



Inverted hierarchical model of neutrino masses revisited

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Received 23 May 2003; accepted 19 June 2003

Editor: T. Yanagida

Abstract

In this Letter we highlight the inherent problems associated with the inverted hierarchical model of neutrinos with only three generations and then suggest possible solutions within the MSSM. We discuss the new parametrization of the solar mixing angle which can identify the light side and dark side of the data. We then argue whether the inverted hierarchical neutrino mass matrix can explain the large mixing angle (LMA) MSW solution of the solar neutrino anomaly in the presence of an appropriate texture of charged lepton mass matrix. In a model independent way we explore such specific form of the charged lepton mass matrix having a special structure in 1–2 block. The contribution to the solar mass splitting arising out of radiative corrections in MSSM, is calculated, thus making the model stable under radiative corrections.

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1. Introduction

The data on solar neutrino oscillation has been traditionally shown on the $(\sin^2 2\theta_{12}, \Delta m_{21}^2)$ plane but a new parametrization on the $(\tan^2 \theta_{12}, \Delta m_{21}^2)$ plane with the usual sign convention $\Delta m_{21}^2 = m_2^2 - m_1^2 > 0$ has been recently suggested [1] to explain the solar data. In this new parametrization the most recent result from the SNO–SuperKamiokande [2] favours the “light side”, $\tan^2 \theta_{12} < 1$, and disfavors the “dark side”, $\tan^2 \theta_{12} > 1$. It has also imposed certain restrictions on the validity of the proposed neutrino mass models which are otherwise allowed in the old parametrization. This can be understood from the definition of leptonic mixing matrix V_{MNS} where $\tan^2 \theta_{12} = |V_{e2}|^2/|V_{e1}|^2 < 1$ implies $|V_{e2}| < |V_{e1}|$ for the usual sign convention $\Delta m_{21}^2 = m_2^2 - m_1^2 > 0$, whereas such restriction does not exist in the definition of $\sin^2 2\theta_{12} = 4|V_{e1}|^2|V_{e2}|^2$.

For our analysis in the present Letter, we take here the most general form of texture of the inverted neutrino masses, which is assumed to be valid at high scale

$$m_{LL} = \begin{pmatrix} \delta & 1 & 1 \\ 1 & \epsilon_1 & \epsilon_2 \\ 1 & \epsilon_2 & \epsilon_1 \end{pmatrix} m_0.$$

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Here we consider two possibilities. In case (i) we take $\delta = \epsilon_1 = \epsilon_2 = 0$, leading to $\Delta m_{21}^2 = 0$. This represents the case of perfect $L_e-L_\mu-L_\tau$ symmetry. In case (ii) we have $\delta \sim \epsilon_1 \sim \epsilon_2 \sim \lambda^3$, leading to $\Delta m_{21}^2 > 0$. The quantum radiative corrections may modify the above two cases and their low-energy values are—case (i): $\Delta m_{\text{sol}}^2 = \delta_{\text{rad}}$ when an appropriate charged lepton mass matrix is taken into consideration [3]; and case (ii): $\Delta m_{\text{sol}}^2 = \Delta m_{21}^2 + \delta_{\text{rad}}$. In MSSM the radiative correction term δ_{rad} is a negative quantity [4] and therefore, it will lead to $\tan^2 \theta_{12} > 1$ in case (i). But in case (ii) if $\Delta m_{21}^2 > \delta_{\text{rad}}$, the radiative stability will still be maintained with $\tan^2 \theta_{21} < 1$. This point will be further examined in present Letter.

2. Formalism and results

Expressing m_{LL} in the basis where the charged lepton mass matrix is diagonal, we have $m'_{LL} = V_{eL} m_{LL} V_{eL}^T$, $m_{LL}^{\text{diag}} = V'_{\nu L} m'_{LL} V_{\nu L}^T$, where $V_{\text{MNS}} = V_{\nu L}^\dagger$, and V_{eL} is the diagonalizing matrix for charged lepton mass matrix. The neutrino flavour eigenstate ν_f is related to the mass eigenstate ν_i by the relation $\nu_f = V_{fi} \nu_i$ and the MNS mixing matrix is given by V_{fi} where $f = \tau, \mu, e$ and $i = 1, 2, 3$. We take the usual convention of the neutrino mass eigenvalues $|m_{\nu 1}| < |m_{\nu 2}|$, and this fixes the ordering of the first two columns of the leptonic mixing matrix V_{MNS} . We present here only one simple case of the left-handed Majorana neutrino mass matrix having $\delta = 0, \epsilon_2 = 0, \epsilon_1 = \lambda^3$ for our demonstration in the present analysis, m_{LL} with $m_0 = 0.05$ eV in Eq. (1); and the neutrino mass eigenvalues $m_i = (-1.4089, 1.4195, 0.01065)m_0, i = 1, 2, 3$, leading to the neutrino mass splittings $\Delta m_{21}^2 = 7.4952 \times 10^{-5}$ eV² and $\Delta m_{23}^2 = 5.0335 \times 10^{-3}$ eV². For our choice of the diagonal charged lepton mass matrix, the MNS mixing matrix is expressed as

