Energy release rates at two perpendicularly meeting cracks by use of the Scaled Boundary Finite Element Method

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Abstract

The recently developed enriched formulation of the Scaled Boundary Finite Element Method is employed for the analysis of the 3D structural situation of two perpendicularly meeting cracks in a homogeneous isotropic continuum. For simplicity, only symmetric deformations and loads are considered. This yields two stress singularities, one being stronger and one being weaker than the classical crack singularity. In both cases, a triangular crack extension shape is assumed as a conservative approximation for the determination of incremental energy release rates. The chosen method is found to show excellent convergence, accuracy and also computational efficiency and, consequently, to be appropriate for this purpose. These findings present a first step towards the comprehensive application of this new method to the analysis and evaluation of 3D fracture mechanics problems including stress singularities.

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1. Introduction

Lately, the structural situation of two perpendicularly meeting cracks has been studied in detail by Hell and Becker (2014). Using the Scaled Boundary Finite Element Method (SBFEM) (Wolf, 2003; Song and Wolf, 1997; Deeks and Wolf, 2002), the stress singularity exponents and their associated deformation modes have been identified. In Hell and Becker (2015a,b) further crack configurations of two meeting cracks with various crack inclinations were considered and, in Hell and Becker (2015c), the complete boundary value problem was solved for the case of a cross ply laminate with two perpendicularly meeting inter-fiber cracks (or matrix cracks) which were subject to a simple temperature decrease. These analyses revealed weakened convergence properties of the SBFEM in the presence of singularities on the discretised boundary, especially in the anisotropic case, and thus provided the motivation for an enriched formulation recently proposed in Hell and Becker (2016).

On the other hand, it was shown that this structural situation exhibits six instead of only three deformation modes which are associated to singular stresses ($\sigma \sim r^{\lambda-1}$ with $\text{Re}(\lambda) < 1$). The deformation modes are depicted in fig. 1 and the associated stress singularity exponents $\text{Re}(\lambda - 1)$ are given in table 1. It reveals that deformation modes $ct_1$ and $co_2$ are associated to stress singularities which are even stronger than the classical crack singularity, i.e. $\text{Re}(\lambda) < 0.5$. 

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In theory, such singularities, which we call hypersingularities, lead to infinite differential energy release rates $G \to \infty$ (e.g. Leguillon and Sanchez-Palencia, 1999), while so-called weak singularities ($0.5 < \text{Re}(\lambda) < 1$) yield $G \to 0$. So in both cases, the Griffith criterion, which is generally employed in Linear Elastic Fracture Mechanics, is not applicable any more. Instead, a criterion involving an integral measure, like the incremental (or averaged) energy release rate $\bar{G}$, could be used as it is done in the concept of Finite Fracture Mechanics (FFM) using a coupled stress and energy criterion (e.g. Hell et al., 2014). Consequently, the first step towards a proper assessment of 3D structural situations, like the one of two perpendicularly meeting cracks, is the determination of incremental energy release rates.

However, especially in the 3D case (e.g. Leguillon, 2014), this can be a rather challenging task as typically many probable neighbouring crack configurations have to be considered. The computational effort can be reduced using the concept of Matched Asymptotics (e.g. Leguillon, 2002), although this approach can lead to other numerical challenges. In recent and computationally expensive studies, Mittelman and Yosibash (2014, 2015) found that crack growth is mainly governed by a maximum normal stress criterion, at least for the simple homogeneous isotropic case. Consequently, in a first step, we will only treat the case of two perpendicularly meeting cracks in a homogeneous isotropic continuum.

Instead of expensive 3D FEM calculations, the SBFEM in conjunction with enriched base functions will be employed and its suitability will be evaluated. The SBFEM is a semi-analytical method which combines the advantages of the Boundary Element Method (BEM) and the Finite Element Method (FEM). Like in the BEM, only the boundary needs to be discretised. On the other hand, the SBFEM is based on the principle of virtual work, like the FEM, and does not need any fundamental solutions. It has proven its high efficiency and accuracy in the presence of stress singularities, especially in 2D fracture mechanics when the singularity is entirely located within the considered domain (e.g. Song, 2006). However, in 3D elasticity problems, there can also be singularities on the discretised boundary itself as in the case of two perpendicularly meeting cracks. Then, as already mentioned, the use of enriched base functions is useful to ensure the good convergence and accuracy of the method.

