$B \to \rho l \nu$ decay and $|V_{ub}|$

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Abstract

$B \to \rho l \nu$ decay is analyzed in the effective theory of heavy quark with infinite mass limit. The matrix element relevant to the heavy to light vector meson semileptonic decays is parametrized by a set of four heavy flavor-spin independent universal wave functions at the leading order of effective theory. The form factors are calculated at the leading $1/m_Q$ order using the light cone sum rule method in the framework of effective theory. $|V_{ub}|$ is then extracted via $B \to \rho l \nu$ decay mode.

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1. Introduction

$B \to \pi(\rho)l\nu$ decays have been studied in the full QCD via sum rule approach [1–6], or using appropriate models [7–9]. In Refs. [10,11], the heavy to light pseudoscalar and vector meson decay matrix elements in the heavy-quark effective theory have been formulated, respectively, up to the order of $1/m_Q$. In Ref. [12], the leading order wave functions of $B \to \pi l \nu$ decay have been calculated in the effective theory of heavy quark by using the light cone sum rule method, and the important CKM matrix element $|V_{ub}|$ has been extracted.

This short Letter is parallel to Ref. [12] and focuses on the calculation of the leading order wave functions of $B \to \rho l \nu$ decay and the extraction of $|V_{ub}|$ by using the light cone sum rules in the effective theory of heavy quark. It is organized as follows: in Section 2, we present the analytic formulae for $B \to \rho l \nu$ decay in the effective theory of heavy quark. In Section 3, the relevant light cone sum rules are derived within the framework of effective theory. In Section 4 we present the numerical results including the extracted value of $|V_{ub}|$. The value of $|V_{ub}|$ is compared with that extracted via $B \to \pi l \nu$ decay. A short summary is drawn in the last section.

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2. Matrix element

The $B \to \rho l \nu$ decay matrix element is generally written in terms of four form factors as

$$\langle \rho(p, \epsilon^*) | \bar{u} \gamma^\mu (1 - \gamma^5) b | B(p + q) \rangle$$

$$= -i(m_B + m_\rho) A_1(q^2) \epsilon^{*\mu} + i \frac{A_2(q^2)}{m_B + m_\rho} (\epsilon^* \cdot (p + q))(2p + q)^\mu$$

$$+ i \frac{A_3(q^2)}{m_B + m_\rho} (\epsilon^* \cdot (p + q)) q^\mu - \frac{2V(q^2)}{m_B + m_\rho} \epsilon^{\mu\alpha\beta\gamma} \epsilon_\alpha^* (p + q)_\beta p_\gamma, \tag{2.1}$$

where p and ϵ^* are the momentum and polarization vectors of the ρ meson, and q is the momentum carried by the lepton pair.

In the effective theory of heavy quark, heavy-quark expansion (HQE) can be applied to the matrix element [10–13,16,17]. The normalization of the matrix elements in full QCD and in the effective theory is [12,13,16,17]

$$\frac{1}{\sqrt{m_B}} \langle \rho(p, \epsilon^*) | \bar{u} \Gamma b | B \rangle = \frac{1}{\sqrt{\bar{\Lambda}_B}} \{ \langle \rho(p, \epsilon^*) | \bar{u} \Gamma Q_v^+ | B_v \rangle + \mathrm{O}(1/m_b) \}, \tag{2.2}$$

where $\bar{\Lambda}_B = m_B - m_b$, and

$$\bar{\Lambda} = \lim_{m_Q \to \infty} \bar{\Lambda}_B \tag{2.3}$$

is the heavy flavor independent binding energy reflecting the effects of the light degrees of freedom in the heavy hadron. Q_v^+ is the effective heavy-quark field.

Based on the heavy-quark symmetry (HQS), one can parametrize the leading order matrix element in the effective theory of heavy quark on the r.h.s. of Eq. (2.2) as [11]

$$\langle \rho(p, \epsilon^*) | \bar{u} \Gamma Q_v^+ | B_v \rangle = -i \, \mathrm{Tr} [\Omega(v, p) \Gamma \mathcal{M}_v] \tag{2.4}$$

with the decomposition of $\Omega(v, p)$:

$$\Omega(v, p) = L_1 (v \cdot p) \slashed{\epsilon}^* + L_2 (v \cdot p)(v \cdot \epsilon^*) + [L_3(v \cdot p)\slashed{\epsilon}^* + L_4(v \cdot p)(v \cdot \epsilon^*)]\hat{p},$$

$$\hat{p}^\mu = \frac{p^\mu}{v \cdot p}. \tag{2.5}$$

$\mathcal{M}_v$ is the well known spin wave function associated with the heavy meson states,

$$\mathcal{M}_v = \sqrt{\bar{\Lambda}} \frac{1 + \slashed{v}}{2} \begin{cases} -\gamma_5, & \text{for pseudoscalar meson,} \\ \slashed{\epsilon}, & \text{for vector meson with polarization vector } \epsilon^\mu. \end{cases} \tag{2.6}$$

L_i ($i = 1, 2, 3, 4$) are the leading order wave functions in the effective theory. Generally, they do not depend on the heavy-quark mass but are functions of the variable $v \cdot p$ and the energy scale μ as well. For the sake of simplicity we do not write down their μ dependence explicitly until the numerical analysis in Section 4.

