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Designing optimal controllers for nonlinear frames by considering the effect of response feedback

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Abstract The effect of response feedback on designing optimal controllers for nonlinear frames has been studied. Different combinations of response feedback have been used in the performance index. The Newmark based nonlinear instantaneous optimal control algorithm has been used as the control algorithm in controlling the response of an eight-story bilinear hysteretic frame subjected to white noise excitation and real earthquakes, and controlled by either eight actuators or a single actuator. While the objective has been to minimize the maximum control force for reducing the maximum drift to below the yielding level, the distributed genetic algorithm (DGA) has been used to determine the proper set of weighting matrices in the performance index. Results show that the performance of the active control system depends on the combination of response feedback, where the velocity feedback has been more effective than acceleration and displacement. Also, although using the full feedback of response in the performance index leads to the design of optimal controllers that require the smallest control force, it is costlier, because it requires more online measurements. Finally, it has been concluded that amongst all possibilities, using only velocity feedback can provide the best results regarding the maximum required control force and online measurement, simultaneously.

1. Introduction

Over the past decades, many theoretical and experimental studies have been undertaken on the application of active control systems to linear and nonlinear structures [1,2]. Also, active control systems have been installed in prototype, full scale structures [1,3–5]. According to these researches, many active control mechanisms and algorithms have been proposed in the literature, most of which have been developed for linear systems. Some examples are classical optimal control, pole assignment, predictive control [6] and instantaneous optimal control [7] methods, as well as intelligent control, such as neural network and fuzzy logic based methods for active and semi-active control [8–12]. However, there are some methods developed to work with nonlinear and hysteretic systems, such as active pulse control [13], instantaneous optimal control for active and hybrid control [14,15], predictive instantaneous optimal control [16] and predictive optimal linear control [17], as well as neural networks and fuzzy logic based control methods for active and semi-active control [18–22]. Joghataie and Mohebbi [23] developed Wilson's-θ based instantaneous optimal control for controlling nonlinear and hysteretic confined masonry walls. Also, Joghataie and Mohebbi [24] proposed the distributed genetic algorithm based nonlinear optimal control algorithm for controlling nonlinear frames. In the methods proposed in [23,24], the full feedback of response has been included in the control law for designing optimal nonlinear controllers, which requires the measurement of displacement, velocity and acceleration of all floors, needing a large number of sensors. In this paper, following the method proposed in [23], the Newmark based instantaneous optimal control algorithm for controlling Multi Degree Of Freedom (MDOF) nonlinear hysteretic frames has been used, while different response feedbacks have been included in the performance index.

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In most of the proposed active control algorithms for linear and nonlinear structures, such as classical optimal control and instantaneous optimal control, a time-dependent quadratic performance index has been defined, which includes the feedback of response and control force [6]. Also, the performance index uses a positive semi-definite weighting matrix for the response and a positive definite matrix for the control force. In previous research different combinations of response feedback, such as velocity–displacement [25], velocity–acceleration [26] and full response feedback (displacement, velocity and acceleration) [23,27], have been used in the performance index. Chang and Yang [25] used the velocity–displacement of the system in the performance index of the instantaneous optimal control algorithm for active control of linear structures, and concluded that to design a controller for which the maximum required control force is minimum, using velocity feedback in the performance index is more efficient than displacement feedback. Since time delay and its reduction is a decisive issue in designing active control systems, it is preferred to use the response components which are easiest to measure. Yang and Li [26] have stated that the measurement of acceleration and velocity of the structural response is easier than displacement, and hence they have proposed a new instantaneous optimal control law that uses the acceleration and velocity responses in the performance index. They concluded that the proposed algorithm works as well as if velocity–displacement feedback is used. To modify the mass, damping and stiffness matrices of the structure, Bahar et al. [27,28] proposed an instantaneous optimal control which uses the displacement, velocity and acceleration response in the performance index, and studied the preliminary effects of some combinations of weighting matrices on the performance of an active tuned mass damper for linear frames. In previous studies, the effect of response feedback in the control law for linear and especially nonlinear frames, has not been studied in depth. In this paper, the design of optimal controllers for nonlinear frames, using the method proposed by Joghataie and Mohebbi [23], has been studied, where different response feedbacks have been included in the performance index to assess their effects on the effectiveness of nonlinear controllers, regarding the maximum required control force and the number of sensors that should be used for response measurement.

