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Full length article

Multi-response optimization and modeling of trim cut WEDM operation of commercially pure titanium (CPTi) considering multiple user's preferences

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In this research work, development of a multi response optimization technique has been undertaken, using traditional utility method in conjunction with the weight assignment concept (for multiple customer's priorities) in trim cut wire electrical discharge machining (WEDM). Pure titanium has been selected as work material for experimentation. The effect of key process parameters such a wire type (zinc coated and uncoated brass wire), pulse on time (T_{ON}), pulse off time (T_{OFF}), peak current (IP), wire feed (WF), servo voltage (SV) and wire offset (W_{OFF}) were investigated on material removal rate (MRR), surface roughness and wire weight consumption (eroded weight of wire after machining) in finish cut WEDM operation. Two different types of wire electrodes were taken for experimental research (uncoated, zinc coated). Further, the variation of the MRR was modeled semi-empirically through dimensional analysis. The developed model is mechanistic, as it can be used by the machinists to predict the MRR over a wide range of input parameters. The optimization of multiple responses has been done for satisfying the priorities of multiple users, in contrast to the traditional multi-response techniques where the optimized process setting is realized without giving any attention to the priorities of different users.

1. Introduction

Wire Electrical Discharge Machining (WEDM) is an electro thermal machining process to machine any material which is electrically conductive regardless of strength and hardness. This process utilizes thin wire (electrode), which follows a programmed path. The material removal takes place by series of electric sparks which erode away a part of material, which is vaporized and melted from the workpiece. Some of the wire electrode material is also eroded. These particles (chips) are flushed away from the machining zone with a stream of de-ionized water flowing through the top and bottom flushing nozzles. The de-ionized water prevents the heat build-up in the workpiece. WEDM can machine any electrically conductive material such as tool steel, aluminum, copper, graphite, exotic space-age alloys including hastaloy, inconel, titanium, tungsten carbide, polycrystalline diamond compacts, Ni based alloys and ceramics. WEDM process enables higher accuracy and surface finish together with reasonable cutting efficiency. WEDM process is generally used in aerospace, automobile, tool and dies industries where accuracy and surface finish have great importance [1]. Fig. 1 represents the WEDM set up used for this research work.

For WEDM process, MRR (material removal rate), surface roughness (SR) and wire weight consumption (WWC) which is weight of eroded wire after each experimental run are the most critical quality characteristics determining the process capability for a given job. Single response optimization process can only focus on an individual quality characteristic at a time. But to satisfy customer's requirements, the quality and productivity must be equally addressed. Multi-response optimization process can target both requirements, such as quality (surface finish of machined components) and productivity (MRR and WWC) for fulfillment of the customer requirements (i.e. the machining industry). A large number of researchers have reported the optimization of multiple, correlated responses of WEDM process using traditional

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Fig. 1. WEDM set up used for experimentation.

optimization techniques such as MRSN (multi response signal to noise) ratio, Principal component analysis (PCA) and grey relation analysis (GRA). Table 1 illustrates the previous research on multi-response optimization of EDM/WEDM process.

1.1. Gaps observed and the expected outcomes

It was observed from the intensive literature review that most of the studies have targeted the optimization of multiple quality characteristics such as MRR, surface roughness (SR) and wire wear ratio through traditional as well as modern optimization techniques such as MRSN (multi response signal to noise), principal weight component analysis, weighted S/N ratio, TOPSIS, Taguchi-grey and Grey fuzzy logic, neural network modeling etc. But most of the authors have assigned the weights (importance) to the multiple quality characteristics on the basis of their own assumptions or past experience. In this research work, various possibilities regarding differences of opinion from customer/end user's point of view have been considered, by applying adjacency matrix [17] in the weight assignment process. Further, to the author's best knowledge, no work has been reported regarding optimization of multiple, correlated responses (such as MRR, SR and wire weight consumption) for trim cut wire electric discharge machining of pure titanium.

It is also important here to mention that the eroded wire collected after machining with WEDM is not reusable as it could affect the dimensional accuracy and the machining efficiency. Hence, the focus of this experimental study is on minimizing the wire consumption from the economical considerations. No investigation for wire consumption has been introduced in the past studies reported in the literature for trim cut WEDM process (for machining pure titanium). It has also been observed that the relevant machining guidelines for trim cut WEDM of pure titanium are not available in most of the manufacturer's catalogues of WEDM. This necessitates the development of appropriate mechanistic models for prediction of the machining speed (MRR) and surface quality (SR) for trim cut WEDM process of pure titanium.

Keeping these facts in view, the investigation is focused at development of a hybrid method for incorporating the priorities of different customers (users) for multi-response optimization in trim cut WEDM operation. Taguchi's method has been employed for design and analysis of the experiments and an attempt has been made to obtain robust process design for multiple performance characteristics, which could be very useful for the machining industry. A mechanistic model has also been developed for prediction of the responses of interest (MRR) over a wide range of input variables.

2. Experimental procedure

A trim cut WEDM operation is generally performed after rough cut to achieve higher surface finish of the machined surface. The trim cutting also enables the complete or partial removal of surface defects such as recast layer, thermal stress and micro-cracks generated during the rough cut operation [16]. In this experimental work, rough and subsequent finish cut WEDM operation for pure titanium was performed on Sprintcut (ELPULS-40A DLX) CNC Wire-EDM machine manufactured by Electronica Machine Tool Limited, India (Fig. 1). The chemical composition of material is given in the Table 2. Zinc coated and uncoated brass wire electrodes (0.25 mm diameter) were used, to investigate the effect of wire electrode material on the machining performance. Rough cut parameters and trim cut parameters were selected on the basis of past literature review and machine manual, while the ranges for these parameters were determined on the basis of pilot experimentation conducted by using one factor at a time approach. Trim cut parameter's level was selected on basis of criteria of low pulse discharge energy level and wire offset value [5]. In rough-cut stage, high pulse discharge energy setting was used, which involved higher level of pulse on time, peak current in combination with lower level of pulse off time and servo-voltage. In rough cut stage, emphasis was given to achieve maximum cutting speed irrespective of the surface finish for realization of higher productivity. While in trim cut, low to moderate pulse discharge energy setting, coupled with lower wire offset was selected. All other factors such as wire offset (0.148 mm), dielectric fluid pressure (WP = 1 unit), pulse peak voltage (VP = 2 unit) were fixed during the rough cut. The parametric setting for initial rough cut is given in Table 3.