Eqs. (2.1)–(2.6) lead to

$$A_1(q^2) = \frac{2}{m_B + m_\rho} \sqrt{\frac{m_B \bar{\Lambda}}{\bar{\Lambda}_B}} \{ L_1(v \cdot p) + L_3(v \cdot p) \} + \cdots,$$

$$A_2(q^2) = 2(m_B + m_\rho) \sqrt{\frac{m_B \bar{\Lambda}}{\bar{\Lambda}_B}} \left\{ \frac{L_2(v \cdot p)}{2m_B^2} + \frac{L_3(v \cdot p) - L_4(v \cdot p)}{2m_B(v \cdot p)} \right\} + \cdots,$$

$$A_3(q^2) = 2(m_B + m_\rho)\sqrt{\frac{m_B \bar{\Lambda}}{\bar{\Lambda}_B}} \left\{ \frac{L_2(v \cdot p)}{2m_B^2} - \frac{L_3(v \cdot p) - L_4(v \cdot p)}{2m_B(v \cdot p)} \right\} + \cdots,$$

$$V(q^2) = \sqrt{\frac{m_B \bar{\Lambda}}{\bar{\Lambda}_B}} \frac{m_B + m_\rho}{m_B(v \cdot p)} L_3(v \cdot p) + \cdots. \tag{2.7}$$

The dots denote higher order $1/m_Q$ contributions which will not be taken into account in the following calculations.

3. Light cone sum rule

The form factors A_1, A_2, A_3 and V (and/or L_i ($i = 1, 2, 3, 4$)) are difficult to be predicted straightforwardly from QCD calculation due to their nonperturbative nature. QCD sum rules, quark models and lattice simulations are the main approaches to evaluate the nonperturbative contributions. For heavy to light decays, the light cone sum rule approach is found to be more reliable than the sum rules based on short distance operator product expansion (OPE) and vacuum condensate input. In the light cone sum rule calculation, the relevant correlation functions are expanded near the light cone, and the nonperturbative contributions are introduced into the treatment through the light cone distribution functions of the mesons.

We now consider the vacuum-ρ correlation function

$$F^\mu(p, q) = i \int d^4 x \, e^{-ip_B \cdot x} \langle \rho(p, \epsilon^*) | T \{ \bar{u}(0)\gamma^\mu (1 - \gamma^5) b(0), \bar{b}(x)i\gamma^5 d(x) \} | 0 \rangle. \tag{3.1}$$

Here the B meson has momentum $P_B = p + q$, whereas p and q are momenta carried by the rho-meson and leptons. Phenomenologically, we insert a complete set of states with B-meson quantum numbers in (3.1),

$$F^\mu(p, q)_{\text{phen}} = \frac{\langle \rho(p, \epsilon^*) | \bar{u}\gamma^\mu (1 - \gamma^5) b | B \rangle \langle B | \bar{b}i\gamma^5 d | 0 \rangle}{m_B^2 - (p + q)^2} + \sum_H \frac{\langle \rho(p, \epsilon^*) | \bar{u}\gamma^\mu (1 - \gamma^5) b | H \rangle \langle H | \bar{b}i\gamma^5 d | 0 \rangle}{m_H^2 - (p + q)^2}. \tag{3.2}$$

Due to Eqs. (2.2), (2.4) and the B decay constant definition [16]

$$\langle 0 | \bar{q} \Gamma Q_v^+ | B_v \rangle = \frac{F}{2} \text{Tr}[\Gamma \mathcal{M}_v], \tag{3.3}$$

at the leading order of HQE, $F^\mu(p, q)_{\text{phen}}$ contributes

$$\frac{m_B \bar{\Lambda}}{m_b \bar{\Lambda}_B} \frac{2F}{2\bar{\Lambda}_B - 2v \cdot k} \left\{ (L_1 + L_3)\epsilon^{*\mu} - L_2 v^\mu (\epsilon^* \cdot v) - (L_3 - L_4)p^\mu \frac{\epsilon^* \cdot v}{v \cdot p} + i\frac{L_3}{v \cdot p}\epsilon^{\mu\nu\alpha\beta}\epsilon_\nu^* p_\alpha v_\beta \right\}$$

$$+ \int_{s_0}^\infty ds \, \frac{\rho(v \cdot p, s)}{s - 2v \cdot k} + \text{subtractions}, \tag{3.4}$$

with k^μ being the heavy hadron's residual momentum, $k^\mu = P_B^\mu - m_b v^\mu$. The integral represents the higher resonance contributions.

The correlator can also be calculated in theory and written as

$$\int_0^\infty ds \, \frac{\rho(v \cdot p, s)_{\text{theory}}}{s - 2v \cdot k} + \text{subtractions}. \tag{3.5}$$

Furthermore the quark–hadron duality assumes the equality of $\rho(v \cdot p, s)_{\text{theory}}$ and the physical spectral density $\rho(v \cdot p, s)$. As a result, Eqs. (3.4) and (3.5) yields

$$\frac{m_B \bar{\Lambda}}{m_b \bar{\Lambda}_B} \frac{2F}{2\bar{\Lambda}_B - 2v \cdot k} \left\{ (L_1 + L_3)\epsilon^{*\mu} - L_2 v^\mu (\epsilon^* \cdot v) - (L_3 - L_4)p^\mu \frac{\epsilon^* \cdot v}{v \cdot p} + i\frac{L_3}{v \cdot p} \epsilon^{\mu\nu\alpha\beta} \epsilon_\nu^* p_\alpha v_\beta \right\}$$

$$= \int_0^{s_0} ds \frac{\rho(v \cdot p, s)}{s - 2v \cdot k} + \text{subtractions.} \tag{3.6}$$

