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Conformal coupling of higher spin gauge fields to a scalar field in AdS_4 and generalized Weyl invariance

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Abstract

The higher spin interaction currents for the conformally coupled scalar in AdS_4 space for both regular and irregular boundary condition corresponding to the free and interacting critical point of the boundary $O(N)$ sigma model are constructed. The explicit form of the linearized interaction of the scalar and spin two and four gauge fields in the AdS_D space using Noether's procedure for the corresponding spin two and four linearized gauge and generalized Weyl transformations are obtained.

1. Introduction

In contrast to the case of usual AdS_5/CFT_4 correspondence [1] where the strong coupling regime of the boundary theory corresponds to the weak coupled string/supergravity theory on the bulk, the AdS_4/CFT_3 correspondence of the critical $O(N)$ sigma model [2] operates at small 't Hooft coupling λ and the corresponding bulk theory is described as a theory of arbitrary even high spins. So it is a theory of Fradkin–Vasiliev type [3]. This case of AdS/CFT correspondence is also of great interest because here dynamical considerations and calculations both in AdS_{d+1} and CFT_d cases are essential on account of the absence of supersymmetry and BPS arguments and because in this case of correspondence perturbative expansions with small coupling constants are mapped on each other. So the essential point of $HS(4)$ and $d = 3$ $O(N)$ sigma model correspondence is that both conformal points of the boundary theory, i.e., unstable free field theory and critical interacting point, in the large-N limit correspond to the same higher spin theory. Moreover as we have learned from [2] these two points are connected on the boundary by a Legendre transformation which corresponds to the different boundary condition (regular dimension $\beta = 1$ or shadow $\beta = 2$) in the quantization of the bulk scalar field. This quantization of the free scalar field in the AdS with different boundary conditions and corresponding multi-trace deformation of the boundary theory were investigated and explored in many papers, we will refer just to the articles [4] most interesting for us.

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In this article we explore these two different boundary conditions of the scalar field from the point of view of the linearized interaction of the scalar field with the high spin fields in $d=4$ *AdS* space. In Section 2 we extend our previous consideration [5] to the case of the $\beta=2$ boundary condition. We show that if the $\beta=1$ case corresponds to the interaction of $HS(4)$ gauge fields with the *conformal* (traceless) conserved higher spin currents constructed from scalar field, then the $\beta=2$ case we can describe in non-contradictory fashion with the *AdS*/*CFT* correspondence using the non-conformal double-traceless currents and gauge fields in Fronsdal's formulation [6]. In the last two sections we explicitly construct a linearized interaction *Lagrangian* of the conformal scalar field with the spin two and four gauge field using Noether's procedure for *gauge* and *generalized Weyl invariance* (some consideration of non-linear gauge invariant coupling of the scalar field on the level of equation of motion one can find in [7]). We show that the interaction of the scalar with the spin four Fronsdal gauge field can be constructed in a non-unique way due to the existence of the gauge invariant combinations of gauge field itself (analogue of Ricci scalar for higher spins). But this ambiguity can be fixed in unique fashion by gauging the analogue of scale invariance in the higher spin case. This is a symmetry with the local tensor parameter permitting to gauge away the trace of a double traceless gauge field and leading to the tracelessness of the corresponding spin four current.

2. Conformal and Fronsdal higher spin currents and AdS_4/CFT_3

In our previous article [5] we considered coupling of the $HS(d+1)$ gauge field with a current constructed from a scalar field in fixed AdS_{d+1} background. Using the ansatz including the AdS_{d+1} curvature corrections we have shown there that all ambiguity in the construction of a spin ℓ *traceless* current from the conformally coupled scalar field reduces to the ambiguity of the set of leading coefficients A_p in the expansion of the current[2]

$$J^{(\ell)}(z;a)=\frac{1}{2}\sum_{p=0}^{\ell}A_p(a\nabla)^{\ell-p}\phi(z)(a\nabla)^p\phi(z)+\frac{a^2}{2}\sum_{p=1}^{\ell-1}B_p(a\nabla)^{\ell-p-1}\nabla_\mu\phi(z)(a\nabla)^{p-1}\nabla^\mu\phi(z)$$
$$+\frac{a^2}{2L^2}\sum_{p=1}^{\ell-1}C_p(a\nabla)^{\ell-p-1}\phi(z)(a\nabla)^{p-1}\phi(z)+O(a^4)+O\left(\frac{1}{L^4}\right), \tag{1}$$

where $A_p=A_{\ell-p}$, $B_p=B_{\ell-p}$, $C_p=C_{\ell-p}$ and $A_0=1$. The tracelessness condition fixes relations between B_p, C_p and A_p in the following way [5]

$$B_p=-\frac{p(\ell-p)}{(D+2\ell-4)}A_p, \tag{2}$$

$$C_p=\frac{-1}{2(D+2\ell-4)}\left[s_t(p+1,\ell,D)A_{p+1}+s_t(\ell-p+1,\ell,D)A_{p-1}\right], \tag{3}$$

$$s_t(p,\ell,D)=\frac{1}{4}p(p-1)D(D-2)+\frac{1}{3}p(p-1)(p-2)(\ell+2D-5). \tag{4}$$