In rough cut stage, a square cut (6 mm × 6 mm) was taken, leaving a path (of 2 mm) to start the finish cut as shown in Fig. 2a. This untrimmed length grips the workpiece in its place securely so that subsequent finish cut can be made easily. After finishing the whole path, the punch was removed from rectangular plate (workpiece). The wire path planning of finish cut and the concept of wire offset are shown in Fig. 2a and b respectively.

In the trim cut stage, seven factors such as wire type (zinc coated and uncoated brass wire), pulse on time (T_{ON}), pulse off time (T_{OFF}), peak current (IP), wire feed (WF), servo voltage (SV) and wire offset (W_{OFF}) were selected for evaluation of their effects on three response variables- MRR, surface roughness (SR) and wire weight consumption (WWC) through experimentation. L18 orthogonal array (OA) was used for planning the experiments. Initial nine experiments (1–9) were performed with uncoated brass wire (for both rough and trim cut) while rest of the nine experiments (10–18) were conducted with zinc coated wire (for both rough and trim cut), as per the design matrix obtained by using L18 orthogonal array (OA). The experiments were conducted with fixed values of wire tension (10 units), dielectric fluid pressure (WP = 1 unit), pulse peak voltage (VP = 2 unit) and de-ionized water as a dielectric fluid. The input parameters and their levels are shown in Table 4. Based on the experimental layout depicted in Table 4, the experiments were performed in random order and each specific run was repeated two times, in order to get a measure of the experimental error. Three machining characteristics namely MRR, SR and WWC were recorded under varying experimental conditions.

During WEDM process, the wire diameter remains constant and the variation in amount of kerf width is negligible as compared to

Table 1
A summary of the previous research on multi-response optimization of EDM/WEDM.

S.no.	Author(s)	Optimization approach	Quality characteristics	Remarks
1	Ramakrishnan and Karunamoorthy [2]	Artificial neural network modeling and Multi-response optimization using multi-response S/N ratio (MRSN) for WEDM of inconel 718 alloy.	MRR and surface finish.	Applied varying proportion of weights to quality characteristics irrespective of difference in customer's opinion.
2	Ramakrishnan and Karunamoorthy [3]	multi-response S/N ratio (MRSN) for WEDM of tool steel.	MRR, surface finish and wire wear ratio.	Applied varying proportion of weights to various responses irrespective of difference in customer's opinion.
3	Sarkar et al. [4]	Additive modeling and Pareto-optimal strategy for WEDM of γ- titanium aluminide alloy.	Machining speed, surface finish and dimensional deviation.	The set of 20 Pareto-optimal solutions could provide guidelines for machining of γ -titanium aluminide alloy. However, the priorities of different users were not considered for arriving at the optimal solutions.
4	Sarkar et al. [5]	Response Surface Modelling technique and Pareto-optimal strategy for trim cut WEDM of γ- titanium aluminide alloy.	Cutting speed, surface finish and dimensional shift.	The technology table could provide useful guidelines for optimum machining of γ titanium aluminide alloy from the perspective of a single user.
5	Tzeng et al. [6]	hybrid method consisting of BPNN and genetic algorithm (GA) for WEDM of pure tungsten.	MRR and surface finish.	The variable priorities of multiple users were not considered.
6	Sahu et al. [7]	DEA approach for optimization of multiple responses in electrical discharge machining of AISI D2 Steel.	MRR and surface roughness and tool wear.	The variable priorities of multiple users were not considered
7	Chakravorty et al. [8]	Modified principal component analysis-based utility theory approach for optimization of correlated responses of EDM process.	MRR, tool wear rate and surface roughness.	Modified PCA based method was found to be yield better results than PCA-based PQLR method. However, the conflicting preferences of multiple customers were not taken into account.
8	Chakravorty et al. [9]	Multi-response optimization using weighted signal-to-noise (WSN) ratio method and utility theory (UT) method in USM process.	Experimental data through literature (MRR, Tool wear rate and surface roughness).	Weigted S/N ratio and utility theory method was found to give better result than GRA and MRSN method.WSN method is preferable than UT method because of less complexity. Same weight-age was given to quality characteristics.
9	Nayak et al. [10]	AHP and TOPSIS method for WEDM for of D-2 tool steel	MRR, surface finish and kerf width	The conflicting preferences of multiple customers were not taken into account.
10	Gopalakannan et al. [11]	Taguchi based grey analysis for EDM of aluminium hybrid MMC.	MRR, electrode wear rate and surface roughness.	The conflicting preferences of multiple customers were not taken into account.
11	Rao and Gopala Krishna [12]	Principal Component Analysis (PCA) for WEDM ZC63/SiCp MMC.	MRR, surface roughness, wire wear ratio and white layer thickness.	The conflicting preferences of multiple customers were not taken into account.
13	Sengottuvel et al. [13]	Desirability Approach and Fuzzy Modelling for EDM of for Inconel 718 alloy.	MRR, tool wear rate and surface roughness.	The proposed fuzzy model provides an easy, more precise selection of EDM input parameters for the required MRR, TWR and SR. However, relative importance of the responses was not established.
14	Lin and Lin [14]	Grey-fuzzy logic for EDM of SKD 11 alloy steel.	MRR, electrode wear rate and surface roughness.	Grey relational coefficient analyzes the relational degree of the multiple responses and subsequently fuzzy logic performs a fuzzy reasoning of the multiple performance characteristics.
15	Assarzadeh and Ghoreishi [15]	neural network modeling for EDM of BD3 steel.	MRR and surface roughness.	Output parameters (MRR and surface roughness) were optimized at each level of machining regime such as finishing (Ra<2 μm), semi finishing (Ra<4.5 μm) and roughing (Ra<7 μm) using augmented Lagrange multiplier (ALM) algorithm.

other parameters such as cutting speed and material thickness. Cutting speed is displayed on the WEDM monitor. Therefore, the MRR for WEDM operation was calculated using Eq. (1), which is shown below:

Table 2
Chemical composition of commercially pure titanium (wt.%).

N	C	Fe	O	Ti	Other elements
0.001	0.06	0.10	0.002	99.82	0.017

Table 3
Rough cut parametric setting [1].

