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## Empirical estimation of grinding specific forces and energy based on a modified Werner grinding model

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Advanced grinding processes include relatively new grinding processes such as Creep Feed, HEDG and VIPER grinding. These processes are more productive than conventional ones as a result of favourable process kinematics. Proper understanding of grinding forces can be useful in designing grinding machine tools and fixtures. Additionally information on specific energy helps in selecting process parameters for achieving optimum output. In the present paper, analysis of the effects of process parameters, tribology, work material and auxiliary equipment on grinding forces and specific energy has been carried out. Existing models have been critically analysed and Werner's specific force model was found to be quite promising for advanced grinding processes. It was found that under specific boundary conditions and environment, similar to advanced grinding processes, this model estimates grinding forces with acceptable accuracy. Werner's model was further analysed and an alternative and slightly modified one was proposed. The proposed models were validated using experimental data from the literature and good agreement between the calculations and the experiments was found.

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**Keywords:** Grinding forces; modelling; empirical**1. Introduction**

Increased focus on reduction of set up, material handling and loading/unloading time has resulted in formation of automated modern grinding cells. Grinding cells replace the need for multiple operations such as milling, turning, conventional grinding etc. by single advanced grinding process, which often incorporates features such as (a) continuous/ intermittent dressing (b) automated material handling and inspection and (c) increased depth of cut for increasing productivity. In order to achieve optimum level of performance these cells are custom made for specific families of parts from same material. In such applications, fixed combination of wheel, dressing method, grinding fluid and grinding fluid application method are used. This evolution is quite evident in aero engine manufacturing sector, where continuous creep feed grinding cells have taken over conventional manufacturing set ups [1]. Advanced

grinding technologies (i.e. creep feed grinding, HEDG, etc.) are established as an attractive alternative to conventional manufacturing processes.

Despite availability of many state-of-the-art models for predicting grinding forces and specific energy, there is a need for simple and practical models for these two significant parameters. The available models are either too complex to use or they lack precision due to their empirical nature. Moreover the characteristics of advanced grinding processes are different from convention grinding processes. Hence there is a need for an exclusive model for advanced grinding technologies, which can be simple, accurate and easy to use.

Within the present paper a number of models were analysed and Werner's specific force model was found to be quite promising for advanced grinding processes. It was found that under specific boundary conditions and environment, Werner's model can estimate grinding forces with acceptable accuracy. The model analysis resulted in slight modifications with regards the

empirical factors. The proposed models were validated using experimental data from the literature and good agreement between the calculations and the experiments was found. The proposed models are simple and accurate and they minimise the need for performing costly trials. With a limited number of experiments, empirical values of proposed coefficients can be determined, which can be afterwards transferred and used for different parts in similar boundary conditions.

**2. Grinding Forces Model**

*2.1. Grinding forces*

Grinding force is the sum of individual forces acting from grits to workpiece during the grinding process. The force basically consists of chip formation force, sliding force and ploughing force and each force originate as a result of the mode of interaction between grit and workpiece. The grinding forces can be separated into a tangential component  $F_t$  and a normal component  $F_n$  or into horizontal component  $F_h$  and a vertical component  $F_v$  (Figure 1). However, since the diameter of the grinding wheel is much larger than the depth of cut, the horizontal component can be assumed to be identical to the tangential one.

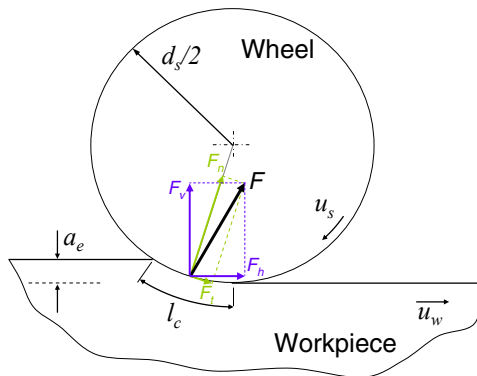


Fig. 1. Relationship between grinding force components

The total grinding force can be represented as the sum of the grinding force exerted for the chip formation, for the plastic deformation (plowing) of the workpiece and for the sliding of the grinding grains on the workpiece surface.

$$F_t = F_{t,sl} + F_{t,ch} + F_{t,pl} \tag{1}$$

where  $F_{t,sl}$ ,  $F_{t,ch}$  and  $F_{t,pl}$  are the tangential force for sliding, for chip formation and for plowing respectively. The forces exerted for chip formation and plowing are characterized as cutting forces.

