Optimization of Planar Five bar Parallel Mechanism via Self reconfiguration Method

HE Guang ping¹, LU Zhen²

¹(1. School of Mechanical and Electrical Engineering, North China University of Technology, Beijing 100041, China)
²(2. School of Automation Science and Electrical Engineering, Beijing University of Aeronautics and Astronautics, Beijing 100083, China)

Abstract: The parallel mechanisms have the disadvantage of small workspace and complication in kinematics and dynamics. An optimizing design for the parallel mechanisms can improve the motion performance relatively, but not guarantee the design results which satisfy the various practical requirements simultaneously. In this paper, a dynamical and optimal synthesis method is proposed for parallel mechanisms based on the dynamical reconfiguration technique. As a specific application, the problem of optimizing the kinematics isotropy of a five bar planar parallel mechanism is studied. The motion of a reconfigurable mechanism can be parted into two phases, the natural motion phase and the reconfiguration phase. The two motion phases can be studied by the same performance evaluation methodology. This points out from both theory and practices a novel method for improving the motion performance of the parallel mechanisms. Simulation by a symmetrical five bar planar parallel manipulator shows some aspects of the investigations.

Key words: self-reconfiguration; parallel mechanisms; underactuated; optimization

The predominant advantages of the parallel mechanisms are higher structural stiffness and larger load-carrying capability with specific position accuracy than those of the serial one⁰. However, the parallel mechanisms have disadvantages such as smaller workspace, lower dexterity, more complication in kinematics and dynamics, and difficulty of motion control in large range of manipulation space. Many researchers have investigated the designing method for maximizing the workspace, optimal synthesis technique in kinematics, force transmission capability, and analysis of the singularity configuration of the parallel mechanisms.

For instance, Liu and Gao plotted the performance atlases of the workspace volume by the CAD method based on the physical model of the solution space, and the atlases can be used to optimizing the design of the mechanisms⁰. Sanchez proposed a
The underactuated serial mechanism naturally is a second-order nonholonomic system, which causes notable complexity in control. In this paper, differently, the parallel underactuated mechanisms are studied. An obvious value in use of the parallel underactuated mechanism is that the mechanism can be reconfigured to different type by switching the passive joint between the free swing and locked models. These reconfiguration styles of the underactuated close-loop mechanism can be selected to implement different tasks\cite{11}. Concretely, the method for optimizing the kinematical performance of the parallel mechanisms is investigated, and the planar five-bar close loop mechanism for the simplest parallel manipulator is selected to help to show the process of the method distinctly, but the result can be extended to general cases.

1 The Kinematical Formulations of Five-bar Parallel Mechanism

A full-revolute joint planar five-bar parallel mechanism is shown in Fig.1 schematically, in which joints $A$ and $E$ are fixed to ground, and joints $B$, $C$ and $D$ are movable. The end-effector is mounted to the joint $C$. Thus the mechanism is a 2-DOF parallel manipulator. The lengths of the links are denoted as $L_0$, $L_1$, $L_2$, $L_3$ and $L_4$ respectively. The actuated joints are supposed to be $\theta_1$ and $\theta_4$, which locate points $A$ and $E$. The symbol $\phi$ denotes the transmission angle of the manipulator.

A system whose input link number is less than the degree of freedoms (DOFs) of the mechanism\cite{8}. From the point of view of kinematics, the underactuated mechanism is uncontrollable. Nevertheless, the motion of the underactuated mechanism can be controlled by the dynamics coupling in the linkage. For a serial mechanism, the underactuated manipulator can move in multi-dimension configuration space driving by less dimension inputs\cite{9}. This research result encourages many scholars to devote themselves to the designing and controlling the nonholonomic constraint mechanical system, which is shown by underactuated mechanism with second-order nonholonomic constraint generally\cite{10}.

1 The Kinematical Formulations of Five-bar Parallel Mechanism

During the past decades, the underactuated mechanisms or manipulators attracted many researchers to it because of the advantages of lightweight or the requirements in fault tolerance technique. The underactuated mechanical system is
tor.