We now calculate the correlator (3.1) in the effective theory of heavy quark. When neglecting higher order $1/m_Q$ corrections Eq. (3.1) can be written as

$$F^\mu(p, q) = i \int d^4x \, e^{-ip_B \cdot x + im_b v \cdot x} \langle \rho(p, \epsilon^*) | T\{\bar{u}(0)\gamma^\mu(1 - \gamma^5)Q_v^+(0), \overline{Q}_v^+(x)i\gamma^5 d(x)\} | 0 \rangle. \tag{3.7}$$

This could then be expanded into a series in powers of the twist of light cone distribution functions. The ρ distribution functions are defined by the following matrix elements [2,4–6]:

$$\langle \rho(p, \epsilon^*) | \bar{u}(0)\sigma_{\mu\nu}d(x) | 0 \rangle = -if_\rho^\perp (\epsilon_\mu^* p_\nu - \epsilon_\nu^* p_\mu) \int_0^1 du \, e^{iup \cdot x} \phi_\perp(u),$$

$$\langle \rho(p, \epsilon^*) | \bar{u}(0)\gamma_\mu d(x) | 0 \rangle = f_\rho m_\rho p_\mu \frac{\epsilon^* \cdot x}{p \cdot x} \int_0^1 du \, e^{iup \cdot x} \phi_\|(u) + f_\rho m_\rho \left(\epsilon_\mu^* - p_\mu \frac{\epsilon^* \cdot x}{p \cdot x} \right) \int_0^1 du \, e^{iup \cdot x} g_\perp^{(v)}(u),$$

$$\langle \rho(p, \epsilon^*) | \bar{u}(0)\gamma_\mu \gamma_5 d(x) | 0 \rangle = \frac{1}{4} f_\rho m_\rho \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p^\alpha x^\beta \int_0^1 du \, e^{iup \cdot x} g_\perp^{(a)}(u). \tag{3.8}$$

$\phi_\perp$ and $\phi_\|$ are twist-2 distribution functions of transversely and longitudinally polarized ρ mesons, respectively, while $g_\perp^{(v)}$ and $g_\perp^{(a)}$ are associated with both twist-2 and twist-3 operators. Higher twist components still lack for more precise analysis, and they are beyond our consideration in this Letter.

Contracting the effective heavy-quark fields into the propagator $\frac{1+\slashed{v}}{2} \int_0^\infty dt \, \delta(-x - vt)$, Eq. (3.7) turns into

$$F^\mu(y, \omega) = \frac{i}{2} \int_0^\infty dt \int_0^1 du \, e^{\frac{it\omega}{2}} e^{-iuyt} \left\{ f_\rho m_\rho p^\mu \frac{\epsilon^* \cdot v}{p \cdot v} \phi_\|(u) + f_\rho m_\rho \left(\epsilon^{*\mu} - p^\mu \frac{\epsilon \cdot v}{p \cdot v} \right) g_\perp^{(v)}(u) \right.$$

$$\left. + \frac{t}{4} f_\rho m_\rho \epsilon^{\mu\nu\alpha\beta} \epsilon_\nu^* p_\alpha v_\beta g_\perp^{(a)}(u) + f_\rho^\perp [(v \cdot p)\epsilon^{*\mu} - (v \cdot \epsilon^*)p^\mu + i\epsilon^{\mu\nu\alpha\beta} \epsilon_\nu^* p_\alpha v_\beta] \phi_\perp(u) \right\} \tag{3.9}$$

with $y \equiv v \cdot p$ and $\omega \equiv 2v \cdot k$.

Performing a Wick rotation of the t axis and using the feature of Borel transformation:

$$\widehat{B}_T^{(\omega)} e^{\lambda\omega} = \delta\left(\lambda - \frac{1}{T}\right), \tag{3.10}$$

we get from (3.9)

$$\widehat{B}_T^{(\omega)} F^\mu(y, \omega)$$

$$= \int_0^1 du \, e^{-\frac{2}{T}uy} \left\{ \frac{\epsilon^* \cdot v}{v \cdot p} p^\mu \left[f_\rho m_\rho (\phi_\|(u) - g_\perp^{(v)}(u)) - f_\rho^\perp (v \cdot p)\phi_\perp(u) \right] \right.$$

$$+ \epsilon^{*\mu}\big[f_\rho m_\rho g_\perp^{(v)}(u) + f_\rho^\perp (v \cdot p)\phi_\perp(u)\big] + \epsilon^{\mu\nu\alpha\beta}\epsilon_\nu^* p_\alpha v_\beta \Big[\frac{-i}{2T} f_\rho m_\rho g_\perp^{(a)}(u) + i f_\rho^\perp \phi_\perp(u)\Big]\bigg\}. \qquad (3.11)$$

Following the approach in [18,19], we can now carry out continuous double Borel transformations on the correlator itself to produce the spectral function $\rho(y, s)$.