*2.2. Discussion on Werner's force model*

In past many grinding force models have been developed. Initial models considered shear strength of work material as an important parameter. Recent work by Chang and Wang [2] considered the random nature of grit distribution as an important criterion. In a recent model developed by Durgumahanti et al. [3] both tangential and normal components of chip formation force, sliding force and ploughing force were mathematically modelled and experimentally validated for conventional grinding process.

One of the most popular model was developed for estimating the normal component of force by Werner [4], is expressed in equation (2):

$$F'_n = K[C_1]^\gamma \left[ \frac{Q'_w}{v_s} \right]^{2\epsilon-1} [a_e]^{1-\epsilon} [d_s]^{1-\epsilon} \tag{2}$$

Where,  $K$  is a proportionality factor,  $\epsilon$  is an exponent taking values from 0.5 to 1 depending on the workpiece material,  $\gamma$  is another exponent taking values from 0 to 1 depending on the grinding parameter and final  $C_1$  is the cutting edge density.  $Q'_w$  is the specific material removal rate,  $v_s$  is the grinding wheel speed,  $a_e$  is the depth of cut and  $d_s$  is the diameter of the grinding wheel.

Chip formation and sliding mechanism are considered as the main sources of grinding force [5]. The ploughing effect was assumed to be negligible, as ploughing forces are much lower compared to other forces, and their contribution in total forces become even less when depth of cut is more [6].

Equation (2) can be used for estimating forces while grinding both easy to grind ( $\epsilon = 1$ ) materials as well as difficult ones ( $\epsilon = 0.5$ ). For the case of the former ones, the equation can be written as:

$$F'_n \propto \frac{Q'_w}{v_s} \tag{3}$$

This indicates that depth of cut will have less effect on specific forces for such materials, and for both conventional and creep feed grinding force will be same. In case of difficult to grind materials, equation (2) can be written as:

$$F'_n \propto \sqrt{a_e \cdot d_s} \tag{4}$$

In this case the force is governed by the length of contact  $l_c$  ( $l_c = \sqrt{a_e d_e}$ ), thus it will be more in creep feed as compared to conventional grinding if rest of the grinding conditions are same.

Werner's force model can be also used to explain the forces change for the case of HEDG; Tawakoli [7] indicates that for constant specific removal rate,

increasing wheel speed results in reduction of specific normal force. Also if specific removal rate is increased, specific normal force increases as well.

Since sliding force depends on the percentage of wear flat area in a wheel; the total grinding force increases with increase in grain wear flat. However, if grinding wheel is continuously or intermittently dressed the percentage of wear flat area is constant and typically less than 4% [6]. In Werner’s model although the increase in wear flat with time is not considered as a factor, however as discussed above this model can be used in case of continuous/intermittent dressed wheels.

Grinding fluid type and method of its application has considerable effect on grinding force. Netterscheid [8] developed a force model in which the above effect has been considered. Werner’s force model does not take into account such a parameter; however it can be applied in many practical situations where, for a given type of grinding set up of material and machine, the type of grinding fluid and its application method remains unchanged.

### 2.3. Modified Werner’s force model

As indicated in the introduction, modern grinding operations share some key characteristics such as continuous and/or intermittent dressing and higher material rates (through the selections of high depth of cuts). Therefore, Werner’s equation can be used as the wear flat area in the grinding wheel remains within a limited value and the effect of ploughing force is insignificant. The basic hypothesis thus tested in the present paper is whether the empirical factors  $K$ ,  $C_1$  and  $\gamma$  can be replaced by a single factor  $K_1$  for such grinding cases. In that case, equation (2) will be replaced by a simpler one with only two empirical factors to be determined:

$$F'_n = K_1 \left[ \frac{Q'_w}{v_s} \right]^{2\epsilon-1} [a_e]^{1-\epsilon} [d_s]^{1-\epsilon} \quad (5)$$

Determining the values of proportionality factor  $K_1$  and exponent  $\epsilon$  in modified version of Werner’s force model is the most important aspect. Validity of the force model predictions greatly depends on these two empirical factors’ value. The above values can be experimentally determined using actual specific normal force measured by dynamometer.

Exponent  $\epsilon$ , as already mentioned, depends the material characteristics. The value of  $K_1$  depends on grinding parameters, material characteristics and type of wheels. Both the values can be calculated using experimental data. The force model contains variable grinding parameters such as wheel speed, work speed, wheel diameter and depth of cut and therefore it is

extremely difficult to derive a perfect numerical value for proportionality factor  $K_1$  and exponent  $\epsilon$ .