![Fig. 1 A five bar parallel mechanism](image)

Referring to Fig. 1, some analytical expressions can be formulated as
\[ \overrightarrow{AB} = [L_1 \cos \theta_1 \quad L_1 \sin \theta_1]^T \]
\[ \overrightarrow{AD} = [L_0 + L_4 \cos \theta_4 \quad L_4 \sin \theta_4]^T \]
and
\[ \overrightarrow{BD} = \overrightarrow{AD} - \overrightarrow{AB} = \left[ \frac{1}{1} \overrightarrow{BF} \right] \]

If it is defined that
\[ H = | \overrightarrow{BD} | \]
where \( | \cdot | \) denotes the modulus of a vector.

Referring to Fig. 1, the following trigonometric functions are obtained as
\[ \cos \angle CBD = \frac{L_4^2 + H^2 - L_3^2}{2L_4H} \]
\[ \sin \angle CBD = \sqrt{1 - \left( \frac{L_4^2 + H^2 - L_3^2}{2L_4H} \right)^2} \]
\[ \cos \angle DBF = \left[ \frac{1}{1} \overrightarrow{BF} \right] \]
\[ \sin \angle DBF = \left[ \frac{1}{1} \overrightarrow{DF} \right] \]
and
\[ \cos \angle CBF = \cos(\angle CBD + \angle DBF) = \frac{1}{1} \overrightarrow{BG} \]

Based on the Eqs. (5)-(9), the length of line segment \( | \overrightarrow{BG} | \) can be resolved as
\[ | \overrightarrow{BG} | = | \overrightarrow{BF} | - | \overrightarrow{DF} | = \frac{L_4^2 + H^2 - L_3^2}{2H^2} - \xi^2 \]

where
\[ \xi = \frac{L_4^2 + H^2 - L_3^2}{2H^2} \]
\[ | \overrightarrow{BF} | = L_0 + L_4 \cos \theta_4 - L_1 \cos \theta_1 \]
and
\[ | \overrightarrow{DF} | = L_4 \sin \theta_4 - L_1 \sin \theta_1 \]

On the other hand,
\[ \sin \angle CBF = \sin(\angle CBD + \angle DBF) = \frac{| \overrightarrow{CG} |}{L_2} \]

Combining Eqs. (5)-(8) and (10), the length of line segment \( | \overrightarrow{CG} | \) can be resolved into
\[ | \overrightarrow{CG} | = | \overrightarrow{DF} | - \xi + | \overrightarrow{BF} | = \frac{L_4^2}{2H^2} - \xi^2 \]

Consider the symmetrical mechanism, that is, \( L_1 = L_4 \) and \( L_2 = L_3 \). A symmetrical mechanism has symmetrical workspace and the kinematics equation is simplified. By the equations given above, the forward kinematics equation for the end-effector (point C) of the mechanism can be written as
\[ x = L_1 \cos \theta_1 + | \overrightarrow{BG} | = L_1 \cos \theta_1 + \frac{1}{2} (L_0 + L_1(\cos \theta_4 - \cos \theta_1)) - \frac{1}{2} \sin \theta_4 - \sin \theta_1 \]
\[ y = L_1 \sin \theta_1 + | \overrightarrow{CG} | = L_1 \cos \theta_1 + \frac{1}{2} L_1(\sin \theta_4 - \sin \theta_1) + \frac{1}{2} L_1 \sin \theta_1 \]

where \( K = \frac{L_2}{\sqrt{2H^2 - \frac{1}{4}}} \) and \( H^2 = L_0^2 + 2L_0L_1(\cos \theta_4 - \cos \theta_1) + 2L_1^2(1 - \cos(\theta_1 - \theta_4)) \)

The forward kinematics Eqs. (13) and (14) can be used to analyze the workspace by point mapping method with computer aided. This is a general method that is not dependent on the special problem.