The unknown A_p we can fix in two different ways: the first possibility is to use the conservation condition for the current. This leads to the recursion relation with the same solution for the A_p coefficients [8] as in the flat $(D=d+1)$-dimensional case

$$A_p=(-1)^p\frac{\binom{\ell}{p}\binom{\ell+D-4}{p+D/2-2}}{\binom{\ell+D-4}{D/2-2}}. \tag{5}$$

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[2] For the investigation of the conservation and tracelessness conditions for general spin ℓ symmetric conformal current $J^{(\ell)}_{\mu_1\mu_2\cdots\mu_\ell}$ we contract it with the ℓ-fold tensor product of a vector a^μ.

For the important case $D=4$ this formula simplifies to

$$A_p=(-1)^p\binom{\ell}{p}^2. \tag{6}$$

The result of our previous consideration [5] was the following: the curvature corrections do not change flat space tracelessness and conservation conditions between leading coefficients and therefore the solution (5) remains valid. The other way to fix this ambiguity is a boundary CFT_3 consideration. The result is [5]

$$A_p=\frac{\mathcal{C}^\ell(-1)^p\binom{\ell}{p}}{2^\ell(\beta)_p(\beta)_{\ell-p}}. \tag{7}$$

Here β is the dimension of the scalar field of the boundary CFT_3, and $\mathcal{C}^\ell$ is the normalization constant of the three point function of two scalars and the spin ℓ current in CFT_3 (we define also the Pochhammer symbols $(z)_n=\Gamma(z+n)/\Gamma(z)$). The expression (7), for $\beta=1$, is in agreement with the previous one (6) obtained from AdS_4 consideration, if we will normalize in (7) $\mathcal{C}^\ell=2^\ell\ell!$. It means that the $\beta=1$ point of boundary CFT_3 (free field conformal point of $O(N)$ vector model) we can describe as a *conformal* $HS(4)$ model in AdS_4. In other words in this case we have to operate in dual higher spin theory with the linearized interaction

$$S_{\mathrm{int}}^{(\ell)\mathrm{conf}}=\frac{1}{\ell}\int d^4x\,\sqrt{g}h^{(\ell)\mu_1\cdots\mu_\ell}J^{(\ell)}_{\mu_1\cdots\mu_\ell}, \tag{8}$$

where the corresponding current is conserved and traceless

$$J^{(\ell)\alpha}_{\alpha\mu_2\cdots\mu_\ell}=0,\qquad \nabla^{\mu_1}J^{(\ell)}_{\mu_1\mu_2\cdots\mu_\ell}=0. \tag{9}$$

For $\beta=2$ we have to change the constraints imposed on (1). For that we turn from conformal higher spins to Fronsdal's [6] formulation where gauge fields and currents are double traceless only

$$S_{\mathrm{int}}^{(\ell)}=\frac{1}{\ell}\int d^4x\,\sqrt{g}h^{(\ell)\mu_1\cdots\mu_\ell}\Psi^{(\ell)}_{\mu_1\cdots\mu_\ell}, \tag{10}$$

$$h^{(\ell)\alpha\beta}_{\alpha\beta\mu_5\cdots\mu_\ell}=0,\qquad \Psi^{(\ell)\alpha\beta}_{\alpha\beta\mu_5\cdots\mu_\ell}=0, \tag{11}$$

$$\delta_0h^{(\ell)}_{\mu_1\cdots\mu_\ell}=\partial_{(\mu_1}\epsilon_{\mu_2\cdots\mu_\ell)},\qquad \epsilon^{\alpha}_{\alpha\mu_4\cdots\mu_\ell}=0,\qquad \left[\nabla^{\mu_1}\Psi^{(\ell)}_{\mu_1\mu_2\cdots\mu_\ell}\right]^{\mathrm{traceless}}=0 \tag{12}$$

and the conservation condition looks a little bit different from the usual one due to the double-tracelessness of the gauge field and current. Then we can realize the double-traceless current $\Psi^{(\ell)}$ using two traceless (but not conserved) currents $J^{(\ell)}$, $\Theta^{(\ell-2)}$ with the same dimension $\ell+2\beta+O(1/N)$ on the boundary [9]. It means that the expansions for these fields start from the following series

$$J^{(\ell)}(z;a)=\frac{1}{2}\sum_{p=0}^{\ell}A^\ell_p(a\nabla)^{\ell-p}\phi(z)(a\nabla)^p\phi(z)+\cdots, \tag{13}$$

$$\Theta^{(\ell-2)}(z;a)=\frac{1}{2}\sum_{p=1}^{\ell-1}B^{\ell-2}_p(a\nabla)^{\ell-1-p}\nabla_\mu\phi(z)(a\nabla)^{p-1}\nabla^\mu\phi(z)+\cdots. \tag{14}$$