### 2.4. Grinding force ratio

Grinding force ratio links the tangential component ( $F_t$ ) of the grinding forces with the normal ones ( $F_n$ ):

$$\lambda = \frac{F_t}{F_n} \quad (6)$$

Value of  $\lambda$  depends on grinding parameters, grinding wheel condition, work material and the environment [9]. For a sharp wheel, it is relatively low, as tangential force component is higher compared to normal force and for dull wheel it is opposite. As shown in section 2.1,  $F_t$  and  $F_n$  can be expressed in terms of their cutting and sliding components. The ratio of the sliding components of the forces is equal to friction coefficient ( $\mu$ ) between wear flat and work. Similarly, the ratio of the cutting components ( $\phi$ ) depends on tip angle of grain [10]. Therefore, the grinding force ratio can be expressed [5] from equation:

$$\lambda = \phi \frac{F_{n,c}}{F_n} + \mu \frac{F_{n,s}}{F_n} \quad (7)$$

Where,  $F_{n,c}$  and  $F_{n,s}$  are the cutting and sliding components of the normal grinding force respectively. Hence it is evident that grinding force ratio depends on both  $\phi$  and  $\mu$ , but in case the chip formation phenomenon is more dominant than sliding; then it will be more influenced by  $\phi$ . Similarly if sliding is more dominant then  $\mu$  will have more dominance grinding force ratio.

Grinding force ratio thus is a highly dynamic entity, however so far no empirical model has demonstrated the relationship between  $\lambda$  and other grinding parameters. The grinding force ratio is found to be range bound (0.20-0.60) [11] and as already mentioned it depends on process parameters when all other grinding conditions remain unchanged. Therefore, in the present paper a multiple linear regression model of  $\lambda$  was established, keeping depth of cut, wheel speed, wheel diameter and work speed as regress:

$$\lambda = Aa_e + Bd_s + Cv_w + Dv_s + E \quad (8)$$

Experimental data can be used for determining the coefficients of the equation using either Matlab or Minitab. In the present study Minitab software was used for estimating the coefficients. The accuracy of a regression is indicated by  $p$  values of various coefficients. If  $p$  value of any coefficient in the model is higher than 0.05 then results may not be accurate enough and this is subsequently reflected in the energy model.

### 3. Specific Energy Model

Specific grinding energy indicates the energy consumption in machining unit volume of the material. This energy is consumed in a number of complex phenomena that occur during grinding process [11], such as due to chip formation, sliding, ploughing, friction between loaded chip and workpiece and friction between wheel bond chip and workpiece. In general, specific energy can be determined using equation:

$$u = \frac{power}{Q'_w} = \frac{F'_t \cdot v_s}{Q'_w} = \frac{\lambda F'_n \cdot v_s}{Q'_w} \tag{9}$$

Combining equations (5) and (9) results in the following empirical equation for the estimation of specific energy:

$$u = \lambda K_1 \left[ \frac{Q'_w}{v_s} \right]^{2\epsilon-2} [a_e d_s]^{1-\epsilon} \tag{10}$$

### 4. Procedure for estimating factors

As indicated the importance of the empirical factors ( $K_1, \epsilon$ ) on the model's predictions is paramount. These two factors have to be determined using experimental results. Exponent  $\epsilon$  depends on the material characteristics, whereas  $K_1$  factor depends on grinding parameters, material characteristics and type of wheel used. Since the model links a number of process parameters with the grinding forces, these two factors should not be determined by only one experimental run, but have to be optimized based on multiple experiments.

For this reason, a two-way sensitivity analysis was conducted as to account for the possibility of two or more different sets of empirical factors to simultaneously satisfy the model with minimum error. This two-way sensitivity analysis is conducted for a specific experimental setup (all grinding parameters kept constant). At least ten different grinding setups had to be checked. The results obtained for all these different grinding setups were combined as to specify the optimum values that result in minimum percentage absolute value difference between estimated and actual specific normal force.

Hence, the recommended steps for using the modified Werner's Force model for given materials and for specific combination of wheel type, dressing method, grinding fluid and grinding fluid application are as following:

1. Conduct initial experiments for various process parameters ( $a_e, d_s, v_w, v_s$ ) combinations measuring  $F'_n$  using a dynamometer.
2. Perform multiple sensitivity analysis as to determine optimum combination of  $K_1$  and  $\epsilon$  for minimum absolute value of  $F'_n$ .

The IDEF0 model for using modified Werner's model is illustrated in the appendix.

### 5. Validation and discussion

For the validation of the proposed empirical model, experimental data from previous studies were used. The main goal was to prove the basic hypothesis for the case of advanced material removal rate grinding processes such as Creep Feed Grinding. In table 1, the data sets used for the validation are presented.

