \ell_3\}.$$  \hspace{1cm} (3)

In Eqs. (2) and (3) $(\ell_0, \ell_3)$ is the length of the crack along the axis $OX_1(OX_3)$. It is assumed that edge-surfaces of the crack have an insignificant initial imperfection and this imperfection is symmetric with respect to the $x_1 = \ell_1/2$ plane and with respect to the plane $x_3 = \bar{h}_1$. Fig. 1 shows schematically the geometry of the plate which contains the mentioned edge-crack. According to Fig. 1, the geometry of the plate containing the band-crack can be imagined easily. The equations of the crack edge-surfaces can be written as follows:

**Case 1.**

$$x_1^+ = \bar{h}_1 + df^+ (x_1), \text{ for } (\ell_1/2 - \ell_0/2) < x_1 < (\ell_1/2 + \ell_0/2), \ 0 < x_3 < \ell_3;$$

$$f^+((\ell_1 - \ell_0)/2) = f^+((\ell_1 + \ell_0)/2) = 0,$$

$$\frac{df^+((\ell_1 - \ell_0)/2)}{dx_1} = \frac{df^+((\ell_1 + \ell_0)/2)}{dx_1} = 0.$$  \hspace{1cm} (4)

**Case 2.**

$$x_1^\pm = \bar{h}_1 + df^\pm (x_1, x_3), \text{ for } (\ell_1/2 - \ell_0/2) < x_1 < (\ell_1/2 + \ell_0/2), \ (\ell_1 - \ell_3) \leq x_3 < \ell_3;$$

$$f^\pm((\ell_1 - \ell_0)/2, x_3) \Big|_{(\ell_1 - \ell_0)/2} = f^\pm((\ell_1 + \ell_0)/2, x_3) \Big|_{(\ell_1 + \ell_0)/2} = 0,$$

$$\frac{df^\pm((\ell_1 - \ell_0)/2, x_3)}{dx_1} \Big|_{(\ell_1 - \ell_0)/2} = \frac{df^\pm((\ell_1 + \ell_0)/2, x_3)}{dx_1} \Big|_{(\ell_1 + \ell_0)/2} = 0.$$  \hspace{1cm} (5)

In (4) and (5), $\epsilon$ is the dimensionless small parameter ($\epsilon \ll 1$) which characterizes the degree of the initial imperfection of the crack edge-surfaces and the upper index “+” (”-”) shows the upper (lower) surface of the crack. We will consider two modes of the initial imperfection of the crack edge-surfaces. For the first mode, denoted as sine-phase, we assume that

$$f^+ = f^-.$$  \hspace{1cm} (6)

However, for the second mode we assume that

$$f^+ = -f^-$$  \hspace{1cm} (7)

and call this the anti-phase mode. Thus, based on the above, we investigate an evaluation of the foregoing initial infinitesimal imperfections of the crack edge-surfaces under compression of the plate along the $OX_1$ axis with
uniformly distributed normal forces with intensity \( p \). This evaluation will be investigated by utilizing the three-dimensional geometrically non-linear equations of the theory of viscoelasticity for the anisotropic body.

The governing field equations of this theory are

\[
\frac{\partial}{\partial \xi_j} \left[ \sigma_{ji} \left( \frac{\partial u_i}{\partial \xi_j} + \frac{\partial u_j}{\partial \xi_i} \right) \right] = 0, \quad \sigma_{ji} = C_{jir0} \varepsilon_i(t) + \int_0^t C_{jir}(t - \tau) \varepsilon_i(\tau) d\tau,
\]

where \( \varepsilon_{rs} = \frac{1}{2} \left( \frac{\partial u_i}{\partial \xi_j} + \frac{\partial u_j}{\partial \xi_i} + \frac{\partial u_i}{\partial \xi_i} \right) \), \( i,j,r,s = 1,2,3 \).

Throughout, repeated indices indicate summation over their ranges. In Eq. (8) the following notation is used: \( \sigma_{ji} \) is a component of the stress tensor, \( \varepsilon_i \) is a component of the displacement vector, \( C_{jir0} \) and \( C_{jir} \) are instantaneous values of the elastic constants and \( C_{jir}(t) \) is a function through which the core of the operators of the constitutive relations of the anisotropic viscoelastic material is determined.

Now we consider formulation of the boundary conditions. For Case 1, these can be written as follows:

\[
\left. u_2 \right|_{\xi_1 = 0} = -u_2 \left|_{\xi_1 = -\epsilon_3} \right. = \left. u_2 \right|_{\xi_1 = -\epsilon_3 - 0} = u_2 \left|_{\xi_1 = -\epsilon_3} \right. = 0,
\]

\[
\left. \sigma_{j1} \left( \frac{\partial u_i}{\partial \xi_0} + \frac{\partial u_i}{\partial \xi_0} \right) \right|_{\xi_0 = 0} = \left. \sigma_{j1} \left( \frac{\partial u_i}{\partial \xi_0} + \frac{\partial u_i}{\partial \xi_0} \right) \right|_{\xi_0 = b} = 0,
\]

