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Regular rotating black holes and the weak energy condition

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ABSTRACT

We revisit here a recent work on regular rotating black holes. We introduce a new mass function generalizing the commonly used Bardeen and Hayward mass functions and extend the recently proposed solutions in order to accommodate a cosmological constant Λ. We discuss some aspects of the causal structure (horizons) and the ergospheres of the new proposed solutions. We also show that, in contrast with the spherically symmetrical case, the black hole rotation will unavoidably lead to the violation of the weak energy condition for any physically reasonable choice of the mass function, reinforcing the idea that the description of the interior region of a Kerr black hole is much more challenging than in the Schwarzschild case.

1. Introduction

The problem of spacetime singularities is an open issue in Physics (see, for instance, [1] for a general discussion and [2] for some cosmological implications). The general and commonly accepted belief is that only a not yet available quantum theory of gravity would be capable of solving them properly. In recent years, without a fully developed and reliable candidate for a quantum theory of gravity, many phenomenological models have been proposed for which the central singularity of a black hole is avoided (see, for a review and motivations, [3]). These non-singular solutions of General Relativity are the so-called regular black holes (BH) and, since there are strict uniqueness theorems for BH solutions of vacuum Einstein–Maxwell equations [4], they will necessarily require some kind of exotic matter/field or internal structure in order to exist. The typical stationary spherically symmetrical regular BH has line element

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \tag{1}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ and $f(r) = 1 - 2m(r)/r$. A mass function of the type

$$m(r) = \frac{M_0}{(1 + (\frac{r_0}{r})^q)^{\frac{p}{q}}} \tag{2}$$

will guarantee an asymptotic flat spacetime for positive p and q. M_0 and r_0 are, respectively, a mass and a length parameters. The well known Bardeen [3] and Hayward [5] BH correspond, respectively, to the choices $p = 3$, $q = 2$ and $p = q = 3$ in the mass function expression (2). The limits of small and large r of (2) are, respectively,

$$m(r) \approx M_0 \left(\frac{r}{r_0}\right)^p \tag{3}$$

and

$$m(r) \approx M_0 \left(1 - \frac{p}{q}\left(\frac{r_0}{r}\right)^q\right). \tag{4}$$

Both Bardeen and Hayward BH are realizations of a quite old idea, introduced by Sakharov and co-workers in the sixties [6] and later improved [7], that spacetime in the highly dense central region of a BH would be de Sitter-like

$$G_{\mu\nu} = -\lambda g_{\mu\nu}, \tag{5}$$

with $\lambda > 0$, which requires

$$f(r) = 1 - \left(\frac{r}{\ell}\right)^2 \tag{6}$$

for $r \approx 0$. Comparing with (3), we see that (6) demands $p = 3$. The only typical requirement on q is to get from (4) an asymptotically Schwarzschild solution, what requires only $q > 0$. We will consider here the general mass function with $p = 3$ and $q > 0$. Fig. 1 depicts some typical cases. Notice that for the mass function (2) with $p = 3$ one has

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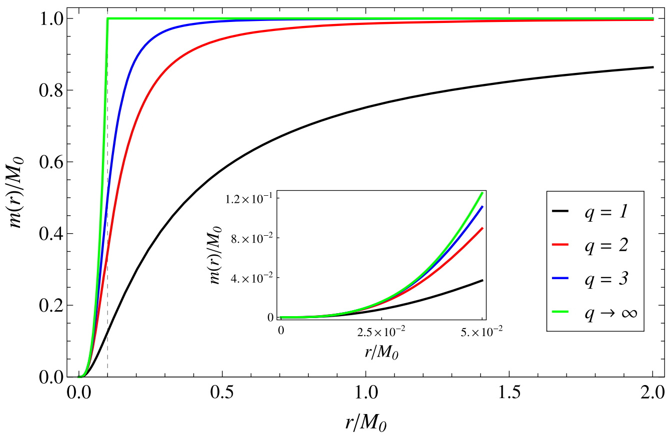

Fig. 1. Mass function (2) with $p = 3$ and $q > 0$. The parameter r_0 is typically assumed to be microscopic ($r_0 \ll M_0$) and, hence, the exterior region of the BH can be very close to the Schwarzschild spacetime. The $q \to \infty$ case corresponds to the usual matching between de Sitter and Schwarzchild solutions in the interior region of the BH. The dashed line indicates $r = r_0$. In this graphic we have used $r_0/M_0 = 10^{-1}$.