The Fronsdal field $\Psi^{(\ell)}$ we can present then as

$$\Psi^{(\ell)}(z;a)=J^{(\ell)}(z;a)+\frac{a^2}{2(D+2\ell-4)}\Theta^{(\ell-2)}(z;a), \tag{15}$$

$$\mathrm{Tr}\,\Psi^{(\ell)}(z;a)=\Box_a\Psi^{(\ell)}(z;a)=\Theta^{(\ell-2)}(z;a). \tag{16}$$

The conservation condition (12) in this representation is

$$\nabla^{\mu} \frac{\partial}{\partial a^{\mu}} \Psi^{(\ell)}(z; a) = \frac{a^2}{2(D+2\ell-6)} \operatorname{Tr} \nabla^{\mu} \frac{\partial}{\partial a^{\mu}} \Psi^{(\ell)}(z; a) \tag{17}$$

or

$$\nabla^{\mu} \frac{\partial}{\partial a^{\mu}} J^{(\ell)}(z; a) + \frac{(a\nabla)\Theta^{(\ell-2)}(z; a)}{(D+2\ell-4)} = \frac{a^2 \nabla^{\mu}(\partial/\partial a^{\mu})\Theta^{(\ell-2)}(z; a)}{(D+2\ell-6)(D+2\ell-4)}. \tag{18}$$

From this we can read off a restriction on the coefficients in (13) and (14)

$$p(D+2p-4)A^{\ell}_{p} + (\ell-p+1)(D+2\ell-2p-2)A^{\ell}_{p-1} + B^{\ell-2}_{p} + B^{\ell-2}_{p-1} = 0. \tag{19}$$

For $D=4$ we get

$$2p^2 A^{\ell}_{p} + 2(\ell-p+1)^2 A^{\ell}_{p-1} + B^{\ell-2}_{p} + B^{\ell-2}_{p-1} = 0. \tag{20}$$

Then after using (7) for $\beta = 2$ we obtain

$$B^{\ell-2}_{p} + B^{\ell-2}_{p-1} = \frac{\mathcal{C}^{\ell}\ell!}{2^{\ell-1}} \frac{(-1)^p(\ell-2p+1)}{(p-1)!(p+1)!(\ell-p)!(\ell-p+2)!}. \tag{21}$$

The solution of this equation fulfilling the boundary conditions

$$B^{\ell-2}_{0} = B^{\ell-2}_{\ell} = 0 \tag{22}$$

is

$$B^{\ell-2}_{p} = \frac{\mathcal{C}^{\ell}\ell!}{2^{\ell-1}}(-1)^p \sum_{k=1}^{p} \frac{(\ell-2k+1)}{(k-1)!(k+1)!(\ell-k)!(\ell-k+2)!}. \tag{23}$$

The latter sum can be proceeded using Pascal's formula for binomials. The result is very elegant

$$B^{\ell-2}_{p} = \frac{\mathcal{C}^{\ell}(-1)^p}{2^{\ell-1}(\ell+1)!} \binom{\ell}{p-1} \binom{\ell}{p+1}. \tag{24}$$

So we show that in contrast to the $\beta = 1$ case where the interaction includes the traceless conformal higher spin currents, the $\beta = 2$ boundary condition necessitates the interaction with the double trace higher spin currents. The connection between these two types of interaction can be described adding local Weyl (in the spin two case) and generalized "Weyl" invariants realizing the conformal coupling of the scalar with the higher spin fields.

3. Linearized spin two gauge and conformal scalar field interaction ($\ell = 2$)

The well-known action for the conformally coupled scalar field in D dimensions in external gravity is

$$S = \frac{1}{2} \int d^D z \sqrt{-G} \left[G^{\mu\nu} \nabla_{\mu}\phi \nabla_{\nu}\phi - \frac{(D-2)}{4(D-1)} R(G)\phi^2 \right]. \tag{25}$$

In this section we restore the linearized form of this action in fixed *AdS* background using a gauging procedure both for the gauge and Weyl symmetry on the linearized level. We do this derivation just for methodical reasons because the final nonlinear answer is known (25). But we would like to extend this consideration to the higher spin case and try to elaborate a linearized construction which works in the case $\ell = 4$ where the final answer is unknown.

