Mode Instability and Nonlinear Superradiance Phenomena in Open Fabry-Perot Cavity

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Abstract—Collective spontaneous emission of discrete modes (i.e., mode superradiance or superfluorescence) in an open Fabry-Perot cavity is considered. The simultaneous emission of counterpropagating coherent pulses is associated with the instability of normal "hot" modes, even in the case of very low, but finite reflection at mirrors, beginning with \( R_{cr} \ll 1 \). A role of nonlinear Bragg backscattering by a self-consistent \((\lambda/2)\)-lattice of population inversion of active two-level medium is discussed. It is shown that the interference of inhomogeneous counter-propagating waves alters essentially the space-time dynamics of inversion exhaustion and the profile of superradiance pulse, compared with the mean field and the unidirectional models. The novel nonlinear equation of mode superradiance in an open cavity is derived. The approximate analytic solution to the equation is found. Various regimes of superradiance are analyzed on the basis of this solution and compared with numerical simulations.

Keywords—Nonlinear optics, Open cavity, Superradiance, Ultrashort pulses, Bragg scattering.

1. INTRODUCTION

In [1–4] we started the close analysis of superradiance (SR) phenomenon, i.e., collective spontaneous emission of an inverted two-level medium, in an open Fabry-Perot cavity. There was found a very small, but finite reflection factor of mirrors, \( R_{cr} \ll 1 \), that limits the unidirectional SR approximation [5–7]. Even a rather bad cavity with

\[
R_{cr} \lesssim R < 1; \quad \ln R_{cr}^{-1/2} \sim \ln (N S L)^{1/4} \cdot \left\{ 1 + \sqrt{1 + \left(\frac{1}{4}\right) \ln (N S L)^{1/4}} \right\}^{-1}
\]

(1)
gives simultaneous emission of correlated forward-backward pulses, this mode SR shortening and intensifying; see Section 2. Here \( N \) stands for the density of atoms in an active sample with cross-section \( S \) and with length \( L, L \gtrsim S/\lambda \gg \lambda = 2\pi c_0/\omega_0 \), where \( \omega_0 \) is the atomic transition frequency, and \( c_0 = c_0 \sqrt{1/2} \) is the velocity of light in a matrix with dielectric constant \( \varepsilon_0 \).

Interference of counter-propagating inhomogeneous waves (Figure 1) produces the nonlinear \((\lambda/2)\)-lattice and large-scale inhomogeneity of population inversion. Thus the homogeneous-mean-field approach fails also; cf. [1–4,8–12]. To interpret correctly the experimental data [13–17] for low-Q cavities and to find the course of mode SR, one needs the new nonlinear theory, taking

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Figure 1. The inhomogeneous mode structure of normalized amplitude of field, $|E|$ (or polarization), in open Fabry-Perot cavity with reflection factor $R = e^{-4}$.

Figure 2. The 1D dynamics of inversion, $n(z, t)$, with due regard for the self-consistent scattering of forward-backward waves inside a sample in the absence of mirrors and reflections at the boundaries. In the numerical calculations presented, we choose $L = 8\lambda$, $\varepsilon_0 = 1$, and start from a small initial amplitude of polarization ($\sim 10^{-2}Nd$) of symmetric counter-propagating waves. The initial $(\lambda/2)$-lattice of inversion is consistent with this standing wave of polarization according to the law of conservation of Bloch vector length.

into account the coherence and the inhomogeneity of waves. There is such a theory [18], and it is presented in Sections 4–9. This theory is based on the fact that the Bragg scattering of forward-
backward SR waves by a self-consistent inversion ($\lambda/2$)-lattice inside any-Q Fabry-Perot cavity cannot play an essential part; see Section 3. This notwithstanding, in [18] we argue that the mode SR process is strongly affected by coherence and inhomogeneity of counter-propagating waves via the self-consistent exhaustion of inversion. It makes the great difference between the case of an open cavity with $R_{cr} \lesssim R \lesssim 1/2$ and the well-known limiting cases $R \to 0$ and $R \to 1$. In particular, for a high-Q cavity ($R \to 1$), our theory reduces to the well-known theory of optical nutation and superfluorescence of homogeneous plane waves [8,9,19]. For a low-Q cavity, the estimate [1] of the threshold (1), $R_{cr}$, has been recently confirmed in [20] and differs considerably from the first estimates [5-7,21] that were put too low (in real SR experiments, $R_{cr} \sim e^{-5}$).

Note that under special coherent initial conditions the inhomogeneous exhaustion of inversion and the nonlinear growth of a ($\lambda/2$)-lattice of the active refractive index may affect the SR dynamics even in the absence of a cavity, $R \to 0$, i.e., without linear forward-backward coupling; cf. [1,22-24]. The example is shown in Figure 2. It represents the numerical solution of the Maxwell-Bloch equations with symmetric initial conditions in the form of a weak standing wave of polarization, $P$, and corresponding inversion, $n = (N_2 - N_1)/N$. (If an active sample is short enough, the similar mode structure arises spontaneously from any weak quasi-homogeneous polarization, even without boundary reflections, and looks like the most unstable mode as in Figure 1.) The pulses emitted from the opposite sides of a sample are identical and correlated (Figure 3), but cannot be described in the mean field approach and in the unidirectional SR approximation. Unfortunately, the coherent initial conditions for counter-propagating waves and the subsequent correlation of forward-backward SR pulses will never arise spontaneously from space-incoherent noises in the absence of real linear reflection.

On the contrary, the necessary conditions originate naturally in an open Fabry-Perot cavity; see [1-12,20,21]. The grounds are no more than the spontaneous growth of some discrete "hot"
modes that prevail over waves with continuous spectra, forming the unidirectional SR, under the very condition (1).

2. MODE SUPERRADIANCE

The hot modes are formed by linearly polarized forward-backward waves in a cavity filled with active two-level atoms (with high-frequency dipole moment $d$). We can easily find the hot modes of order zero, i.e., without taking into account the backscattering inside a cavity. They are discrete due to the boundary conditions, $E_{\text{ref}} = -R^{1/2}E_{\text{inc}}$, for the amplitudes of reflected and incident waves at the mirrors. For definiteness, we take the minus sign and suppose the reflection factor, $R$, to be real-valued and the same for both mirrors, the field amplitude being symmetric and having local minimum at the mirrors (Figure 1). So, for a given (symmetric) profile of inversion, $n(z)$, the 1D Maxwell-Bloch equations (see, e.g., [1-12]) have the following set of linearly independent solutions with dimensionless amplitudes, $a_m$, and integer index, $m$:

$$
\{E \text{ or } P\}_m(z,t) = \text{Re}[\{E \text{ or } P\}_m(z)\exp(-i\omega_m t)], \quad P_m = \left(\frac{\mu_0}{2\pi}\right)(\tilde{\mu} - 1)E_m; \quad (2)
$$

$$
E_m = a_m \pi N d \{\exp[i\Psi_m(z)] - (-1)^m \exp[-i\Psi_m(z)]\},
$$

$$
\Psi_m(z) = \frac{\omega_m}{c_0} \int_0^z \mu(z',\omega_m) \, dz'; \quad (3)
$$

$$
\omega_m \tilde{\mu}(\omega_m) = \left(m\pi - i \ln R^{-1/2}\right) \frac{c_0}{L}; \quad \mu(\omega_m) = 1 + In(z) \frac{\omega_0}{(\omega_m - \omega_0)}. \quad (4)
$$

Here we use the small factor $I = 2\pi N d^2/h\omega_0 \ll 1$. The $\tilde{\mu}$ over the refractive index $\tilde{\mu}$ (as well as over the inversion, $n$) means the averaging over the length, $L$, of a cavity. The resonance unstable modes have positive imaginary parts of complex frequencies, $\omega_m$, coming from the solutions of the characteristic equation (4):
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\[ \omega_m - \omega_0 \approx - \left[ \left( 1 + \hat{\nu} \right) \frac{L\omega_0}{c_0} - m\pi + i \ln \frac{R^{-1/2}}{} \right] \frac{c_0}{2L} + \left\{ \left[ \left( 1 + \hat{\nu} \right) \frac{L\omega_0}{c_0} - m\pi + i \ln \frac{R^{-1/2}}{} \right]^2 \frac{c_0^2}{4L^2} - \omega_0^2 \hat{\nu} \right\}^{1/2}; \quad |\omega_m - \omega_0| \ll \omega_0; \quad (5) \]

\[ \text{Im}(\omega_m) \leq \hat{\omega} = -\left( \ln R^{-1/2} \right) \frac{c_0}{2L} + \left( \ln R^{-1/2} \right)^2 \frac{c_0^2}{4L^2} + \omega_0^2 \hat{\nu} \right\}^{1/2} \leq \omega_0 (\hat{\nu})^{1/2}. \]

Obviously, in comparison with the unidirectional SR from the same sample of a given length, \( L \), the presence of an open cavity (1) shortens the duration of the main SR pulse (up to \( \sim 1/\hat{\omega} \)), thus increasing its power.