$$\lambda = \frac{6M_0}{r_0^3}. \tag{7}$$

Since r_0 is typically assumed to be microscopic ($r_0 \ll M_0$), the core density described by the central de Sitter solution (5) may be effectively very high, possibly in the regime where quantum gravity effects should come out. For the spherically symmetrical case (no rotation), several mass functions interpolating between the de Sitter core ($r \approx 0$) and the asymptotically flat infinity ($r \to \infty$) lead to physically reasonable regular black holes since, despite of violating the strong energy condition as required by the singularities theorem [1], they do obey the weak energy condition and, hence, might be in principle formed from a physically reasonable matter content.

In the recent work [8], Bambi and Modesto explore the Newman–Janis algorithm [9] in order to construct rotating regular BH with Bardeen and Hayward mass functions. One of their conclusions is that for these two commonly used mass functions, the weak energy condition (WEC) is violated due to the rotation of the black hole. Despite of being physically problematic due to the violations of WEC and, hence, to the presence of negative energy density content somewhere, such solutions as those ones introduced by Bambi and Modesto are certainly interesting from a phenomenological point of view, since astrophysical bodies, the main data sources for exploring BH physics, typically have nonvanishing angular momentum. In the present paper, we extend the Bambi and Modesto solutions for the case where a cosmological constant Λ is present, a situation which could be useful, for instance, to the studies involving rotating black holes and the AdS/CFT conjecture [10]. We consider mass functions of the type (2) with $p = 3$ and $q > 0$, but some of our conclusions are valid for any physically reasonable functions, *i.e.*, functions compatible with the behavior (5)–(6) near the origin. We discuss some aspects of the causal structure (event, cosmological, and Cauchy horizons) and the ergospheres of the new proposed solutions. We show also that the violation of WEC is indeed generic and unavoidable for rotating BH, irrespective of the used mass terms, with the only requirement that they behave as $m(r) \propto r^3$ for $r \to 0$, which is necessary to have a behavior similar to (5) and hence to have an extremely dense central region, but free of singularities. Our result is another indication that the description of the interior region of the Kerr BH is a much more challenging problem than in the Schwarzschild case.

2. The regular Kerr black hole with cosmological constant

We will not follow here the same approach (the Newman–Janis algorithm [9]) used by Bambi and Modesto in [8], but rather we will employ the so-called Synge g-method: assume $g_{\mu\nu}$, calculate and interpret $T_{\mu\nu}$. For an early application of Synge g-method to the problem of the interior of the Kerr black hole, see [11]. Our main goal is to extend Bambi and Modesto solutions for the case where a cosmological constant Λ is present, and for other mass functions as well. We envisage two possible coordinate systems to explore here. The first one is related the so-called Kerr–Schild ansatz with cosmological constant (see, for instance, [12])

$$ds^2 = ds^2_\Lambda + H\left(l_\mu dy^\mu\right)^2, \tag{8}$$

where ds^2_Λ is a pure anti-de Sitter (AdS, $\Lambda < 0$) or de Sitter (dS, $\Lambda > 0$) metric, H is a smooth function, and l_μ stands for a null vector. By introducing the $(\tau, r, x = \cos\theta, \varphi)$ spheroidal coordinates, Eq. (8) can be decomposed as

$$\begin{aligned} ds^2_\Lambda = &-\frac{(1-\frac{\Lambda}{3}r^2)\Delta_x}{\Xi}d\tau^2 + \frac{\Sigma}{(1-\frac{\Lambda}{3}r^2)(r^2+a^2)}dr^2 \\ &+ \frac{\Sigma}{(1-x^2)\Delta_x}dx^2 + \frac{(r^2+a^2)(1-x^2)}{\Xi}d\varphi^2 \end{aligned} \tag{9}$$

and

$$l_\mu dy^\mu = \frac{\Delta_x}{\Xi}d\tau + \frac{\Sigma}{(1-\frac{\Lambda}{3}r^2)(r^2+a^2)}dr - \frac{a(1-x^2)}{\Xi}d\phi, \tag{10}$$

where

$$\Delta_x = 1 + \frac{\Lambda}{3}a^2x^2, \qquad \Sigma = r^2 + a^2x^2, \qquad \Xi = 1 + \frac{\Lambda}{3}a^2. \tag{11}$$