$$V_{\text{MNS}} = \begin{pmatrix} \frac{1}{\sqrt{2}} + \alpha & \frac{1}{\sqrt{2}} - \alpha & 0 \\ \frac{1}{2} - \beta & -(\frac{1}{2} + \beta) & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} - \beta & -(\frac{1}{2} + \beta) & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (1)$$

where $\alpha = 0.001335$ and $\beta = 0.00094$. Here $\tan^2 \theta_{12} = \left| \frac{1}{\sqrt{2}} - \alpha \right|^2 / \left| \frac{1}{\sqrt{2}} + \alpha \right|^2 < 1$ for the usual convention $\Delta m_{21}^2 > 0$. In case of exact $L_e-L_\mu-L_\tau$ symmetry as in case (i) where $\delta = \epsilon_{1,2} = 0$ and $\Delta m_{21}^2 = 0$, and the above V_{MNS} takes the form with $\alpha = 0$ and $\beta = 0$. In the above example the $V_{\text{MNS}} (= V_{\nu L}^\dagger)$ obtained from the m_{LL} alone fails to explain the LMA MSW solution as it predicts the maximal solar mixings, and any small deviation in the texture of m_{LL} will hardly affect the maximal value of $\tan^2 \theta_{12}$. The last hope is that there could be a significant negative contribution to θ_{12} from V_{eL} obtained from the diagonalisation of the charged lepton mass matrix m_l having special structure in 1–2 block. We wish to examine here how $\theta_{12} = (\theta_{12}^\nu - \theta_{12}^e)$ can resolve the LMA MSW solar neutrino mixing scenario.

In a model independent way we parametrize the charged lepton mixing V_{eL} as

$$V_{eL} = \begin{pmatrix} \bar{c}_{12} & \bar{s}_{12} & 0 \\ -\bar{s}_{12} & \bar{c}_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (2)$$

This gives a special form in the 1–2 block. We can reconstruct the symmetric charged lepton mass matrix using Eq. (2) from the relation, $m_l = V_{eL}^\dagger m_l^{\text{diag}} V_{eL}$, where we consider $V_{eL} = V_{eR}$ for symmetric matrix. The MNS mixing matrix $V_{\text{MNS}} = V_{eL} V_{\nu L}^\dagger$ is now calculated as

$$\begin{pmatrix} \left(\frac{1}{\sqrt{2}} + \alpha \right) \bar{c}_{12} + \left(\frac{1}{2} - \beta \right) \bar{s}_{12} & \left(\frac{1}{\sqrt{2}} - \alpha \right) \bar{c}_{12} - \left(\frac{1}{2} + \beta \right) \bar{s}_{12} & -\frac{\bar{s}_{12}}{\sqrt{2}} \\ -\left(\frac{1}{\sqrt{2}} + \alpha \right) \bar{s}_{12} + \left(\frac{1}{2} - \beta \right) \bar{c}_{12} & -\left(\frac{1}{\sqrt{2}} - \alpha \right) \bar{s}_{12} - \left(\frac{1}{2} + \beta \right) \bar{c}_{12} & -\frac{\bar{c}_{12}}{\sqrt{2}} \\ \frac{1}{2} - \beta & -\left(\frac{1}{2} + \beta \right) & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (3)$$

Table 1

Few representative cases for the values of the quantities in Eqs. (4) and (5) obtained for the different values of Δ_{23} as input

Case	Δ_{23}	$\sin^2 2\theta_{23}$	$ V_{e3} $	$\tan^2 \theta_{21}$	$ m_{ee} $ (eV)
I	0.02	0.98	0.10	0.66	0.014
II	0.04	0.96	0.14	0.55	0.020
III	0.05	0.95	0.16	0.52	0.022
IV	0.06	0.94	0.17	0.49	0.024