Table 1. Brief of data sets used for validation

Data set No.	Reference	Grinding Mode	Process	Dressing
1	[12]	D/S	CF	(I) C
2	[13]	D/S	CF	I
3	[13]	U/S	CF	I
4	[14]	D/S	CF	I
5	[14]	U/S	CF	I
6	[15]	D/S	CF	(I) I

U: Up, D: Down, S: Surface, CF: Creep Feed, C: Continuous, I: Intermittent

Both energy and force models were validated for all six sets of data. For determining the value of proportionality factor  $K_1$ , exponent  $\epsilon$  and grinding force ratio, random samples were used as experimental data from each data set. Afterwards, the specific forces and energy was calculated. Indicatively, for data set 1, the average error between the actual and estimated forces was determined to be 4.33%. Pearson correlation coefficient ( $r$ ) between estimated and actual specific normal force was calculated to be 0.98 (figure 2) using:

$$r = \frac{\sum_{i=1}^n (F'_n(est)_i - \bar{F}'_n(est))(F'_{n_i} - \bar{F}'_n)}{\sqrt{\sum_{i=1}^n (F'_n(est)_i - \bar{F}'_n(est))^2} \sqrt{\sum_{i=1}^n (F'_{n_i} - \bar{F}'_n)^2}} \tag{11}$$

This high value of  $r$  proves that model is quite reliable with regards the grinding forces for the specific data set. In similar manner, the specific energy was calculated and the maximum error determined was found equal to 4.69% with Pearson correlation coefficient equal to 0.977.

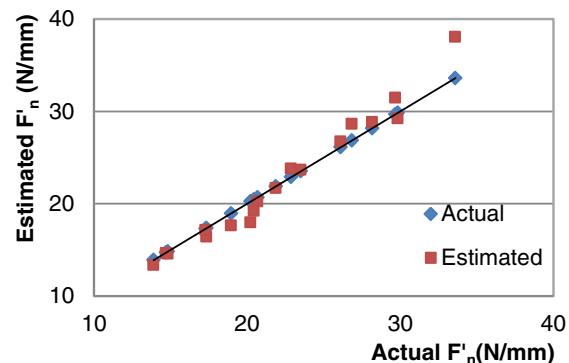


Fig. 2. Comparison between actual and estimated  $F'_n$

In table 2, the validation of all data sets is summarized. It can be concluded that both models predict grinding forces and specific forces with acceptable accuracy. In case of the modified force model; the maximum average error is 10.68%, whereas the maximum average error in energy model is 16.90%.

Models' estimations for data set 1 present the least error as compared to other data sets. This can be probably justified due to the continuous mode of dressing, whereas in other cases dressing is intermittent.

For the specific energy model, the average error is higher as compared to force model. The high value of error in case of specific energy model can be attributed to grinding force ratio ( $\lambda$ ). The value of  $\lambda$  was obtained using multiple linear regression. The accuracy of a regression is indicated by  $p$  values of various coefficients. As mentioned, if  $p$  value of any coefficient in the model is higher than 0.05 then results may not be very accurate and this is directly reflected in the specific energy model.

Table 2. Summary of results (where PCC stands for Pearson correlation coefficient)

Data set No.	Modified Force		Specific Energy	
	% Error	PCC (r)	% Error	PCC (r)
1	4.33	0.98	4.36	0.97
2	10.64	0.95	9.70	0.91
3	10.68	0.95	16.90	0.95
4	8.39	0.99	8.23	0.98
5	7.49	0.98	7.61	0.99
6	5.05	0.98	7.05	0.96

Another goal was to investigate whether there are specific trends with regards the empirical factors of the models. The value of  $\epsilon$  depends on material properties and it was found to be range bound (between 0.59 and 0.74). A lower value indicating poor and higher value indicating good grind-ability of the material.

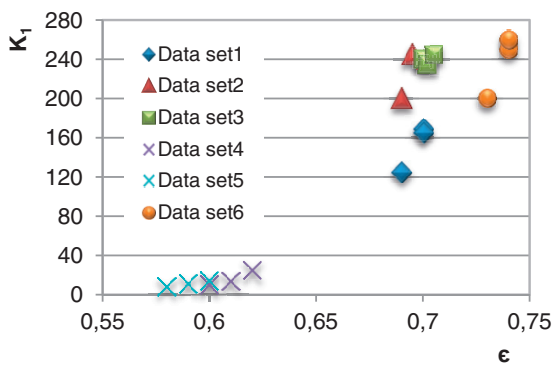


Fig. 3. Analysis of values of  $K_1$  and  $\epsilon$

Figure 3 proves that there are families of empirical sets between the coefficient values and the grinding setups. From the available data sets, three major groups were identified as can be seen in table 3.