\[
\left. \sigma_{21} \left( \frac{\partial u_i}{\partial \xi_1} + \frac{\partial u_i}{\partial \xi_1} \right) \right|_{\xi_1 = 0} = \left. \sigma_{21} \left( \frac{\partial u_i}{\partial \xi_1} + \frac{\partial u_i}{\partial \xi_1} \right) \right|_{\xi_1 = -\epsilon_3} = \frac{p}{2}, \quad i = 1,2,3,
\]

\[
\left. \sigma_{31} \left( \frac{\partial u_i}{\partial \xi_1} + \frac{\partial u_i}{\partial \xi_1} \right) \right|_{\xi_1 = 0} = \left. \sigma_{31} \left( \frac{\partial u_i}{\partial \xi_1} + \frac{\partial u_i}{\partial \xi_1} \right) \right|_{\xi_1 = -\epsilon_3 - \epsilon_3} = 0.
\]

\[
\left( \sigma_{21} \left( \frac{\partial u_i}{\partial \xi_1} + \frac{\partial u_i}{\partial \xi_1} \right) \right) x_2^+ = h_4 + ef^+(\xi_1),
\]

\[
\left( \xi_1 - \xi_1^0 \right) / b < x_2 < \left( \xi_1 - \epsilon_3 \right) / \epsilon_3,
\]

\[
0 < x_3 < \epsilon_3 \]

\( n_i^+ = 0 \).

In Eq. (9), \( n_i^+ \) is the orthonormal component of the unit normal vector of the considered surfaces (i.e. the crack edge-surfaces). The other notations used in Eqs. (8) and (9) are conventional. Almost the same conditions hold for Case 2, but in the latter case, the condition written for \( u_2 \) at \( x_3 = \epsilon_3 \) is dropped and the last condition in (9) is satisfied for the interval \( x_3 \in (\xi_3 - \epsilon_3, \xi_3) \). Thus, we assume that in Case 1 the displacement of all the ends’ surfaces along the plate thickness are equal to zero. However, in Case 2, this condition is satisfied only for the ends at \( x_1 = 0 \) and \( x_1 = \epsilon_1 \) and \( x_3 = 0 \). The end \( x_3 = \epsilon_3 \) is free.

### 3. Solution method

To simplify the analysis, we consider only the solution procedure for Case 2, from which, using a particular case, the solution procedure for Case 1 can be obtained. First, using the equations of the crack edge-surfaces given in (5), i.e. the equation \( x_2^2 = h_4 + ef^2(\xi_1, \xi_3) \), we derive the following expressions for \( n_i^+ \):

\[
\begin{align*}
n_i^+ &= \pm \varepsilon_0^i \left( \frac{\partial u_i}{\partial \xi_1} \right) + \pm \varepsilon_0^i \left( \frac{\partial u_i}{\partial \xi_3} \right) + \varepsilon_0^i \left( \frac{\partial u_i}{\partial \xi_1} \right), \quad j = 1,2,3, \\
\end{align*}
\]

where \( \delta_0^i \) (\( j = 1,2,3, i = 1,2,3 \)) is a Kronecker symbol.

Assume that \( \varepsilon^2 \left( \frac{\partial u_i}{\partial \xi_1} \right)^2 + \left( \frac{\partial u_i}{\partial \xi_3} \right)^2 \ll 1 \). According to this, the expression (10) can be represented in the series form in terms of the small parameter \( \varepsilon^2 \):

\[
\begin{align*}
n_i^+ &= \sum_{k=0}^{\infty} \varepsilon^{2k+1} n_i^{(k)}(x_1, x_3), \quad n_i^{(0)} = \pm 1 + \sum_{k=1}^{\infty} \varepsilon^{2k} n_i^{(k)}(x_1, x_3), \\
n_i^{(2k)} &= \sum_{k=0}^{\infty} \varepsilon^{2k+1} n_i^{(k)}(x_1, x_3). \\
\end{align*}
\]

In Eq. (11), the explicit expressions of the coefficients \( n_i^{(k)}(x_1, x_3) \), \( n_i^{(2k)}(x_1, x_3) \) and \( n_i^{(2k)}(x_1, x_3) \) are too long so they are not given here. At the same time, these expressions can be easily obtained by employing the well-known power series expansion of the expressions given in (10). Note that Eqs. (10) and (11) are written for Case 2 and by substituting \( \partial f / \partial \xi_3 = 0 \) the corresponding expressions for Case 1. According to Akbarov (1998), Akbarov and Yahioglu (2001), Akbarov et al. (1997), Akbarov and Rzayev (2002a,b, 2003) and Rzayev and Akbarov (2002), the sought values are represented in series form in terms of \( \varepsilon^2 \) as follows:

\[
\begin{align*}
\left\{ \sigma_{ji}, \partial f / \partial \xi_1; \partial f / \partial \xi_3 \right\} &= \sum_{k=0}^{\infty} \varepsilon^{2k} \left\{ \sigma_{ji}^{(k)}, \partial f / \partial \xi_1; \partial f / \partial \xi_3^{(k)} \right\}, \\
\end{align*}
\]

After substituting Eq. (12) into Eqs. (8) and (9) and comparing identical powers of \( \varepsilon^2 \), we obtain a corresponding closed system of equations and boundary conditions to describe each approximation. Owing to the linearity of the mechanical relations in Eq. (8) and the end conditions for displacements in Eq. (9), these relations and conditions will be satisfied for each approximation in Eq. (12) separately. The remaining relations obtained from Eqs. (8) and (9) for every \( q \)th approximation, contain the values of all the previous approximations. At the same time, whilst satisfying the last edge-surface conditions in Eqs. (8) and (9), we employ the boundary form perturbation technique, according to which the values of each approximation in Eq. (12) are expanded in series in the vicinity of \( (x_1, \xi_3 = 0, x_3) \) and \( (x_1, \xi_3 = 0, x_3) \) and using expression (11), the corresponding conditions on the crack edge-surfaces are also obtained for the first and subsequent approximations.