Table 1. Stress singularity exponents obtained from a Richardson extrapolation of the $enr$SBFEM results for the structural situation of two perpendicularly meeting cracks in a homogeneous isotropic continuum.

<table>
<thead>
<tr>
<th>Deformation mode $\phi$</th>
<th>co1</th>
<th>co2</th>
<th>cs1 / cs2</th>
<th>ct1</th>
<th>ct2</th>
</tr>
</thead>
<tbody>
<tr>
<td>singularity exponent Re($\lambda - 1$)</td>
<td>-0.61759</td>
<td>-0.32701</td>
<td>-0.49225</td>
<td>-0.66154</td>
<td>-0.20794</td>
</tr>
</tbody>
</table>
and a conical (i.e. scaled polar) coordinate system has to be defined for every such line singularity with polar coordinates \( \eta \) surface spanned by \( \eta \).

In eq. (2), \( F \) are similar to element stiffness matrices in the well-known FEM and are accordingly assembled. Eq. (2) contains an

2. The Scaled Boundary Finite Element Method with enriched base functions (enrSBFEM)

The fundamental requirement for the applicability of the SBFEM is the geometric scalability of the whole boundary value problem, i.e. it must be possible to connect any point on the boundary to a chosen scaling centre by a straight line (scaling ray), without this line meeting the boundary at any other point. This also allows for locating the scaling centre on the boundary itself so that part of the boundary can be represented by scaling rays. Indeed, this exceptional case can beneficially be exploited in the analysis of V-notches, multi-material corners, etc. A scaled boundary coordinate system is introduced with a scaling coordinate \( \xi \) (running from the scaling centre to the boundary) and boundary coordinates \( \eta_1 \) and \( \eta_2 \) (fig. 2). Additionally, if line singularities are present and shall be considered in an enrichment, a conical (i.e. scaled polar) coordinate system has to be defined for every such line singularity with polar coordinates \( r \) and \( \varphi \) on the boundary and also a scaling coordinate \( \xi \) (\( \xi = 0 \) at the scaling centre and \( r = 0 \) at the line singularity).

Fig. 2. Plane, 4-node scaled boundary finite element forming a square-based pyramid (scaling centre \( S \) with \( \xi = 0 \) at origin of a global Cartesian coordinate system \( x, y, z \)). An isoparametric scaled boundary coordinate system \( \xi, \eta_1, \eta_2 \) and a conical coordinate system \( \xi, r, \varphi \) are present, the latter such that \( r = 0 \) at the crack front (singularity line) and \( \varphi = \pm \pi \) at the crack faces. Enriched nodes are circled in red. (Hell and Becker, 2016)

If the scalability condition is fulfilled, the displacement field can be expressed by a separation of variables representation. This standard formulation can be supplemented with an enrichment to better fulfill local boundary conditions at line singularities (second summand in the following representation):

\[
\mathbf{u}(\xi, \eta_1, \eta_2) = \mathbf{N}(\eta_1, \eta_2) \mathbf{u}(\xi) + \mathbf{N}_b(\eta_1, \eta_2)(\mathbf{F}(r, \varphi) - \mathbf{N}(\eta_1, \eta_2) \mathbf{F}(r_k, \varphi_k)) \mathbf{b}(\xi)
\]

(1)