Only not too short and not too long cavities will have real interest to us:

\[ 1 + R^{-1/2} \ln R^{-1/2} \ll \frac{2L}{\lambda} \ll \sqrt{\frac{I}{I}} \ll \sqrt{\frac{I}{I}} \quad (6) \]

The left inequality guarantees, in particular, the smoothness of mode amplitudes, \( E_m \) or \( P_m \), on the wavelength scale and the negligible reflection from a jump of refractive index at the boundary of an active medium (in comparison with reflection from a mirror). The right inequality ensures the causality around a sample, so that the spontaneous forming of hot mode structure is possible during a period of the order of the inverse (maximum) growth rate, \( 1/\omega_0 I^{1/2} \), of plane waves in a homogeneous infinite medium. According to the equations (4)-(6), for an arbitrary inversion profile, \( n(z) \), the refractive index is close to unity. It is true even in the case of the most unstable mode in exact resonance, \( m_\text{res} = (1 + \hat{\nu})L\omega_0/c_0 \), when the difference \( \mu_\text{res} - 1 \) is pure imaginary. This fact predetermines the weak efficiency (9) of the nonlinear scattering of forward-backward waves in each hot mode starting from spontaneous (quantum) noise under the condition of quasi-homogeneous initial inversion, \( \hat{n} \approx n(t = 0) \approx \text{const} \).

3. POWERLESS OF BRAGG SCATTERING

We will show qualitatively that in any-Q Fabry-Perot cavity the self-consistent (\( \lambda/2 \))-lattice of inversion never causes the effective backscattering of counter-propagating SR waves that exceeds the reflections at mirrors of a cavity. To be in favour of Bragg scattering, let us maximize the initial inversion, \( n(t = 0) \approx 1 \), and suppose that the (\( \lambda/2 \))-lattice is formed by the center resonance mode (3)-(5) only, \( m = m_\text{res} \approx 2L/\lambda \). It gives the deepest ideal modulation of refractive index, \( \mu(z,t) = 1 - i[n(z)/\hat{n}(t)]\ln R^{-1/2} \), with the maximum growth rate (5), \( 2\tilde{\omega} \), where

\[ n(z) \approx 1 - \frac{P_m^2}{2N^2d^2} = 1 - \frac{1}{8} \left[ \left( \ln R^{-1/2} \right) \frac{\varepsilon_0 c_0}{\omega_0 L N d} \right]^2 |E_\text{m}|^2 \exp \left[ \int_0^t \tilde{\omega}(t') dt' \right]; \]

\[ \hat{n}(t) \approx 1 - \frac{1}{8} \left( R^{-1/2} - R^{1/2} \right) \left( \ln R^{-1/2} \right) \frac{\varepsilon_0 c_0}{\omega_0 L} |\alpha_\text{m}|^2 \exp \left[ \int_0^t \tilde{\omega}(t') dt' \right] \quad (7) \]

\[ |E_\text{m}|^2 \approx (2\alpha_m \pi N d^2) \left[ \sin^2 (2\pi z) + \cos^2 (2\pi z) \right], \]

(sin or cos lies with odd or even mode number, \( m \gg 1 \)).

In the adiabatic way (7), we have to consider the moment, \( t = T \approx t_\text{delay} \), limiting the linear SR theory, when the population inversion just becomes zero and the amplitude of polarization just reaches the maximum value, \( P_m \approx Nd \), at the sides of a cavity (precisely, at a quarter wavelength’s distance from the mirrors, \( n(\pm L/2 \mp \lambda/4, T) = 0 \)). At this moment, on the one hand, the (\( \lambda/2 \))-lattice provides the strongest scattering, the amplitude of sine modulation of inversion profile being equal to \( 2/(R^{-1/2} + 2 + R^{1/2}) \leq 1/2 \). On the other hand, the average
inversion (8) decreased not more than half as much, so that the linear theory of unperturbed hot modes (Section 2) is still valid approximately, provided the distributed backscattering turns out to be negligible.

To see the latter, we rewrite the wave equation for the complex amplitude of electric field (2) with the frequency \( \omega_m = \omega_0 + i \tilde{\omega}(T) \) in the equivalent form of the coupling equations for the complex amplitudes of inhomogeneous counter-propagating waves [25]:

\[
\frac{d^2 \mathcal{E}}{dz^2} + \left[ \frac{\mu(z) \omega_m}{c_0} \right]^2 \mathcal{E} = 0 \quad \Longleftrightarrow \quad \begin{cases} 
\frac{d e_1}{dz} = e_2(2\mu)^{-1} \left( \frac{\partial \mu}{\partial z} \right) \exp(-2i\Psi_m), \\
\frac{d e_2}{dz} = e_1(2\mu)^{-1} \left( \frac{\partial \mu}{\partial z} \right) \exp(2i\Psi_m);
\end{cases} \quad (8)
\]

\[
\mathcal{E}(z) = \alpha_m \pi N d |\mu(z)|^{-1/2}\{e_1(z) \exp[i\Psi_m(z)] + e_2(z) \exp[-i\Psi_m(z)]\}.
\]

The last expression agrees with the preceding formulae (7) if one takes the zero-order mode structure with \( e_1 = 1, e_2 = -(-1)^m \), and \( \beta \approx 1 \). (We may consider the center resonance mode only because other modes (5), with \( \omega_m \neq \omega_0 + i \tilde{\omega} \), are slow in growing, and their weak \((\lambda/2)\)-lattices cannot change the scattering process up to the crucial moment, \( T \).)

Now, all are ready to estimate the efficiency of Bragg backscattering by means of the perturbation method. Namely, we have to fix one wave in equation (8), e.g., propagating to the right, \( e_1(z) = 1 \), and to find the amplitude of the backscattered wave, propagating to the left, \( \tilde{e}_2(z) \), under the zero boundary condition at the right facet, \( \tilde{e}_2(z = L/2) = 0 \). The approximative integration of the wave equation, by taking into account the inequality (6), leads to the following outgoing amplitude at the left facet:

\[
e_2 \left( z = -\frac{L}{2} \right) \approx (-1)^m \left\{ \frac{1}{2} + \left[ \frac{1}{4} + \tilde{n} \left( \frac{2\pi L}{\lambda \ln R^{-1/2}} \right)^2 \right]^{1/2} \right\} \frac{1 - R^{1/2}}{2\tilde{n}(1 + R^{1/2})} < \frac{1}{2} \quad (9).
\]

It is just the maximum efficiency of self-consistent Bragg coupling between counter-propagating SR waves in an open cavity.

A rather high-Q cavity, leading to oscillation SR [1,18,19], gives

\[
\ln R^{-1/2} \approx R^{-1/2} - 1 \lesssim \frac{2\pi I^{1/2} L}{\lambda} \Rightarrow \tilde{e}_2 \left( z = -\frac{L}{2} \right) \approx (-1)^m \left( \frac{\pi}{2\tilde{n} I^{1/2}} \right) I^{1/2} \frac{L}{\lambda}, \quad (10)
\]

so that \( |\tilde{e}_2| \ll 1 \). In the opposite case, a rather low-Q cavity, leading to single-pulse SR [1,18,19], gives

\[
\ln R^{-1/2} \gtrsim \frac{8\pi I^{1/2} L}{\lambda} \Rightarrow \tilde{e}_2 \left( z = -\frac{L}{2} \right) \approx (-1)^m \frac{(1 - R^{1/2})}{2\tilde{n}(1 + R^{1/2})}. \quad (11)
\]

For \( R \gtrsim 1/2 \) we have again \( |\tilde{e}_2| \ll 1 \). The limiting efficiency, \( |\tilde{e}_2(z = -L/2)| \approx 1/2 \), is realized for a very low-Q cavity only, when \( R \ll 1/2 \) and \( 1 - \tilde{n}(T) \approx 1/\ln R^{-1} \ll 1 \).

Thus, for any-Q cavity the scattering does not considerably alter the amplitudes of counter-propagating waves, \( e_1 = 1 \) or \( e_2 = -(-1)^m \), presented in the zero-order hot mode (3) \( \approx (8) \). Taking into account also the opposite signs of \( e_2 \) and \( \tilde{e}_2 \), we see that the self-consistent backscattering can only slow down a little the SR dynamics and slightly decrease the SR intensity. Hence, the leading edge (\( t < T \)) of the mode SR pulse is practically unaffected by the nonlinear Bragg scattering, which is unable to prevail over the linear reflections at mirrors. The same is true for the major part of the trailing edge (\( t > T \)), as it results from the close nonlinear analysis; see Sections 8 and 9. It can be shown that even in the optimum case of a quite low-Q cavity with \( R \sim e^{-2} \), the self-consistent backscattering makes a correction to the duration of SR pulse and to its amplitude of the order of 10% only. In particular, the Bragg scattering by the self-consistent \((\lambda/2)\)-lattice is too weak to produce the substantial nonlinear self-locking of a SR pulse inside an active sample, whatever a cavity will be.
So, we establish the central point of the nonlinear theory of mode SR in any-Q cavity (1). Namely, the resonance backscattering of forward-backward waves by a self-consistent ($\lambda/2$)-lattice is always weak and cannot affect essentially the SR pulse. At the same time the interference of inhomogeneous counter-propagating waves, phased in by reflections at mirrors, influences considerably the inversion dynamics and changes strongly the course of collective spontaneous emission. The reasonable neglect of Bragg backscattering allows us to describe analytically the nonlinear stage of mode SR.