We start from the massive free scalar action in the fixed *AdS* external metric[3]

$$S_0(\phi) = \frac{1}{2}\int d^D z\,\sqrt{-g}\big[\nabla_\mu\phi\nabla^\mu\phi + \lambda\phi^2\big]. \tag{27}$$

For getting an interaction with linearized gravity using the gauging procedure we have to variate S_0 with respect to $\delta^1_\varepsilon\phi = \varepsilon^\mu(z)\nabla_\mu\phi$

$$\delta^1_\varepsilon S_0 = \int d^D z\,\sqrt{-g}\nabla^{(\mu}\varepsilon^{\nu)}\left[\nabla_\mu\phi\nabla_\nu\phi - \frac{g_{\mu\nu}}{2}\big(\nabla_\alpha\phi\nabla^\alpha\phi + \lambda\phi^2\big)\right] \tag{28}$$

and solving (we assume that ε^μ and $h^{\mu\nu}$ have the same infinitesimal order) the equation

$$\delta^1_\epsilon S_0(\phi) + \delta^0_\varepsilon S_1\big(\phi, h^{(2)}\big) = 0, \qquad \delta^0_\varepsilon h^{(2)}_{\mu\nu} = 2\nabla_{(\mu}\varepsilon_{\nu)}, \tag{29}$$

we immediately find the following cubic interaction linear in the gauge field

$$S_1\big(\phi, h^{(2)}\big) = \frac{1}{2}\int d^D z\,\sqrt{-g}h^{(2)\mu\nu}\left[-\nabla_\mu\phi\nabla_\nu\phi + \frac{g_{\mu\nu}}{2}\big(\nabla_\mu\phi\nabla^\mu\phi + \lambda\phi^2\big)\right]. \tag{30}$$

Note that here we used many times partial integration which means that we admit that all fields or at least parameters of symmetry are zero on the boundary, otherwise we would have to check all symmetries taking into account some boundary terms and their variations also. It is clear that for constructing the local interaction on the bulk we can use partial integrations without watching the boundary effects.

So we see that gauge invariance

$$\delta^1_\varepsilon\phi(z) = \varepsilon^\mu(z)\nabla_\mu\phi(z), \qquad \delta^0_\varepsilon h^{(2)}_{\mu\nu}(z) = 2\nabla_{(\mu}\varepsilon_{\nu)}(z) \tag{31}$$

in this linear approach does not fix the free parameter λ and the corresponding spin two Noether current (energy–momentum tensor)

$$\Psi^{(2)}_{\mu\nu}(\phi, \lambda) = -\nabla_\mu\phi\nabla_\nu\phi + \frac{g_{\mu\nu}}{2}\big(\nabla_\mu\phi\nabla^\mu\phi + \lambda\phi^2\big) \tag{32}$$

is conserved but not traceless. But we can fix this problem having noted that there is one more gauge invariant combination of two derivatives and one $h_{\mu\nu}$ field

$$r^{(2)}\big(h^{(2)}(z)\big) = \nabla_\mu\nabla_\nu h^{(2)\mu\nu} - \nabla^2 h^{(2)\mu}_\mu - \frac{D-1}{L^2}h^{(2)\mu}_\mu, \qquad \delta^1_\varepsilon r^{(2)}\big(h^{(2)}\big) = 0. \tag{33}$$

It is of course the linearized Ricci scalar-but at this moment it is important for us that there is only one gauge invariant combination of $h^{(2)}_{\mu\nu}(z)$, two scalars $\phi(z)$ and two derivatives

$$\int d^D z\,\sqrt{g}r^{(2)}\big(h^{(2)}\big)\phi^2, \tag{34}$$

[3] We will use *AdS* conformal flat metric, curvature and covariant derivatives commutation rules of the type

$$\begin{aligned}
&ds^2 = g_{\mu\nu}\,dz^\mu\,dz^\nu = \frac{L^2}{(z^0)^2}\eta_{\mu\nu}\,dz^\mu\,dz^\nu, \qquad \eta_{z^0z^0} = -1, \qquad \sqrt{-g} = \frac{1}{(z^0)^{d+1}},\\
&[\nabla_\mu, \nabla_\nu]V^\rho_\lambda = R_{\mu\nu\sigma}{}^\rho V^\sigma_\lambda - R_{\mu\nu\lambda}{}^\sigma V^\rho_\sigma,\\
&R_{\mu\nu\lambda}{}^\rho = -\frac{1}{(z^0)^2}\big(\eta_{\mu\lambda}\delta^\rho_\nu - \eta_{\nu\lambda}\delta^\rho_\mu\big) = -\frac{1}{L^2}\big(g_{\mu\lambda}\delta^\rho_\nu - g_{\nu\lambda}\delta^\rho_\mu\big),\\
&R_{\mu\nu} = -\frac{D-1}{(z^0)^2}\eta_{\mu\nu} = -\frac{D-1}{L^2}g_{\mu\nu}, \qquad R = -\frac{D(D-1)}{L^2}.
\end{aligned} \tag{26}$$

which we can add to our linearized action with one more free parameter. So finally we can write the most general gauge invariant action in this approximation of the first order in the gauge field