It follows from well-known mechanical considerations that for comparatively rigid composites under determination of the zeroth approximation we can use the relation \( \delta_0^i = \delta_0^i, \) and according
to the boundary conditions for the zeroth approximation obtained from conditions (8) and (9), the stresses in the zeroth approximation are determined as follows:

$$\sigma_{ij}^{(0)} = p, \quad \varepsilon_{ij}^{(0)} = 0, \quad \text{for } i \neq j.$$  \hspace{1cm} (13)

Moreover, according to the relation \(\delta_{ij} + \partial \sigma_{ij}^{(0)}/\partial x_i = \delta_{ij}\), and expression (12) we obtain the following equilibrium equations and geometrical relations for the first approximation

$$\frac{\partial \sigma_{ij}^{(1)}}{\partial x_i} + \sigma_{ij}^{(0)} \frac{\partial u_j^{(1)}}{\partial x_i} = 0, \quad \varepsilon_{ij}^{(1)} = \frac{1}{2} \left( \frac{\partial u_i^{(1)}}{\partial x_j} + \frac{\partial u_j^{(1)}}{\partial x_i} \right).$$  \hspace{1cm} (14)

Consider the boundary conditions for the first approximation obtained from Eq. (9). For Case 2 these conditions are

$$u_2^{(1)}|_{x_3=0} = u_2^{(1)}|_{x_3=\ell_3} = 0, \quad u_1^{(1)}|_{x_3=0} = u_1^{(1)}|_{x_3=\ell_3} = 0,$$

$$\sigma_{1i}^{(1)}|_{x_3=0} = \sigma_{1i}^{(1)}|_{x_3=\ell_3} = 0,$$

$$\sigma_{3i}^{(1)}|_{x_3=0} = \sigma_{3i}^{(1)}|_{x_3=\ell_3} = 0,$$

$$q_i^{(1)}|_{x_3=\ell_3} = h_A \pm 0.$$

By integrating over domain \(\Omega\) we obtain the following integral operators

$$\sigma_{ij}^{(0)} = A_{ij} \varepsilon_{ij}^{(0)}, \quad i, j = 1, 2, 3, \quad \sigma_{ij}^{(0)} = 2A_{ij} \varepsilon_{ij}^{(0)},$$

$$\varepsilon_{ij}^{(0)} = 2A_{ij} \sigma_{ij}^{(0)}, \quad \varepsilon_{ij}^{(0)} = 2A_{ij} \sigma_{ij}^{(0)}, \quad q = 0.1, 2, \ldots.$$  \hspace{1cm} (16)

In Eq. (16) \(A_{ij} \ldots A_{ij}\) are the following integral operators

$$A_{ij} \phi(t) = A_{ij} \phi(t) + \int_0^t A_{ij}(t-\tau) \phi(\tau) d\tau,$$

$$ij = 11, 22, 33, 12, 13, 23, 44, 55, 66.$$  \hspace{1cm} (17)

The meaning of the notation used in Eqs. (16) and (17) is obvious.

In a likewise manner, the corresponding equations and boundary conditions for the second and subsequent approximations can also be obtained. Thus, the investigation of buckling delamination around a rectangular edge-crack contained within a thick rectangular plate is reduced to the solutions to series-boundary-value problems such as (14)–(17). As in papers by Akbarov and Rzayev (2002a,b, 2003) and others, by direct verification it is proven that the linear equations in (14)–(17) coincide with the corresponding equations for TDLT presented by Guz (1999).

After determination of the stress-deformation state in the considered plate (using the solution procedure described above) it is necessary to select the stability loss criteria. According to Hoff (1954), for the stability loss criterion we will assume the case where the size of the initial imperfection starts to increase and grows indefinitely with the external compressive forces (for the elastic plate) or with duration of time for the viscoelastic plate under considered fixed finite values of these forces. From this criterion, the critical force or the critical time will be determined.

The investigations, which are not detailed here, indicated that the values of the critical force or of the critical time can be determined only within the framework of the zeroth and first approximations. The second and subsequent approximations do not change the values of the critical parameters. Taking these subsequent approximations into account improves only the accuracy of the stress distribution in the plate. Since our aim is to investigate the stability loss (i.e. to determine the values of the critical parameters), we restrict ourselves to consideration of the zeroth and first approximations.