On the other hand, in those active control algorithms that use a performance index to determine the control force, the control law includes response and control force weighting matrices. Some papers have discussed criteria and qualitative approaches for the selection of weighting matrices based on trial and error [27] or some simplifier assumptions [25,29]. Such techniques of determining the weights are not systematic, and the mitigation of response to a specified desired level generally requires extensive numerical analysis. Joghataie and Mohebbi [24] used optimization techniques, such as the Genetic Algorithm (GA) [30,31], to determine the weighting matrices. In their method, which has the significant advantage of being systematic and also requiring smaller control forces, the weighting matrices are determined through solving an optimization problem using a Distributed Genetic Algorithm (DGA) [32,33]. The same method has been used in this paper. However, practical limitations have been considered in selecting the form of the weighting matrices, and hence only diagonal forms have been studied.

In the following sections, first, the equations and algorithm for a Newmark based nonlinear instantaneous optimal control algorithm will be explained briefly. Next, a brief explanation of the Distributed GA (DGA) will be presented, followed by an example and conclusions.

2. Newmark based nonlinear instantaneous optimal control algorithm

In this paper, for the active control of a nonlinear, n degree of freedom (n-DOF) structure, following the methods proposed by Joghataie and Mohebbi [23,24], the Newmark based nonlinear instantaneous optimal control has been used, which is reported briefly in this section. Defining t = time, k = integration time step, $\ddot{X}_g$ = ground acceleration, $\mathbf{X}$, $\dot{\mathbf{X}}$ and $\ddot{\mathbf{X}}$ = displacement, velocity and acceleration vectors, respectively, $\mathbf{M} = n \times n$ mass matrix, $\mathbf{D} = n \times m$ location matrix of actuators, $\mathbf{e} = [-1, -1, \ldots, -1]^T$ = n-dimensional ground acceleration transformation vector, $\mathbf{u}(t)$ = m-dimensional control force vector and Δ = variation of a parameter or a vector between $(k-1)$ and k time steps. The response of a nonlinear structure at time $(k)\Delta t$ is obtained by solving the set of equations of motion at times $(k-1)\Delta t$ and $(k)\Delta t$ using the Newmark method [24], as follows:

$$\mathbf{X}_k = \mathbf{X}_{k-1} + \Delta\mathbf{X}_k, \tag{1}$$

$$\dot{\mathbf{X}}_k = (1 - a_5)\,\dot{\mathbf{X}}_{k-1} - a_6\ddot{\mathbf{X}}_{k-1} + a_4\Delta\mathbf{X}_k, \tag{2}$$

$$\ddot{\mathbf{X}}_k = (1 - a_3)\,\ddot{\mathbf{X}}_{k-1} - a_2\dot{\mathbf{X}}_{k-1} + a_1\Delta\mathbf{X}_k, \tag{3}$$

$$\Delta\mathbf{X}_k = \mathbf{K}_{n_k}^{*-1}\Delta\mathbf{F}_k, \tag{4}$$

$$\mathbf{K}^*_{n_k} = a_1\mathbf{M} + a_4\mathbf{C}^*_{k-1} + \mathbf{K}^*_{k-1}, \tag{5}$$

$$\begin{aligned}\Delta\mathbf{F}_k \;=\; & \Delta\mathbf{P}_k + \mathbf{M}\left(a_2\dot{\mathbf{X}}_{k-1} + a_3\ddot{\mathbf{X}}_{k-1}\right)\\ & + \mathbf{C}^*_{k-1}\left(a_5\dot{\mathbf{X}}_{k-1} + a_6\ddot{\mathbf{X}}_{k-1}\right),\end{aligned} \tag{6}$$

varies at each time step. $\mathbf{K}^*_{n_k}$ where

$$\Delta\mathbf{P}_k = (\mathbf{Me}\ddot{X}_{g_k} + \mathbf{Du}_k) - (\mathbf{Me}\ddot{X}_{g_{k-1}} + \mathbf{Du}_{k-1}), \tag{7}$$

$$a_1 = \frac{1}{\delta\,(\Delta t)^2}, \tag{8a}$$

$$a_2 = \frac{1}{\delta\Delta t}, \tag{8b}$$

$$a_3 = \frac{1}{2\delta}, \tag{8c}$$

$$a_4 = \frac{\gamma}{\delta\Delta t}, \tag{8d}$$

$$a_5 = \frac{\gamma}{\delta}, \tag{8e}$$

$$a_6 = \Delta t\left(\frac{\gamma}{2\delta} - 1\right), \tag{8f}$$

where γ and δ are Newmark parameters [34]. Also, $\mathbf{C}^*$ and K^* are tangential damping and stiffness matrices, respectively.