4. TO THE SELF-CONSISTENT NONLINEAR THEORY

Now, let us motivate the necessity of the self-consistent nonlinear theory. Usually, the problem of collective spontaneous emission, i.e., superradiance (SR), is considered for a long homogeneous sample of two-level medium, e.g., for a cylinder, with cross-section $S$ and length $L \gg \lambda = 2\pi c_0/\omega_0$. Here $\omega_0$ is the frequency of an atomic transition, $c_0 = c_0^{-1/2}$ is the velocity of light in a medium with a background dielectric constant $\varepsilon_0$. Starting from the time zero, $t = 0$, a pump creating atomic inversion is turned off and, in particular, there is no external resonance field. The emission is begun with zero-point fluctuations of electric field, $E_0$, and polarization, $P_0 \sim (NSL)^{-1/2}Nd$, where $N$ is the density of atoms with dipole moment $d$. The electromagnetic waves propagating along the long axis of a sample, $z$, are enhanced, spontaneous emission of high-frequency atomic dipoles become synchronized. As a result, the narrow-directional, powerful pulse of SR (i.e., superfluorescence) is formed in a medium via induced atomic emission. Its duration is shorter than the time of incoherent relaxation of atomic polarization, $T_2$, and the time of spontaneous radiation of an isolated atom, $T_1$.

In the absence of linear coupling (backscattering) of counter-propagating waves, the regime of unidirectional SR takes place and the main SR pulses from the opposite sides of a sample are emitted independently. Due to initial spontaneous fluctuations, the forward-backward pulses do not correlate, and their delay times change by chance from shot to shot, with a standard deviation as much as a duration of a pulse. The amplitudes of counter-propagating waves may be different by many times, and their phases are random. That is why the regular inversion ($\lambda/2$)-lattice does not appear, and there is no nonlinear Bragg scattering produced self-consistently; see e.g., [5-12,26-28].

However, one may take the initial conditions for polarization and inversion to be like a regular lattice (even weak and inhomogeneous) with the period of half a wavelength, $\lambda/2$. Then, according to numerical calculations, the SR dynamics change essentially, and the lattice regularity is self-consistently kept due to interference of counter-propagating waves [10-12,18]. It results in the change of a SR pulse duration and in the correlation of forward-backward pulses. Notwithstanding, these facts do not testify to the efficiency of Bragg scattering yet. They are rather to associate with the artificial initial conditions. The extreme case of such forced synchronization, concerted with the linear backscattering of resonance waves, is given in [10-12] and called the “collective SR”.

More natural cases are bound up with spontaneous growth of a regular (inhomogeneous) structure of polarization and inversion without special initial conditions. It is possible when there is the linear coupling of counter-propagating waves due to reflections (from the boundaries or distributed within a sample). The interest in the role of reflections in the SR theory came into being long ago; see, e.g., [5-7,9]. Recently it was confirmed by experimental works [13-17]. The theory of SR under the condition of “given” Bragg diffraction of forward-backward pulses in a crystal is presented in [29-34].

In what follows, we consider the simplest situation, namely, a superradiant medium in an open symmetrical Fabry-Perot cavity with the reflection factor of mirrors $R$ (determined over the intensity); see Figure 4. (The extension to the case of different right and left mirrors, $R_r \neq R_l$, turns into replacing $R$ by $(R_rR_l)^{1/2}$ in all formulas below, with the exception of those for the
complicated spatial structure of field and polarization (16),(30),(31),(38); cf. [10–12].) Bearing in mind the leading role of mirrors, we run into SR of discrete “hot” modes that were analyzed in detail in [2–4] for the first time (the qualitative estimates were made also in [21]). The problem of competition of this SR regime with a unidirectional one was solved in [1,22] (the analogous results were obtained independently in [20]). In particular, there was estimated the critical reflection factor (1), $R_{cr}$, necessitating dominance of mode SR over unidirectional SR.

\[ R^{1/4} \]
\[ n, E \]

**Figure 4. Spatial structure of**

(i) normalized field amplitudes of inhomogeneous counter-propagating waves in a hot mode, $|E(z)|$ (dashed curve),

(ii) homogeneous lattice of population inversion of two-level atoms induced by this mode, $n(z)$ (solid curve), and

(iii) inhomogeneous envelope of inversion (dash-dot curve) in the low-Q Fabry-Perot cavity with length $L = 8\lambda$ and reflection factor of mirrors $R = e^{-4}$. The amplitude of polarization, $P$, of a resonance hot mode is given five times as small as the maximum possible value, $\max P = AN_0$.

It was argued that for $R > R_{cr}$ the regular spatial structure of inversion and polarization will appear from quantum noises. It follows the structure of the most unstable mode of a cavity and causes the correlation between counter-propagating waves. Note that the novel SR dynamics is peculiar to a very low-Q cavity, since for typical SR experiments the value of critical reflection factor is very small, $R_{cr} \sim e^{-5} \ll 1$. (The first estimates [5–7,21] of reflections providing SR insensitivity to the backscattering were put too low.)

The present linear theory [1,20] does not allow one to find the profile of SR pulse and to evaluate its duration and delay time correctly. What one needs is the nonlinear theory taking into account strongly inhomogeneous structures of inversion, $\Delta N(z,t) = N_2 - N_1$, polarization, $P(z,t)$, and field, $E(z,t)$ (Figure 4). Such a theory is developed in Sections 7–9. (The preceding approaches [23,24] are based on the approximation of weak forward-backward coupling, both nonlinear and linear, being not self-consistent for the mode SR on the whole.) To make the presentation more physical, we remember the main properties of hot modes in the simplified form in Section 5, and calculate the preliminary values of duration and intensity of the mode SR pulse, estimating velocity of the relaxation wave front of inversion, in Section 6.
5. DISSIPATIVE INSTABILITY OF NATURAL MODES

Keeping in mind the Fabry-Perot cavity, let us consider the plane waves that have linear polarization and propagate between mirrors in the direction ±z. In the absence of ohmic dissipation of field and incoherent relaxation of polarization and inversion, the Maxwell-Bloch equations take the following form in dimensionless variables (see, e.g., [1-12]):

\[
\begin{align*}
\frac{\partial^2 a}{\partial \tau^2} - \frac{\partial^2 a}{\partial \zeta^2} &= -2\varepsilon_0^{-1} \frac{\partial^2 p}{\partial \tau^2}, \quad a \equiv \frac{E}{2\pi d\Delta N_0}; \\
\frac{\partial^2 p}{\partial \tau^2} + p &= -2\varepsilon_0 I_n a, \quad p \equiv \frac{P}{d\Delta N_0}; \\
\frac{\partial n}{\partial \tau} &= 2\varepsilon_0 I_n \frac{\partial p}{\partial \zeta}, \quad n \equiv \frac{\Delta N}{\Delta N_0}; \quad 0 \leq \tau \equiv \omega_0 t,
\end{align*}
\]

\[
-\frac{B}{2} \leq \zeta \equiv \frac{\omega_0 \phi}{c_0} \leq \frac{B}{2}; \quad B \equiv \frac{L\omega_0}{c_0} \gg \pi, \quad I \equiv \frac{2\pi d^2 \Delta N_0}{\hbar \omega_0 \varepsilon_0} \ll 1.
\]

Here \(\Delta N_0 = N_2(\zeta, 0) - N_1(\zeta, 0)\) is an initial density of inversion. For definiteness, it is assumed to be homogeneous along the axis of a cavity. The nonlinear coupling of field and polarization oscillations due to induced modulation of inversion (14) is weak because the parameter \(I\) is usually small.

The linear theory means the approximation of fixed inversion (at the initial period of SR formation). Then, under the condition \(n(\tau) = \bar{n} = \text{const}\), the general solution of equations (12), (13) is the superposition of linear independent hot modes, which look like

\[
\begin{align*}
\left\{ a \right\}_m &= \frac{1}{4} \left\{ a \right\}_m \exp(-i\Omega_m \tau) \left\{ \exp \left[ i\Omega_m \int_0^\zeta \mu(\zeta', \Omega_m) d\zeta' \right] \\
&\quad -(-1)^m \exp \left[ -i\Omega_m \int_0^\zeta \mu(\zeta', \Omega_m) d\zeta' \right] \right\} + \text{c.c.} \simeq \frac{1}{4} \left\{ \frac{\alpha}{\rho} \right\}_m \exp(-i\Omega_m \tau) \\
&\quad \times \left\{ \exp \left[ \frac{(im + \ln R^{-1/2})\zeta}{B} \right] - (-1)^m \exp \left[ -\frac{(im + \ln R^{-1/2})\zeta}{B} \right] \right\} + \text{c.c.};
\end{align*}
\]

\[
\Omega_m \tilde{\mu}(\Omega_m) = \frac{(m\pi - i \ln R^{-1/2})}{B}; \quad \rho_m = \varepsilon_0 (\tilde{\mu} - 1) \alpha_m; \quad \mu \simeq 1 + \frac{nI}{(\Omega - 1)}; \quad \tilde{\mu}(\Omega_m) \simeq 1 + \frac{\bar{n}I}{(\Omega_m - 1)}, \quad \bar{n} = B^{-1} \int_{-B/2}^{B/2} n(\zeta) d\zeta.
\]

Here \(\mu(\zeta, \Omega_m)\varepsilon_0^{1/2}\) is a local refractive index at the discrete mode frequency \(\Omega_m\). (In the general case of inhomogeneous inversion, \(n = n(\zeta)\), all undermentioned results concerning hot modes are valid also; see Section 2.) In the dispersive equation (17), the value, \(\tilde{\mu}(\Omega_m)\) plays the part of an average refractive index for \(m\)-mode. In the equation (18) we use the resonance approach since only the modes keeping up the resonance between two-level atoms and electromagnetic field \((m - B/\pi = 0, \pm1, \pm2, \ldots, \ll B/\pi \equiv 2L/\lambda)\) are of interest. Among these modes with dimensionless frequencies \(\Omega_m' \equiv \text{Re} \Omega_m \simeq 1\), we find unstable modes [2-4], that have positive growth rates, \(\Omega_m'' > 0\):
Contrary to the electromagnetic modes in a cold cavity, the hot modes (19) in an open cavity with inverted active medium have negative energy and are associated with the so-called polarization waves [1-4]. Their instability has dissipative origin, i.e., it is based on the energy loss due to emission into the ambient space.