According to the foregoing considerations, the stresses in the zeroth approximation have already been determined by the expression (13). Now we consider determination of the values of the first approximation for which it is necessary to solve the problems (14)–(17). For this purpose we use the principle of correspondence by using the Laplace transform

$$\tilde{\phi}(s) = \int_0^\infty \phi(t) e^{-st} dt$$  \hspace{1cm} (18)

with the parameter \(s > 0\) for the Eqs. (14)–(17). So, replacing \(\sigma_{ij}^{(0)}, \varepsilon_{ij}^{(0)}, u_i^{(1)}, \varepsilon_{ij}^{(0)}\) and \(A_{ij}\) in (14)–(17) by \(\sigma_{ij}^{(0)}, \varepsilon_{ij}^{(0)}, u_i^{(1)}\) and \(A_{ij}\), respectively, we obtain the corresponding equations and boundary conditions with respect to the Laplace transform of values for the first approximation. For the solution to the problems corresponding to the Laplace transform of the sought values we employ the Finite Element Method (FEM).

4. FEM modeling of the considered problems

It is known that for FEM modeling employing the Ritz method, it is necessary to construct the functional, the Euler equations of which are the Eqs. (14)–(17), rewritten for the Laplace transform of the corresponding sought functions. For the realization of this construction, the Eqs. 14, 15, 16, 17 must be self-adjoint ones. In the monograph by Guz (1999), it is proven that the equations of the TDLTs are the self-adjoint ones. According to this statement, we construct the following functionals for the problems under consideration.

In Case 2, the Eqs. (14)–(17) for the Laplace transform of the sought functions are the Euler equations of the functional

$$\Pi(\vec{u}_1^{(1)}, \vec{u}_2^{(1)}, \vec{u}_3^{(1)}) = \frac{1}{2} \int \int \left[ \left( \sigma_{ij}^{(1)} + \sigma_{ij}^{(0)} \frac{\partial u_i^{(1)}}{\partial x_j} \right) \frac{\partial u_j^{(1)}}{\partial x_i} + \frac{\partial u_i^{(1)}}{\partial x_j} \frac{\partial \varepsilon_{ij}^{(1)}}{\partial x_k} + \frac{\partial \varepsilon_{ij}^{(1)}}{\partial x_k} \frac{\partial u_k^{(1)}}{\partial x_j} \right]_{ij} d\Omega dx d\Omega dx.$$  \hspace{1cm} (19)

Note that the corresponding functional for Case 1 is obtained from the functional (19) by replacement of the integrating interval \((\ell_3 - \ell_3, 0, \ell_3)\) with the interval \((0, \ell_3)\) in the last two terms in the Eq. (19).

By direct verification, it is proven that the equations and boundary conditions with respect to the forces in Cases 1 and 2 are obtained from the equation \(\Pi(\vec{u}_1^{(1)}, \vec{u}_2^{(1)}, \vec{u}_3^{(1)}) = 0\).

The virtual work principle in conjunction with the FEM technique is employed. The FEM grid over domain \(\Omega\) is revised until
convergence is achieved. Realization of the corresponding computer programs has been made using the package FTN77.

In the paper by Akbarov et al. (2007) it was established that under investigation of the buckling delamination around the cracks contained in a plate, the numerical results on the critical parameter obtained by the use of the singular type finite elements in the vicinity of the crack tips coincided (with very high accuracy) with those obtained by the use of the ordinary type finite elements in the vicinity of the crack tips. According to this statement, in the present investigation, the finite elements containing the crack tips (fronts) are also ordinary brick elements. In this way, we simply establish FEM modeling of the problems under consideration.

Thus, by employing the FEM algorithm detailed above we calculate the values of the Laplace transform of the sought values. The values of the original of the sought functions are determined by the use of the method by Schapery (1966).

5. Numerical results and discussion

We assume that the plate material consists of two alternating layers whose materials are isotropic and homogeneous and that these layers are located in the Ox1OX2 plane. The reinforcing layers are supposed to be pure elastic with mechanical characteristics $E^{(2)}$ (Young’s modulus) and $v^{(2)}$ (Poisson coefficient). The material of matrix layers is supposed to be linearly viscoelastic with the operators

$$E^{(1)} = E^{(1)}_0 \left[ 1 - \omega_0 R_c^2 (\omega_0 - \omega_\infty) \right],$$
$$v^{(1)} = v^{(1)}_0 \left[ 1 + \frac{1 - 2v^{(1)}_0}{2v^{(1)}_0} \omega_0 R_c^2 (\omega_0 - \omega_\infty) \right],$$

(20)

where $E^{(2)}_0$ and $v^{(2)}_0$ are the instantaneous values of Young’s modulus and of Poisson coefficient, respectively; $\omega_0$ and $\omega_\infty$ are the rheological parameters of the matrix material and $R_c$ is the fractional-exponential operator of Rabotnov (1977) which is determined by

$$R_c^2 (\omega) = \int_0^\infty R_c (\beta, \beta - \tau) \phi (\tau) d\tau,$$
$$R_c (\beta, \tau) = \tau^{(n-1)/2} \frac{n^2 \Gamma (1 + n)}{\Gamma (1 + n/2)},$$

(21)

In Eq. (21), $\Gamma (\cdot)$ is the Gamma function.