The constant a will be later interpreted as the rotation parameter, but notice that it is present already in the pure AdS or dS metric (9) due to the use of spheroidal coordinates. Notice that our construction requires $\Xi > 0$, leading to the restriction

$$\Lambda > -\frac{3}{a^2}, \tag{12}$$

and we will adopt this hypothesis hereafter. We will return to this point in the causal structure analysis in the next section. We assume also rotational symmetry, and hence $H = H(r, x)$. A particularly convenient choice is

$$H(r, x) = \frac{2m(r)r}{\Sigma}. \tag{13}$$

For this case, with a r-dependent mass functions $m(r)$, one can introduce the usual Boyer–Lindquist coordinates (t, r, x, ϕ) by means of the following coordinates transformation

$$d\tau = dt + \frac{\Sigma H}{(1-\frac{\Lambda}{3}r^2)\Delta_r}dr, \tag{14}$$

$$d\varphi = d\phi - \frac{\Lambda}{3}adt + \frac{a\Sigma H}{(r^2+a^2)\Delta_r}dr, \tag{15}$$

where

$$\Delta_r = \left(r^2+a^2\right)\left(1-\frac{\Lambda}{3}r^2\right) - 2rm(r). \tag{16}$$

The metric (8) will then take the Boyer–Lindquist form

$$
\begin{aligned}
ds^2 = &-\frac{1}{\Sigma}\left(\Delta_r - \Delta_x a^2\left(1-x^2\right)\right)dt^2 \\
&- \frac{2a}{\Xi\Sigma}\left[\left(r^2+a^2\right)\Delta_x - \Delta_r\right]\left(1-x^2\right)dt\,d\phi \\
&+ \frac{\Sigma}{\Delta_r}dr^2 + \frac{\Sigma}{(1-x^2)\Delta_x}dx^2 + \frac{1}{\Xi^2\Sigma}\left[\left(r^2+a^2\right)^2\Delta_x \right.\\
&\left.- \Delta_r a^2\left(1-x^2\right)\right]\left(1-x^2\right)d\phi^2, \qquad (17)
\end{aligned}
$$

which is much more convenient for the analysis of the causal structure of the spacetime, and we will adopt it hereafter. We will come back in the last section to the case of possible mass functions of the type $m = m(r, x)$, for which the above coordinate transformations are not defined.

For r-dependent mass functions $m(r)$, the curvature scalar for the metric (8) or (17) reads simply

$$
R = 2\frac{rm''(r) + 2m'(r)}{r^2 + a^2x^2} + 4\Lambda, \qquad (18)
$$

and, in spite of its simplicity, this expression retracts rather well the behavior of the singularities of the spacetime. For generic $m(r)$, we have a singularity at $x = 0$ (equatorial plane) for $r \to 0$. The situation is equivalent to the divergence of the Kretschmann scalar on a "ring" for the usual Kerr spacetime (constant m, $\Lambda = 0$). In order to avoid such singularities for non-constant m, the numerator of (18) must vanish as r^α for small r, with $\alpha \geq 2$, which is satisfied for any mass function behaving as $m(r) \approx M_0(\frac{r}{r_0})^3$ for $r \to 0$. Moreover, for mass functions of this type, we have an "effective" de Sitter core (5) on the equatorial plane ($x = 0$) and the corresponding Kretschmann scalar will read

$$
\begin{aligned}
K = &96\left(\frac{M_0}{r_0^3}\right)^2 \frac{r^4(r^8 + 4a^2x^2r^6 + 11a^4x^4r^4 - 2a^6x^6r^2 + 6a^8x^8)}{(r^2+a^2x^2)^6} \\
&+ 32\frac{M_0}{r_0^3}\frac{\Lambda r^2}{r^2+a^2x^2} + \frac{8}{3}\Lambda^2, \qquad (19)
\end{aligned}
$$

from where one can deduce straightly all the cases considered by Bambi and Modesto in [8] for $\Lambda = 0$. The inclusion of the cosmological constant Λ does not alter their conclusions about the avoidance of the central singularity.