Factor	Value
Pulse on time (T_{ON})	0.9 μs
Pulse off time (T_{OFF})	7 μs
Peak current (IP)	200 Amp
Wire feed (WF)	8 m/min
Wire tension (WT)	850 gm
Servo voltage (SV)	50 V

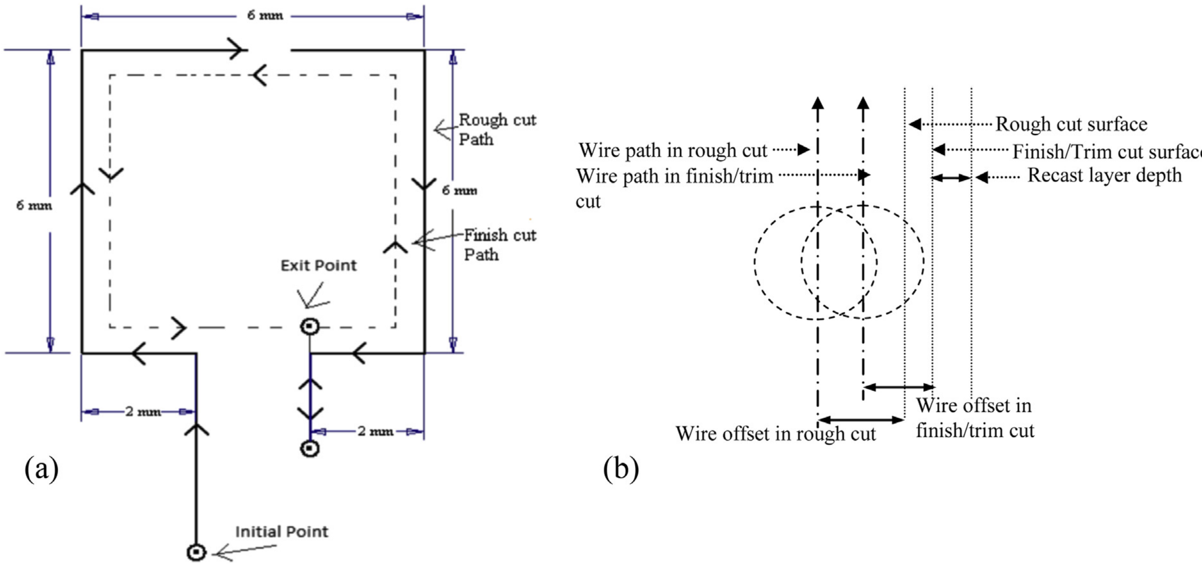

Fig. 2. (a) Wire path profile during machining. (b) Wire offset in rough cut and trim cut.

$$\text{MRR (mm}^2\text{/min)} = \text{cutting speed (mm/min)} \times \text{thickness of material (mm) [2]} \quad (1)$$

A roughness tester (Mitutoyo make) was used for measurement of average surface roughness (Ra) of the machined surface. The cut off length (λc) and the sampling number were chosen as 0.8 mm and 5 respectively. Three independent readings were taken on each surface of machined component and an average was computed. Eroded wire after completion of each experiment was obtained from the collection spool and weighted by weighing machine (SHIMADZU electronic balance with 0.01 gm LC). Table 5 depicts the experimental results.

3. Multi-response optimization

3.1. Utility concept methodology

Any process having multiple quality characteristics is accessed by prospective users according to their unique set of priorities. The multiple process characteristics are evaluated in combination as a composite index in utility concept methodology. This composite index represents overall utility of the process. The overall utility could be assumed to be the sum of the utilities of individual quality characteristics of particular process. Thus if ρ_i is the measure of effectiveness of an attribute i and there are n attributes representing the process quality, the overall utility function can be expressed as

$$U(\rho_1, \rho_2 \ldots\ldots\ldots, \rho_n) = f[U_1(\rho_1), U_2(\rho_2) \ldots\ldots U_n(\rho_n)] \quad (2)$$

where $\mathrm{Ui}(\rho_i)$ is the utility of the ith attribute. The overall utility function is defined as

$$U(\rho_1, \rho_2 \ldots\ldots\ldots\ldots, \rho_n) = \sum \mathrm{Ui}(\rho_i) \quad (3)$$

where i varies from 1 to n.

The quality characteristics may be assigned weights based on the priorities set by end users for the individual utility index. The weights for multiple quality characteristics (MRR, SR and WWC) have been determined in ‘weights computation’ section. The overall utility function thus can be expressed as

$$U(\rho_1, \rho_2 \ldots\ldots, \rho_n) = \sum_{i=1}^{n} \omega\mathrm{iUi}(\rho\mathrm{i}) \quad (4)$$

where ωi is the weight assigned to the attribute i and the sum of the weights assigned to all the attributes is equal to 1. The utility function is considered to be “larger-the-better” type characteristic for optimization.

Table 4
Factors and their levels for trim cut.

Factor	Level-1	Level-2	Level-3
Wire type	1 (Uncoated Brass wire)	2 (Zn coated brass wire)	–
Pulse on time (T_{ON})	0.2 µs	0.35 µs	0.5 µs
Peak current (IP)	40 A	60 A	80 A
Wire offset (W_{OFF})	0.07 mm	0.09 mm	0.11 mm
Wire feed (WF)	6 m/min	8 m/min	10 m/min
Pulse off time (T_{OFF})	18 µs	26 µs	36 µs
Servo-voltage (SV)	65 V	75 V	85 V

Table 5
Experiment design matrix (L18 OA) and results.

Exp. no.	Wire type	T_{on}	IP	W_{off}	WF	T_{off}	SV	MRR (mm^2/min)	SR (µm)	WWC (gm)
1	1	1	1	1	1	1	1	10.791	1.23	115.93
2	1	1	2	2	2	2	2	32.010	1.44	74.825
3	1	1	3	3	3	3	3	45.711	1.42	70.84
4	1	2	1	1	2	2	3	20.613	1.35	90.905
5	1	2	2	2	3	3	1	28.009	1.66	85.685
6	1	2	3	3	1	1	2	92.392	2.02	45.695
7	1	3	1	2	1	3	2	36.618	1.82	40.66
8	1	3	2	3	2	1	3	91.786	2.22	22.72
9	1	3	3	1	3	2	1	58.321	2.21	21.225
10	2	1	1	3	3	2	2	73.841	1.51	34.785
11	2	1	2	1	1	3	3	14.550	1.27	86.88
12	2	1	3	2	2	1	1	106.700	2.00	24.895
13	2	2	1	2	3	1	3	66.688	1.97	34.825
14	2	2	2	3	1	2	1	176.904	2.48	11.325
15	2	2	3	1	2	3	2	59.412	1.92	45.7
16	2	3	1	3	2	3	1	171.084	2.47	11.665
17	2	3	2	1	3	1	2	128.767	2.80	21.24
18	2	3	3	2	1	2	3	117.370	2.71	20.125

3.2. Determination of utility value

The preference scale for each quality characteristic is constructed for determining the overall utility value for the process, which is characterized by a number of quality characteristics. Thereafter; these scales are assigned weights to obtain an overall utility value. The preference scale is a logarithmic scale. The minimum acceptable quality level for each quality characteristic is set out at preference number of 0 and the best available quality is assigned a preference number of 9 [18].