We introduce the dimensionless rheological parameter $\omega = \omega_\infty/\omega_0$ and the dimensionless time $t' = \omega_0^{1/2} t$ and assume further that $\eta^{(2)} = 0.5$ where $\eta^{(2)}$ is the volume fraction of filler. It is known that in the continuum approach, this layer–composite material can be taken as viscoelastic transversally isotropic material with effective mechanical properties whose isotropy axis lies on the Ox3 axis. It is also known that these normalized mechanical properties are determined by the well-known operations described for example, in a monograph by Cristensen (1979).

For concrete numerical investigations, suitable initial imperfection modes of the crack edge-surfaces can be selected as follows:

For Case 1:
The sine-phase initial imperfection mode

$$f^+(x_1) = f^-(x_1) = h_A + \ell_{10} \sin^2 \left( \frac{\pi (x_1 - \ell_{10})}{\ell_{10}} \right),$$

and the anti-phase initial imperfection mode

$$f^+(x_1) = h_A + \ell_{10} \sin^2 \left( \frac{\pi (x_1 - \ell_{10})}{\ell_{10}} \right).$$

For Case 2:
The sine-phase initial imperfection mode

$$f^+(x_1) = f^-(x_1) = h_A + A \sin^2 \left( \frac{\pi (x_1 - \ell_{10})}{\ell_{10}} \right),$$
$$\times \left( \frac{\pi (x_1 - \ell_{10})}{\ell_{10}} \right) \sin^2 \left( \frac{\pi (x_1 - \ell_{10})}{2\ell_{10}} \right),$$

(24)

and the anti-phase initial imperfection mode

$$f^+(x_1) = h_A \pm A \sin^2 \left( \frac{\pi (x_1 - \ell_{10})}{\ell_{10}} \right),$$
$$\times \left( \frac{\pi (x_1 - \ell_{10})}{2\ell_{10}} \right) \sin^2 \left( \frac{\pi (x_1 - \ell_{10})}{\ell_{10}} \right).$$

(25)

Fig. 1 illustrates schematically the anti-phase initial imperfection mode. It is easy to image the sine-phase initial imperfection mode. Thus, we turn to the analysis of the numerical results and first, we consider pure elastic stability loss buckling delamination which takes place at $t = 0$ and $t = \infty$. In these cases the absolute critical values of the averaged strain $\delta = |p|/E$ are calculated for selected values of the problem parameters, where $E$ is the instantaneous value of the effective modulus of elasticity in the direction of the Ox1 axis and $\bar{E} = E_0 (1 - \eta^{(2)}) + E_0^{(2)} \eta^{(2)}$. We consider separately the numerical results obtained in Case 1 and in Case 2 and assume that $h/\ell_{10} = 0.157$ and $v^{(2)}_0 = \nu^{(2)} = 0.3$. At the same time, the parameter $\gamma = \ell_3/\ell_{10}$ is introduced.

5.1. Numerical results obtained in Case 1

In the case considered, we assume that $h_0 = h_A = h/2$. First, we analyze the numerical results regarding the pure elastic buckling delamination which takes place at $t = 0$ and $t = \infty$. We denote the critical values for $\delta$ obtained at $t = 0$ and at $t = \infty$ through $\delta_{\omega,0} = |p_{\omega,0}/\bar{E}|$ and $\delta_{\omega,\infty} = |p_{\omega,\infty}/\bar{E}|$, respectively.

Table 1 shows the values of $\delta_{\omega,0}$ (upper number) calculated for various values of $E^{(2)}/E_0^{(1)}$ and $\gamma$ for $\ell_{10}/\ell_1 = 0.5$ in the anti-phase initial imperfection mode. At the same time Table 1 shows the values of the reduced critical forces $|p_{\omega,0}/E_0^{(1)}$ (lower number). It follows from the data given in this table that the values of $\delta_{\omega,0}$ decrease, but the values of $|p_{\omega,0}/E_0^{(1)}$ increase with $E^{(2)}/E_0^{(1)}$ and the order of the critical strains is quite realistic.

According to the well-known mechanical consideration, the numerical results obtained in the case under consideration must approach the corresponding results obtained in the paper by Akbarov and Rzayev (2002a) with $\gamma$. Because in this paper, the plane-strain state (i.e. the case where $\gamma = \infty$) is considered. The results given in Table 1 agree with this prediction. This statement also validates the reliability of the algorithm and packed programs used in the present investigation. These packed programs have been composed by the authors. Moreover, it follows from Table 1 that the values of $\delta_{\omega,0}$ and $|p_{\omega,0}/E_0^{(1)}$ decrease monotonically with $\gamma$. This conclusion also agrees with the mechanical considerations.

We analyze below the results related to the critical values of the averaged strain $\delta$ only, because the values of $|p_{\omega,0}/E_0^{(1)}$ and $|p_{\omega,\infty}/E_0^{(1)}$ can be calculated from the relation $|p_{\omega,0}/E_0^{(1)} = \delta_{\omega,0} \times \left( (1 - \eta^{(2)}) + E^{(2)}/E_0^{(1)} \eta^{(2)} \right)$ and $|p_{\omega,\infty}/E_0^{(1)} = \delta_{\omega,\infty} \times \left( (1 - \eta^{(2)}) + E^{(2)}/E_0^{(1)} \times \eta^{(2)} \right)$, respectively.