$$\rho(y, s) = \widehat{B}_{1/s}^{(-1/T)} \widehat{B}_T^{(\omega)} F^\mu(y, \omega)$$

$$= \frac{1}{2y}\bigg\{ p^\mu \frac{\epsilon^* \cdot v}{v \cdot p}\big[f_\rho m_\rho\big(\phi_\parallel(u) - g_\perp^{(v)}(u)\big) - f_\rho^\perp (v \cdot p)\phi_\perp(u)\big] + \epsilon^{*\mu}\big[f_\rho m_\rho g_\perp^{(v)}(u)$$

$$+ f_\rho^\perp (v \cdot p)\phi_\perp(u)\big] + \epsilon^{\mu\nu\alpha\beta}\epsilon_\nu^* p_\alpha v_\beta \Big[\frac{-i}{4y} f_\rho m_\rho\Big(\frac{\partial}{\partial u} g_\perp^{(a)}(u)\Big) + i f_\rho^\perp \phi_\perp(u)\Big]\bigg\}\bigg|_{u=\frac{s}{2y}}. \qquad (3.12)$$

Eq. (3.12) can be easily derived from (3.11) by first writing $1/T$ as a derivative of the exponent in (3.11) over u, and then using the method of integration by parts over u.

We then get from Eqs. (3.6) and (3.12)

$$L_1(y) = \frac{m_b \bar{\Lambda}_B}{m_B \bar{\Lambda}} \frac{1}{4F} e^{2\bar{\Lambda}_B/T} \int\limits_0^{s_0} ds\, e^{-s/T} \frac{1}{y} f_\rho m_\rho \Big[g_\perp^{(v)}(u) + \frac{1}{4}\Big(\frac{\partial}{\partial u} g_\perp^{(a)}(u)\Big)\Big]\bigg|_{u=\frac{s}{2y}},$$

$$L_3(y) = \frac{m_b \bar{\Lambda}_B}{m_B \bar{\Lambda}} \frac{1}{4F} e^{2\bar{\Lambda}_B/T} \int\limits_0^{s_0} ds\, e^{-s/T} \Big[\frac{-1}{4y} f_\rho m_\rho \Big(\frac{\partial}{\partial u} g_\perp^{(a)}(u)\Big) + f_\rho^\perp \phi_\perp(u)\Big]\bigg|_{u=\frac{s}{2y}},$$

$$L_4(y) = \frac{m_b \bar{\Lambda}_B}{m_B \bar{\Lambda}} \frac{1}{4F} e^{2\bar{\Lambda}_B/T} \int\limits_0^{s_0} ds\, e^{-s/T} \frac{1}{y} f_\rho m_\rho \Big[\phi_\parallel(u) - g_\perp^{(v)}(u) - \frac{1}{4}\Big(\frac{\partial}{\partial u} g_\perp^{(a)}(u)\Big)\Big]\bigg|_{u=\frac{s}{2y}}, \qquad (3.13)$$

and $L_2(y)$ equals zero in the present approximation since no twist-2 distribution functions contribute to it. For this reason, the form factors A_2 and A_3 have the same absolute value but opposite signs at the order considered in this Letter.

4. Numerical results

The functions $\phi_\perp$ and $\phi_\parallel$ give the leading twist distributions in the fraction of total momentum carried by the quark in transversely and longitudinally polarized mesons, respectively. They have a nontrivial scale dependence which can be described by the renormalization group method [4]. These distribution functions can be expanded in Gegenbauer polynomials $C_n^{3/2}(x)$ whose coefficients are renormalized multiplicatively. Namely, writting their scale dependence explicitly, we have

$$\phi_{\perp(\parallel)}(u, \mu) = 6u(1 - u)\Big[1 + \sum_{n=2,4,\dots} a_n^{\perp(\parallel)}(\mu) C_n^{3/2}(2u - 1)\Big],$$

$$a_n^{\perp(\parallel)}(\mu) = a_n^{\perp(\parallel)}(\mu_0)\Big(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\Big)^{(\gamma_n^{\perp(\parallel)} - \gamma_0^{\perp(\parallel)})/(2\beta_0)}, \qquad (4.1)$$

where $\beta_0 = 11 - (2/3)n_f$, and the one loop anomalous dimensions are [20,21]

$$\gamma_n^\parallel = \frac{8}{3}\Big(1 - \frac{2}{(n+1)(n+2)} + 4\sum_{j=2}^{n+1} \frac{1}{j}\Big), \qquad \gamma_n^\perp = \frac{8}{3}\Big(1 + 4\sum_{j=2}^{n+1} \frac{1}{j}\Big). \qquad (4.2)$$

The coefficients $a_n^{\perp}$ and $a_n^{\parallel}$ themselves are nonperturbative parameters, and have been calculated using sum rule methods in Ref. [4]. In the following discussions we will use the values [4]

$$a_2^{\perp}(1\text{ GeV}) = 0.2 \pm 0.1, \qquad a_2^{\parallel}(1\text{ GeV}) = 0.18 \pm 0.10 \tag{4.3}$$

and $a_n^{\perp(\parallel)} = 0$ for $n \neq 2$.