Preference number (Pi) is calculated by following equation.

$$P_i = \mathrm{A}\ \log\,[\rho_i/\rho'] \tag{5}$$

where ρ_i is the value of quality characteristic i, ρ' is the minimum acceptable value of characteristic i and A is a constant whose value can be determined as given under:

$$A = 9/[\log\ \rho^*/\rho'] \tag{6}$$

A is chosen such that $P_i = 9$ at $\rho i = \rho^*$, where ρ^* is the most desirable (optimum) value of ρi.

3.3. Construction of preference scale

The preference scales for calculating overall utility was constructed using Eqs. (5) and (6). Since MRR is considered to be a "higher-the-better" type characteristic, so the smallest value obtained from the experimentation could be selected as the minimum acceptable value (ρ'). The other quality characteristics such as surface roughness and wire weight consumption are considered to be "lower-the-better" type characteristics, so for these, largest value was selected as minimum acceptable value (ρ') from the experimental results (Table 5). ρ^* is the most desirable value of quality characteristics (MRR, SR and WWC) obtained from the single response optimization. The preference scale data is tabulated in Table 6.

In next step, weights are assigned to the quality characteristics depending upon the requirements of end users. The weights are assigned to satisfy the following condition.

$$\sum_{i=1}^{n} \omega \mathrm{i} = 1 \tag{7}$$

3.4. Weights computation

MRR, SR and WWC are considered to be the critical quality characteristics of WEDM process. Every customer or end user might assign different priorities to the machining characteristics. Here, the priorities of three different users (industries) have been considered. These priorities have been represented in graphical form, for each customer separately. Fig. 3(a)–(c) represents customer's priorities for each of the machining characteristics considered. Following methodology has been adopted for computation of weights on the basis of multiple user's preferences [17].

3.4.1. Formation of the adjacency matrix

The relationship shown in Fig. 3 has been transformed in the matrix form. This matrix is termed as adjacency matrix [17].

$$\mathrm{AB}_n = [\mathrm{ab}_{xy}]_{M\times M}\ (x, y = 1, 2, \ldots, M) \tag{8}$$

where n is the number of users and M is the number of quality characteristics. ab_{xy} represents the dominance of x over y in a matrix of $M \times M$.

Table 6
Preference scale for MRR, SR and WWC.

	MRR	SR	WWC
Optimal value of quality characteristics (ρ^*)	202.28 mm^2/min	0.712 µm (1.23)	7.551 gm (20.125)
Minimum acceptable value (ρ')	11.12 mm^2/min (10.791)	2.8 µm (2.81)	117 gm (115.93)
Preference scale (P)	7.143 log[Xi/11.12]	−15.134 log[Xi/2.80]	−7.561 log[Xi/117]

AB₁=

	SR	MRR	WWC	
	0	1	1	→ SR
	0	0	1	→ MRR
	0	0	0	→ WWC

AB₂=

	SR	MRR	WWC	
	0	0	1	SR
	1	0	1	MRR
	0	0	0	WWC

AB₃=

	SR	MRR	WWC	
	0	0	1	SR
	0	0	1	MRR
	0	0	0	WWC

3.4.2. Dominance matrix

The dominance matrix (ψ^n) is prepared with the help of adjacency matrix (AB). It is represented by the following equation [17],

$$\psi^n = \mathrm{AB}_n^1 + \mathrm{AB}_n^2 + \mathrm{AB}_n^3 + \ldots\ldots + \mathrm{AB}_n^m + \mathrm{AB}_n^{M-1} \tag{9}$$

ξ_m^n is the sum of elements in each row of dominance matrix. The dominance matrix is assessed in the following manner.

$$\xi_m^n = \sum_{j=1}^{M} \mathrm{ab\ mj} \tag{10}$$

$$\psi^1 = \begin{vmatrix} 0 & 1 & 1 & = & 2 \\ 0 & 0 & 1 & = & 1 \\ 0 & 0 & 0 & = & 0 \end{vmatrix}$$

$$\psi^2 = \begin{vmatrix} 0 & 0 & 1 & = & 1 \\ 1 & 0 & 1 & = & 2 \\ 0 & 0 & 0 & = & 0 \end{vmatrix}$$

$$\psi^3 = \begin{vmatrix} 0 & 0 & 1 & = & 1 \\ 0 & 0 & 1 & = & 1 \\ 0 & 0 & 0 & = & 0 \end{vmatrix}$$

Hence,

$$\psi^1 = \begin{bmatrix} 2 & 1 & 0 \end{bmatrix}, \psi^2 = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix} \text{ and } \psi^3 = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$$

Dominance matrix for first user (ψ^1) signifies the importance of various responses as, SR has dominance (importance) over the other two responses, MRR over one response (WWC) and WWC does not possess dominance over any of the other responses considered. Similar computation can be performed for other users on by considering their respective dominance matrices.

3.4.3. Relative degree of performance

Relative degree of performance exhibits the relative importance of the responses considered, in the scaled form (i.e. 0–1) by taking

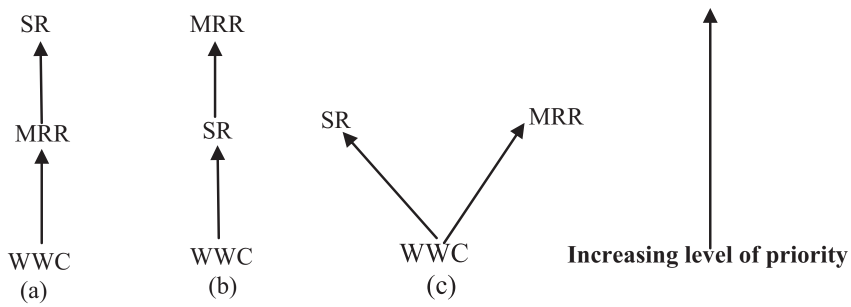

Fig. 3. Representation of the priorities of three different users in graphical form.