In order to analyze the matter content associated with (17), we introduce the usual orthonormal tetrads [13]

$$
e_\mu^{(a)} = \begin{pmatrix} \sqrt{\mp(g_{tt} - \Omega g_{t\phi})} & 0 & 0 & 0 \\ 0 & \sqrt{\pm g_{rr}} & 0 & 0 \\ 0 & 0 & \sqrt{g_{xx}} & 0 \\ g_{t\phi}/\sqrt{g_{\phi\phi}} & 0 & 0 & \sqrt{g_{\phi\phi}} \end{pmatrix}, \qquad (20)
$$

which corresponds to the standard locally non-rotating frame, with $\Omega = \frac{g_{t\phi}}{g_{\phi\phi}}$ being interpreted as the angular velocity of the BH. Notice that (12) suffices to assure $g_{xx} > 0$ and, since

$$
g_{\phi\phi} = \frac{(r^2+a^2)(r^2+a^2x^2)(1+\frac{\Lambda}{3}a^2) + 2a^2rm(r)(1-x^2)}{(1+\frac{\Lambda}{3}a^2)^2(r^2+a^2x^2)}, \qquad (21)
$$

it also assures a regular and positive $g_{\phi\phi}$. Moreover, from

$$
\begin{aligned}
&g_{tt} - \Omega g_{t\phi} \\
&= -\frac{1}{3}\frac{(r^2+a^2x^2)(1+\frac{\Lambda}{3}a^2)[(r^2+a^2)(1-\frac{\Lambda}{3}r^2) - 2rm(r)]}{(r^2+a^2)(r^2+a^2x^2)(1+\frac{\Lambda}{3}a^2) + 2a^2rm(r)(1-x^2)}
\end{aligned}
\qquad (22)
$$

we see that the condition (12) also assures that $g_{tt} - \Omega g_{t\phi}$ does not diverge and has a opposite sign of

$$
g_{rr} = \frac{r^2+a^2x^2}{(r^2+a^2)(1-\frac{\Lambda}{3}r^2) - 2rm(r)}. \qquad (23)
$$

Finally, the signs in (20) must be selected in accordance with the considered region. The spacetime (17) has generically two or three horizons (see next section) located at the zeros of $g^{rr} = g_{rr}^{-1}$. The innermost corresponds to a Cauchy horizon, and since we are considering BH solutions, a event horizon will be necessarily present. The region outside the event horizon and the region inside the Cauchy horizon correspond to the choice $(-,+)$, respectively, in the components $e_0^{(0)}$ and $e_1^{(1)}$. In this case, $e_\mu^{(0)}$ is timelike. On the other hand, the choice $(+,-)$ corresponds to the regions contained between the Cauchy and the event horizons, where $e_\mu^{(1)}$ will be timelike. Our main argument here is based on the behavior of the energy–momentum tensor near the origin and, hence, inside the Cauchy horizon. The expressions for the components of the energy–momentum tensor in the orthonormal tetrads frame $T^{(a)(b)} = \frac{1}{8\pi}e_\mu^{(a)}e_\nu^{(b)}G^{\mu\nu}$ are rather cumbersome, but they simplify considerably for $x = \pm 1$ (the "poles" along the rotation axis). In particular, for $x = \pm 1$, $T^{(a)(b)}$ is diagonal. One can check that $T^{(0)(3)}$ does vanish for $x = \pm 1$ by a direct calculation, but this result could also be advanced from the fact that smooth "tangential" flows as the ones corresponding to $T^{(0)(3)}$ must vanish along the symmetry axis. The non-vanishing $T^{(a)(b)}$ components for $x = \pm 1$ in the region outside the event horizon or inside the Cauchy horizon read simply

$$
T^{(0)(0)} = \frac{2r^2m'(r)}{8\pi(r^2+a^2)^2} + \frac{\Lambda}{8\pi} = -T^{(1)(1)}, \qquad (24)
$$

$$
T^{(2)(2)} = -\frac{2a^2m'(r) + r(r^2+a^2)m''(r)}{8\pi(r^2+a^2)^2} - \frac{\Lambda}{8\pi} = T^{(3)(3)}. \qquad (25)
$$

For the region inside the Cauchy horizon, $e_\mu^{(0)}$ is timelike and the WEC on the "poles" reads $T^{(0)(0)} \geq 0$ and $T^{(0)(0)} + T^{(i)(i)} \geq 0$, $i = 1, \dots, 3$. In the present case, the condition for $i = 2$ or 3 requires $m'(r) \leq 0$ near the origin for $a \neq 0$, a rather unnatural requirement for any mass function in this context. In fact, for our paradigmatic case $m(r) \propto r^3$ one has

$$
T^{(0)(0)} + T^{(2)(2)} = T^{(0)(0)} + T^{(3)(3)} \propto -\frac{12r^2a^2}{(r^2+a^2)^2} \qquad (26)
$$

for $r \approx 0$ and $x = \pm 1$, from where one can see that the violation of WEC cannot be prevented if $a \neq 0$, irrespective of the value of Λ and the details of $m(r)$ far from the origin.