$$\begin{aligned} S^{\mathrm{GI}}\big(\lambda,\xi,\phi,h^{(2)}\big) = {} & \frac{1}{2}\int d^D z\sqrt{-g}\big[\nabla_\mu\phi\nabla^\mu\phi+\lambda\phi^2\big] \\ & + \frac{1}{2}\int d^D z\sqrt{-g}h^{(2)\mu\nu}\left[-\nabla_\mu\phi\nabla_\nu\phi+\frac{g_{\mu\nu}}{2}\big(\nabla_\mu\phi\nabla^\mu\phi+\lambda\phi^2\big)\right] \\ & + \xi\int d^D z\sqrt{-g}\left[\nabla_\mu\nabla_\nu h^{(2)\mu\nu}-\nabla^2 h^{(2)\mu}_{\mu}-\frac{D-1}{L^2}h^{(2)\mu}_{\mu}\right]\phi^2. \end{aligned} \tag{35}$$

Then we search for the additional local symmetry permitting to remove the trace of the gauge field $h_{\mu\nu}$ and therefore leading to the traceless conformal spin two current. The natural choice here is of course Weyl invariance and we will define local Weyl transformation in linear approximation in the following way

$$\delta^1_\sigma\phi(z)=\Delta\sigma(z)\phi(z),\qquad \delta^0_\sigma h^{(2)}_{\mu\nu}(z)=2\sigma(z)g_{\mu\nu}, \tag{36}$$

where Δ is the conformal weight (one more additional free parameter to fit) of the scalar field. The important point here is that when we impose on the gauge invariant action (35) conformal (Weyl) invariance (36) we obtain the condition

$$\begin{aligned} \frac{\delta}{\delta\sigma(z)}S^{\mathrm{GI}}\big(\lambda,\xi,\phi,h^{(2)}\big) = {} & \left[\Delta\lambda+\frac{\lambda D}{2}-\frac{2\xi D(D-1)}{L^2}\right]\sigma\phi^2+\left[\Delta-1+\frac{D}{2}\right]\sigma\nabla_\mu\phi\nabla^\mu\phi \\ & + \left[2\xi(1-D)-\frac{\Delta}{2}\right]\nabla^2\sigma\phi^2=0 \end{aligned} \tag{37}$$

with the unique solution for all free constants

$$\Delta=1-\frac{D}{2},\qquad \xi=\frac{1}{8}\frac{D-2}{D-1},\qquad \lambda=\frac{D(D-2)}{4L^2}. \tag{38}$$

So finally we come to the gauge and conformal invariant action

$$S^{\mathrm{WI}}(\phi,h_{\mu\nu})=S_0(\phi)+S_1^{\Psi^{(2)}}\big(\phi,h^{(2)}\big)+S_1^{r^{(2)}}\big(\phi,h^{(2)}\big), \tag{39}$$

where

$$S_0(\phi)=\frac{1}{2}\int d^D z\sqrt{-g}\left[\nabla_\mu\phi\nabla^\mu\phi+\frac{D(D-2)}{4L^2}\phi^2\right], \tag{40}$$

$$S_1^{\Psi^{(2)}}\big(\phi,h^{(2)}\big)=\frac{1}{2}\int d^D z\sqrt{-g}h^{(2)\mu\nu}\left[-\nabla_\mu\phi\nabla_\nu\phi+\frac{g_{\mu\nu}}{2}\left(\nabla_\mu\phi\nabla^\mu\phi+\frac{D(D-2)}{4L^2}\phi^2\right)\right], \tag{41}$$

$$S_1^{r^{(2)}}\big(\phi,h^{(2)}\big)=\frac{1}{8}\frac{D-2}{D-1}\int d^D z\sqrt{-g}\left[\nabla_\mu\nabla_\nu h^{(2)\mu\nu}-\nabla^2 h^{(2)\mu}_{\mu}-\frac{D-1}{L^2}h^{(2)\mu}_{\mu}\right]\phi^2, \tag{42}$$

which is of course the linearized action (25) and can be obtained from that after expansion near to the AdS_D background $G_{\mu\nu}(z)=g_{\mu\nu}+h^{(2)}_{\mu\nu}(z)$ in the first order on $h^{(\ell)}_{\mu\nu}$.

4. Solution for spin four

Now we start from action (40) to apply Noether's method for the following transformation of the scalar field with a traceless third rank symmetric tensor parameter

$$\delta^1_\epsilon\phi=\epsilon^{\mu\nu\lambda}\nabla_\mu\nabla_\nu\nabla_\lambda\phi,\qquad \epsilon^{\alpha}_{\alpha\mu}=0. \tag{43}$$

First of all we have to calculate $\delta_1 S_0$. For brevity we introduce the notation (and in a similar way for any other tensor)

$$\tilde{\epsilon}^{\mu\nu} = \nabla_\lambda \epsilon^{\lambda\mu\nu}, \qquad \tilde{\tilde{\epsilon}}^{\mu} = \nabla_\nu \nabla_\lambda \epsilon^{\nu\lambda\mu}. \tag{44}$$

