Forecasting by exponential smoothing, the Box and Jenkins procedure and spectral analysis. A simulation study

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1. INTRODUCTION

In this introduction, we shall try to shed light on some basic ideas that lurk behind the title, and on the purpose of this study.
We shall indicate the main conclusions, details of which will be given in a later chapter.

1.1. Forecasting

When speaking about forecasting, one has to make a distinction between naive and causal prediction. The latter is performed by using a model composed of one or more equations, every such equation relating a dependent variable to one or more explanatory variables.
The former only takes time into account as an explanatory variable, be it explicitly or implicitly by relating the series to one or more of its lagged versions.
The assumption behind this is in fact stationarity. One presumes the dependent variable to behave in the future as it did in the past.
Defining this reasoning more precisely, the dependent variable is in fact correlated with hidden variables, that are on their turn correlated with time. The assumption therefore is that the correlation of these unknown variables with time will stay the same.
As a consequence, naive forecasts will nearly always miss turning-points. The advantage of naive models is, despite the fact that they don't give information concerning the underlying structure, that they are, relatively speaking, computationally much easier and faster to perform.
One could however expect causal predictions to outperform naive forecasts.
We write deliberately "could expect", since building a causal model is a difficult and longwinded task, behind which lurk many risks.
One has to overcome problems, such as the choice of the functional form, specification of the variables and often complicated estimation procedures.
When this matter is carried to a successful conclusion, the even more difficult task of prediction stands to be solved. Indeed, one is usually obliged to use forecasts of exogenous variables, that are based on subjective grounds. It therefore could well be that the resulting causal forecasts are worse than the naive predictions. However, one should be careful not to overlook the advantages of structural analysis. Indeed, the knowledge of the impact of the decision or explanatory variables on the dependent variables can sometimes be of more importance than making a good or easy forecast. Naive models are not so much a substitute for causal models, but imply a totally different attitude towards analysing a time series.

1.2. The purpose of this study-simulation

As the title indicates, the purpose of this study is to compare forecasts made by exponential smoothing, the Box and Jenkins procedure and by spectral analysis.
Bhansali [1] has made a similar study comparing spectral analysis with a technique called the "Regression-Akaike" method. His main conclusion is that the former performs quite well, especially when predicting more than one step ahead.
There are, in our opinion, however two drawbacks to his paper. First of all, he simulates samples of size 1.000. Theoretically, in order to prove asymptotic properties this is necessary, but conclusions concerning prediction, drawn out of such a study are of little importance to economists, since they will never be confronted with time series of that length.
It is then very much the question whether the spectral method will do equally well, using short series, since it is a well known fact that spectral analysis needs quite large samples.
Secondly, the underlying generating processes are of simple form. Indeed, the processes used are of the autoregressive (AR) and moving average (MA) type. Therefore he can make use of a regression technique called the "Regression-Akaike Method", which is computationally straightforward. If however, as we did, a slight complication is introduced, such as autocorrelation of the disturbance term in an AR-scheme, a mixture of both previously mentioned types results. These so called mixed autoregressive moving average (ARMA) processes however, according to Box and Jenkins [3], are very often encountered in reality, and do not lend themselves to such a simple estimation procedure.
The Box and Jenkins procedure, which is applicable to this kind of models, does in fact involve subjective reasoning at some stages.
This already explains our intention to use the Box and Jenkins and the spectral method.
Both methods are statistically refined, but involve a great deal of computation. It could therefore be argued

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whether the game is worth the candle. That is why we have planned to compare these two procedures with exponential smoothing, which is, we believe the most simple and commonly used method for forecasting. If indeed it should turn out to be only slightly worse, then one might question those more complicated methods.
A word has to be said about the method used for comparing these procedures, namely simulation. Before resorting to simulation, one has to be certain that the problem cannot be solved analytically, since the former is expensive and involves a great deal of work.
It can be shown [3, p. 107] that the constant model underlying the first order exponential smoothing, is equivalent to an integrated moving average of first order [IMA (1,1)], if the disturbance terms are uncorrelated. Therefore exponential smoothing will do worse or equally well as the Box and Jenkins procedure, according to whether the generating process is or is not an IMA (1,1).
In the case under study however, the disturbance terms are correlated. This being the case, it is impossible to draw conclusions from analytical arguments.
As for the Box and Jenkins procedure, compared to spectral analysis, it can be shown [1,2] that the asymptotic mean squared error of the former is, in general, smaller than the asymptotic mean squared error of the latter, if the coefficients of the ARMA process are exactly known. In practice however, these are rarely known, and are generally estimated from the data.
The above reasoning implies that for the general case no analytical comparison is possible, such that one is obliged to resort to simulation.