Notice that for $r \gg r_0$, one has

$$
R \approx 4\Lambda + \frac{6M_0}{r_0^3}\frac{1-q}{1+\frac{a^2x^2}{r^2}}\left(\frac{r_0}{r}\right)^{q+3} \qquad (27)
$$

for mass functions of the type (2) with $q > 0$. We see from Eq. (27) that the larger the value of q, the faster the solution converges to the vacuum solution for large r, as one would indeed expect. The $q = 1$ case corresponds to a Reissner–Nordstrom-like solution for which the energy–momentum tensor is traceless and, hence, only the cosmological constant counts for the Ricci scalar R. From (27), one can estimate, for instance, the deviations from the usual Kerr solution. For a BH with $r_0 = \varepsilon M_0$, $\varepsilon < 1$, the curvature deviations in the external region ($r > M_0$) have an upper bound given by ε^{q+1}/M_0^2. The exterior regions of a regular rotating and a Kerr BH can be almost indistinguishable for high values of q.

3. Spacetime causal structure of the regular solutions

From the metric in the Boyer–Lindquist coordinates (17), we can find out the roots of g^{rr}, which will provide the radii of the horizons. The pertinent equation is $\Delta_r = 0$, which leads to

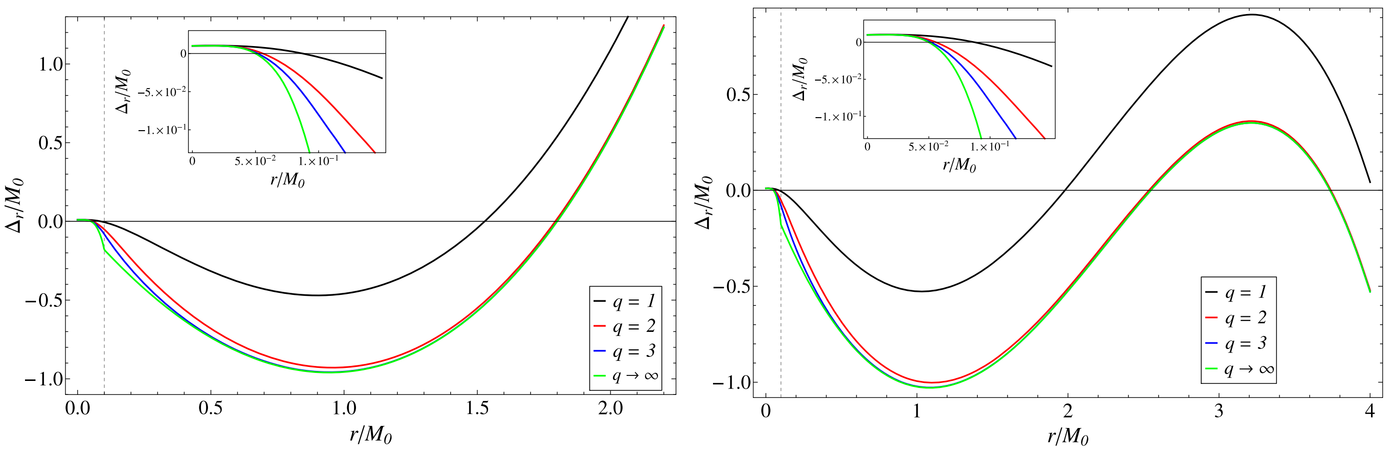

Fig. 2. The horizons corresponding to the zeros of Δ_r. Left: AdS case ($\Lambda < 0$), right: dS case ($\Lambda > 0$). The dashed line indicates $r = r_0$. In these graphics we have used $p = 3$, $r_0/M_0 = a/M_0 = \pm\Lambda M_0^2 = 10^{-1}$.

$$F(r) \equiv \frac{r^2 + a^2}{2r}\left(1 - \frac{\Lambda}{3}r^2\right) = m(r). \tag{28}$$