For the inverse kinematics problem of the parallel mechanisms, there are a few simple mechanisms which can be obtained analytically. Generally, it is required to depend on a numerical method. For the sake of measuring the kinematics performance of parallel mechanisms, the relationship between the input velocity and the output velocity is set up by differentiating the Eqs. (13) and (14). The differential equation is as follows
\[ A \dot{\theta} + B \dot{x} = 0 \]
where \( \theta = [\theta_1 \quad \theta_4]^T \) and \( X = [x \quad y]^T \). Eq. (15) can be detailed as
The Eq. (15) can be rewritten as
\[
\begin{align*}
    a_{11}0_1 + a_{12}0_2 + b_{11}x + b_{12}y & = 0 \\
    a_{21}0_1 + a_{22}0_2 + b_{21}x + b_{22}y & = 0
\end{align*}
\] (16)

Once more,
\[
a_{11} = -\frac{1}{2}L_1KH^4\sin\theta_1 + L_1K^2H^4\cos\theta_1 + L_0L_1\frac{L_3}{2}(\sin\theta_4 - \sin\theta_1)\sin\theta_1 + L_1^2L_2\sin\theta_1 - \sin(\theta_1 - \theta_4)
\]
\[
a_{12} = -\frac{1}{2}L_1KH^4\sin\theta_4 - L_1(KH^4\cos\theta_4 - L_0L_1\frac{L_3}{2}(\sin\theta_4 - \sin\theta_1)\sin\theta_4 - L_1^2L_2\sin\theta_1)\sin(\theta_1 - \theta_4)
\]
\[
a_{21} = -\frac{1}{2}L_1KH^4\cos\theta_1 - L_0L_1L_2\frac{L_0}{2}(\cos\theta_4 - \cos\theta_1)\sin\theta_1 - L_1^2L_2\frac{L_0}{2}(\cos\theta_4 - \cos\theta_1)\sin(\theta_1 - \theta_4) + L_1K^2H^4\sin\theta_1
\]
\[
a_{22} = \frac{1}{2}L_1KH^4\cos\theta_4 + L_0L_1L_2\frac{L_0}{2}(\cos\theta_4 - \cos\theta_1)\sin\theta_4 - L_1^2L_2\frac{L_0}{2}(\cos\theta_4 - \cos\theta_1)\sin(\theta_1 - \theta_4) - L_1K^2H^4\sin\theta_4
\]

and
\[
b_{11} = -KH^4, b_{12} = 0, b_{21} = 0, b_{22} = -KH^4
\]

Obviously, the matrix \( A \) and \( B \) have the forms as
\[
A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}
\]

The Eq. (15) can be rewritten as
\[
X = -B^{-1}A0
\] (17)

where matrix \( J = -B^{-1}A \) is the Jacobian matrix of the manipulator. Several measure indexes such as manipulability and condition number are related to the matrix \( J \), and can be used to evaluate the kinematics performance of the mechanism. The condition number is defined as
\[
C(J) = \frac{\|J\| \cdot \|J^{-1}\|}{\|J\| \cdot \|J^{-1}\|} \tag{18}
\]

where \( \| \cdot \| \) denotes norm. In the next section the condition number will be used to analyze the motion performance of the five-bar parallel mechanism.

2 Kinematics Performance Analysis of Planar Five bar Parallel Mechanism

Generally, the kinematics performance analysis of parallel mechanism includes workspace, manipulability and force transmission capability. The optimal synthesis methods proposed by many researchers can improve the motion performance of the parallel mechanisms relatively. In fact, these different measure indexes depict different aspects of motion performance of the mechanism, and they are relevant and incompatible. An optimal design is applicable to one kind of tasks but not for the others, and the mechanism is unchangeable once it is designed. Therefore, the performance of a single model mechanism is limited to be adapted to different task. For confirming this issue, some simulation results are given as follows.

The relationship between the shape of workspace and the lengths of links is investigated firstly. Fig. 2 shows some calculation results with different parameters selected and shown in Cartesian space by Eqs. (13) and (14). The unit lengths of the axes \( x \) and \( y \) are arbitrary. Fig. 2(a) has a better shape than Fig. 2(b) in the full workspace. An small workspace leads to difficulty in motion planning and control of the system.