1.3. Main conclusions

Before going over to the study into more detail, we would like to state the main conclusions.
Although we were quite critical with regard to Bhansali's [1] results, in the particular ARMA (1,1) model we tested, having a small sample size and a more complicated generating process, they seem to remain valid.
Although we cannot generalize to other models than the ones tested, in the light of Bhansali's [1] study, we are inclined to presume that the results hold for other simple processes.
However, only further research can confirm this assumption.
Indeed, we found one of the two tested spectral methods to be the best method to put forward. It was superior or at least equivalent to the other methods.
Only for larger forecasting periods (greater than four), proved the Box and Jenkins procedure to be superior.
Exponential smoothing is clearly inferior to the other methods, since for all tested forecasting periods at least one method proves to be better or at least equivalent.

2. THE EXPERIMENT AND THE FORECASTING RESULTS

2.1. Preliminary remarks

a. Since there is contradiction in the literature concerning certain definitions, in order to avoid all misunderstanding, we shall define some concepts as they are used in this study.
Let $\{y_t, t=\ldots,-1,0,1,\ldots\}$ be a stochastic process with discrete time parameter. Assuming that this process has been observed over the period $T=\{1,2,\ldots,t\}$, the following definitions can be set forward.
Forecast error (FE) = $\hat{y}_{t+r} - y_{t+r}$; $\hat{y}_{t+r}$ being the point forecast for period $t+r$ made at time t, y_{t+r} being the observed value of the stochastic process.
Expected squared error (ESE) = $E(\hat{y}_{t+r} - y_{t+r})^2$.
Mean squared error (MSE) = $\frac{1}{n}\sum_{j=1}^{n}(\hat{y}_{t+r,j} - y_{t+r,j})^2$
The MSE is the simulated value for the ESE.
We therefore simulate the process n times over a period $\{1, 2, \ldots, t+r\}$ and take the average of the values $(\hat{y}_{t+r} - y_{t+r})$ over the n realisations.

b. *(Covariance) stationary time series*
In what follows we shall consider a generating process which is (covariance) stationary.
The stochastic process $\{x_t,\ t \in T\}$ is called covariance stationary, if for $m = 1, 2$, $E\{x_t^m\}$ exists and
$E\{x_{t_1}, x_{t_2}, \ldots, x_{t_m}\} = E\{x_{t_1+\tau}, \ldots, x_{t_m+\tau}\}$
for all $t_1, \ldots, t_m, \tau \in T$.

One could argue whether stationarity of the data is a reasonable assumption, since most economic time series are not stationary.
However, as Box and Jenkins state [3, chapter 4], many, if not most, series are nonstationary but homogeneous, which means that they are non-stationary only in level and/or slope, but that they can be transformed to stationary series by simple differencing.
Even non-stationarity of the variance can sometimes be accounted for by taking a logarithmic transformation.
Anyway, leaving the question undecided, neither of the three methods applies to series that cannot be transformed into stationary time series by one of the two previously mentioned methods.
Therefore we can state that non-stationarity is not a relevant factor in the comparison between the different methods. This being the case, we have not built in non-stationarity in the generating process.

c. We will not go into the theoretical aspects of the various forecasting techniques. The interested reader is referred to the bibliography at the end of this paper. The most relevant references are [1], [2], [3], [5], [10] and [29].

2.2. The results

2.2.1. THE EXPERIMENT

We have applied the three techniques to data generated by the following underlying process :

$$x_t - 0.5\, x_{t-1} = u_t + 0.6\, u_{t-1}$$

We thus generated a realisation $\{x_t,\ t = 1,\ldots, 80\}$ of the time series $\{x_t,\ t = \ldots, -1, 0, 1, \ldots\}$.
In order to be able to draw statistical meaningful conclusions, one has to have a sample containing several of such realisations.
We generated 50 replicates, each replicate being different by using a different stream of stochastic variates u_t.
From each replicate we used 75 observations to predict x_{76} up to x_{80}.