For $a \neq 0$, the left-handed side of (28) behaves as a^2/r for $r \approx 0$. On the other hand, for $\Lambda \neq 0$, it goes as $-\Lambda r^4/3$ for large r. Since the metric in the Boyer–Lindquist coordinates (17) is independent of t and ϕ, we have two explicit Killing vectors $\xi_t = \frac{\partial}{\partial t}$ and $\xi_\phi = \frac{\partial}{\partial \phi}$. Since $g_{\phi\phi} > 0$ by hypothesis due to the condition (12), ξ_ϕ is spacelike everywhere. For ξ_t, we have

$$|\xi_t|^2 = g_{tt} = -\frac{(r^2 + a^2x^2)(1 - \frac{\Lambda}{3}(r^2 + a^2(1 - x^2))) - 2rm(r)}{r^2 + a^2x^2}. \tag{29}$$

A Killing horizon corresponds to a surface with a null type tangent Killing vector. Thus, one can locate Killing horizons by setting (29) to zero, and the pertinent equation is similar to (28), namely

$$F_x(r) \equiv \frac{r^2 + a^2x^2}{2r}\left(1 - \frac{\Lambda}{3}\left(r^2 + a^2\left(1 - x^2\right)\right)\right) = m(r). \tag{30}$$

Notice that

$$F(r) - F_x(r) = \frac{a^2(1 - x^2)}{2r}\left(1 + \frac{\Lambda}{3}a^2x^2\right), \tag{31}$$

and from (12), we have $F(r) \geq F_x(r)$. The cases of AdS and dS are qualitatively different and we will treat them separately. Notice, however, that for $x = \pm 1$, the Killing and event horizons coincide ($F(r) = F_x(r)$). The situation is identical to the Kerr solution. On the other hand, outside the symmetry axis, the Killing and ordinary horizons do not coincide, giving origin to the ergoregions as in the usual Kerr solution.

3.1. Asymptotically AdS case

For the AdS case ($\Lambda < 0$), $F(r)$ has a global minimum located at

$$r^2_{\mathrm{min}} = \frac{1}{6\Lambda}\left(3 - a^2\Lambda - \sqrt{\left(3 - a^2\Lambda\right)^2 - 36a^2\Lambda}\right). \tag{32}$$

The existence of a black hole here requires two zeros for (28), the inner (Cauchy, $r = r_-$) and the outer (event, $r = r_+$) horizons. A sufficient condition for this is $F(r_{\mathrm{min}}) < m(r_{\mathrm{min}})$. It is not a surprise that certain combinations of parameters do not correspond effectively to black holes, a similar behavior is observed already for the simplest cases with $a = \Lambda = 0$ [5]. The algebraic expressions for the roots are quite involved and we will omitted them. Fig. 2 depicts some typical cases for the Cauchy and event horizons. Since $F(r) \geq F_x(r)$, the condition $F(r_{\mathrm{min}}) < m(r_{\mathrm{min}})$ also assures two roots for (29), which will correspond to the Killing horizons $r = S_-$ and $r = S_+$. For $x^2 \neq 1$, we can divide the spacetime structure in five regions

$$0 < S_- < r_- < r_+ < S_+ < \infty. \tag{33}$$

This situation is depicted in Fig. 3. The region between r_+ and S_+ is the ergosphere, with the same properties of the usual ergosphere in the Kerr solution. In the present case, however, we also have an interior ergosphere, which corresponds to the region limited by S_- and r_-.

3.2. Asymptotically dS case

The dS case ($\Lambda > 0$) is more involved. The function $F(r)$ can have up to two critical points. Besides r_{min} given by (32), we have also

$$r^2_{\mathrm{max}} = \frac{1}{6\Lambda}\left(3 - a^2\Lambda + \sqrt{\left(3 - a^2\Lambda\right)^2 - 36a^2\Lambda}\right). \tag{34}$$

In order to have r_{min} and r_{max} properly defined, one needs $\Lambda < 3(7 - 4\sqrt{3})/a^2$. This condition, in addition to $F(r_{\mathrm{min}}) < m(r_{\mathrm{min}})$ and $F(r_{\mathrm{max}}) > m(r_{\mathrm{max}})$, is sufficient to guarantee three zeros for (28), which corresponds to the inner ($r = r_-$), event ($r = r_+$), and cosmological ($r = r_c$) horizon. The situation where the cosmological and event horizon coincide corresponds to a rotating Nariai solution, see [14]. We will focus here only the situations containing black holes and, hence, we assume that all the necessary conditions are met.