into account the preference of each user. It is shown by following equation. [17],

$$\eta_m^n = \frac{1+\xi_m^n}{\text{MAX } m = 1\ldots M(1+\xi_m^n)} \quad (11)$$

Relative degree of performance is presented in the following form.

$$\eta_n = (\eta_1^n,\ \eta_2^n,\ \eta_3^n, \ldots \eta_M^n) \quad (12)$$

Thus,

$$\eta_1 = (1, 0.66, 0.33)$$

$$\eta_2 = (0.66, 1, 0.33)$$

$$\eta_3 = (1, 1, 0.5)$$

3.4.4. Relative importance rating

Relative importance rating shows overall rating of each of the quality characteristics, considering the preference of each user. It is calculated by following equation [17].

$$\boldsymbol{\Phi} = \frac{\sum_{n=1}^{N} \eta_m^n}{\text{MAX } m = 1\ldots M\left(\sum_{n=1}^{N} \eta_m^n\right)} \quad (13)$$

The representation of the relative importance rating is done as

$$\Phi = (\Phi_1, \Phi_2, \Phi_3 \ldots\ldots \Phi_M) \quad (14)$$

Thus,

$$\Phi = (2.66/2.66,\ 2.66/2.66,\ 1.16/2.66) = (1, 1, 0.436)$$

3.4.5. Weights calculation

The weight for each quality characteristic is calculated by following equation.

$$\omega_m = \frac{\Phi}{\sum_{m=1}^{M} \Phi} \quad (15)$$

Weights for SR, MRR and WWC have been computed as 0.410, 0.410 and 0.178 respectively by using Eq. (13) in conjunction with Eq. (15).

The overall utility is calculated thereafter by the following relation.

$$\text{Ui} = \sum_{i=1}^{n} \omega \text{iPi} \quad (16)$$

In present case, overall utility index is computed from the following relation.

$$U(n,R) = [P_{\text{MRR}}(n,R) \times \omega_{\text{MRR}} + P_{\text{SR}}(n,R) \times \omega_{\text{SR}} + P_{\text{WWC}}(n,R) \times \omega_{\text{WWC}}] \quad (17)$$

where n is total number of trials, R = replication no. of a particular trial run

After finding out the overall utility index values for all of the experimental runs (Table 7), the data (Table 7) was analyzed using MINITAB 16 Software, for statistical optimization as per the standard Taguchi method. The utility data was considered as "larger-the-better" type characteristic for S/N ratio analysis. Fig. 4 shows the mean effect plot for S/N data.

The optimal parametric setting for multi-response optimization of the trim cut WEDM operation of commercially pure titanium was found to be as (Table 8).

ANOVA was performed on the utility data which revealed that wire type (26.93%), wire offset (46.66%) and pulse on time (6.05%) are the most influencing parameters affecting the variation in the overall utility index (see Table 9). Although, all of the parameters considered were found to be statistically significant at 95% confidence level, as the p-value obtained has been less than 0.05 for all the parameters (Table 9).

4. Confirmation experiments

The Taguchi approach has been applied for prediction of the mean (optimal value) for all the performance characteristics and determination of the confidence intervals for the predicted means. Two confirmation experiments were performed at the optimal

Table 7
Utility data.

S. no.	R1	R2	S/N Ratio
1	2.255	2.273	7.098
2	3.387	3.428	10.64
3	3.917	3.923	11.86
4	2.896	2.920	9.271
5	2.757	2.780	8.844
6	4.105	4.136	12.29
7	3.293	3.290	10.34
8	4.252	4.297	12.61
9	3.742	3.751	11.47
10	4.764	4.806	13.59
11	2.643	2.734	8.586
12	4.668	4.707	13.41
13	3.918	3.951	11.89
14	5.230	5.200	14.34
15	3.701	3.694	11.35
16	5.141	5.186	14.25
17	4.105	4.127	12.28
18	4.109	3.981	12.13

setting (Table 8) and the mean value was computed. The average values of the performance characteristics obtained through the confirmation experiments (two runs) must be within the 95% confidence interval, CI_{CE} (fixed number of confirmation experiments).

The optimum value of utility index (μ) has been predicted for the optimal levels of the process parameters which were found significant. The estimated utility mean was computed for the parametric setting summarized in Table 8.

CI_{CE} is given by following equation.

$$CE_{CE} = \sqrt{F\alpha(1,f_e)\left\{\frac{1}{\eta_{\text{eff}}}+\frac{1}{R}\right\}V_e} \tag{18}$$

where $F\alpha\,(1, f_e)$ = the F-ratio at a confidence level of $(1-\alpha)$ against DOF 1 and error degrees of freedom (f_e).

V_e = error variance

$$\eta_{\text{eff}} = \frac{N}{1+\text{Total DOF used in estimating mean}} \tag{19}$$

N = total number of experiments

R = No. of replicates for the confirmatory experiments

Predicted confidence interval for confirmation experiment is given as,

$$\mu_{\text{mean}} - CI_{CE} < \mu_{\text{mean}} < \mu_{\text{mean}} + CI_{CE} \tag{20}$$

By putting the values of μ_{mean} and CI_{CE} in above equation we get,

$$5.481 < \mu_{\text{mean}} < 6.349 \tag{21}$$

Two confirmation experiments were conducted at optimal parametric setting. The average values reported are following.

MRR = 178.90 mm^2/min, SR = 2.21 μm and WWC = 11.79 gm.

The utility value for the above confirmation experiments was calculated using Eq. (17) which was found to be 5.517. Since the value of utility index falls within 95% CI limits, the optimized results are validated.

Table 8
Optimized parametric setting for trim cut WEDM.

Parameter	Value
Pulse on time (T_{ON})	0.5 μs
Peak current (IP)	80 A
Wire offset (W_{off})	0.11 mm
Wire feed rate (WF)	8 m/min
Pulse off time (Toff)	26 μs
Servo-voltage (SV)	75 V

Table 9
ANOVA of utility raw data.

Factor	DOF[a]	Seq SS[b]	Adj MS[c]	F-ratio	p-value	Percent-contribution (%)
Wire type	1	6.4695	6.4695	131.09	0.000	**26.93**
Pulse on time	2	1.4542	0.7271	14.73	0.000	**6.05**
Peak current	2	0.7291	0.36455	7.39	0.004	3.03
Wire offset	2	11.2073	5.60365	113.54	0.000	**46.66**
Pulse off time	2	1.1814	0.5907	11.01	0.000	4.91
Wire feed	2	1.0863	0.54315	11.97	0.000	4.52
Servo-voltage	2	0.8010	0.4005	8.12	0.002	3.33
Error	22	1.0858	0.7271			4.57
Total	35	24.0146				

The parameters in bold font indicates the strong influence on utility raw data.
[a] Degree of freedom.
[b] Sequential sums of squares.
[c] Adjusted mean of squares.