Table 2 shows the ratio $\delta_{\omega,0}/\delta_{\omega,\infty}$ obtained for various $\gamma$ and $\ell_{10}/\ell_1$ for $\omega = 1.0, E^{(2)}/E_0^{(1)} = 50$ in the anti-phase initial imperfection.
mode (23). These results indicate that the values of $\delta_{cr,0}$ as well as the values of $\delta_{cr,\infty}$ decrease with $\ell_{t0}/\ell_1$.

Figs. 2 and 3 show the graphs of the dependencies among $\delta_{cr,0}$, $\delta_{cr,\infty}$ and $\ell_{t0}/\ell_1$ for the anti-phase and sine-phase initial imperfection modes, respectively. Note that these graphs are constructed for various $E^{(2)}/E_0^{(1)}$ for $r = 1$ and $\omega = 1$. It follows from these results that the values of $\delta_{cr,0}$ and $\delta_{cr,\infty}$ obtained for the anti-phase initial imperfection mode (23) are significantly greater than the corresponding values of $\delta_{cr,0}$ and $\delta_{cr,\infty}$ obtained for the sine-phase initial imperfection mode (22). The difference between the values of critical forces obtained for the sine-phase and anti-phase initial imperfection modes decreases with $\ell_{t0}/\ell_1$, i.e. with the crack length along the $\alpha x_1$ axis.

The numerical results, which are not given here, show that in Case 1 the buckling delamination mode coincides with the initial imperfection mode.

Now we assume that the material of the plate is viscoelastic with the rheological parameters $\alpha = -0.5$; $\omega = 0.5$; 1.0 and 2.0. Consider the numerical results obtained for the critical time $t_{cr,0}^{(2)}$. These results are given in Table 3 for the case where $r = 1$ and $E^{(2)}/E_0^{(1)} = 1$ for various $\omega$ under sine-phase initial imperfection mode. Note that the external compressive force must satisfy the following relation so that the viscoelastic buckling delamination of the plate considered will occur

$$|\delta_{cr,\infty}| < \delta < |\delta_{cr,0}|.$$ (26)

Therefore, from the results obtained, illustrated in Table 3, the values of $\delta$ are selected according to relation (26). It can be seen from Table 3 that the values of $t_{cr,0}^{(2)}$ increase with the rheological parameter $\omega$.

### 5.2. Numerical results obtained in Case 2

Within the foregoing assumptions and notation, we analyze the numerical results obtained in Case 2. As in the previous case, first we consider the pure elastic buckling delamination of the rectangular plate containing a rectangular edge-crack, the edge-surfaces of which have the initial imperfection described by the Eqs. (24) and (25). Note that in Case 2 the crack geometry is characterized not only by the initial imperfection mode and parameters $h_{ui} / \ell_1$, $\ell_{t0}/\ell_1$, but also with the parameter $\ell_{t0}/\ell_1$ through which the crack depth along the $\alpha x_3$ axis is estimated.

Thus, we consider the graphs given in Figs. 4 and 5, which show the dependencies among $\delta_{cr,0}$, $\delta_{cr,\infty}$ and $\ell_{t0}/\ell_1$ for the anti-phase and sine-phase initial imperfection modes, respectively. Note that these graphs are constructed for various $E^{(2)}/E_0^{(1)}$ under $h_{ui} / \ell_1$.

![Fig. 2. The graphs of the dependence among $\delta_{cr,0}$, $\delta_{cr,\infty}$ and $\ell_{t0}/\ell_1$ for various values of $E^{(2)}/E_0^{(1)}$ in the anti-phase buckling mode for the plate with a band-crack (Case 1) under $h_{ui} / \ell_1 = 0.075$, $\omega = 1$ and $r = 1.0$.](image-url)

### Table 1
The values of $\delta_{cr,0}$ (upper number) and $|\delta_{cr,\infty}|$ (lower number) under $\ell_{t0}/\ell_1 = 0.5$ for various values of $r$ and $E^{(2)}/E_0^{(1)}$.

<table>
<thead>
<tr>
<th>$\gamma = t_{cr} / \ell_1$</th>
<th>$E^{(2)}/E_0^{(1)}$</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0500</td>
<td>0.0429</td>
<td>0.0613</td>
<td>0.0223</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>0.0497</td>
<td>0.0413</td>
<td>0.0588</td>
<td>0.0202</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.0500</td>
<td>0.0429</td>
<td>0.0690</td>
<td>0.0223</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.0500</td>
<td>0.0429</td>
<td>0.0613</td>
<td>0.0223</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.0497</td>
<td>0.0413</td>
<td>0.0588</td>
<td>0.0202</td>
<td></td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.0497</td>
<td>0.0413</td>
<td>0.0590</td>
<td>0.0197</td>
<td></td>
</tr>
</tbody>
</table>

### Table 2
The values of $\delta_{cr,0}/\delta_{cr,\infty}$ under $\omega = 1$ and $\ell_{t0}/\ell_1 = 0.5$ for various values of $r$ and $E^{(2)}/E_0^{(1)} = 50$.