The functions $g_{\perp}^{(v)}$ and $g_{\perp}^{(a)}$ describe transverse polarizations of quarks in the longitudinally polarized mesons. They receive contributions of both twist-2 and twist-3. And the twist-2 contributions are related to the longitudinal distribution $\phi_{\parallel}(u, \mu)$ by Wandzura–Wilczek type relations [4,5]:

$$g_{\perp}^{(v),\text{twist-2}}(u, \mu) = \frac{1}{2}\left[\int_0^u dv\, \frac{\phi_{\parallel}(v, \mu)}{1 - v} + \int_u^1 dv\, \frac{\phi_{\parallel}(v, \mu)}{v}\right],$$

$$g_{\perp}^{(a),\text{twist-2}}(u, \mu) = 2\left[(1 - u)\int_0^u dv\, \frac{\phi_{\parallel}(v, \mu)}{1 - v} + u\int_u^1 dv\, \frac{\phi_{\parallel}(v, \mu)}{v}\right]. \tag{4.4}$$

For the energy scale μ to be used in the sum rules (3.13), we use

$$\mu_b \sim \sqrt{m_B^2 - m_b^2} \approx 2.4\text{ GeV}, \tag{4.5}$$

which is an appropriate choice of scale set by the typical virtuality of the beautiful quark [2].

The values of the hadron quantities f_{ρ}, $f_{\rho}^{\perp}$, $\bar{\Lambda}_B$, $\bar{\Lambda}$ and F are needed in order to perform the sum rule numerical analysis. The decay constant f_{ρ} has been measured in experiments [22,23]. $f_{\rho}^{\perp}$ is the tensor coupling defined by

$$\langle 0|\bar{u}\sigma_{\mu\nu}d|\rho^+(p, \epsilon)\rangle = i(\epsilon_{\mu}p_{\nu} - \epsilon_{\nu}p_{\mu})f_{\rho}^{\perp} \tag{4.6}$$

and its value has been calculated in Ref. [4]. $\bar{\Lambda}_B$, $\bar{\Lambda}$ and F are associated with heavy mesons and are parameters in the effective theory of heavy quark. Their values have been estimated consistently in Ref. [16] by sum rules in the framework of effective theory. We use for these parameters the following values,

$$f_{\rho^{\pm}} = (195 \pm 7)\text{ MeV}, \qquad f_{\rho^0} = (216 \pm 5)\text{ MeV}, \qquad f_{\rho}^{\perp} = (160 \pm 10)\text{ MeV},$$
$$\bar{\Lambda}_B \approx \bar{\Lambda} = 0.53\text{ GeV}, \qquad F = (0.30 \pm 0.06)\text{ GeV}^{3/2}. \tag{4.7}$$

Combining (2.7) and (3.13), the form factors A_1, A_2, A_3 and V can be calculated as functions of T, q^2 and s_0. In Fig. 1, we present our results for the form factors at the zero momentum transfer point $q^2 = 0$, which shows the variations of these form factors with respect to the Borel parameter T at different threshold energy s_0. The T range of interest should be similar to that in the light cone sum rule analysis for the $B \to \pi l \nu$ decay [12], i.e., $T \approx 2.0$ GeV. It can be seen from Fig. 1 that, in general consideration, the good stability of the form factors exists at the threshold $s_0 = 2.1 \pm 0.6$ GeV. With such a threshold energy, we are now in the stage to evaluate the q^2 dependence of the form factors. It should be intuitive and convenient to represent these form factors in an algebraic representation. We parametrize each form factor in terms of a set of three parameters as follows,