5. Mathematical modeling using Buckingham's π theorem

It was considered to develop a model for prediction of the quality characteristics (MRR) by including the process parameters such as pulse on time (T_{ON}), pulse off time (T_{OFF}), peak current (IP), wire offset (W_{OFF}) and servo voltage (SV). All of the input parameters selected for developing the model have been found to be significant for MRR, as determined from single response optimization. The present work uses the technique of dimensional analysis for modeling. The theory of dimensional analysis is the mathematical theory which is purely algebraic. The applicability of dimensional analysis to a certain situation is based on the hypothesis that the solution of the problem is expressible by means of a dimensionally homogeneous equation in terms of specified variables.

Buckingham's π theorem states that if there are n variables in a problem and these variables contain m primary dimensions (for example M, L, T, and I), the equation relating all the variables will have (n−m) dimensionless groups. Buckingham referred to these groups as π groups. The final equation obtained has the following form

$$\pi_1 = f(\pi_1, \pi_2 \ldots \pi_{n-m}) \tag{22}$$

This method offers the advantage of being simpler than the method of solving simultaneous equations for obtaining the values of the indices (the exponent values of the variables). Following conditions must be satisfied for application of this method.

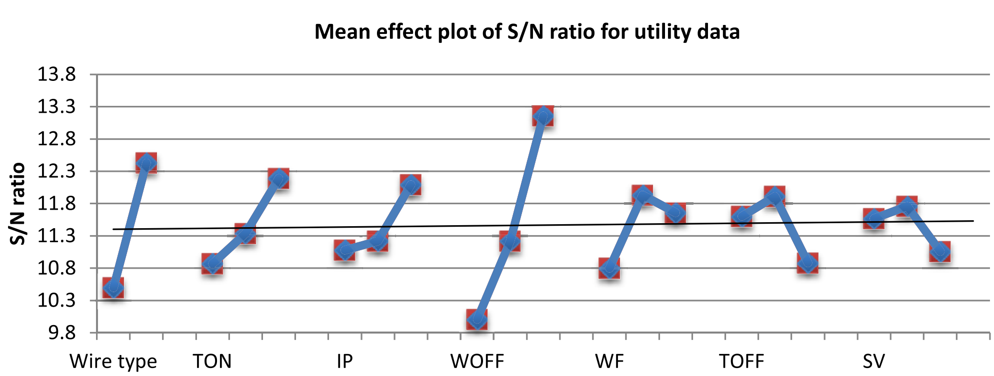

Fig. 4. Mean effect plot of S/N ratio for utility data.

1. Each of the fundamental dimensions must appear in at least one of the m variables
2. It must not be possible to form a dimensionless group from one of the variables within a recurring set. A recurring set is a group of variables forming a dimensionless group.

After applying the dimensional analysis, MRR can be given by following equation.

$$MRR = f(T_{on}, T_{off}, IP, W_{off}, SV) \tag{23}$$

where,
MRR = Material removal rate
T_{on} = Pulse on time
T_{off} = Pulse off time
IP = Peak current
W_{off} = Wire offset
SV = Servo voltage

The dimensions and units for process variables and quality characteristics are give in the Table 10.

In this case, number of variables $n = 6$ (i.e. MRR, T_{on}, T_{off}, IP, W_{off}, SV)

No. of fundamental dimensions = m = 4 (i.e., [M], [L], [T], [I])

By Buckingham's theorem,

No. of dimensionless groups = n−m = 6−4 = 2.

The dimensionless term can be expressed in following form,

$$[MRR]^a[T_{on}]^b[T_{off}]^c[IP]^d[W_{off}]^e\,[SV]^f = [\text{Dimensional less term}] \tag{24}$$

By putting dimensions from Table 10,

$$[M^0\,L^2T^{-1}]^a[M^0\,L^0T]^b[M^0\,L^0T]^c[M^0\,L^0T^0I]^d[M^0\,LT^0]^e\,[M^1L^2T^{-3}I^{-1}]^f = [M^0L^0T^0I^0] \tag{25}$$

The set of simultaneous equations is as follows:
(By equating powers of same dimensional unit)

$$f = 0 \tag{26.1}$$

$$2a + e + 2f = 0 \tag{26.2}$$

$$-a + b + c - 3f = 0 \tag{26.3}$$

$$d - f = 0 \tag{26.4}$$

The simultaneous equation can be converted in to matrix form of [A] × [X] = [B].where,

$$[A] = \begin{vmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -1 \end{vmatrix} \quad [X] = \begin{vmatrix} c \\ d \\ e \\ f \end{vmatrix}$$

By assuming $a = 1$ and $b = 0$ and subsequently putting these values in above simultaneous equations, we get

Table 10
Variables, units and dimensions used in model.

Variable	Units	Symbol	Dimensions
Material removal rate	mm^2/min	MRR	$M^0\,L^2T^{-1}$
Pulse on time	µs	T_{on}	$M^0\,L^0T$
Pulse off time	µs	T_{off}	$M^0\,L^0T$
Peak current	A	IP	$M^0\,L^0T^0I$
Wire offset	mm	W_{off}	$M^0\,LT^0$
Servo-voltage	V	SV	$M^1L^2T^{-3}I^{-1}$

$$B = [0 \quad -2 \quad 1 \quad 0]^T$$

But,

$$X = [A]^{-1} \times [B]$$

Subsequently, we get

$$X = [1 \quad 0 \quad -2 \quad 0]^T$$

Putting the values of X in Eq. (25),

$a = 1, b = 0, c = 1, d = 0, e = -2, f = 0$

Now, dimensionless term can be written as

$$\pi_1 = [MRR]^1\,[T_{off}]^1\,[W_{off}]^{-2} \tag{27}$$

Similarly, by assuming $b = 1$and $a = 0$; repeating the above procedure, we get

$c = -1, d = 0, e = 0, f = 0$

Dimensionless term can therefore be written as

$$\pi_2 = [T_{on}]\,[T_{off}]^{-1} \tag{28}$$

The above calculated values of powers of dimensional analysis are represented in tabular form (see Table 11).