Taking into account the imaginary part of mode frequencies, one can prove that the complex refractive index is close to unity even in exact resonance, \( m = (1 + \tilde{n})B/\pi \):

\[
\mu_{\text{res}} - 1 \equiv \frac{\rho}{\varepsilon_0 \alpha} = -i \left[ \frac{K}{2} + \left( \frac{K^2}{4} + \tilde{n}I \right)^{1/2} \right]; \quad K \equiv B^{-1} \ln R^{-1/2} \ll 1.
\]  

Using the last inequality we mean that \( \ln R^{-1/2} \sim 1 \) (in addition to \( B \gg \pi \)). Formally, we are not bound to limit the range of possible values of reflection factor, \( 0 < R < 1 \), but actually, in the mode SR problem, we are interested in the narrower range, say, \( 0.1 \lesssim \ln R^{-1/2} < \ln R_{\alpha}^{-1/2} \).

The left inequality excludes high-Q cavities (\( R \rightarrow 1 \)), which prevent SR emission into the ambient space. The right inequality excludes low-Q cavities (\( R \rightarrow 0 \)), which do not influence the SR process [1-4,9]; see equation (1). Usually \( \ln R_{\alpha}^{-1/2} \sim 3 \), so that only a few modes near the resonance (\( |m - B/\pi| < 3 \)) have the growth rate of the order of the maximum, \( \tilde{\Omega}'' \), and can take part in the SR generation. Moreover, only one center mode with the number \( m \approx B/\pi \) plays the leading part, if initial conditions are quite homogeneous around a sample and initial amplitudes of all the modes are approximately the same. Indeed, it is easy to see that owing to the inequality

\[
\left( 1 - \frac{\Omega''_{m \pm 1}}{\Omega''_m} \right) \ln \left( \frac{P_{\text{max}}}{P_0} \right) \sim \left[ 1 + \pi^{-2} \left( \ln R^{-1/2} \right)^2 \right]^{-1} \ln (\Delta N_{0d}SL)^{1/2} \gg 1,
\]

the amplitude of the center mode surpasses all the other modes essentially up to the moment of maximum SR intensity when the center mode polarization has gone to the maximum value:

\[
P_{\text{max}} \sim \Delta N_{0d} \gg P_0 \sim (\Delta N_{0d}SL)^{-1/2} \Delta N_{0d}.
\]

The last statement is made under the usual assumption

\[
I \lesssim \frac{2}{B^2} \ll \frac{1}{2\pi^2}, \quad \text{i.e.,} \quad 3L_c \gtrsim L \gg \lambda,
\]  

where \( L_c = c_0 / 2\omega_0 I^{1/2} \) is the so-called "cooperative length"; see e.g., [1-12]. In other words, we assume that the light (with velocity \( c_0 \)) has time to run over a cavity during the period \( 1/\omega_0 I^{1/2} \), which is the order of the inverse growth rate of homogeneous plane waves in a boundless inverted medium and is equal to the inverse maximum growth rate of hot modes (19). The assumption (21) is the necessary condition of the mode SR regime that implies spontaneous forming and growth of quite independent hot modes.

Running ahead of Sections 8 and 9, we will focus more attention on the two additional useful relations simplifying the final results. The first one is

\[
\ln R^{-1/2} \gtrsim BI^{1/2} \Rightarrow \tilde{\Omega}'' \approx \frac{\tilde{n}IB}{\ln R^{-1/2}}.
\]
It excludes the oscillation regime of mode SR that is inefficient and possible only in a rather high-Q cavity with \( R \gtrsim 1/2 \). The condition (22) means that a reflection factor is low enough and field dissipation via outgoing radiation is quite efficient to diminish the growth rates of modes (19) appreciably as compared with the maximum value, \( I^{1/2} \).

On the other hand, in what follows concerning low-Q cavities \( (R \ll 1) \), we set the lower limit to a reflection factor by means of the second relation:

\[
R^{-1/2}(\pi + \ln R^{-1/2}) \ll B.
\]

This inequality allows us, in the first place, to neglect the reflection factor \(|\mu - 1|^2/4\), owing to the jump of refractive index at the boundaries of an active sample, as compared with the reflection factor of mirrors, \( R^{1/2} \). (Cf. [27] where the latter was taken zero, \( R = 0 \), and had to be replaced with the critical factor (1) \( R_{cr} \), in order to compare to the maximum active factor (20), \(|\mu_{res} - 1|^1/2\), thus excluding totally the mode SR regime in the case \( B > R_{cr}^{-1/2} \ln R_{cr}^{-1/2} \).) In the second place, it guarantees smoothness of mode amplitudes on the wavelength scale. In particular, as we shall show, it guarantees weakness of distributed backscattering due to the self-consistent inhomogeneous structure of refractive index at the nonlinear stage. In fact, the inequality (23) concerns very-low-Q cavities, and according to the estimate (1) it can exclude very short ones only, with the distance between mirrors as short as one hundred wavelengths.

### 6. ESTIMATION OF SR PULSE DURATION AND INTENSITY IN A VERY LOW-Q CAVITY

Let us estimate qualitatively the change-over of the nonlinear stage, namely, the change of intensity and duration of the mode SR pulse due to inhomogeneity of field envelope in the case \( R \ll 1 \) when the difference from the homogeneous mean field approach is the most considerable. Note that the scale of this inhomogeneity is much longer than a wavelength, \( \Delta z \sim L/\ln R^{-1/2} \gg A \), thus the associated additional reflections are negligible. Moreover, according to [18] and the results of Section 3 and 8, the Bragg backscattering due to the nonlinear small-scale \((\lambda/2)\)-lattice of atomic population inversion may be neglected also.

However, the cooperation of weak reflections from boundaries and strong amplification inside an active sample for \( \ln R^{-1/2} > 2 \) (see the mode structure in Figure 4 and the formula (16)) results in the great difference between the amplitudes of forward and backward propagating waves around a cavity, with the exception of the narrow center region, \( |z| \lesssim L/\ln R^{-1/2} \). Therefore, the saturation nonlinearity implements a waste of population inversion firstly at the sides of a sample, giving there \( n < 1/2 \) and even \( n < 0 \), while the principal central part of inverted medium remains unaffected by the nonlinear process and keeps longer the linear character of the dissipative SR instability. It makes longer the mode SR pulse, in contrast with the well-known prediction of the homogeneous mean field approach. Roughly speaking, we can define the duration of SR pulse, \( \Delta t = \Delta \tau/\omega_0 \), as the period of emission with intensity of the order of its maximum value. Let us show that the duration of the mode SR pulse has to be as long as several inverse values of the maximum growth rate, \( 1/\omega_0 \tilde{\Gamma}'' \) (though it remains shorter than the delay time, \( t_d = \tau_d/\omega_0 \)).

Using the linear theory (16)–(19) and taking an arbitrary point inside a cavity, \( \zeta > 0 \), we may estimate that moment, \( \tau \) \((\tau_2 < \tau < \tau_4 + \Delta \tau)\), when the dimensionless amplitude of polarization of the center mode \( P_m(\zeta, \tau) \), \( m \approx \pi/B \), with the greatest growth rate, reaches the maximum possible value, i.e., unity:

\[
\left(\frac{1}{2}\right) \rho_m \exp \left[ \tilde{\Gamma}'' \tau + \frac{(\ln R^{-1/2}) \zeta}{B} \right] \sim 1.
\]

This relation gives the coordinate as an explicit function of time:

\[
z \sim \left[ \ln(2\rho_m^{-1}) - \frac{t_2 \pi d^2 \Delta N_0 \tilde{\omega}_0 L}{c_0 \hbar \epsilon_0 \ln R^{-1/2}} \right] \frac{L}{\ln R^{-1/2}}.
\]

(24)
The point of sign-flipping of inversion, \( n(z(t)) = 0 \), i.e., the relaxation wave front, moves deep into a cavity approximately in accordance with the same law (24), since the atomic inversion is connected with the polarization via the conservation law for the Bloch vector length. It is obvious, that the period of the front running, \( z = z(t) \), over the half of a cavity, \( L/2 \), is of the order of the SR pulse duration:

\[
\Delta t \sim \frac{c_0 \hbar \epsilon_0 (\ln R^{-1/2})^2}{2 \pi d^2 \Delta N_0 \omega_0 L}.
\]

(The formula (25) results from the equation (24) with the following average value of population inversion: \( \tilde{n} \approx 1/2 \).) The corresponding velocity of the relaxation wave front coincides with the group velocity of dissipatively unstable polarization waves in the same but boundless medium that is supplied with additional (effective) ohmic losses, \( \sigma = (\ln R^{-1/2})c/2\pi L \), playing the part of radiation losses outward of a cavity [1]:

\[
v_g \approx \frac{c_0 \tilde{n} IL^2}{(\ln R^{-1/2})^2}.
\]

Under the condition (21) of the negligible delay effect, we find that the group velocity is small, \( v_g \ll c_0 \), and, hence, we may use the energy balance relation without taking into account the energy of electromagnetic field within a cavity. Thus, assuming again \( \tilde{n} \approx 1/2 \), we obtain the rough estimate of the average SR intensity, i.e., the average density of electromagnetic energy flux in a one-way pulse:

\[
\tilde{F} \sim \frac{\hbar \omega_0 \Delta N_0 v_g}{2} \approx \left( \frac{\pi c_0}{2 \epsilon_0} \right) \left( \frac{d \Delta N_0 \omega_0 L}{c_0 \ln R^{-1/2}} \right)^2.
\]