$$F(q^2) = \frac{F(0)}{1 - a_F q^2/m_B^2 + b_F(q^2/m_B^2)^2}, \tag{4.8}$$

where $F(q^2)$ can be any one of $A_1(q^2)$, $A_2(q^2)$, $A_3(q^2)$ and $V(q^2)$. The parameters $F(0)$, a_F and b_F can be fitted from the sum rule results (3.13). The results at $s_0 = 2.1$ GeV are presented in Table 1. The form factors as functions of the momentum transfer q^2 are also shown in Fig. 2.
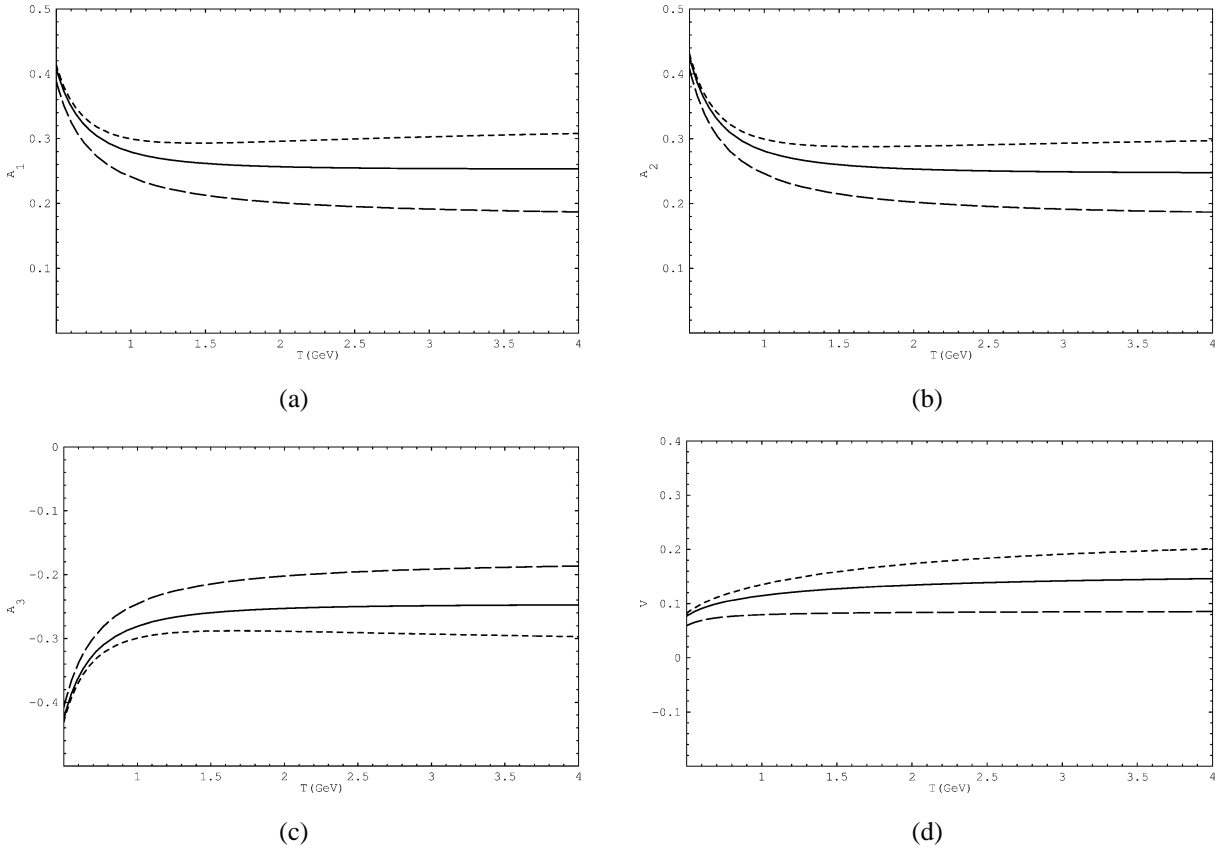
Fig. 1. Variation of form factors with the Borel parameter T for different values of the continuum threshold s_0. The dashed, solid and dotted curves correspond to $s_0 =$ 1.5, 2.1 and 2.7 GeV, respectively. Considered here is at the momentum transfer $q^2 = 0$ GeV^2.

Table 1
Results of the three parameter fit (4.8) for the $B \rightarrow \rho l \nu$ decay form factors. These data are fitted from the sum rules (3.13) at the Borel parameter $T = 2.0$ GeV and the threshold energy $s_0 = 2.1$ GeV

	$F(0)$	a_F	b_F
A_1	0.257	0.352	−0.239
A_2	0.253	1.090	0.202
A_3	−0.253	1.090	0.202
V	0.134	1.027	−0.223

When the lepton masses are neglected, the differential decay width of $B \rightarrow \rho l \nu$ with respect to the momentum transfer q^2 is [5]

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \lambda^{1/2} q^2 \left(H_0^2 + H_+^2 + H_-^2 \right) \tag{4.9}$$
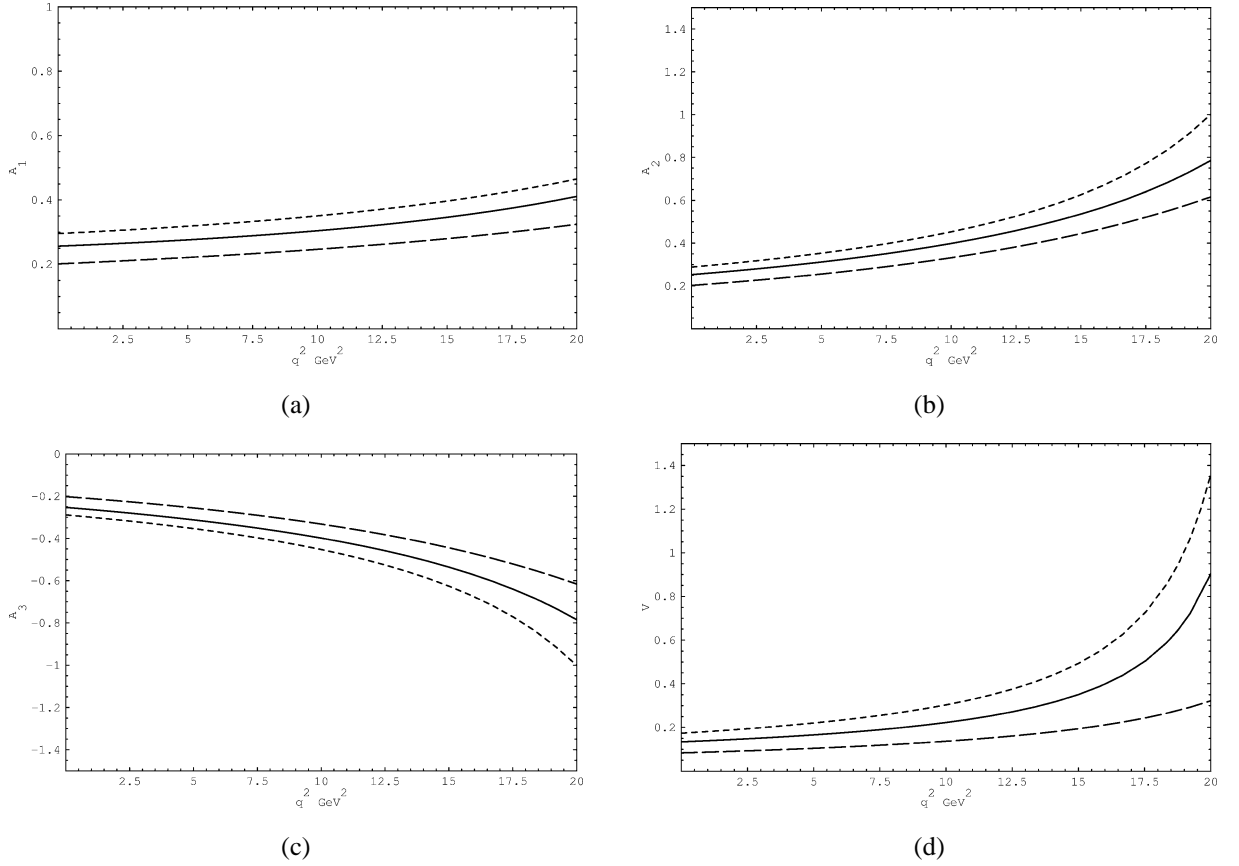
Fig. 2. Form factors obtained at the fixed Borel parameter $T = 2.0$ GeV. The dashed, solid and dotted curves correspond to $s_0 = 1.5$, 2.1 and 2.7 GeV, respectively.