Fig. 2 The workspace of the five bar parallel mechanism

Furthermore, consider the motion performance of the planar five bar parallel mechanism. The performance is measured by the condition number given in Eq. (18). The condition number can be calculated by the maximal value of the Jacobian matrix divided by the minimal one. Thus, the condition number satisfies the relationship with \( 1.0 \leq C(J) < \infty \). If the condition number is close to 1.0, it is implied that the configuration of the mechanism is isotropy while the mechanism has better manipulation accuracy, dexterity and singularity avoidance. In Fig. 3 two cases with different
link lengths selected are plotted. It is shown that the mechanism has better manipulability because of the smaller condition number when the parameters are selected as $L_0 = 9.0$, $L_1 = 4.0$ and $L_2 = 8.5$.

Referring to the Fig. 2 to Fig. 4, it can be found that the mechanism has better force transmission capability but worse kinematics isotropy when the link lengths are selected as $L_0 = 8.0$, $L_1 = 4.0$ and $L_2 = 8.5$. Contrarily, when the link lengths are selected as $L_0 = 9.0$, $L_1 = 4.0$ and $L_2 = 8.5$, the mechanism has better kinematics isotropy but worse force transmission capability. Therefore, the different performance index of the parallel mechanism is conflicted in synthesis problem generally. A mechanism with unchangeable structure parameters is limited itself to fit different works.

3 The Self-reconfiguration Method for Mechanism Synthesis

The underactuated redundant mechanism can be reconfigured to satisfy different requirements of the multiformal tasks. For a specific manipulation task, one can synthesize the mechanism optimally so that the mechanism has the best adaptability. This ideal can be confirmed by the simulation results mentioned above. That is, when the length of
the \( L_0 \) is adjusted dynamically, the mechanism has the both. Therefore, the reconfiguration technique is a new kind of optimization method for parallel mechanism synthesis.

As an example, the reconfiguration method by using the planar five-bar parallel mechanism is studied. For adjusting to the length of the \( L_0 \), a passive sliding joint is added to the point \( A \) or \( E \). Suppose the passive additional joint is mounted to point \( E \) and denoted as \( F \), as shown in Fig. 5. The entire passive joints including \( B \), \( C \), \( D \), and \( F \) are mounted with brakes, which may switch the passive joints in both the locked state and the free-swing state. When the state of the passive joints is switched, some characteristics such as the DOF, function and type of the mechanism will be changed. A different switch route will result in a different mechanism type. For the planar six-bar parallel mechanism, the entire reconfigurable models are listed in Table 1.

Table 1  The entire reconfigurable models of the mechanism plotted in Fig 5 ( 0 free, 1 locked, PJ Passive Joint)  

<table>
<thead>
<tr>
<th>No.</th>
<th>Status of PJs</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>F</th>
<th>DOFs</th>
<th>Type</th>
<th>Controllability</th>
<th>Style</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 0 0 0 0 0</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Underactuated</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0 0 0 0 1 2</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Full actuated</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0 0 1 0 0 2</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Full actuated</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0 0 1 1 1 1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Overactuated</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0 1 0 0 2 0</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Overactuated</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0 1 0 1 1 1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Overactuated</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0 1 1 0 1 1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Overactuated</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0 1 1 1 0 0</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>Three bar</td>
<td>No</td>
<td>Structure</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1 0 0 0 0 2</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Full actuated</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1 0 0 1 1 1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Overactuated</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>1 0 1 0 1 1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Overactuated</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>1 0 1 1 0 3</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>Three bar</td>
<td>No</td>
<td>Structure</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>1 1 0 0 1 1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Overactuated</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>1 1 0 1 0 3</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>Three bar</td>
<td>No</td>
<td>Structure</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1 1 1 0 0 3</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>Three bar</td>
<td>No</td>
<td>Structure</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>1 1 1 1 0 3</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>Three bar</td>
<td>No</td>
<td>Structure</td>
<td></td>
</tr>
</tbody>
</table>

Considering the task of adjusting to the length of link \( L_0 \), and referring to the Table 1, one can find three cases marked by the numbers in Table 1 of the first column with No. 3, 5 and 9, which satisfy the conditions that the mechanism has two DOFs and the sliding joint \( F \) is free. The three models of the reconfigured mechanisms are drawn in Fig. 6, which are redundant mechanisms for the reconfiguration task by reason that the output is one-dimensional but the inputs have two DOFs. Under this kind of models, the length of the fixed link \( L_0 \) can be adjusted dexterously. Not only the reconfiguration is feasible, but also the motion can be optimized by appending some limits such as time, energy and position accuracy for the notable capability of the redundant mechanism.