The intensity of a pulse becomes close to the maximum value at that moment, \( t_d \), when the saturation nonlinearity just wastes the population inversion at the very side of an active sample. This fact allows us to estimate the delay time,

\[
t_d \sim \frac{\Delta t \ln(\Delta N_0 L^2 \lambda R^{1/2})}{\ln R^{-1/2}},
\]

if we insert the coordinate, \( z = L/2 \), into the equation (24) and put the initial polarization amplitude to be small, \( \rho_m \sim (\Delta N_0 L^2 \lambda)^{-1/2} \ll 1 \) according to the well-known diffraction-fluctuation reasons [2–16]. So, in a low-Q cavity with \( R \ll 1 \), the delay time considerably exceeds the pulse duration (25), \( t_d > \Delta t \), if \( \Delta N_0 > 1/L^2 \lambda R \), i.e., even in the case of a rather weak initial inversion. The last inequality is practically guaranteed by the principal initial suggestion, \( R > R_{cr} \); see the equation (1). Independently from the estimation (27), one can also find the same density of field energy flux at the delay moment, \( t = t_d \), by means of the formula (16) and the relation \( \alpha_m \approx i \rho_m / \epsilon_0 K \):

\[
F(t_d) \approx \frac{c_0 \epsilon_0}{8\pi} (1 - R)|\pi d \Delta N_0 \alpha_m R^{-1/4}|^2 \exp \left[ 2 \int_0^{t_d} \hat{n}''(\tau) \, d\tau \right] \sim \tilde{F}.
\]

The trailing edge of a pulse (\( t > t_d \)) is expected to be more gentle than the leading edge due to reabsorption of radiation propagating from the active center region of a sample through its deactivated sides. However, the precise calculation of the mode SR profile needs the rigorous nonlinear theory which is developed below.

7. MODE-TYPE ANSATZ AND SPATIAL STRUCTURE OF BLOCH ANGLE

The foregoing qualitative analysis of peculiarities of the dissipative instability of hot modes (16) indicates the following form of solution of the equations (12)–(14), which is approximately valid.
both at linear and nonlinear SR stages in an open any-Q cavity:

\[ a(\zeta, r) = \left( \frac{1}{2} \right) A(\tau) \exp(-i\tau) \sinh \left( \zeta + \frac{B}{2} \right) + \int_0^\zeta \kappa d\zeta' + \text{c.c.} \]

\[ \equiv A(\tau) \left[ \cos \tau \cdot \sinh \left( \zeta + \frac{B}{2} \right) \cdot \sin \tau \cdot \sin \left( \zeta + \frac{B}{2} \right) \cdot \cosh \int_0^\zeta \kappa d\zeta' \right], \]  

\[ p(\zeta, \tau) = \left( \frac{1}{2} \right) \sin \varphi(\zeta, \tau) \exp(-i\tau) \cdot \left( \frac{\sin \left( \zeta + B/2 - i \int_0^\zeta \kappa d\zeta' \right)}{\sin \left( \zeta + B/2 - i \int_0^\zeta \kappa d\zeta' \right)} \right) + \text{c.c.} \]

\[ \equiv \sin \varphi(\zeta, \tau) \left[ \cos \tau \cdot \sinh \left( \zeta + \frac{B}{2} \right) \cdot \cosh \int_0^\zeta \kappa d\zeta' - \sin \tau \cdot \cos \left( \zeta + \frac{B}{2} \right) \cdot \sinh \int_0^\zeta \kappa d\zeta' \right] \]

\[ \times \left[ \sin^2 \left( \zeta + \frac{B}{2} \right) + \sinh^2 \int_0^\zeta \kappa d\zeta' \right]^{-1/2}, \]

\[ n(\zeta, \tau) = \cos \varphi(\zeta, \tau); \quad B^{-1} \int_{-B/2}^{B/2} \kappa(\zeta, \tau) d\zeta \equiv K = B^{-1} \ln R^{-1/2}. \]

To motivate this ansatz, we start with the note that the last identity guarantees the fulfillment of the boundary conditions for the field reflecting at the mirrors. The field structure (30) is fixed by a local (unknown) amplification factor, \( \kappa(\zeta, \tau) \equiv \Im \mu = Kn(\zeta, \tau)/\tilde{n}(\tau) \), and suits well the simultaneous action of several resonance modes (16). The slow varying amplitude factor, \( A(\tau) \), includes the exponential growth at the linear stage, that is assumed to have the growth rate of the order of the maximum value, \( \tilde{\Omega}'' \), the quantity \( L\omega_0/\pi\nu_0 \simeq m \) being integer for definiteness. The inhomogeneous structure of the field ansatz (30) takes into account only the local modulation of field with the spatial period \( \lambda \), which is approximately the same for all resonance modes. It is quite enough, according to the estimates presented below. That is why we neglect both the influence of an active medium on the phase change of a wave running through over a cavity (because \( \text{Re} \tilde{\mu}B \ll \pi \)) and the difference \( \sim 2\pi \) between these phase changes for several neighboring modes that can take part in the forming of SR pulse.

As for the polarization, its spatial structure can be essentially different from that of the linear field modes due to additional space-varying normalization in the ansatz (31). It pays for the nonlinear self-consistency between atomic inversion and polarization via the law of conservation of the Bloch vector length. The equation for the Bloch angle, \( \varphi \), results from the high-frequency averaging of the equation for inversion (14):

\[ \frac{\partial \varphi}{\partial \tau} = \varepsilon_0 IA(\tau) \left[ \sin^2 \left( \zeta + \frac{B}{2} \right) + \sinh^2 \int_0^\zeta \kappa d\zeta' \right]^{1/2} \]

\[ \times \left\{ 1 + \frac{\sin(\zeta + B/2) \cdot \cos(\zeta + B/2)}{\sin^2(\zeta + B/2) + \sinh^2 \int_0^\zeta \kappa d\zeta' \cdot \int_0^\zeta \frac{\partial \kappa}{\partial \tau} d\zeta'} \right\}. \]

Using the following equation (35) and the inequality (21), we can neglect the amendment to 1 inside the curly brackets as it is approximately equal to \(-2i \cos \varphi \). The same equation (33) with \{ = 1 results also from the equation for polarization (13) by equating zero with the coefficients in front of the time-periodic functions \( \sin \tau \) and \( \cos \tau \) there. This procedure will be correct if one can neglect the small derivatives of the second order, \( \frac{\partial^2 \sin \varphi}{\partial \tau^2} \), \( \frac{\partial^2 \pi}{\partial \tau^2} \), \( \frac{\partial \varphi}{\partial \tau} \left( \frac{\partial \pi}{\partial \tau} \right) \), and require
formally that

\[ \varepsilon_0 A(\tau) \cos \varphi \cdot \left\{ \begin{array}{l}
\text{ch} \int_0^\zeta \kappa \, d\zeta' \sin \left( \zeta + \frac{B}{2} \right) \\
\text{sh} \int_0^\zeta \kappa \, d\zeta' \cos \left( \zeta + \frac{B}{2} \right)
\end{array} \right\} \]

\[ \gg \sin 2\varphi \cdot \left[ \sin^2 \left( \zeta + \frac{B}{2} \right) + \text{sh}^2 \int_0^\zeta \kappa \, d\zeta' \right]^{-1/2} \cdot \left\{ \begin{array}{l}
\text{sh} \int_0^\zeta \kappa \, d\zeta' \cos \left( \zeta + \frac{B}{2} \right) \\
\text{ch} \int_0^\zeta \kappa \, d\zeta' \sin \left( \zeta + \frac{B}{2} \right)
\end{array} \right\} . \]

In fact, it is enough to consider the last inequalities simplified by averaging over a cavity. Then, performing their addition and subtraction term-by-term with subsequent reduction and integration with the weight \([\sin^2(\zeta + B/2) + \text{sh}^2 \int_0^\zeta \kappa \, d\zeta']\), we jump to the sole necessary condition of our derivation of the mode SR equations:

\[ |A(\tau)| \gg \left( \frac{4K}{\varepsilon_0 \text{sh} KB} \right) \left| \int_{-B/2}^{B/2} \sin \varphi \cdot \left[ \sin^2 \left( \zeta + \frac{B}{2} \right) + \text{sh}^2 K\zeta \right]^{1/2} \, d\zeta \right| . \]

It is written down under the assumption \(\kappa \simeq K\), justified below via the equation (37). We will not come again to this inequality since, applying to the following equations (38),(39) and to the inequality \(K \gg I\), one can satisfy oneself that the inequality (34) is valid in all cases of interest, both during the linear stage \((0, t_d - 1/w_0\Omega'')\), when \(|A| \ll \max |A|\), and during the emission of the main pulse \((t_d - 1/w_0\Omega'', t_d + \Delta t)\), when \(|A| \sim \max |A|\). Of course, the foregoing statement implies the reasonably homogeneous initial conditions, e.g., spontaneous ones, that evenly start all principal unstable modes (19). Otherwise, under specific (say, periodic) initial synchronization of active atoms along a cavity, it may turn out that such inequalities are broken down and the nonlinear SR dynamics is not described by the present mode-like theory; cf. [10-12].