The finalized mathematical model for MRR can be represented using Eqs. (27) and (28) as,

$$MRR = C \times \frac{T_{on} \times \left(W_{off}\right)^2}{T_{off}{}^2} \tag{29}$$

where, C is a constant of proportionality. The value of 'C' was determined by performing experiments by "changing one factor at a time approach" i.e. by varying pulse on time (T_{on}) and keeping remaining parameters at fixed level. MRR was computed experimentally for the various values of pulse on time. The experimental data is presented in Table 12.

It is indicated from the above Eq. (29) that MRR is highly dependent on T_{on}, T_{off} and W_{off}. As the square power is associated with two parameters (T_{off}, W_{off}), these are highly significant for MRR in trim cut operation. The developed mathematical model shows that if T_{on}, W_{off} are increased and T_{off} is decreased, MRR will get increased and vice versa. This conclusion can be confirmed from ANOVA analysis done in multi-response optimization section which indicated these parameters to be highly significant for MRR.

The experimentally obtained values of MRR were put in Eq. (29), along with the values of input parameters, to compute corresponding values of constant C. Afterwards, an equation for correlation between C and the factor being varied (T_{on}) was determined using regression analysis (Fig. 5). This equation could be used for prediction of 'C' for any given value of T_{on}.

$$C = \left(0.4932 - 2.000T_{on} + 3.864T_{on}{}^2 - 2.602T_{on}{}^3\right) \tag{30}$$

Subsequently, the value of C (Eq. (30)) was put in Eq. (29). Hence, the final equation for MRR is

Table 11
Dimensional coefficients.

	π_1	π_2
a	1	0
b	0	1
c	1	−1
d	0	0
e	−2	0
f	0	0

Table 12
Experimental values of MRR v/s T_{on}.

Parameter	Level value (μs)	MRR (mm^2/min)
Pulse on time (T_{on})	0.2	32.76
	0.3	36.26
	0.4	42.93
	0.5	47.24
	0.6	53.14

$$MRR = \left(0.4932 - 2.000T_{on} + 3.864T_{on}^{2} - 2.602T_{on}^{3}\right) \times \frac{T_{on} \times \left(W_{off}\right)^{2}}{T_{off}^{2}} \tag{31}$$

Further, to predict the MRR over a range of process parameters, the Eq. (31) was used. The different values of T_{on} (as mentioned in Table 12) were used along with the fixed values of other parameters (T_{off} = 26 μs, W_{off} = 0.09 mm, IP = 60 A, SV = 75 V, WF = 8 m/min). Comparison between predicted and experimental MRR is depicted in form of bar chart as shown in Fig. 6, where *R* (Pearson correlation coefficient) is 0.994 which indicates a very good agreement of the predictions of the model with the experimental results.

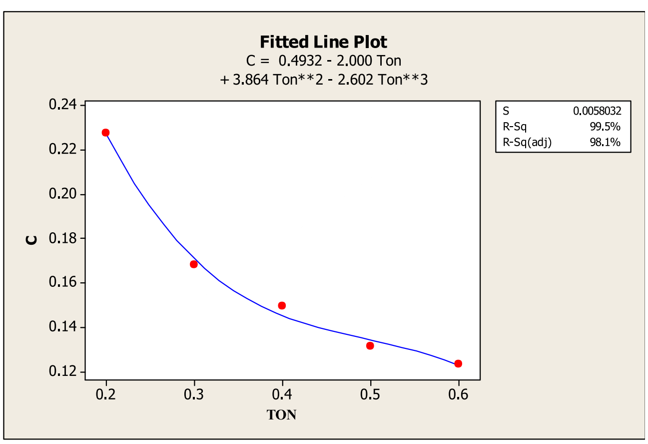

Fig. 5. Fitted line plot between T_{ON} and constant C.

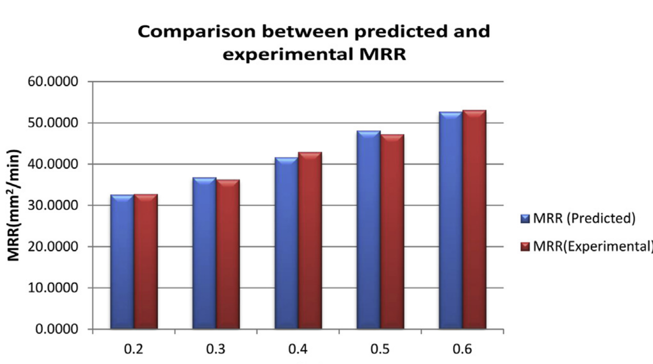

Fig. 6. Comparative bar chart between predicted and experimental values of MRR for T_{ON}.

Table 13
Experimental values of MRR v/s W_{off}.

Parameter	Level value (mm)	MRR (mm^2/min)
Wire offset (W_{off})	0.07	20.17
	0.08	28.83
	0.09	35.91
	0.1	41.15
	0.11	52.48

Similarly, the above procedure was repeated by varying wire offset (W_{off}) and keeping other parameters at certain fixed level (T_{on} = 0.3 μs, T_{off} = 26 μs, IP = 60 A, SV = 75 V, WF = 8 m/min). Table 13 depicts experimental values of MRR under the influence of wire offset parameter. The equation for MRR is as following.

$$MRR = \left(1.612 + 58.37W_{off} - 645.6W_{off}^{2} + 2348W_{off}^{3}\right) \times \frac{T_{on} \times \left(W_{off}\right)^{2}}{T_{off}^{2}} \tag{32}$$

Comparison between predicted and experimental MRR under the effect of wire offset parameter (W_{off}) is depicted in form of bar chart as shown in Fig. 7.

Further the analysis of mean error (ME), root mean square error (RMSE) and average error of prediction (AEP) was performed for predicting the accuracy of model from following equation.

$$\text{Mean error (ME)} = (1/n)\sum_{i=1}^{n}(Xi - \bar{X}) \tag{33}$$

$$\text{Root mean square error (RMSE)} = (1/n)^{0.5}\sum_{i=1}^{n}(Xi - \bar{X}) \tag{34}$$

$$\text{Avg. error of prediction (AEP)} = (1/n)\sum_{i=1}^{n}\frac{Xi}{Xi - \bar{X}} \tag{35}$$

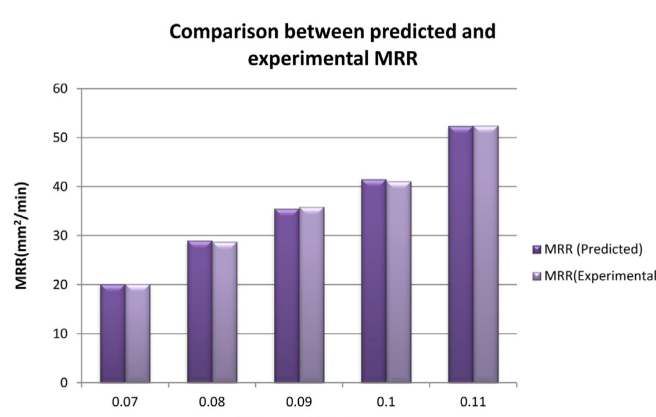

Fig. 7. Comparative bar chart between predicted and experimental values of MRR for W_{off}.