Then after variation of (40) we obtain

$$\begin{aligned}\delta^1_\epsilon S_0(\phi) = \int dx^4 \sqrt{-g}\Big\{ & -\nabla^{(\alpha}\epsilon^{\mu\nu\lambda)}\nabla_\mu\nabla_\alpha\phi\nabla_\nu\nabla_\lambda\phi + \frac{3}{2}\tilde{\epsilon}^{\nu\lambda}\nabla_\nu\nabla_\alpha\phi\nabla_\lambda\nabla^\alpha\phi - \frac{1}{2}\tilde{\epsilon}^{\nu\lambda}\nabla^2(\nabla_\nu\phi\nabla_\lambda\phi) \\ & + \frac{1}{8L^2}\big[3D(D+2)-8\big]\tilde{\epsilon}^{\nu\lambda}\nabla_\nu\phi\nabla_\lambda\phi \\ & - \nabla^{(\alpha}\tilde{\tilde{\epsilon}}^{\lambda)}\bigg[-\nabla_\mu\phi\nabla_\nu\phi + \frac{g_{\mu\nu}}{2}\bigg(\nabla_\mu\phi\nabla^\mu\phi + \frac{D(D-2)}{4L^2}\phi^2\bigg)\bigg]\Big\}.\end{aligned} \tag{45}$$

We see that we can introduce an interaction with the spin four gauge field $h^{(4)}_{\mu\nu\alpha\beta}$ in the minimal way if we will deform the transformation law for the spin two field. The solution for the equation

$$\delta^1_\epsilon S_0(\phi) + \delta^0_\epsilon\big[S_1^{\Psi^{(2)}}\big(\phi, h^{(2)}\big) + S_1^{\Psi^{(4)}}\big(\phi, h^{(4)}\big)\big] = 0 \tag{46}$$

is

$$\begin{aligned}S_1^{\Psi^{(4)}}\big(\phi, h^{(4)}\big) = \frac{1}{4}\int dx^4 \sqrt{-g}\Big[& h^{(4)\mu\nu\alpha\beta}\nabla_\mu\nabla_\nu\phi\nabla_\alpha\nabla_\beta\phi - 3h^{(4)\alpha\mu\nu}_{\alpha}\nabla_\mu\nabla_\beta\phi\nabla_\nu\nabla^\beta\phi + h^{(4)\alpha\mu\nu}_{\alpha}\nabla^2(\nabla_\mu\phi\nabla_\nu\phi) \\ & - \frac{3D(D+2)-8}{4L^2}h^{(4)\alpha\mu\nu}_{\alpha}\nabla_\mu\phi\nabla_\nu\phi\Big],\end{aligned} \tag{47}$$

$$\delta^0_\epsilon h^{(4)\mu\nu\alpha\beta} = 4\nabla^{(\mu}\epsilon^{\nu\alpha\beta)}, \qquad \delta^1_\epsilon\phi = \epsilon^{\mu\nu\alpha}\nabla_\mu\nabla_\nu\nabla_\alpha\phi, \tag{48}$$

$$\delta^0_\epsilon h^{(4)\alpha\mu\nu}_{\alpha} = 2\tilde{\epsilon}^{\mu\nu}, \qquad \delta^0_\epsilon h^{(2)\mu\nu} = 2\nabla^{(\mu}\tilde{\tilde{\epsilon}}^{\nu)}. \tag{49}$$

So we obtain the following gauged action with linearized interaction with both spin two and spin four gauge fields and linearized usual Weyl invariance

$$S^{\mathrm{GI}}\big(\phi, h^{(2)}, h^{(4)}\big) = S^{\mathrm{WI}}\big(\phi, h^{(2)}\big) + S_1^{\Psi^{(4)}}\big(\phi, h^{(4)}\big), \tag{50}$$

$$\delta^0 h^{(4)\mu\nu\lambda\alpha} = 4\nabla^{(\mu}\epsilon^{\nu\lambda\alpha)}, \qquad \delta^0 h^{(2)\mu\nu} = 2\nabla^{(\mu}\varepsilon^{\nu)} + 2\nabla^{(\mu}\tilde{\tilde{\epsilon}}^{\nu)} + 2\sigma g_{\mu\nu}, \tag{51}$$

$$\delta^1\phi = \varepsilon^\mu\nabla_\mu\phi + \epsilon^{\mu\nu\lambda}\nabla_\mu\nabla_\nu\nabla_\lambda\phi + \bigg(1 - \frac{D}{2}\bigg)\sigma\phi, \tag{52}$$

where $S^{\mathrm{WI}}(\phi, h^{(2)})$ can be read from (39)–(42) and we note that on this linearized level usual Weyl transformation does not affect the spin four part of the action but the spin four gauge transformation deforms the gauge transformation for spin two gauge field.