The functions \( \mathbf{u}(\xi) \) and \( \mathbf{b}(\xi) \) are assumed to be free in the scaling coordinate \( \xi \), \( \mathbf{N}(\eta_1, \eta_2) \) are shape functions in the boundary coordinates, \( \mathbf{F}(r, \varphi) \) are the enrichment functions in the introduced local polar coordinates. We discretise the surface spanned by \( \eta_1 \) and \( \eta_2 \) with isoparametric elements with bilinear shape functions for \( \mathbf{N}(\eta_1, \eta_2) \). The nodal values of the enrichment functions \( \mathbf{F}(r_k, \varphi_k) \) are subtracted from the enrichment so that \( \mathbf{u}(\xi) \) constitute the actual displacements at the scaling rays. For the enrichment functions, we use the classical decomposition of the analytical crack displacement field also known from the popular XFEM (e.g. Fries and Belytschko, 2010). In contrast to most XFEM applications, cracks are modelled by inserting double nodes at the crack faces, here. For computational efficiency, the plateau method (Laborde et al., 2005) is adopted, i.e. the same enrichment coefficients \( \mathbf{b}(\xi) \) are assigned to all enriched nodes associated to one crack. Consequently, the enrichment leads to only 12 additional degrees of freedom (DOF) per crack tip on the boundary. Please also note that \( \mathbf{N}_b(\eta_1, \eta_2) \) are shape functions employed as blending functions, i.e. they take the value 1 at enriched nodes but 0 at all other nodes. According to Ventura et al. (2009), this conserves the important partition of unity property of the enriched formulation. Substitution of the enriched displacement representation (1) into the virtual work equation and numerical integration over the boundary domain yields

\[
\int_{\xi=0}^{1} \delta \mathbf{u}^T(\xi) \left[ \xi^2 \mathbf{E}_0 \mathbf{u}(\xi)_{,\xi} + \xi \left( 2 \mathbf{E}_0 - \mathbf{E}_1 + \mathbf{E}_1^T \right) \mathbf{u}(\xi)_{,\xi} + \left[ \mathbf{E}_1^T - \mathbf{E}_2 \right] \mathbf{u}(\xi) \right] d\xi = 0 \quad \text{→ differential equation system of Cauchy–Euler type}
\]

\[
\delta \mathbf{u}^T(\xi) \left[ -p_{\xi=1} + \left[ \mathbf{E}_0 \mathbf{u}(\xi)_{,\xi} + \mathbf{E}_1^T \mathbf{u}(\xi) \right]_{\xi=1} \right].
\]

(2)

In eq. (2), \( \mathbf{p} \) is a nodal loads vector at the discretised boundary and \( (\cdot)_{,\xi} \) denotes derivatives in \( \xi \). The matrices \( \mathbf{E}_0, \mathbf{E}_1, \mathbf{E}_2 \) are similar to element stiffness matrices in the well-known FEM and are accordingly assembled. Eq. (2) contains an
ordinary differential equation system (DES) of Cauchy-Euler type and a linear equation system (LES). The DES can be converted into a quadratic eigenvalue problem that in turn can be linearised and solved by standard eigenvalue solvers for unsymmetric matrices. Its solution leads to a power law function series for the displacements:

\[ u(\xi) = \sum_{i=1}^{n} K_i \xi^{\lambda_i} \Phi_i \longrightarrow \sigma \sim \xi^{\text{Re}(\lambda_i)-1}. \] (3)

This directly includes the deformation modes \( \Phi_i \) (eigenvectors of the quadratic eigenvalue problem) and their associated decay rates \( \lambda_i \) (eigenvalues) which are explicitly connected with the stress singularity exponents \( \text{Re}(\lambda_i - 1) \).

This solution can be employed in the LES for enforcing the boundary conditions on the discretised boundary, finally yielding the generalized stress intensity factors \( K_i \).

3. Energy release rate at two perpendicularly meeting cracks

3.1. The “pre-cracked” configuration

In this work, we aim at revealing the validity and the advantages of the proposed method (enrSBFEM) for the analysis of 3D crack situations and, in particular, for the determination of associated incremental energy release rates \( \bar{G} \). As a consequence, only a few exemplary cases will be considered in this work. The “pre-cracked” configuration is the one of the two perpendicularly meeting cracks in a homogeneous steel block prior to crack extension. For simplicity and in order to reduce the computational effort, we only consider deformations being symmetric to the crack faces, for now. As depicted in fig. 3a, this reduces the model size to only a quarter of the original size and appropriate symmetry boundary conditions have to be defined at the faces marked in grey. The scaling centre is chosen to be located at the crack interaction point which is marked by a red dot. Then, only the front, upper and lower faces need to be discretised as shown in fig. 3b for \( n_e = 12 \) elements along the long edge of the body. The locations of the cracks are marked by the red lines and the enriched nodes are marked by red circles. Symmetry boundary conditions are applied by removing the corresponding displacement DOFs from the quadratic eigenvalue problem. The same has to be done with the DOFs of the symmetric enrichment functions for the displacement direction perpendicular to the symmetry plane and with the DOFs of the antisymmetric enrichment functions for the other two displacement directions.