Finally, we obtain the other two equations,

\[ \int_0^\zeta \frac{\partial \kappa}{\partial \tau} \, d\zeta' \sin(2\zeta + B) \simeq -4I \cos \varphi \cdot \left[ \sin^2 \left( \zeta + \frac{B}{2} \right) + \text{sh}^2 \int_0^\zeta \kappa \, d\zeta' \right] ; \]

\[ \varepsilon_0 \left[ \frac{dA}{d\tau} + A \cdot \left( \kappa + \frac{1}{2} \int_0^\zeta \frac{\partial \kappa}{\partial \tau} \, d\zeta' \cdot \text{sh} \int_0^\zeta 2\kappa \, d\zeta' \right) \right] \left[ \sin^2 \left( \zeta + \frac{B}{2} \right) + \text{sh}^2 \int_0^\zeta \kappa \, d\zeta' \right]^{1/2} , \]

from the field equation (12), again by neglecting the time derivatives of the second order as well as the space derivative \(\frac{\partial \kappa}{\partial \tau}\) and by equating zero with the coefficients in front of the time-periodic functions \(\sin \tau\) and \(\cos \tau\), with subsequent combination of the results. In both equations (35) and (36), we put the expression in the curly brackets, defined as in the equation (33), to be equal unity, \(\{ \} = 1\).

The solution of the equation (35), \(\kappa = \kappa(\tau)\), has an explosive character, i.e., after a finite time it can become infinite at some sections inside a cavity, especially around the points where \(\sin(2\zeta + B) \to -0\). However, the length of these sections that are “blown up” remains much less than the full length of a cavity, if we set the evolution equations a time limit of the order of several inverse growth rates, say \(\sim 10/\Omega''\) (see the formula (19)), that is enough for the nonlinear theory of the main SR pulse. This possibility of ignoring the “explosion” in the equation (35) is based on the stock of strength of the inequality (23) under the conditions (21). And what is more, excluding the “explosion” we may use the simplest approximation,

\[ \sin^2 \left( \zeta + \frac{B}{2} \right) + \text{sh}^2 \int_0^\zeta \kappa \, d\zeta' \simeq \sin^2 \left( \zeta + \frac{B}{2} \right) + \text{sh}^2 K\zeta , \]

around the major part of the active medium inside a cavity.
Thus, following the perturbation method we can exclude the derivative \( \frac{\partial}{\partial K} \) from the equation (36) by means of the refined equation (35), and then forget the latter and, instead of it, use the simplest approximation \( \kappa \approx K = \text{const} \) in all other formulae henceforth. The foregoing statement allows one to integrate the equation (33) and to find the spatial structure of the Bloch angle, with unknown time-varying factor only:

\[
\varphi \simeq \Theta(t) \cdot \left[ \sin^2 \left( \zeta + \frac{B}{2} \right) + \text{sh}^2 K \zeta \right]^{1/2} ; \quad \frac{d\Theta}{dt} \equiv \varepsilon_0 I A. \tag{38}
\]

The last identity introduces the amplitude of Bloch angle, \( \Theta(t) \), that does not depend on a coordinate, \( \zeta \). The given initial values of this amplitude, \( \Theta \), and its derivative, \( \frac{d\Theta}{dt} \), fix completely the initial distributions of field (30), polarization (31), and inversion (32) as well as their further time-space dynamics. Hence, the differences from the factorized structure of the Bloch angle (38), being weak at the initial linear stage of the spontaneous forming of modes under the conditions (21) and (23), remain weak also at the nonlinear stage in the process of the induced forming of the main part of a SR pulse. It is true in the course of that period, when the light still has time to run through over a cavity during the characteristic interval of inversion change. At the nonlinear stage the latter interval is less or of the order of the value \( v_g^{-1} L / \ln R^{-1/2} \) for \( R \ll 1 \) (see Section 6).

8. NOVEL POTENTIAL FUNCTION IN THE NONLINEAR PENDULUM EQUATION FOR MODE SR

So, we obtain the analytic formulas (38), (30)-(32) (with \( \kappa \approx K \)) that describe the main part of mode SR in an open cavity. The central point of the paper is the nonlinear pendulum-type equation that presents the evolution law of the amplitude of Bloch angle:

\[
d\frac{d\Theta}{dt^2} + \left( \frac{d\Theta}{dt} \right) \left[ K - I \nu(\Theta) \right] \simeq -l \frac{\partial V}{\partial \Theta}; \quad V = \frac{2K}{\text{sh} KB} \int_{B/2}^{B/2} \cos \left\{ \Theta \left[ \sin^2 \left( \zeta + \frac{B}{2} \right) + \text{sh}^2 K \zeta \right]^{1/2} \right\} d\zeta. \tag{39}
\]

This equation follows from the averaging of the equation (36) over a cavity. This procedure is inevitable as the field amplitude, \( A(\tau) \), is assumed to be independent on a coordinate, \( \zeta \). Without averaging, the ansatz (30)-(32) would contradict the fundamental equations (12)-(14). The effective potential, \( V(\Theta) \), is the final extract from the inhomogeneous field structure forced by the boundary conditions, i.e., by mirrors. The correction, \( I \nu(\Theta) \), to the decaying rate, \( K \), in the equation (39) may be associated with the self-consistent Bragg scattering due to \( (\lambda/2) \)-lattice of inversion inside a cavity. The straightforward calculations give

\[
\nu \simeq \left( \frac{4K}{\text{sh} KB} \right) \int_{K/2}^{K/2} \left[ \frac{\text{sh}(2K \zeta)}{\sin(2\zeta + B)} \right] \left[ \sin^2 \left( \zeta + \frac{B}{2} \right) + \text{sh}^2 K \zeta \right] \times \\
\times \cos \left\{ \Theta \left[ \sin^2 \left( \zeta + \frac{B}{2} \right) + \text{sh}^2 K \zeta \right]^{1/2} \right\} d\zeta \simeq 2K^2 \frac{(2G - \tilde{G})}{\text{sh} KB} \\
+ \left( \frac{8KG}{\pi \text{sh} KB} \right) \left\{ \int_{K/2}^{K/2} \left[ \cos(\Theta \text{sh} u) \text{ch} 4u - \left( \frac{1}{4} \right) \Theta \sin(\Theta \text{sh} u) \cdot \text{ch} u \cdot \text{sh} 4u \right] du \\
+ \int_{0}^{K(B-\pi)/2} \left[ \cos(\Theta \text{ch} u) \text{ch} 4u + \left( \frac{1}{4} \right) \Theta \sin(\Theta \text{ch} u) \cdot \text{sh} u \cdot \text{sh} 4u \right] du \right\} \\
+ \left( \frac{4KG}{\pi \text{sh} KB} \right) \left\{ \int_{K/2}^{K/2} \left[ \cos(\Theta \text{sh} u) \text{ch} 2u + \left( \frac{1}{4} \right) \Theta \sin(\Theta \text{sh} u) \cdot \text{ch} u \cdot \text{sh} 2u \right] du \\
+ \int_{0}^{K(B-\pi)/2} \left[ \cos(\Theta \text{ch} u) \text{ch} 2u + \left( \frac{1}{4} \right) \Theta \sin(\Theta \text{ch} u) \cdot \text{ch} u \cdot \text{sh} 2u \right] du \right\}
\]
+ \int_0^{K(B-\pi)/2} \left[ \cos(\Theta \Sigma u) \Sigma 2u + \left( \frac{1}{2} \right) \Theta \sin(\Theta \Sigma u) \cdot \Sigma u \cdot \Sigma 2u \right] du ;

G \equiv \left( \frac{1}{4} \right) \int_{-\pi/2}^{\pi/2} x(\sin x)^{-1} dx = 1 - \frac{1}{9} + \frac{1}{25} - \cdots \simeq 0.92;

\tilde{G} \equiv \left( \frac{1}{2} \right) \int_{-\pi/2}^{\pi/2} x \cot x \ dx = \left( \frac{\pi}{2} \right) \left[ 1 - \frac{\pi^2}{36} - \left( \frac{\pi^2}{60} \right)^2 - \cdots \right] \simeq 1.1.

The estimation of the foregoing expression shows that \( \nu(\Theta) \ll K \) under the conditions (21), (23). Hence, we really can neglect the self-consistent Bragg scattering of counter-propagating waves in accordance with the qualitative estimates in Section 3 and \cite{18}. This statement and the simplified equation (39) with \( \nu \simeq 0 \) are true, in particular, for a high-Q cavity with almost homogeneous mode structure, when \( R \gtrsim 1/2 \).

According to the equation (39), the peculiarities of nonlinear SR dynamics in the regime of dissipative instability of discrete modes are completely defined by the effective potential, \( V(\Theta) = \bar{n}(\tau)(2KB/\Sigma KB) \), that is associated with the instant value of mean (averaged) inversion, \( \bar{n}(\tau) \). The familiar homogeneous-mean-field approach \cite{5-18,28}, with the potential \( V \propto \cos \Theta \), is not suitable for any reflection factor, \( R \), and never correct.

In a high-Q cavity (\( R \gtrsim 1/2 \)), the potential is reduced to the Bessel function of zero order: \( V(\Theta) \simeq 2\bar{n} \simeq (4/\pi)J_0(\Theta) \). In the case of weak dissipation of field, \( K \ll R^{1/2} \), when the good mirrors prevent efficient emission outward from a cavity, the solution of the equation (39) is known as the optical nutation; cf. \cite{19,35}. It is the oscillation regime of mode SR in which the derivative of the second order, \( \frac{d^2\Theta}{dt^2} \), plays an essential part.