Now we turn to the construction of the conformal invariant coupling of the scalar field with the spin four gauge field in a similar way as in the case of spin two. For this we note first that here we can construct also the gauge invariant combination of two derivatives and $h^{(4)\mu\nu\alpha\beta}$. This is the following traceless symmetric second rank tensor

$$r^{(4)\alpha\beta} = \nabla_\mu\nabla_\nu h^{(4)\mu\nu\alpha\beta} - \nabla^2 h^{(4)\mu\alpha\beta}_{\mu} - \nabla^{(\alpha}\nabla_\nu h^{(4)\beta)\mu\nu}_{\mu} - \frac{3(D+1)}{L^2}h^{(4)\alpha\beta\mu}_{\mu}, \tag{53}$$

$$\delta^1_\epsilon r^{(4)\alpha\beta} = 0, \qquad r^{(4)\alpha}_{\alpha} = 0. \tag{54}$$

This is the analogue of the Ricci scalar in the spin four case and we can construct using this tensor *two* additional gauge invariant combinations of the same order.

$$S_1^{r^{(4)}}\big(\xi_1, \xi_2, \phi, h^{(4)}\big) = \xi_1\int d^D z\sqrt{-g}\, r^{(4)\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + \xi_2\int d^D z\sqrt{-g}\,\nabla_\mu\nabla_\nu r^{(4)\mu\nu}\phi^2. \tag{55}$$

Then we can define the *generalized* "Weyl" transformation for the scalar and spin four gauge field with the second rank symmetric traceless parameter $\chi^{\mu\nu}(z)$

$$\delta^0_\chi h^{(4)\mu\nu\alpha\beta}(z) = 12\chi^{(\mu\nu}(z)g^{\alpha\beta)}, \qquad \delta^1_\chi \phi(z) = \tilde{\Delta}\chi^{\alpha\beta}(z)\nabla_\alpha\nabla_\beta\phi(z), \tag{56}$$

where we introduced the "conformal" weight $\tilde{\Delta}$ for the scalar field. Computing the following χ variations

$$\begin{aligned}
&\delta^1_\chi S_0(\phi) + \delta^0_\chi S_1^{\Psi^{(4)}}\big(\phi, h^{(4)}\big) \\
&\quad = \int \bigg\{ (\tilde{\Delta}-1)\nabla^{(\alpha}\tilde{\chi}^{\beta)}\Psi^{(2)}_{\alpha\beta}\bigg(\phi, \frac{D(D-2)}{4L^2}\bigg) - \bigg(\tilde{\Delta} + \frac{3D}{2} + 3\bigg)\chi^{\alpha\beta}\nabla_\alpha\nabla_\mu\phi\nabla_\beta\nabla^\mu\phi \\
&\qquad + \frac{\tilde{\Delta}+D+3}{2}\nabla^2\chi^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi - \frac{1}{L^2}C(\tilde{\Delta}, D)\chi^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi + \frac{D(D-2)}{8L^2}\tilde{\tilde{\chi}}\phi^2\bigg\}\sqrt{-g}\,d^D z,
\end{aligned} \tag{57}$$

$$C(\tilde{\Delta}, D) = (\tilde{\Delta}-1)(D-1) + \frac{\tilde{\Delta}}{4}D(D-2) + (D+4)\bigg(\frac{3D(D+2)}{8} - 1\bigg), \tag{58}$$

$$\begin{aligned}
&\delta^0_\chi S_1^{r^{(4)}}\big(\phi, h^{(4)}\big) \\
&\quad = \xi_1 \int \bigg[2D\nabla^{(\alpha}\tilde{\chi}^{\beta)}\Psi^{(2)}_{\alpha\beta}\bigg(\phi, \frac{D(D-2)}{4L^2}\bigg) - (D-2)\tilde{\tilde{\chi}}\nabla_\alpha\phi\nabla^\alpha\phi - 2(D+3)\nabla^2\chi^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi \\
&\qquad - \frac{2}{L^2}(D+3)(3D+4)\chi^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi\bigg]\sqrt{-g}\,d^D z \\
&\quad - \bigg[\xi_1\frac{D^2(D-2)}{4L^2} + \xi_2\frac{12(D+1)(D+2)}{L^2}\bigg]\int d^D z\sqrt{-g}\,\tilde{\tilde{\chi}}\phi^2 - \xi_2 4(D+1)\int d^D z\sqrt{-g}\,\nabla^2\tilde{\tilde{\chi}}\phi^2
\end{aligned} \tag{59}$$

we see again that for obtaining a "Weyl" invariant interaction we have to deform the gauge and usual Weyl transformation of the spin two gauge field $h^{(2)}_{\mu\nu}$

$$\delta^0_\chi h^{(2)}_{\mu\nu} = 2(1 - \tilde{\Delta} - 2D\xi_1)\nabla^{(\mu}\tilde{\chi}^{\nu)} + 2\xi_1\tilde{\tilde{\chi}}g_{\mu\nu}. \tag{60}$$