Of course, the analysis of this symmetric configuration only yields two deformation modes with singular stresses, namely the symmetric crack opening modes co1 and co2. The convergence of the corresponding eigenvalues \( \lambda \) is illustrated in fig. 3c. It shows the results from the standard (dots) and enriched (squares) formulation of the SBFEM and clearly indicates the superior convergence of the enrSBFEM results towards the extrapolated values (black lines; from Richardson extrapolation). The comparably poor accuracy obtained for \( n_e = 6 \) using the enrSBFEM results from the fact that this mesh lacks any fully enriched elements.

Being subject to a purely vertical loading, none of the two stress singularities are active. In fact, such a vertical loading would only result in a simple homogeneous strain and stress field. On the other hand, almost any in-plane
loading (in the plane spanned by the crack fronts) would trigger the hypersingularity associated to deformation mode \( c_1 \). This is the reason why this deformation mode will mainly be considered in the following.

The trend of the stress field associated to deformation mode \( c_1 \) is rather easy to guess. There are the line singularities at the crack fronts with the classical stress singularity exponent \( \text{Re}(\lambda - 1) = -0.5 \). At the same time, there is the 3D stress singularity with exponent \( \text{Re}(\lambda - 1) = -0.61759 \). Thus, the stresses decay with growing distance from the crack front on the one hand but, on the other hand, the crack stress intensity factors must grow steadily (towards infinity) when moving towards the crack interaction point.

3.2. The “post-cracked” configuration

Up to this point, only the initial “pre-cracked” structural situation of two meeting cracks has been considered. But, for the determination of an incremental energy release rate, a subsequent “post-cracked” configuration after crack extension has to be taken into account as well. In the framework of FFM in connection with a coupled stress and energy criterion, any crack in the overstressed domain, i.e. the domain in which a stress criterion is fulfilled, is admissible. This is a rather weak restriction so that typically further assumptions are necessary to limit the number of possible “post-cracked” configurations. We start from the premise that subsequent cracks are plane and originate at the point of maximum stress, which is assumed to be located at the crack interaction point. Following the results of Mittelman and Yosibash (2015), the crack extension direction in homogeneous isotropic continua is determined by the maximum normal stress criterion. As this would result in a curved crack surface here, we are going to presume a crack shape similar to the one shown in fig. 4a (shaded in red) as a hopefully acceptable compromise. The presumed newly generated crack surface gets quite tight at the line singularities, so we assume the energy released due to these parts of the additional crack area to be small in comparison. This approach may become reasonable when taking a look at the virtual crack closure integral (e.g. Anderson and Anderson, 2005) which, in essence, simply is half of the integrated product of stresses in the state before crack growth (\( 1 \)) and the displacements after crack growth (\( 2 \)) over the newly generated crack area. At the tight parts of the newly generated crack surface close to the line singularity and far from the crack interaction point, comparably small displacements and, consequently, only a small share of released energy can be expected. Leguillon (2014) gives references to similar actually observed crack shapes at the corner of thermally loaded electronic chips, which supports our assumptions. Moreover, he compares incremental energy release rates of the triangular crack extension shape with a more “realistic” non-convex quadrangular crack extension shape and found them to be very similar. In fact, the triangular crack extension shape even yielded higher incremental energy release rates and, consequently, conservative results. Although this was for a very different structural situation (rotated bimaterial beam in four-point bending), we understand this as another supporting example and confine our further considerations to the crack extension shape of a simple isosceles triangle as depicted in fig. 4b.