<table>
<thead>
<tr>
<th>$\gamma = t_{cr} / \ell_1$</th>
<th>$\ell_{t0}/\ell_1$</th>
<th>0.5</th>
<th>0.4</th>
<th>0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0500</td>
<td>0.0211</td>
<td>0.0328</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>0.0497</td>
<td>0.0206</td>
<td>0.0221</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.0497</td>
<td>0.0206</td>
<td>0.0221</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.0497</td>
<td>0.0206</td>
<td>0.0221</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.0497</td>
<td>0.0206</td>
<td>0.0221</td>
<td></td>
</tr>
</tbody>
</table>

### Table 3
The values of $t_{cr,0}^{(2)}$ under $r = 1.0$, $E^{(2)}/E_0^{(1)} = 1$ and $\alpha = -0.5$ for various values $\omega$.

| $\delta$ | $|\delta_{cr,0}|$ | $|\delta_{cr,\infty}|$ | $\ell_{t0}/\ell_1$ | 0.5 | 1 | 2 |
|----------|------------------|---------------------|-------------------|----|---|---|
| 0.042    | 0.0089           | 0.0102              | 0.0139            |
| 0.041    | 0.0021           | 0.0026              | 0.0047            |
| 0.040    | 0.0045           | 0.0063              | 0.1523            |
This prediction is proven by the graphs of the dependence among $\delta_{\alpha,0}$, $\delta_{\alpha,\infty}$ and $\ell_{30}/\ell_1$ for various values of $E^{(2)}/E_0^{(3)}$ in the anti-phase buckling mode for the plate with an edge-crack (Case 2) under $h_{d}/\ell_1 = 0.075$, $\omega = 1$, $\ell_{30}/\ell_1 = 0.5$ and $\gamma = 1$.

$\ell_1 = 0.075$, $\omega = 1$, $\ell_{30}/\ell_1 = 0.5$ and $\gamma = 1$. It follows from the results that, as in Case 1, the values of $\delta_{\alpha,0}$ and $\delta_{\alpha,\infty}$ decrease with crack length along the $Ox_1$ axis.

Note that a large number of numerical results (which are not illustrated here) show that in the case under consideration the delamination mode coincides with the initial imperfection mode. This result occurs for both Cases 1 and 2.

Figs. 6 and 7 show the influence of the ratio $\ell_{30}/\ell_1$ on the values of $\delta_{\alpha,0}$ and $\delta_{\alpha,\infty}$ in the anti-phase and sine-phase initial imperfection modes, respectively, under $\omega = 1$ and $\ell_{30}/\ell_1 = 0.5$. As can be predicted, it follows from the graphs that the values of the critical strains increase monotonously with a decrease in the depth of the crack along the axis $Ox_2$, i.e. with a decrease in the ratio $\ell_{30}/\ell_1$.

According to the mechanical considerations, the values of $\delta_{\alpha,0}$ and $\delta_{\alpha,\infty}$ obtained in Case 2 must approach the corresponding values of those obtained in Case 1. This prediction is proven by comparison of the results obtained under $\ell_{30}/\ell_1 = 0.7$ (Figs. 6 and 7) with the corresponding results given in Table 1.

So far we have assumed that $h_{d} = h/2$ (i.e. $h/\ell_1 = 0.15$; $h_{d}/\ell_1 = 0.075$). Now we consider the influence of the ratio $h_{d}/\ell_1$ on the values of $\delta_{\alpha,0}$ and $\delta_{\alpha,\infty}$. This influence is illustrated by the data given in Table 4 calculated for $\gamma = 1$, $\ell_{30}/\ell_1 = 0.5$ and $\omega = 1$ for

Table 4

<table>
<thead>
<tr>
<th>$h_{d}/\ell_1$</th>
<th>Initial imperfection modes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Anti-phase $E^{(2)}/E_0^{(3)}$</td>
</tr>
<tr>
<td>$\gamma = 1$, $\ell_{30}/\ell_1 = 0.5$ and $\omega = 1$</td>
<td></td>
</tr>
<tr>
<td>0.0750</td>
<td>0.0184</td>
</tr>
<tr>
<td>0.0625</td>
<td>0.0164</td>
</tr>
<tr>
<td>0.0500</td>
<td>0.0144</td>
</tr>
<tr>
<td>0.0375</td>
<td>0.0125</td>
</tr>
<tr>
<td>0.0250</td>
<td>0.0105</td>
</tr>
<tr>
<td>0.0125</td>
<td>0.0085</td>
</tr>
</tbody>
</table>
various $E^2/E_0^{(1)}$. It follows from the data that the values of $\delta r_0$ and $\delta r_\infty$ decrease as the ratio of $h_0/l'_1$ decreases. This result also agrees with known results on the stability loss of the rectangular plates.

Now we consider how changing the parameter $\gamma (= \xi'_{l'/1})$ affects the values of $\delta r_0$ and $\delta r_\infty$. This effect is demonstrated by the results shown in Table 5 obtained in the case where $h_0 = h/2$, $l_0/l'_1 = 0.15$, $\xi_0/l'_1 = 0.2$ and $E^2/E_0^{(1)} = 1$. It follows from these results that the values of critical force approach a certain asymptotical value with $\gamma$, because the effect of the boundary conditions given at the end $x_3 = 0$ on the values of $\delta r_0$ and $\delta r_\infty$ decays with $\gamma$.