Table 14
Error analysis of model results.

MRR- Process parameter	ME	RMSE	AEP
MRR- T_{on}	0.068	0.583	0.595
MRR- W_{off}	−0.04	0.258	8.20

where Xi is the predicted value from model and $\overline{X}$ is experimental value.

The above values have been reported in Table 14. The magnitude of various error terms is found to be negligible which indicates the high level of accuracy of the model predicted values.

Thus, the developed model is validated and could be very useful on the shop floor for predicting the value of MRR over a range of input parameters.

6. Conclusions

The following conclusions might be drawn from this experimental research.

1. All the input parameters considered for the experimental investigation were found to be statistically significant for their effects on the overall utility index. However, three parameters namely Wire type, pulse on time and wire offset were established as the most significant from the analysis of their contributions in the variation of utility data.
2. The weights for MRR, SR and WWC were found to be 0.410, 0.410 and 0.178 respectively, while considering the preferences of multiple users.
3. The Multi-response optimization through the proposed method yielded a single optimal solution for all the responses considered as; wire type-zinc coated brass wire,T_{ON}-0.5 μm, IP-80 A, W_{off}-0.11 mm, WF- 8 m/min, T_{off}-26 μm, and SV- 75 V.
4. The confirmation experiments results validated the predicted optimal values of the responses considered in the study. Also, the optimized process setting could be used for realizing optimal solutions for satisfying the priorities of multiple users.
5. A mechanistic model was developed and validated for prediction of MRR over a wide range of input parameters such as T_{on} and W_{off}. The model could be highly useful for the machinists on the shop floor for trim cut WEDM operation of pure titanium.
6. The indexes of accuracy such as mean error (ME), root mean square error (RMSE) and average error of prediction (AEP) reflected the exceptional level of accuracy of the developed model.

References

[1] R. Chalisgaonkar, J. Kumar, Optimization of WEDM process of pure titanium with multiple performance characteristics using Taguchi's DOE approach and utility concept, Front. Mech. Eng. 8 (2) (2013) 201–214, http://dx.doi.org/10.1007/s11465-0.

[2] R. Ramakrishnan, L. Karunamoorthy, Modeling and multi-response optimization of Inconel 718 on machining of CNC WEDM process, J. Mater. Process. Technol. 207 (2008) 343–349.

[3] R. Ramakrishnan, L. Karunamoorthy, Multi response optimization of wire EDM operations using robust design of experiments, Int. J. Adv. Manuf. Technol. 29 (2006) 105–112.

[4] S. Sarkar, S. Mitra, B. Bhattacharyya, Parametric analysis and optimization of wire electrical discharge machining of γ-titanium aluminide alloy, J. Mater. Process. Technol. 159 (2005) 286–294.

[5] S. Sarkar, M. Sekh, S. Mitra, B. Bhattacharyya, Modeling and optimization of wire electrical discharge machining of gamma-TiAl in trim cutting operation, J. Mater. Process. Technol. 205 (2008) 376–387.

[6] C.J. Tzeng, Y.K. Yang, M.H. Hseigh, M.C. Jeng, Optimization of wire electrical discharge machining of pure Tungsten using neural network and response surface methodology, Proc. Inst. Mech. Eng. Part B 225 (2010) 841–852.

[7] J. Sahu, C. Mohanty, S. Mahapatra, A dea approach for optimization of multiple responses in electrical discharge machining of AISI D2 steel, Procedia Eng. 51 (2013) 585–591.

[8] R. Chakraborty, S. Gauri, S. Chakraborty, A modified principal component analysis-based utility theory approach for optimization of correlated responses of EDM process, Int. J. Eng. Sci. Technol. 4 (2) (2012) 34–45.

[9] R. Chakraborty, S. Gauri, S. Chakraborty, Optimization of multiple responses of ultrasonic machining (USM) process: a comparative study, Int. J. Ind. Eng. Comput. 4 (2013) 285–296.

[10] B. Nayak, S. Mahapatra, Multi-response optimization of WEDM process parameters using the AHP and TOPSIS method, Int. J. Theor. Appl. Res. Mech. Eng. (IJTARME) 2 (3) (2013) 2319–3182. ISSN.

[11] S. Gopalakannan, T. Senthivelan, S. Ranganathan, Statistical optimization of EDM parameters on machining of aluminium hybrid metal matrix composite by applying taguchi based grey analysis, J. Sci. Ind. Res. 72 (2013) 358–365.

[12] T. Rao, A. Gopala Krishna, Simultaneous optimization of multiple performance characteristics in WEDM for machining ZC63/SiCp MMC, Adv. Manuf. (1) (2013) 265–275.

[13] P. Sengottuvel, S. Satishkumar, D. Dinakaran, Optimization of multiple characteristics of EDM parameters based on desirability approach and fuzzy modeling, Procedia Eng. 64 (2013) 1069–1078.

[14] J.L. Lin, C.L. Lin, The use of grey-fuzzy logic for the optimization of the manufacturing process, J. Mater. Process. Technol. 160 (2005) 9–14.

[15] S. Assarzadeh, M. Ghoreishi, Neural network based modeling and optimization of the electro-discharge machining process, Int. J Adv. Manuf. Technol. 39 (2008) 488–500, http://dx.doi.org/10.1007/s00170-007-1235-1.

[16] K. Jangra, Study of un-machined area in intricate machining after rough cut in WEDM, Int. J. Industrial Eng. Comput. 3 (2012) 887–892.

[17] E. Nahm Yooni, H. Ishikawa, I. Masato, New rating methods to prioritize customer requirements in QFD with incomplete customer preferences, Int. J. Adv. Manuf. Technol. 65 (2013) 1587–1604, http://dx.doi.org/10.1007/s00170-012-4282-1.

[18] V. Gupta, P.N. Murthy, An Introduction to Engineering Design Method, Tata McGraw-Hill, New Delhi, 1980.