This “post-cracked” configuration can also be analysed using the SBFEM, although it needs a little more effort. The scalability requirement is fulfilled if the scaling centre is relocated from the former crack interaction point \( S^{(1)} \) (where the two crack fronts perpendicularly met) to the new crack interaction point \( S^{(2)} \) (where the crack front of the triangular crack extension meets one of the original crack fronts). Additionally, the domain must be partitioned into two identical (but one rotated) parts as depicted in fig. 4c in which the two new crack interaction points \((S_1^{(2)}, S_2^{(2)})\) are marked by red dots.
Fig. 5. Exemplary boundary mesh of left part of the “post-cracked” configuration with \( n_e = 12 \) elements along the long edge: (a) front view, (b) back view. Red lines/surfaces mark crack faces and red circles mark enriched nodes. (c) Comparison of the convergence of the first three eigenvalues \( \lambda \) which are not associated to rigid body motions for the standard and the enriched formulation of the SBFEM.

Fig. 6a,b show the employed boundary mesh of such a part with \( n_e = 12 \) elements along the long edge of the body from the front and the back respectively. The boundary faces being coincident with the ones of the “pre-cracked” model are identically meshed. Again, the red lines mark crack faces and the red circles mark enriched nodes. This configuration yields 3 stress singularities with the following extrapolated singularity exponents \( \text{Re}(\lambda_1 - 1) = -0.5574 \), \( \text{Re}(\lambda_2 - 1) = -0.2983 \) and \( \text{Re}(\lambda_3 - 1) = -0.1569 \). In Fig. 5c, the values obtained from the standard and enriched formulation of the SBFEM are given and again the excellent convergence properties of the \( \text{enrSBFEM} \) are confirmed.

In the next step, we want to check the convergence properties of a complete boundary value problem. Accordingly, two of the introduced parts (fig. 5a,b) are assembled as depicted in fig. 6a. Please note that, in contrast to the former illustrations, we now only look at the considered body from the back. The boundary stiffness matrices are calculated and assembled as well and the resulting body is subjected to a simple vertical load. This would only have led to a simple homogeneous stress field in the “pre-cracked” configuration, but here, naturally leads to a crack opening of the added triangular crack extension. Fig. 6b illustrates the convergence of the magnitude of the displacements \( ||\mathbf{u}||_2 \) at the upper corner opposite to the symmetry planes. In Fig. 6c, the convergence of the strain energy \( \Pi_i \) is shown in a double logarithmic plot. Again, it becomes evident that the enriched formulation of the SBFEM clearly excels the standard one, usually by at least an order of magnitude. Please note that already for only two elements along the diagonal of the crack extension faces (\( n_e = 12 \)), i.e. as soon as there is at least 1 row of fully enriched elements around the crack extension front, the error in both displacement and strain energy is about 0.1% or less when using the \( \text{enrSBFEM} \).

Finally, we are going to calculate the incremental energy release rates for two special cases. In the first one, the boundary displacements and enrichment function coefficients of deformation mode co1 (“pre-cracked” configuration)
Table 2. Strain energies $\Pi^{(2)}$ of the "post-cracked" configuration (after crack extension by $\Delta A = 1/8 \text{mm}^2$) and corresponding incremental energy release rates $\bar{G}$ at co1 and co2 displacement boundary conditions and with domain dimensions (1 mm x 1 mm x 2 mm). Please note: 3D GSIFs of deformation modes co1 and co2 are each chosen such that the strain energy in the "pre-cracked" configuration is $\Pi^{(1)} = 1 \text{Nmm}$.

<table>
<thead>
<tr>
<th>$n_e$</th>
<th>$\Pi^{(2)}_{\text{co1}} \text{[Nmm]}$</th>
<th>$\bar{G}_{\text{co1}} \text{[N/mm]}$</th>
<th>$\Pi^{(2)}_{\text{co2}} \text{[Nmm]}$</th>
<th>$\bar{G}_{\text{co2}} \text{[N/mm]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>0.861565018965</td>
<td>1.1075</td>
<td>0.9748</td>
<td>0.2014</td>
</tr>
<tr>
<td>18</td>
<td>0.858974097947</td>
<td>1.1282</td>
<td>0.9732</td>
<td>0.2141</td>
</tr>
<tr>
<td>24</td>
<td>0.858033402253</td>
<td>1.1357</td>
<td>0.9727</td>
<td>0.2187</td>
</tr>
</tbody>
</table>