For $\Lambda > 0$ we can have also up to three Killing horizons, namely S_-, S_i, and S_+. The spacetime can be divided into six regions for $x^2 \neq 1$

$$0 < S_- < r_- < r_+ < S_i < S_+ < r_c. \tag{35}$$

The internal ergosphere is limited by S_- and r_-, as in the AdS case. The external ergosphere consist effectively in two regions, namely those ones limited by r_+ and S_i, and by S_+ and r_c. These regions are disjointed for $x^2 \neq 1$.

4. Final remarks

We have here extended the recent work of Bambi and Modesto [8] where regular rotating BHs were introduced. Our solutions ac-

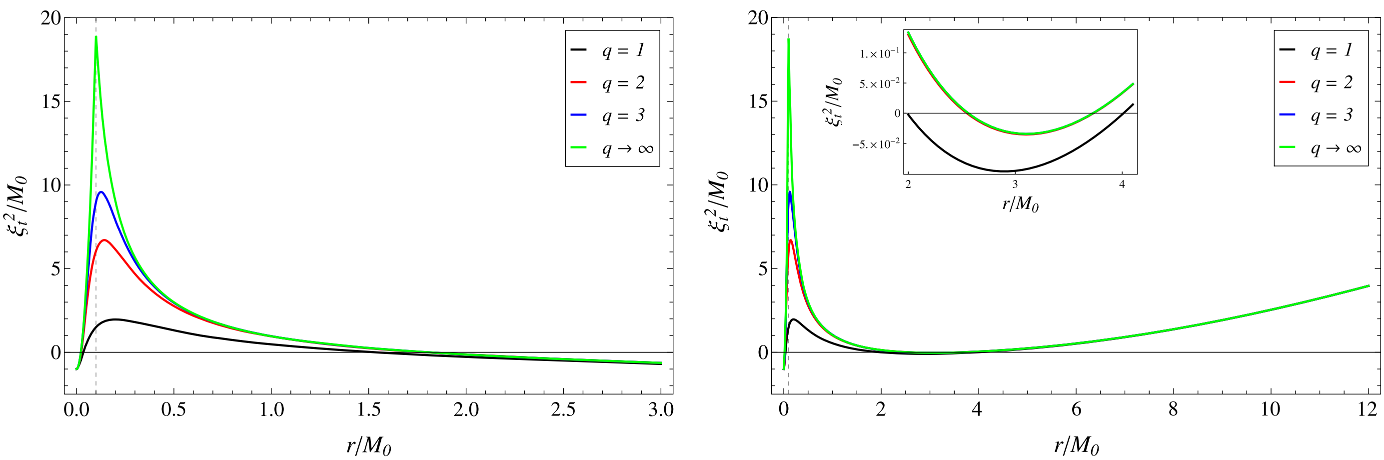

Fig. 3. The Killing horizons corresponding to the zeros of (29), on the equatorial plane ($x = 0$). Left: AdS case ($\Lambda < 0$), right: dS case ($\Lambda > 0$). The dashed line indicates $r = r_0$. In these graphics we have used $p = 3$, $r_0/M_0 = a/M_0 = \pm\Lambda M_0^2 = 10^{-1}$.

commodate a cosmological constant Λ and we have also introduced a more general mass function. We have shown that the black hole rotation will unavoidably lead to the violation of the weak energy condition (WEC) for any physically reasonable mass function. Despite of the violations of WEC, solutions as those ones introduced by Bambi and Modesto and extended here are important not only from a phenomenological point of view, but it could also contribute to the study of possible violations of the cosmic censorship conjecture in quasi-extremal black holes [15]. These points are now under investigation.

We finish by noticing that the case of a r-dependent rotation parameter a in the Boyer–Lindquist metric (17), as discussed in [8], will give origin to nonvanishing shear components (namely T^{rx}) in the energy–momentum tensor, challenging the physical interpretation of the matter content of such solutions, as advanced early in the work [16]. The same occurs if one allows a mass function of the type $m(r, x)$. In this case, moreover, one cannot obtain a Boyer–Lindquist metric from the Kerr–Schild ansatz since the coordinate transformations (14)–(15) are not properly defined, leading to an extra ambiguity: the solutions with $m(r, x)$ of the form (8) and (17) are inequivalent, and both correspond to energy–momentum tensors with nonvanishing shear components. It is not clear how to interpret physically BH solutions with r-dependent rotation parameter a and/or with mass functions of the type $m(r, x)$.

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