Some characteristics

1. The u_t are uncorrelated normal stochastic variates with expected value equal to zero, and variance equal to one [4 ; 20, p. 90].
2. Since, because of stationarity, $E(x_t) = E(x_{t-1})$ and $E(u_t) = 0$, $E(x_t) = 0$. For every series we have put the starting value equal to its expected value $x_0 = 0$.
3. The variance $\sigma_x^2 = E[x_t - E(x_t)]^2$, which after some computation gives $\sigma_x^2 = 2.61$.

2.2.2. EXPONENTIAL SMOOTHING

We applied the constant model $x_t = a + v_t$ to the data.

$$\sigma_v^2 = \sigma_x^2 = 2.61 \qquad \rho = d = \frac{E(x_t\, x_{t-1})}{\sigma_x^2} = 0.73$$

It can be shown [27], assuming a first order autoregressive scheme for the disturbance term, that the expected squared error for a forecast r periods ahead is :

$$ESE = E[\hat{y}_{t+r} - y_{t+r}]^2 = \sigma_u^2 \left[1 + \frac{\alpha}{2-\alpha} + \frac{2d\,(1-\alpha)\alpha}{(2-\alpha)(1-d+d\alpha)} - 2d^r \frac{\alpha}{1-d+d\alpha}\right] \qquad (1)$$

d being the autocorrelation coefficient of the disturbance terms.
In the case of correlated disturbance terms, it is actually possible to find a value of α that minimizes the ESE for given d and r.
Using expression (1), with d = 0.73, we found optimal α's of approximately 0.83, 0.55, 0.1, 0.1, and 0.1 for respectively r = 1, 2, 3, 4 and 5. Substituting these α's in (1), we find

ESE =				
	1.40	r = 1	2.84	r = 4
	2.32	r = 2	2.95	r = 5
	2.62	r = 3		

Since the u_t are normally distributed, we assume $\hat{x}_{t+r}$, the forecast for x_{t+r} made at time t, to be normally distributed.

Then $\sum_{j=1}^{50} \frac{(\hat{x}_{t+r,j} - x_{t+r,j})^2}{ESE}$ is χ^2-distributed with 49 degrees of freedom.

$$P\left\{\frac{ESE}{50}\,\chi^2_{\frac{\alpha}{2},49} < \frac{1}{50}\sum_{j=1}^{50}(\hat{x}_{t+r,j} - x_{t+r,j})^2 < \frac{ESE}{50}\,\chi^2_{1-\frac{\alpha}{2},49}\right\} = 1-\alpha .$$

In other words, with a probability of 95 %, in order for the model to be appropriate the MSE should fall within the regions :

r = 1	0.89 < MSE < 1.98
r = 2	1.48 < MSE < 3.29
r = 3	1.70 < MSE < 3.76
r = 4	1.81 < MSE < 3.99
r = 5	1.88 < MSE < 4.18

2.2.3. THE BOX AND JENKINS PROCEDURE

It can be shown [3] that the ESE for a forecast r periods ahead is :

$$ESE(r) = (1 + c_1^2 + \ldots + c_{r-1}^2)\,\sigma_u^2 ,$$

which gives for the consecutive forecasting periods :

ESE =	
1	r = 1
2.21	2
2.51	3
2.59	4
2.61	5

Again we can find the 95 % confidence interval on the MSE :

r = 1	0.64 < MSE < 1.42
2	1.41 < MSE < 3.14
3	1.61 < MSE < 3.56
4	1.66 < MSE < 3.68
5	1.67 < MSE < 3.71

After running through the three stages of the Box and Jenkins procedure, we finely computed the MSE, which can be found in table 1.

2.2.4. SPECTRAL ANALYSIS

It can be shown [1, 2] that the distribution of the forecast errors of the spectral prediction method is only known asymptotically. Therefore, working with a finite sample, we cannot set up confidence intervals for the MSE.
Applying the forecasting formula's of Bhansali [1, 2], we obtained the MSE for both spectral methods (see table 1).

Given the mean squared errors for the consecutive forecasting periods of the three considered methods, we are now ready to make a comparison between them.

2.3. Comparison of the results

2.3.1. INTRODUCTION

We give the results for the applied methods.