Now we obtain the following parameters

$$\tan^2 \theta_{21} = \frac{\left| \left(\frac{1}{\sqrt{2}} \bar{c}_{12} - \frac{1}{2} \bar{s}_{12} \right) - \alpha (\bar{c}_{12} + \bar{s}_{12}) \right|^2}{\left| \left(\frac{1}{\sqrt{2}} \bar{c}_{12} + \frac{1}{2} \bar{s}_{12} \right) + \alpha (\bar{c}_{12} - \bar{s}_{12}) \right|^2},$$

$$\sin^2 2\theta_{23} = \bar{c}_{12}^2, \quad |V_{e3}| = \left| \frac{\bar{s}_{12}}{\sqrt{2}} \right|, \quad |m_{ee}| \simeq |m_1| \sqrt{2} \bar{s}_{12} \bar{c}_{12}.$$

If $\Delta_{23} = 1 - \sin^2 2\theta_{23}$, is the small deviation from the completely maximal value of the atmospheric neutrino mixings, then the above relations take the following new forms:

$$\tan^2 \theta_{21} = |1 - \sqrt{8\Delta_{23}} + 3\Delta_{23}|, \quad (4)$$

$$|V_{e3}| = \sqrt{\frac{\Delta_{23}}{2}}, \quad |m_{ee}| = |m_1| \sqrt{2\Delta_{23}}. \quad (5)$$

Where α in Eq. (4) has been suppressed as it contributes insignificantly to the solar mixing.

The Table 1 shows that in Cases II and III, solar mixings together with solar mass splitting $\Delta m_{21}^2 = 7.49 \times 10^{-5} \text{ eV}^2$ are in excellent agreement with the best fit $\tan^2 \theta_{21} = 0.56$ obtained in KamLAND experiment [8]. The values of $|V_{e3}|$ are also in accord with the CHOOZ experiment.

Finally we check the stability of the neutrino mass matrix under RGEs. Following the standard procedure [4,5] we express m_{LL} in terms of K , the coefficient of the dimension five neutrino mass operator in a scale-dependent manner $m_{LL}(t) = v_u^2(t)K(t)$ where $t = \ln(\mu)$ and $v_u(t)$ is the scale-dependent [5] vacuum expectation value (VEV) $v_u = v_0 \sin \beta$, $v_0 = 174 \text{ GeV}$. In the basis where the charged lepton mass matrix is made diagonal, we can write the above expression as $m'_{LL}(t) = v_u^2(t)K'(t)$ where $K'(t) = V_{eL}K(t)V_{eL}^T$ is the coefficient of the dimension five neutrino mass operators in the basis where the charged lepton mass matrix is diagonal. The approximate analytical solution of the relevant RGEs in MSSM, is given by the following form

$$m'_{LL}(t_0) = R \cdot \text{diag}(1, 1, \alpha) \cdot m'_{LL}(t_u) \cdot \text{diag}(1, 1, \alpha). \quad (6)$$

We have the expressions $R = e^{\frac{9}{10}I_{g_1}(t_0)} e^{\frac{9}{2}I_{g_2}(t_0)} \simeq 1$, $\alpha = e^{-t_\tau(t_0)} = 1 - r$ where the approximate solution is [4–6]

$$r = 1 - (m_t/M_u)^{(1+\tan^2 \beta)(m_\tau/2\sqrt{2}\pi v)^2}$$

with $v = 245.4 \text{ GeV}$. The low-energy solar mass splitting Δm_{sol}^2 can be expressed in terms of high energy solar mass splitting Δm_{21}^2 and the radiative correction effect δ_{rad} as [7]

$$\Delta m_{\text{sol}}^2 = \Delta m_{21}^2 + \delta_{\text{rad}}, \quad \delta_{\text{rad}} = -4r|m_1|^2 \sin^2 2\theta_{23} \cos 2\theta_{21}. \quad (7)$$

In MSSM the quantity r is positive and thus we have $\Delta m_{\text{sol}}^2 < \Delta m_{21}^2$. For example, we obtain the numerical values $\delta_{\text{rad}} = -0.96 \times 10^{-5} \text{ eV}^2$ corresponding to the Case III in Table 1. The mass ratio $|m_2|/|m_1|$ also decreases with energy scale. The model is stable under radiative corrections.

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