Then solving the symmetry condition

$$\delta^1_\chi S_0(\phi) + \delta^0_\chi\big(S_1^{\Psi^{(2)}}\big(\phi, h^{(2)}\big) + S_1^{r^{(2)}}\big(\phi, h^{(2)}\big) + S_1^{\Psi^{(4)}}\big(\phi, h^{(4)}\big) + S_1^{r^{(4)}}\big(\phi, h^{(4)}\big)\big) = 0 \tag{61}$$

we obtain again a unique solution for all three free parameters

$$\tilde{\Delta} = -3 - \frac{3}{2}D, \tag{62}$$

$$\xi_1 = -\frac{1}{8}\frac{D}{D+3}, \tag{63}$$

$$\xi_2 = \frac{1}{64}\frac{D(D-2)}{(D+1)(D+3)}. \tag{64}$$

Thus we constructed the linearized action for a scalar field interacting with the spin two and four field in a conformally invariant way

$$S^{\mathrm{WI}}\big(\phi, h^{(2)}, h^{(4)}\big) = S^{\mathrm{WI}}\big(\phi, h^{(2)}\big) + S_1^{\Psi^{(4)}}\big(\phi, h^{(4)}\big) + S_1^{r^{(4)}}\big(\phi, h^{(4)}\big), \tag{65}$$

$$\delta^1\phi = \varepsilon^\mu\nabla_\mu\phi + \epsilon^{\mu\nu\lambda}\nabla_\mu\nabla_\nu\nabla_\lambda\phi + \Delta\sigma\phi + \tilde{\Delta}\chi^{\mu\nu}\nabla_\mu\nabla_\nu\phi, \tag{66}$$

$$\delta^0 h^{(2)\mu\nu} = 2\nabla^{(\mu}\varepsilon^{\nu)} + 2\nabla^{(\mu}\tilde{\tilde{\epsilon}}^{\nu)} + 2(1 - \tilde{\Delta} - 2D\xi_1)\nabla^{(\mu}\tilde{\chi}^{\nu)} + 2\sigma g_{\mu\nu} + 2\xi_1\tilde{\tilde{\chi}}g_{\mu\nu}, \tag{67}$$

$$\delta^0 h^{(4)\mu\nu\alpha\beta} = 4\nabla^{(\mu}\epsilon^{\nu\lambda\alpha)} + 12\chi^{(\mu\nu}g^{\alpha\beta)}. \tag{68}$$

This interaction has an additional local symmetry permitting to gauge away the trace of spin two and four gauge fields. So we can say that this is a linearized interaction for *conformal higher spin theory* of the type discussed in [9,10]. Unfortunately at the moment we can present only the spin four case in a complete form. But the general spin ℓ case in *AdS* can be considered in a similar but more complicated way and will be presented in future publications.

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References

[1] J.M. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231, Int. J. Theor. Phys. 38 (1999) 1113, hep-th/9711200.
[2] I.R. Klebanov, A.M. Polyakov, Phys. Lett. B 550 (2002) 213, hep-th/0210114.
[3] E.S. Fradkin, M.A. Vasiliev, Phys. Lett. B 189 (1987) 89;
E.S. Fradkin, M.A. Vasiliev, Nucl. Phys. B 291 (1987) 141;
M.A. Vasiliev, Int. J. Mod. Phys. D 5 (1996) 763, hep-th/9611024;
M.A. Vasiliev, hep-th/0304049.
[4] E. Witten, hep-th/0112258;
S.S. Gubser, I.R. Klebanov, Nucl. Phys. B 656 (2003) 23, hep-th/0212138;
I.R. Klebanov, E. Witten, Nucl. Phys. B 556 (1999) 89, hep-th/9905104.
[5] T. Leonhardt, R. Manvelyan, W. Rühl, hep-th/0401240.
[6] C. Fronsdal, Phys. Rev. D 20 (1979) 848;
C. Fronsdal, Phys. Rev. D 18 (1978) 3624.
[7] E. Sezgin, P. Sundell, hep-th/0305040;
E. Sezgin, P. Sundell, JHEP 0207 (2002) 055, hep-th/0205132.
[8] D. Anselmi, Class. Quantum Grav. 17 (2000) 1383, hep-th/9906167.
[9] A.Y. Segal, Nucl. Phys. B 664 (2003) 59, hep-th/0207212.
[10] E.S. Fradkin, V.Y. Linetsky, Nucl. Phys. B 350 (1991) 274;
E.S. Fradkin, V.Y. Linetsky, Phys. Lett. B 231 (1989) 97;
E.S. Fradkin, V.Y. Linetsky, Mod. Phys. Lett. A 4 (1989) 731;
E.S. Fradkin, V.Y. Linetsky, Ann. Phys. 198 (1990) 293.