Table 1. MSE for the various forecasting techniques

	Smoothing	Box and Jenkins	Spectral 1	Spectral 2
r = 1	1.96	1.59	0.92	1.55
r = 2	3.21	2.06	1.01	0.17
r = 3	3.55	2.28	4.73	2.17
r = 4	3.25	2.82	2.49	0.32
r = 5	3.24	2.82	2.34	4.23

Clearly we could make a comparison between the results in the above table.
Since the MSE's are random variables however, the question arises whether the observed differences between the methods are significant or due to pure chance.
To answer this question, we shall have to apply certain statistical techniques. Some problems arise.

1. For the unbiased forecasting methods, - exponential smoothing and the Box and Jenkins procedure -, the mean squared error is equal to the estimated variance of the forecast errors.
This would lead us to believe the F-statistic $\frac{ns_1^2}{ns_2^2}$ is appropriate for comparing these methods.
However, since both methods apply a kind of weighted average to past observations to derive the forecasts, it is very likely the MSE's will be correlated.

Therefore $\frac{MSE1}{MSE2} = \frac{\chi_1^2}{\chi_2^2}$ is a ratio of two correlated χ^2-distributions, the distribution of which is not known. As a consequence we have to look for another method of comparison. The Pitman-test [22; 15, pp. 454-463] is appropriate for setting up confidence intervals for the distribution of the ratio of the variances of two related normal distributions.

2. For comparing the spectral technique with other methods, another problem arises. Indeed, since the forecast errors using the spectral method are biased, the MSE is no longer an estimate of the variance, but the variance plus the bias squared. Indeed :

$$E(\hat{x}_{t+r} - x_{t+r})^2 = \sigma^2 + [E(\hat{x}_{t+r}) - x_{t+r}]^2$$

Since the MSE is an estimate of $E(\hat{x}_{t+r} - x_{t+r})^2$, it is not an estimate of the variance.
Therefore the Pitman-test cannot be applied. In this case we shall use the sign test [25].

2.3.2. COMPARISON

In order not to burden the text with unnecessary comparisons, we shall go to work in the following way.
First we shall compare the Box and Jenkins procedure with exponential smoothing using the Pitman-test. If the Box and Jenkins procedure proves to be, as suspected, superior or at least as good, we drop the smoothing technique for further competition.
Secondly, we do the same with the two spectral methods, using the sign test. Finally we compare the left-over techniques.

2.3.2.1. Exponential smoothing versus Box and Jenkins

The Pitman-test provides us with confidence limits on $1 = \frac{\sigma_1^2}{\sigma_2^2}$, where σ_i^2 are the variances of two correlated normal distributions. The confidence limits for $\sigma^2_{SMOOTH}/\sigma^2_{B\&J}$ are given in table 2 for $a = 5\,\%$.

Table 2. Confidence limits for $\sigma^2_{SMOOTH}/\sigma^2_{B\&J}$

$a = 5\,\%$	LOWER LIMIT	UPPER LIMIT
r = 1	0.86	1.77
r = 2	1.10	2.20
r = 3	1.17	2.05
r = 4	0.92	1.44
r = 5	0.93	1.44

As one can see, only for r = 2,3 is σ_1^2/σ_2^2 significantly different from 1.
For these cases the Box and Jenkins procedure is superior. But even for r = 4, 5 the lower limit is seen to be close to 1, implying that the Box and Jenkins method is almost better.
If no general ranking can be made, one thing we can certainly state, is that the B&J procedure is superior or at least equivalent to smoothing.
This justifies our dropping of the exponential smoothing technique in the search of the best forecasting method.

2.3.2.2. Spectral method 1 versus spectral method 2

As explained above, we can no longer apply the Pitman-test for this comparison. Instead we used the sign test [25].
This test compares two methods by counting the number of times one method was closer to the true value than the other. Under the hypothesis that the methods are equivalent, with 5 % significance, and n = 50, the confidence limits for the number of times that method 2 forecasts closer to the true value than method 1, lie within the region [18, 32].

The actual comparison is given in table 3.

Table 3. Number of times spectral method 2 was closer than method 1.
Comparison method 2/method 1

r = 1	14	worse
r = 2	38	better
r = 3	29	equivalent
r = 4	28	equivalent
r = 5	10	worse

One must bear in mind that the sign test does not account for the magnitude of the difference between the two methods, only for the sign.
Therefore we must kind of subjectively consider the information contained in both tables 1 and 3.