with the helicity amplitudes

$$H_\pm = (m_B + m_\rho)A_1(q^2) \mp \frac{\lambda^{1/2}}{m_B + m_\rho} V(q^2),$$

$$H_0 = \frac{1}{2m_\rho\sqrt{q^2}} \left\{ (m_B^2 - m_\rho^2 - q^2)(m_B + m_\rho)A_1(q^2) - \frac{\lambda}{m_B + m_\rho} A_2(q^2) \right\} \tag{4.10}$$

and

$$\lambda \equiv \left(m_B^2 + m_\rho^2 - q^2\right)^2 - 4m_B^2 m_\rho^2. \tag{4.11}$$

So, with the meson masses $m_B = 5.28$ GeV, $m_\rho = 0.77$ GeV and the maximum momentum transfer $q_{\max}^2 = m_B^2 + m_\rho^2 - 2m_B m_\rho$, the integrated width of $B \to \rho l \nu$ turns out to be

$$\Gamma(B \to \rho l \nu) = (10.6 \pm 4.0)|V_{ub}|^2 \text{ ps}^{-1}. \tag{4.12}$$

The error in Eq. (4.12) results from the variation of the threshold energy $s_0 = 1.5$–2.7 GeV.

On the other hand, the branching fraction of $B^0 \to \rho^- l^+ \nu$ is measured by CLEO Collaboration [24], $\text{Br}(B^0 \to \rho^- l^+ \nu) = \left(2.57 \pm 0.29^{+0.33}_{-0.46} \pm 0.41\right) \times 10^{-4}$. With the world average of the B^0 lifetime [25], $\tau_{B^0} = 1.56 \pm 0.06$ ps,

one has

$$\Gamma\left(B^0 \rightarrow \rho^- l^+ \nu\right) = (1.65 \pm 0.80) \times 10^{-4} \text{ ps}^{-1}. \tag{4.13}$$

From Eqs. (4.12) and (4.13) we get

$$|V_{ub}| = (3.9 \pm 0.6 \pm 0.5) \times 10^{-3}, \tag{4.14}$$

where the first (second) error corresponds to the experimental (theoretical) uncertainty. Here the theoretical uncertainty is mainly considered from the threshold effects. In general, both the higher twist distribution functions and the QCD radiative corrections may modify the parameters and the sum rule results in Eq. (3.13). The modification from higher distribution functions is hard to calculate at present because little is known about the high distribution functions themselves. It is found from the sum rule analysis that the two-loop QCD perturbative corrections may enlarge the constant F by about 25%, and increase $\bar{\Lambda}$ at the same time [16]. To be consistent, one should also include the QCD corrections for the correlation function at the same order. As the higher order QCD corrections to the correlator have not been considered here, we should take the two-loop QCD corrections of the decay constant F as the possible theoretical uncertainty from the high order QCD corrections. Including such an uncertainty, our final result for $|V_{ub}|$ is

$$|V_{ub}| = (3.9 \pm 0.6 \pm 0.7) \times 10^{-3}. \tag{4.15}$$

This value may be compared with the one obtained in Ref. [12] from $B \rightarrow \pi l \nu$ decay by using the same approach,

$$|V_{ub}| = (3.4 \pm 0.5 \pm 0.5) \times 10^{-3}. \tag{4.16}$$

The coincidence of (4.15) and (4.16) within their errors proves the consistency of our light cone sum rule calculations of heavy to light semileptonic B decays in the framework of the effective theory of heavy quark. We also notice that the theoretical error in (4.15) is larger than that in (4.16), which is not out of expectation since we only include the leading twist distribution functions in the $B \rightarrow \rho l \nu$ calculation. This reflects the importance of a more complete study on the ρ distribution functions.