In the case of interest (22), \( K \gtrsim R^{1/2} \), when the mirrors are bad enough, the nutation is suppressed by the dissipation of field via outgoing radiation \cite{1,10-12}. So, the equation (39) describes the single-pulse regime of mode SR. (For \( R \gtrsim 1/2 \) the latter looks like the standard regime of Dicks SR in the homogeneous mean field approach, because the potential \( V(\Theta) \sim J_0(\Theta) \) is similar to \( \cos \Theta \) for \( \Theta \lesssim \pi \).) Thus, the derivative of the second order does not play any part, and we can simplify and solve the equation (39),

\[
\frac{d\Theta}{d\tau} \simeq -IK^{-1} \frac{dV}{d\Theta}. \tag{40}
\]

In the case of the Bessel function, \( \frac{dV}{d\Theta} \simeq -4(4/\pi)J_1(\Theta) \), for \( R \gtrsim 1/2 \) the solution may be written in the form of the following inverse function:

\[
\tau(\Theta) \simeq \left( \frac{\pi K}{4f} \right) \int_0^\Theta \frac{d\Theta}{J_1(\Theta)} ; \quad 0 < \Theta(0) \leq \Theta < 3.83 \ldots, \tag{41}
\]

the pulse amplitude (40) reaches the maximum value at the moment \( \tau = \tau_d \), when the amplitude of Bloch angle is \( \Theta = \Theta_d \sim \pi/2 \).

The equation (40) is approximately valid also in the case of an open low-Q cavity (\( R \lesssim 1/2 \)) since \( |\frac{d^2\Theta}{dt^2}| \lesssim K|\frac{d\Theta}{dt}| \) due to the inequalities (11) and \( |\frac{dV}{d\Theta}| \lesssim 1 \), the latter resulting from the definition (39) of the potential \( V(\Theta) \). However, the potential profile is different here and strongly depends on the reflection factor \( R \); see Figure 5. First of all, the maximum magnitude of potential decreases, \( V(0) \ll 2 \), though there is no change in the falling down law for small angles \( \Theta \), when \( \bar{n} \simeq 1 \):

\[
V \simeq 4R^{1/2}(1 - R)^{-1} \ln R^{-1/2} - \frac{\Theta^2}{2}, \quad \Theta \lesssim \frac{\pi}{2} (1 + R^{-1/2})^{-1}. \tag{42}
\]

Then, the local minimums and maximums become smoother with decreasing \( R \), and for \( R \ll e^{-4} \) they practically disappear. It corresponds to the very weak standing-wave structure of field modes, when there are no stagnant zones of atomic inversion, thanks to the strong difference
between amplitudes of inhomogeneous counter-propagating waves. Therefore, the SR oscillation practically disappears for a low-Q cavity, when \( \ln R^{-1/2} > 2 \gtrsim J^{1/2}B \). Moreover, for \( R < e^{-4} \) the incorporating of description of mode SR oscillation via taking into account the derivative of the second order, \( \frac{d^2\Theta}{dz^2} \), in the equation (39) may be incorrect. Indeed, this oscillation may be substantial only in the case of violation both of the inequality (21) and the equation (39), when the length of an active sample is much greater than the cooperative length, \( L \gg 3L_c \). If so, the mirrors play the zero part; and the SR, including its tailing oscillation, is subordinated to the usual unidirectional regime; cf. [1,20].

\[ 1.25 \]
\[ 0.75 \]
\[ 0.25 \]
\[ -0.25 \]
\[ -0.75 \]
\[ 2 \]
\[ 1 \]
\[ 0.5 \]
\[ -0.5 \]
\[ 0 \]
\[ 0.5 \]
\[ 1 \]
\[ 1.5 \]
\[ 2 \]

Figure 5. Effective potential, \( V(\Theta) \), and the derivative, \( \frac{6V}{6\Theta} \), versus the amplitude of Bloch angle (28), \( \Theta \), in the pendulum equation (29) for \( L = 8A \) and decreasing reflection factor of mirrors: (0) \( R = e^{-1/2} \), (a) \( R = e^{-2} \), (b) \( R = e^{-4} \), (c) \( R = e^{-6} \), (d) \( R = e^{-8} \).

In the limit of a very bad cavity \( (R \lesssim e^{-4}) \) for \( \Theta \gtrsim \pi R^{1/4} \), one may neglect the weak oscillation of potential, \( V(\Theta) \), originated from the local oscillation of inversion, \( n(\zeta) \), on the length scale smaller than \( L/\ln R^{-1/2} \). (These oscillations will occur after exhaustion of inversion at the sides of a cavity.) In this approximation under the conditions (21),(23), the mode SR regime is described by means of the McDonald function of zero order:

\[
V \approx \left( \frac{4}{\sinh KB} \right) \int_0^{KB/2} \cos(\Theta \sinh u) \, du \\
\approx 8R^{1/2}K_0(\Theta) \approx 8R^{1/2} \left\{ \left[ -\gamma - \ln \left( \frac{\Theta}{2} \right) \right] \left( 1 + \frac{\Theta^2}{4} \right) + \frac{\Theta^2}{4}, \quad \pi R^{1/4} \lesssim \Theta \lesssim 1, \right. \\
\left. \left( \frac{\pi}{2G} \right)^{1/2} \exp(-\Theta), \quad \Theta \gtrsim \frac{\pi}{2}, \right.
\]

where \( \gamma = 0.577 \ldots \). The steepest falling down of the potential is reached at the point \( \Theta = \Theta_d \approx \pi R^{1/4} \) that corresponds to the delay time, \( \tau = \tau_d \); see the equation (28). Here the derivative, \( \frac{d\Theta}{dz} \), takes the maximum value and the amplitude of field as well as the SR intensity becomes the most strong. In this case, the approximated equation (40) and the solution (38), (30)-(32) are true for sure, because \( |\frac{d^2\Theta}{dz^2}| \ll K|\frac{d\Theta}{dz}| \) under the condition (11) and the structure of inversion change during characteristic interval, \( v_g^{-1}L/\ln R^{-1/2} \) that exceeds the time of light running through over a cavity, \( 2L/c_0 \).
9. MAIN FEATURES OF NONLINEAR SR IN A LOW-Q CAVITY

Thus, the inequalities (21)−(23) allow us to describe approximately both linear and nonlinear stages of single-pulse mode SR in any-Q cavity by means of the simple equation (40) instead of the equation (39). So, the general solution of the single-pulse mode SR problem takes the following form of inverse monotone function:

$$\tau(\Theta) \simeq - \left( \frac{K}{I} \right) \int_{\Theta(0)}^{\Theta} \left( \frac{\partial V}{\partial \Theta'} \right)^{-1} d\Theta'. \tag{44}$$

Inverting it, one finds the oscillograms of the field amplitude and the Bloch angle (38) and, consequently, the space-time dynamics of field, polarization, and population difference between atomic levels in an active medium. Remember that the condition (21) is the necessary condition of mode SR and that violation of the condition (22) in a high-Q cavity with \( \ln R^{-1/2} \ll I^{1/2} L / 2L_c \) results in the oscillation regime. In the latter case the solution (44) (more precisely, (41)) is approximately valid only for the first half of the main SR pulse. The description of the subsequent oscillation weakly relaxing with the frequency of the order of the cooperative frequency of an active medium, \( 2\omega_0 I^{1/2} \), requires the solution of the complete equation (39); see [10–12].

It is clear from this discussion that there is the universal estimate of the maximum intensity, i.e., the density of electromagnetic energy flux through one mirror,

$$F(t_d) \sim \frac{(1 - R)}{4} c e_0^{1/2} \left( \frac{A 2 \pi d \Delta N_0}{R^{1/4}} \right)^2 \equiv \frac{\pi c_0}{8e_0} (1 - R) \left( \frac{d\Delta N_0 d\Theta/dr}{IR^{1/4}} \right)^2 \tag{45}$$

for the main SR pulse in an any-Q cavity. This estimate is based on the equation (39) and on the corresponding expression (19) for the growth rate. The latter gives the linear relation,

$$\Theta^{-1} \frac{d\Theta}{dr} \simeq \left( I + \frac{K^2}{4} \right)^{1/2} - \frac{K}{2} \simeq \frac{I}{(I + K^2)^{1/2}},$$

that is practically correct up to the maximum of the main pulse, more precisely, up to the value \( \Theta \sim \pi/2(1 + R^{-1/2})^{1/2} \). In the case of a high-Q cavity (\( R \gg 1/2 \)), the most interesting single-pulse regime (22) places the lower limit on the pulse duration of mode SR, \( \Delta t \), of the order of the inverse growth rate, \( (\omega_0 I/K)^{-1} \), that cannot be smaller than the period of optical nutation:

$$\Delta t \sim \frac{(\ln R^{-1/2})}{\omega_0 I B} \gtrsim \frac{\pi}{\omega_0 I^{1/2}}. \tag{46}$$

In the case of a low-Q cavity with reflection factor \( R < e^{-4} \), it is easy to substantiate all the qualitative conclusions from Section 3. Indeed, the solution (44) demonstrates that the SR pulse has a slanting flat-topped profile with the maximum intensity (27) = (29), the delay time (28), \( t_d \sim \Delta t \ln(\Delta N_0 L^2 \lambda R^{1/2}) \), and the characteristic duration (25), \( \Delta \tau \sim \Delta t \ln R^{-1/2} \), that is of the order of several inverse growth rates, \( (\omega_0 I/K)^{-1} \). In general, the numerical analysis of the equation (39) and the solution (44) is necessary to obtain quantitative results. The appropriate examples are presented in Figure 6.