Consider the numerical results attained for the critical time, i.e. for $r'_c$. As in Case 1, in obtaining these results, we assume that the values of the strain $\delta$ caused by the external compressive force satisfy the inequality-relation (26).

Table 6 shows the values of $r'_c$ obtained for various $\xi_0/l'_1$ and $\delta$ under $E^2/E_0^{(1)} = 10$, $\gamma = 1$, $\omega = 0.5$, $\gamma = 1$ and $\xi_0/l'_1 = 0.5$. In the quantitative sense these results agree with the corresponding

<table>
<thead>
<tr>
<th>$\gamma = \xi'_{l'/1}$</th>
<th>Initial imperfection modes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Anti-phase</td>
</tr>
<tr>
<td>$\gamma = 1$</td>
<td>0.2018</td>
</tr>
<tr>
<td>$\gamma = 1.5$</td>
<td>0.1931</td>
</tr>
<tr>
<td>$\gamma = 2$</td>
<td>0.1902</td>
</tr>
<tr>
<td>$\gamma = 3$</td>
<td>0.1882</td>
</tr>
<tr>
<td>$\gamma = 4$</td>
<td>0.1876</td>
</tr>
<tr>
<td>$\gamma = 5$</td>
<td>0.1873</td>
</tr>
<tr>
<td>$\gamma = 6$</td>
<td>0.1872</td>
</tr>
</tbody>
</table>

The effects of $l_0/l'_1$ and $\delta$ on $r'_c$ for $E^2/E_0^{(1)} = 10$, $\omega = 1$, $\gamma = 1$ and $\xi_0/l'_1 = 0.5$.

<table>
<thead>
<tr>
<th>$l_0/l'_1$ (imper. mode)</th>
<th>$\delta$</th>
<th>$r'_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6 (anti-phase)</td>
<td>0.02909</td>
<td>0.4518</td>
</tr>
<tr>
<td>0.5 (anti-phase)</td>
<td>0.03273</td>
<td>0.0131</td>
</tr>
<tr>
<td>0.4 (anti-phase)</td>
<td>0.03818</td>
<td>0.0078</td>
</tr>
<tr>
<td>0.6 (sine-phase)</td>
<td>0.01727</td>
<td>0.2977</td>
</tr>
<tr>
<td>0.5 (sine-phase)</td>
<td>0.01782</td>
<td>0.0915</td>
</tr>
<tr>
<td>0.4 (sine-phase)</td>
<td>0.01818</td>
<td>0.0381</td>
</tr>
<tr>
<td>0.6 (anti-phase)</td>
<td>0.01764</td>
<td>0.4281</td>
</tr>
<tr>
<td>0.5 (anti-phase)</td>
<td>0.01800</td>
<td>0.2033</td>
</tr>
<tr>
<td>0.4 (anti-phase)</td>
<td>0.01891</td>
<td>0.0239</td>
</tr>
<tr>
<td>0.4 (sine-phase)</td>
<td>0.01764</td>
<td>0.7770</td>
</tr>
<tr>
<td>0.02000</td>
<td>0.3658</td>
<td></td>
</tr>
</tbody>
</table>

Table 7 The effects of $\omega$ and $x$ on $r'_c$ for $E^2/E_0^{(1)} = 10$, $l_0/l'_1 = 0.5$, $\gamma = 1$ and $\xi_0/l'_1 = 0.5$.

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>$x$</th>
<th>$r'_c$</th>
<th>$\delta = 0.03636$ (anti-phase)</th>
<th>$\delta = 0.01891$ (sine-phase)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.5</td>
<td>0.0447</td>
<td>0.0259</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-0.5</td>
<td>0.0910</td>
<td>0.0436</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.2771</td>
<td>0.0878</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.0851</td>
<td>0.0605</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-0.5</td>
<td>0.0447</td>
<td>0.0259</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-0.7</td>
<td>0.0089</td>
<td>0.0036</td>
<td></td>
</tr>
</tbody>
</table>

Finally, we note that the foregoing conclusions have a general character and do not depend on the type of material of the plate. The influence of the mechanical and rheological properties of the plate material on the values of the critical parameters is illustrated by the numerical results given in Figs. 2–7 and in Tables 1–7.

6. Conclusion

According to the foregoing analyzes, the following main conclusions can be drawn:

- The values of the critical strain as well as the values of the critical time decrease with the length of the crack along the $Ox_1$ and $Ox_3$ axes (Fig. 1);
- The approaching of the crack location to the free surface plane of the plate causes the decrease in the values of the critical parameters;
- The results obtained in Case 2 approach the corresponding results obtained in Case 1 with the length of the edge-crack in the direction of the $Ox_3$ axis;
- With the length of the rectangular band-crack along the $Ox_3$ axis, the results obtained for the critical parameters approach the results obtained for the corresponding plane-strain state problem studied in the paper by Akbarov and Rzayev (2002a);
- The numerical results obtained and analyzed in the present paper can be taken as standard for estimation of the accuracy (in the qualitative and quantitative senses) of the corresponding numerical results obtained within the scope of the approximate plate and bar theories;
- For the case under consideration, the mode of buckling delamination around the rectangular band-crack (Case 1) as well as the mode of buckling delamination around the rectangular edge-crack (Case 2) depends only on the initial imperfection mode of the crack’s edges.

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References