The estimate in Eq. (4.15) is in agreement with that derived from full QCD calculation [2], and it is also close to the combined result from the analyses based on different models and treatments on $B \rightarrow \pi(\rho)l\nu$ transitions,

$$|V_{ub}| = \left(3.25 \pm 0.14^{+0.21}_{-0.29} \pm 0.55\right) \times 10^{-3}, \tag{4.17}$$

which is given by CLEO [24].

5. Summary

We have studied $B \rightarrow \rho l \nu$ decay within the framework of effective theory of heavy quark. In the effective theory, the relevant matrix elements can be expanded in powers of the inverse of the heavy-quark mass. At the leading order approximation, the form factors concerned in this decay are related to four universal wave functions, which are independent of the heavy-quark mass m_Q. Though the HQS loses some predictive power in the heavy to light decays, it would be helpful for relating different heavy to light decay channels. For example, $B \rightarrow \rho l \nu$ and $D \rightarrow \rho l \nu$ decays are characterized at the leading order of $1/m_Q$ by the same set of wave functions, L_i ($i = 1, 2, 3, 4$).

The form factors for $B \rightarrow \rho l \nu$ have been calculated in the effective theory using the light cone sum rule approach. The important CKM matrix element $|V_{ub}|$ has been extracted by comparing the values of integrated width obtained from sum rule calculations and from the experimental measurements. The result is

$$|V_{ub}| = (3.9 \pm 0.6 \pm 0.7) \times 10^{-3}. \tag{5.1}$$

This result agrees with both the values extracted from the full QCD calculations and that from the $B \to \pi l \nu$ decay by using the same approach within the framework of the effective theory of heavy-quark. This calculation has further shown the reliability of the heavy-quark expansion and the predictive power of light cone sum rule approach in studying heavy to light exclusive decays. We have used $B \to \rho l \nu$ decay, as an example, for concret discussion. However, the method is general and the main formulae in this Letter can be used to other heavy to light vector semileptonic decays after trival modifications such as simple replacement of some parameters.

In this Letter, we have considered only the leading twist-2 distribution functions of ρ meson. Both the higher twist and loop corrections and higher order $1/m_Q$ contributions should be included for a more accurate estimation of $|V_{ub}|$. It is noted that higher order $1/m_Q$ corrections may have different forms in the usual heavy-quark effective theory and the new framework of heavy quark effective field theory [13–17] due to the antiquark contributions.

Acknowledgements

References

[1] V.M. Belyaev, A. Khodjamirian, R. Rückl, Z. Phys. C 60 (1993) 349.
[2] A. Khodjamirian, R. Rückl, WUE-ITP-97-049, MPI-PhT/97-85, hep-ph/9801443.
[3] A. Khodjamirian, R. Rückl, S. Weinzierl, C.W. Winhart, O. Yakovlev, Phys. Rev. D 62 (2000) 114002.
[4] P. Ball, V.M. Braun, Phys. Rev. D 54 (1996) 2182.
[5] P. Ball, V.M. Braun, Phys. Rev. D 55 (1997) 5561.
[6] P. Ball, V.M. Braun, Phys. Rev. D 58 (1998) 094016.
[7] M.A. Ivanov, Y.L. Kalinovsky, C.D. Roberts, Phys. Rev. D 60 (1999) 034018.
[8] C.D. Roberts, S.M. Schmidt, Prog. Part. Nucl. Phys. 45 (2000) S1.
[9] M.A. Ivanov, Y.L. Kalinovsky, C.D. Roberts, Preprint ANL-PHY-9594-TH-2000, hep-ph/0006189.
[10] G. Burdman, Z. Ligeti, M. Neubert, Y. Nir, Phys. Rev. D 49 (1994) 2331.
[11] C.S. Huang, C. Liu, C.T. Yan, Phys. Rev. D 62 (2000) 054019.
[12] W.Y. Wang, Y.L. Wu, hep-ph/0105154, to be published in Phys. Lett. B.
[13] W.Y. Wang, Y.L. Wu, Y.A. Yan, Int. J. Mod. Phys. A 15 (2000) 1817.
[14] Y.A. Yan, Y.L. Wu, W.Y. Wang, Int. J. Mod. Phys. A 15 (2000) 2735.
[15] Y.L. Wu, Y.A. Yan, Int. J. Mod. Phys. A 16 (2001) 285.
[16] W.Y. Wang, Y.L. Wu, Int. J. Mod. Phys. A 16 (2001) 377.
[17] W.Y. Wang, Y.L. Wu, hep-ph/0012240, to be published in Int. J. Mod. Phys. A.
[18] V.A. Nesterenko, A.V. Radyushkin, Phys. Lett. B 115 (1982) 410;
     V.A. Nesterenko, A.V. Radyushkin, Sov. Phys. JETP Lett. 35 (1982) 488.
[19] P. Ball, V.M. Braun, Phys. Rev. D 49 (1994) 2472.
[20] D.J. Gross, F. Wilczek, Phys. Rev. D 9 (1974) 980.
[21] M.A. Shifman, M.I. Vysotsky, Nucl. Phys. B 186 (1981) 475.
[22] M.A. Benitez et al., Phys. Lett. B 239 (1990) 1.
[23] L. Montanet et al., Phys. Rev. D 50 (1994) 1173.
[24] B.H. Behrens et al., CLEO Collaboration, Phys. Rev. D 61 (2000) 052001.
[25] Particle Data Group, Phys. Rev. D 54 (1996) 1.