2.3.2.3. Spectral methods versus Box and Jenkins

Here again we are obliged to resort to the sign test. The results are given in tables 4 and 5.

Table 4. Number of times spectral method 1 was closer than B & J.
Comparison spectral 1/B & J

r = 1	26	equivalent
r = 2	34	better
r = 3	16	worse
r = 4	37	better
r = 5	17	worse

Table 5. Number of times spectral method 2 was closer than B & J
Comparison spectral 2/B & J

r = 1	19	equivalent
r = 2	43	better
r = 3	18	equivalent
r = 4	42	better
r = 5	14	worse

2.3.2.4. Summary

Summarizing the information contained in tables 1, 3, 4 and 5, we would be inclined to set up the following table.

Table 6. Summary of results

r	Best	Second best
1	–	–
2	spectral 2	spectral 1
3	spectral 2/B&J	–
4	spectral 2/spectral 1	–
5	B&J/smoothing	–

2.4. Conclusions and final remarks

2.4.1. CONCLUSIONS

1) The exponential smoothing technique is clearly inferior to the other methods, since for all forecasting periods at least one method proves to be better or at least equivalent. This possibly results from the fact that we assumed, as is commonly done, a first order autoregressive model for the disturbance term, whereas in the Box and Jenkins procedure we do not restrict ourselves to such a simple model. However, to our knowledge, more complex models would make the use of exponential smoothing impossible in practice.

2) For small forecasting periods the second spectral model would seem to us the best method to put forward. It is superior or at least equivalent to the other methods.
A drawback however is the fact that the forecast errors are biased. This bias will most probably decrease, the larger the sample at hand, since Bhansali [2] proved the forecast errors to be asymptotically unbiased.
Another possible improvement due to a larger sample could be the fact that M = number of lags we estimate the covariance function for would be larger. As a consequence $N = \frac{M}{4}$ = the number of spectral estimates and forecasting coefficients would be greater too.
Bhansali [2] proved that the asymptotic variance of the forecast errors decreases for N increasing, as long as $N \leqslant \frac{M}{4}$.
Therefore it seems reasonable that the MSE would decrease too.
Therefore we suggest the spectral method 2 to be an excellent forecasting technique for time series with daily data, since a large sample is then already obtained with one year of observations.

3) For larger forecasting periods ($r > 4$), we would stick to the Box and Jenkins procedure.

2.4.2. FINAL REMARKS

1) One must bear in mind that the conclusions drawn are obtained from a simulation study. Therefore these conclusions hold only for the model studied. It is not possible to make extensions to allow for other generating processes. This doesn't mean that a simulation study has no value. Indeed, it was mentioned that simple ARMA-processes may represent many economic time series. Therefore the conclusions drawn are not so specific after all.
But one might investigate the influence on the results of for example other values of σ_u^2, larger ARMA-schemes with greater lags, or another starting value. All these are parameters that the experimenter has under his control, and might influence the results.

2) Another parameter that has to be supplied is the number of replicates. For some simulation experiments stopping rules exist [20].
In the study at hand the number of replicates only

influences the degree of accuracy of the mean squared error. Indeed, the mean squared error (for the unbiased cases) is $\frac{ESE}{50}\chi^2_{49}$ distributed, the variance of which is solely dependent on the number of degrees of freedom, and hence on the number of replicates. By a desired level of accuracy of β %, we mean that $(1-\alpha)$ %, e.g. 95 %, of the sampled MSE's fall within the interval

$$[E(MSE) - (\frac{1-\beta}{2})E(MSE),\quad E(MSE) + (\frac{1-\beta}{2})E(MSE)]$$

From the desired level of accuracy, one can derive the number of replicates needed. For 50 replicates, $\beta = 20$ %.
Taking into account the expensiveness of computer time, we have felt the increase in the level of accuracy, and hence the simplification obtained for comparing the methods, not to be offset by the increase in costs and time. Indeed, if the level of precision was say 80 %, the figures in table 1 would be subject to little or no variation. Hence, the figures would be suitable for a direct comparison, without running any further statistical tests. Running the statistical tests made our comparison more longwinded and the conclusions less firm, but we saved costs of computer time.
As always, a balance must be reached between additional costs and additional revenue.

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