Fig. 7. Amplified deformed boundary mesh of "post-cracked" configuration for (a) co1 and (b) co2 displacement boundary conditions (back view).

are applied to the assembled “post-cracked” configuration. Doing so, the 3D generalized stress intensity factor (GSIF) of deformation mode co1 is assumed to be such that it yields a strain energy of $\Pi^{(1)}_i = 1 \text{Nmm}$ for the symmetric model of two perpendicularly meeting cracks (cf. fig. 3) with dimensions 1 mm x 1 mm x 2 mm. Then, the incremental energy release rate only depends on the “post-cracked” configuration. The same procedure is performed for deformation mode co2. The resulting incremental energy release rates for a crack extension area $\Delta A = 1/8 \text{mm}^2$ are given in table 2. On the first sight, the results, especially for $\bar{G}$, seem to not exactly match the excellent convergence properties of the enrSBFEM observed in fig. 6 for the case of a constant vertical loading. But it has to be noted that the relative difference of the strain energies still is $\Pi^{(2)}(n_e = 12)/\Pi^{(2)}(n_e = 24) < 0.5\%$. Moreover, fixing the strain energy of the “pre-cracked” configuration to $\Pi^{(1)} = 1$ may also have a spurious influence.

Fig. 7a and b show the corresponding amplified deformations for the co1 and the co2 case respectively. It can be seen that, in the co1 case, the crack faces of the crack extension considerably deflect in mode I (opening) and III (twisting), while the deflections are significantly smaller and mainly in mode II (shearing) in the co2 case. This observation matches the observed difference in the calculated incremental energy release rates $\bar{G}_{\text{co1}}$ and $\bar{G}_{\text{co2}}$.

4. Summary and Conclusions

It was the goal of this work to reveal the validity and the advantages of the enrSBFEM for the analysis of 3D crack situations and, furthermore, for the determination of corresponding incremental energy release rates $\bar{G}$. As a first test case, the structural situation of two perpendicularly meeting cracks in a homogeneous isotropic continuum under symmetry boundary conditions along the crack ligaments was considered. A plane triangular crack extension shape was selected by consideration of simple plausibility criteria and recent results from literature (Leguillon, 2014; Mittelman and Yosibash, 2015) including that the crack extension direction is mainly influenced by a maximum normal stress criterion. Then, the incremental energy release rates $\bar{G}$ for displacement boundary conditions of symmetric singularity deformation modes co1 and co2 were calculated for a 1mm x 1mm x 2mm block (cf. fig. 4 and 7). The variation of the stress singularity when approaching a free surface was not considered because it was assumed to have a negligible effect on the results, on the one hand. On the other hand, when the boundary displacements and crack field coefficients are prescribed, there is no free surface anyway.
Of course, the authors are aware that the considered examples might be considered to be of rather academic nature. For example, deformation mode co2 alone would probably not lead to a crack extension starting from the crack interaction point as the associated point singularity is actually weaker than the line singularities along the crack fronts. The consideration of a hypersingularity (deformation mode co1) with a theoretically infinite differential energy release rate (but finite incremental energy release rate) might also be questioned.

Nevertheless, the obtained results were checked regarding their convergence and found to be accurate within about 0.1% from the extrapolated values. Incidentally, the enriched formulation of the SBFEM was shown to be clearly superior to the standard formulation regarding convergence and accuracy. Moreover, the presented calculations can be seen as a first step towards the application of the enrSBFEM to the concept of Matched Asymptotics, which seems especially promising facing the overwhelming variety of generally admissible crack extensions in 3D fracture mechanics problems within the framework of FFM (Leguillon, 2002; Hell et al., 2014; Leguillon, 2014). Indeed, the authors expect the enrSBFEM to be especially suited for this purpose as infinite domains can quite simply be realised using this method. Finally, in ongoing works, the enrSBFEM has been extended to the anisotropic case of a crack impinging on an interface which clears the way for the application of the method to the practically relevant situation of two meeting inter-fiber cracks in a composite laminate.

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