In the limiting case \( R \ll e^{-4} \), the integral in the solution (44) can be calculated analytically:

$$\int_{\Theta(0)}^{\Theta} \left( \frac{\partial V}{\partial \Theta'} \right)^{-1} d\Theta' \simeq - \frac{1}{8R^{1/2}} \left\{ \begin{array}{ll}
8R^{1/2} \ln \left( \frac{\Theta}{\Theta(0)} \right), & \text{for } \Theta(0) \leq \Theta \lesssim (\frac{\pi}{2}) R^{1/4}, \\
\{\Theta^2 - [\Theta(\tau_0)]^2\}, & \pi R^{1/4} \lesssim \Theta \lesssim 1, \\
\{\exp(\Theta) - \exp[O(1)]\}, & \Theta > \frac{\pi}{2}.
\end{array} \right. \tag{47}$$
Figure 6. Characteristic profiles of normalized intensity, \( s_0 F(t)/c_0 d^2 N_0^2 = e_0^2 A^2 R^{-1/2}(1 - R) - \left[\frac{\partial^2}{\partial t^2}\right] R^{-1/2}(1 - R) \), of SR pulse emitted through a mirror of a low-Q cavity. They are found via numerical solution of the equation (29) for the same set of reflection factors, \( R \), as in Figure 5. The dashed curves show the approximate solutions (34), which are practically equal to the exact ones in the cases (b), (c), and (d). For definiteness, the initial values \( \frac{d\Theta}{dr} = 0, \Theta(0) = 10^{-2} \) and the parameters \( I = 10^{-4}, L = 8\lambda \) are chosen.

It gives the patent expressions for the derivative, \( \frac{d\Theta}{dr} \), and the field amplitude, \( A(\tau) \), in the three ranges of Bloch angle amplitude (38) indicated in the relation (47). These ranges correspond to the front, the smoothly slanting plateau, and the tail of a SR pulse:

\[
\frac{d\Theta}{dr} \approx \frac{IR^{1/2}}{K} \begin{cases} 
R^{-1/2}\Theta, & \Theta(0) \leq \Theta \leq \left(\frac{\pi}{2}\right) R^{1/4}, \\
8\Theta^{-1}, & \pi R^{1/4} \leq \Theta \leq 1, \\
8\exp(-\Theta), & \Theta \geq \frac{\pi}{2}.
\end{cases} \tag{48}
\]

We see the alteration of the type of solution from the exponential growth at the front, \( t < t_d \), to the root-decreasing (\( \propto 1/\sqrt{t - t_0} \)) at the plateau, \( t_d \leq t \leq t_d + \Delta t \), smoothly turning into the logarithmically weak tail (see Figure 6d). This approximate mode-like theory does not seek to describe the far tail of the SR pulse for large Bloch angles \( \Theta \gg 1 \), because the tail corresponds to the exponentially small average inversion, \( \tilde{n} \propto \exp(-\Theta) \), and to the exponentially large time, when the spatial structure of inversion can be strongly jagged and the separation of variables (38) is inadmissible.

In the general case of not very high-Q cavity, under the conditions (22) and (23), we can state only that quite total transfer of atomic population inversion around the whole volume of a cavity in the SR emission is impossible, while the oscillation of the SR intensity is almost inconspicuous. This statement differs substantially from that of the homogeneous mean field approach; cf. [5-18,28]. According to Figure 6, the profile of a pulse becomes more and more sharp as the reflection factor increases from \( R \sim R_{cr} \ll 1 \) up to \( R \sim 1/2 \). However, for \( R \geq 1/2 \) the profile does not depend on the reflection factor of mirrors (see the solution (41)) as the large-scale inhomogeneity of the field amplitude in modes of an open cavity becomes unessential. At the same time the intensity of the mode SR pulse (45) continues increasing, and its duration (46) continues shortening with further increasing of the reflection factor, \( R \rightarrow 1 \).
To be precise here, let us recall the limiting case (22) when single-pulse SR is violated and replaced by the long optical nutation. The corresponding limit,

\[(\ln R_{\text{opt}}^{-1/2})^2 \sim \left(\frac{1}{2}\right) IB^2 \equiv \frac{\pi d^2 AN \omega_0 L^2}{c^2 h} \lesssim 1, \quad (49)\]
determines the optimum reflection factor, \(R_{\text{opt}} \gg e^{-4}\), that ensures the maximum intensity (45) and the minimal duration (46) of mode SR:

\[
\max F(t_d) \simeq \frac{\hbar \omega_0 AN C_0 (1 - R_{\text{opt}}^{1/2})}{3}, \quad \min \Delta t \simeq \frac{2}{\omega_0 I^{1/2}} \quad (50)
\]

(cf. [10–12] and pay attention to the tacit condition, \(T_2 \gg \Delta t\), which means that the time of incoherent relaxation is large enough and will not hurt the optimal mode SR).

Note that the limiting estimates (50) can be obtained both from the ones of Section 6 for mode SR and from the well-known estimates for unidirectional SR in the absence of mirrors. However, the latter way is only fit for the particular case \(IB^2 \sim 2\), i.e., \(R_{\text{opt}} \sim e^{-2}\), when the velocity of the relaxation front (26) is formally equal to \(v_g \approx \omega_0 L^2\) and the length of an active sample is approximately three times the cooperative length, \(L \sim 3L_c\). In the case \(IB^2 \gg 2\), i.e., \(L \gg 3L_c\), the mode SR is impossible in accordance with the discussion of the inequality (21) in Section 5, and we come to the stochastic unidirectional SR, independent of mirrors; cf. [1–4]. In the opposite case \(IB^2 \ll 2\), i.e., \(L \ll 3L_c\), only a high-Q cavity with reflection factor \(R_{\text{opt}} \gtrsim 1/2\) may be optimal, so we cannot apply both the unidirectional model and the analysis of Section 6, where we meant a low-Q cavity with a small reflection factor \(R \ll 1\).

The point of Sections 6 and 9 is that using rather bad mirrors with reflection factors in the range \(R_{\text{cr}} \lesssim R \ll R_{\text{opt}}\) for a short enough cavity, \(L \ll 3L_c\), increases the intensity and shortens the duration of mode SR essentially in comparison with their values for unidirectional SR in the absence of mirrors. Besides, mode SR has strictly single-pulse character (44), unlike the self-similar oscillation character of unidirectional SR. What is more, the formulæ (38)–(41) show that the installation of mirrors with the optimal reflection factor, \(R \sim R_{\text{opt}}\), at the facets of a short sample allows one to obtain the limiting values of intensity and duration of a pulse (50), whereas for unidirectional SR one would obtain these values only in a long sample with the length \(L \sim 3L_c\) (for the same parameters of an active medium). Thus, the application of mirrors lengthens a sample effectively and gives significant gain in the size of active medium and in the power of preliminary pumping.

If the quality of mirrors is too high \((R > R_{\text{opt}})\) the SR regime becomes inefficient and turns into the oscillation regime described by the pendulum-type equation (39). Its solutions are essentially different from the well-known self-similar ones of the sine-Gordon equation describing relaxation oscillation of unidirectional SR; cf., e.g., [1,5–7,10–12,19,20,26,35]. The characteristic duration of the first powerful pulses of mode SR in the oscillation regime is, at the most short, \(\min \Delta t \sim 2/\omega_0 I^{1/2}\), but their intensity is many times, \((1 - R_{\text{opt}}^{1/2})/(1 - R^{1/2}) \gg 1\), smaller than the maximum value, \(\max F(t_d)\), in the formula (50).

10. CONCLUSIONS

We establish the new nonlinear equation (39) describing mode SR in an open any-Q Fabry-Perot cavity (1). The effect of interference between counter-propagating waves in the coherent process of nonstationary inhomogeneous exhaustion of population inversion of two-level medium (enhancing these waves) is analyzed consistently for the first time. According to the analytic solution (44) in the most interesting single-pulse regime, the profile of the mode SR pulse strongly depends on the reflection factor of mirrors, \(R\), and on the parameters of an active sample. By means of the rigorous theory, we find the maximum intensity and the minimal duration of the main pulse of mode SR as well as the optimal reflection factor, \(R_{\text{opt}}\), for a Fabry-Perot cavity.
It is demonstrated that the installation of a rather short sample of inverted atomic medium into the suitable low-Q Fabry-Perot cavity allows one to obtain the limiting intensity and duration of the main pulse of mode SR. These values would be achievable only in a very long sample if the mirrors were absent. The detailed description of oscillation SR dynamics in a high-Q cavity with reflection factor $R > R_{\text{opt}}$ reveals its optical-nutation character, while it is still mode SR. The differences between the homogeneous mean field approach, the unidirectional model without linear backscattering and the limit of a high-Q cavity have been discussed from the unique point of view.

The nonlinear theory of mode SR indicates the optimal conditions, the limiting possibilities and the advantages of using low-Q cavities for the SR generation of powerful ultrashort pulses. These advantages are still little studied experimentally, even in the optical range of spectrum. They are worth paying attention to especially in the ultrashort, X-range of spectrum, where high-Q cavities and pulse generation by means of usual lasing are not applied successfully. In conclusion, the coherent interaction between inhomogeneous counter-propagating waves, phase-matched by the reflections from the Fabry-Perot cavity mirrors, significantly influences the depletion of the inversion of the active media in the cavity and strongly alters the intensity, duration, and profile of a superradiance pulse.

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