As a concluding remark, the proposed models offer simple and realistic solutions for assessing specific normal force and specific grinding energy in advanced grinding processes. However this model is applicable only in special cases where features such as

- Continuous dressing/Intermittent,
- Automated material handling and inspection,
- And increased depth of cut is used

The accuracy of energy model is limited by quality of regression model of grinding force ratio.

Table 3. Group of empirical factor values

No.	Data sets	$K_1$	$\epsilon$	Work Material	Wheel Type	Grinding Type
1	DS4, DS5	11.0	0.59	Heat Treated Steel	Alumina Vitrified	CF-S
2	DS1, DS2, DS3	0.70	169-240	Ni rich alloy	Alumina Vitrified	CF-S
3	DS6	0.74	250	Low alloy Steel	Alumina Vitrified	CF-S

## 6. Conclusions

Within the present paper, the well-established Werner's force model was validated and modified for a number of specific grinding processes. The dynamic behaviour of grinding force ratio was identified and modelled using multiple regression analysis. Based on these two models a new model for specific energy was proposed.

A new method for estimating the empirical factors for the grinding forces model was proposed and validated using existing experimental data. In most of the cases the average error for both force and energy model was found to be within acceptable accuracy limits.

Therefore, this research work offers simple but accurate models for determining specific normal force and specific grinding energy in advanced grinding processes. These models can be used in advanced grinding cells for designing machine tools, grinding fixtures and for selecting process parameters, and with less effort it can give good results.

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## Appendix A. IDEF0 Model for estimating Specific Normal Force

The IDEF0 model for using modified Werner's model is illustrated in the following Figure A.1.

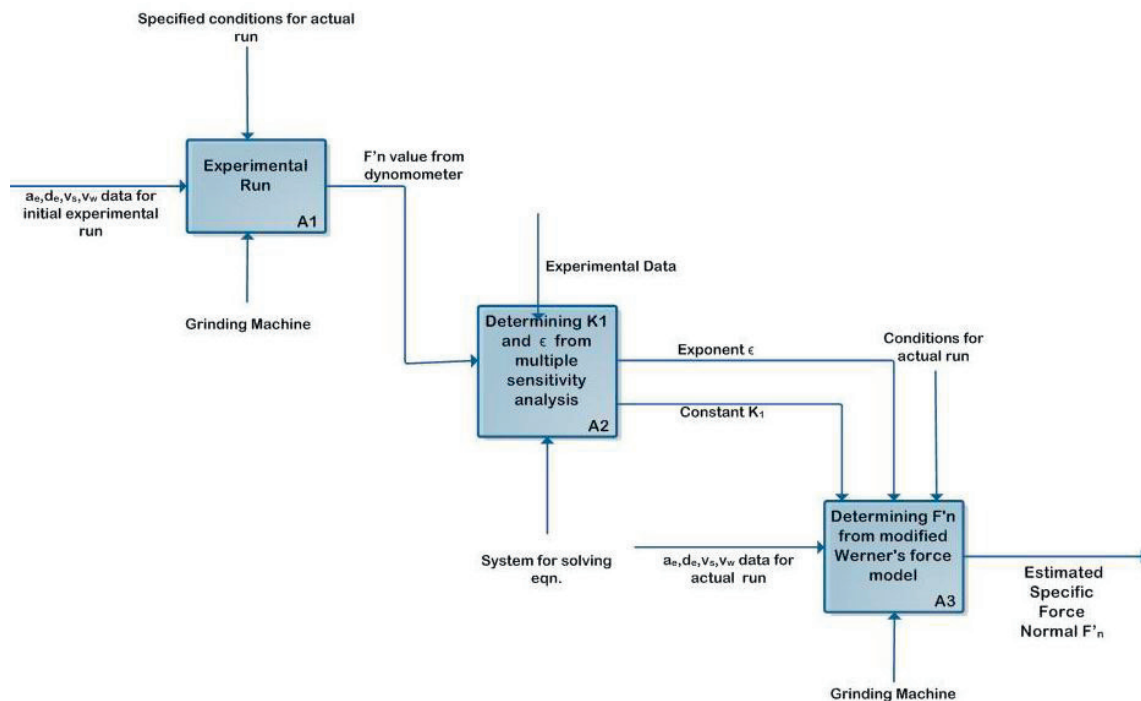


Figure A.1: IDEF0 model for estimating Specific Normal Force.