Another type of optional models is marked by the numbers in Table 1 in the first column with 7, 11 and 13, which satisfy the condition that the mechanism has only one DOF and the sliding joint \( F \) is free, by reason that the DOFs of these models all are one while there are two inputs. These models are belongs to a style of over-actuated mechanisms actually. For avoiding the conflict in the input space, one of the actuated joints can be closed.
Thus, the length of link $L_0$ also can be adjusted by the three reconfigured mechanism models, which are crank-sliding linkages and are plotted in Fig. 7 schematically. Referring to the three models of the crank-sliding linkages plotted in Fig. 7, it is shown that the manipulabilities of the linkages are different. The model (a) has the best transmission angle $\beta$ among the three models where the input is selected to be $\theta_1$ and the joint $\theta_4$ is unactuated. A large transmission angle $\beta$ leads to an easy manipulation.

![Fig. 7 The three models of the four-bar mechanism reconfigured by six-bar mechanism](image)

The models listed in the Table 1 with even serial numbers are not considered because of the sliding joint been locked. The first model in Table is shown by an underactuated case, which is uncontrollable in kinematics, and is not considered too. The model marked by 15 is a structure actually because of the zero mobility in mechanism.

Based on the analysis above, it is shown that the synthesis method based on the mechanism reconfiguration is abundant in choices of motion planning and reveals some convenience for control. This method is composed of two motion phases, the natural manipulation phase and the reconfiguration phase. For the simple four-bar linkages, an optimal choice can be chosen based on the transmission angle measure from all the feasible reconfiguration models. However, for the more complex mechanisms, a general method is needed. Fortunately, there are many studies showing that the Jacobian matrix is a key quantity for investigating the mobility of the mechanism. Generally, the kinematics formulation of a $n$ DOFs mechanism in $m$ dimension workspace can be expressed as $X = J(\theta)$. The vector $X \in \mathbb{R}^m$ represents the end-effector task coordinates, $\theta \in \mathbb{R}^n$ denotes the generalized coordinates of the mechanism in actuated space. The velocity equation can be expressed as $X = J\dot{\theta}$, where $J$ is the Jacobian matrix. The mobility and motion performance of the mechanism can be investigated by a uniform methodology despite that the reconfigurable mechanism has multiple models. The method proposed for the non-reconfigurable mechanisms is fit for the metamorphic mechanism too, no more than the Jacobian matrix is changed according to the specific reconfiguration model.

4 Conclusions

The different measure indexes of the general mechanisms are conflicted and relevant. A optimal design can only improve the performance of the mechanism relatively. An underactuated redundant mechanism can reconfigure itself and lead into multiple models in mechanism type. The motion can be decomposed into two phases, the nature motion phase and the reconfiguration phase. The later can be used to adjust the structure parameters of the mechanism so that the motion performance can be changed in extensive range. This methodology for synthesizing the parallel mechanism is feasible for the compatibility in algorithm, and shows a dynamical synthesis method but controllable in kinematics for mechanism reconfiguration task.

References


Biographies:

HE Guangping Born in 1972, he received B. S., M. S. and Ph. D. degree from Beijing University of Aeronautics and Astronautics in 1994, 1997 and 2002 respectively. He is an associate professor of North China University of Technology currently. His main study interest includes robotics and nonlinear dynamics and control. Tel: 010-88802835, E-mail: guangpinghe@sohu.com

LU Zhen Professor at the Department of Electrical Engineering, Beijing University of Aeronautics and Astronautics, He got his doctoral degree in 1987 from Beijing Institute of Aeronautics and Astronautics. His research interests are mechanism, robotics and bionics. He is the senior member of Chinese Mechanism Engineering Society and Chinese Aeronautics Society, the vice chair of Permanent Commission of HMM IFToMM, member of the Committee of Chinese Intelligent Robots Society and the Committee of robotics IFToMM. He has supervised 17 doctoral candidates. He has published 90 papers and works. Tel: 010-82313993, E-mail: zhenluh@buaa.edu.cn