2.1. Performance index

In instantaneous optimal control, the performance index at time step k includes feedback of the system response and control force. To assess the effect of displacement, velocity and acceleration response on the performance of the control system, the full feedback of the system response and control force have been included in the performance index as:

$$J_k = \frac{1}{2}\left(\mathbf{X}_k^T\mathbf{Q}_1\mathbf{X}_k + \dot{\mathbf{X}}_k^T\mathbf{Q}_2\dot{\mathbf{X}}_k + \ddot{\mathbf{X}}_k^T\mathbf{Q}_3\ddot{\mathbf{X}}_k + \mathbf{u}_k^T\mathbf{R}\mathbf{u}_k\right), \tag{9}$$

where $\mathbf{Q}_1$, $\mathbf{Q}_2$ and $\mathbf{Q}_3$ are $n \times n$ positive semi-definite weighting matrices corresponding to the penalty for large displacements, velocities and accelerations, and $\mathbf{R}$ is a $m \times m$ positive definite matrix representing the cost of applying large forces [6].

2.2. Determination of control force vector

In instantaneous optimal control, at each time step k the control force $\mathbf{u}_k$ is determined by minimizing the performance index J_k, which has been defined in Eq. (9), at the same step. To this end, the equations of motion, Eqs. (1)–(3), are considered as constraints, and the Hamiltonian of the optimization problem is formed according to Chang and Yang [25], as follows:

$$\begin{aligned} H_k = {} & \frac{1}{2}\left(\mathbf{X}_k^T\mathbf{Q}_1\mathbf{X}_k + \dot{\mathbf{X}}_k^T\mathbf{Q}_2\dot{\mathbf{X}}_k + \ddot{\mathbf{X}}_k^T\mathbf{Q}_3\ddot{\mathbf{X}}_k + u_k^T\mathbf{R}u_k\right) \\ & + \boldsymbol{\lambda}_1^T\left(\mathbf{X}_k - \mathbf{X}_{k-1} - \Delta\mathbf{X}_k\right) \\ & + \boldsymbol{\lambda}_2^T\left[\dot{\mathbf{X}}_k - (1-a_5)\,\dot{\mathbf{X}}_{k-1} + a_6\ddot{\mathbf{X}}_{k-1} - a_4\Delta\mathbf{X}_k\right] \\ & + \boldsymbol{\lambda}_3^T\left[\ddot{\mathbf{X}}_k - (1-a_3)\,\ddot{\mathbf{X}}_{k-1} + a_2\dot{\mathbf{X}}_{k-1} - a_1\Delta\mathbf{X}_k\right], \qquad (10) \end{aligned}$$

where $\boldsymbol{\lambda}_{1,2,3}$ = Lagrangian multipliers. The necessary conditions for minimizing the performance index, $J(t)$, are:

$$\begin{aligned} \frac{\partial H_k}{\partial \mathbf{X}_k^T} = \frac{\partial H_k}{\partial \dot{\mathbf{X}}_k^T} = \frac{\partial H_k}{\partial \ddot{\mathbf{X}}_k^T} = \frac{\partial H_k}{\partial \mathbf{u}_k^T} = \frac{\partial H_k}{\partial \boldsymbol{\lambda}_1^T} \\ = \frac{\partial H_k}{\partial \boldsymbol{\lambda}_2^T} = \frac{\partial H_k}{\partial \boldsymbol{\lambda}_3^T} = 0. \qquad (11) \end{aligned}$$

Substituting Eq. (10) into Eq. (11) and after some rearrangements, the control force is determined as follows:

$$\mathbf{u}_k = -\mathbf{R}^{-1}\mathbf{D}^T\mathbf{K}_{n_k}^{*-T}\left(\mathbf{Q}_1\mathbf{X}_k + a_4\mathbf{Q}_2\dot{\mathbf{X}}_k + a_1\mathbf{Q}_3\ddot{\mathbf{X}}_k\right), \qquad (12)$$

where superscript $(-T)$ means the transpose of the inverse matrix.

Since the control force in Eq. (12) depends on the feedback of response and the weighting matrices, it is desired to assess the effect of selecting different combinations of response feedback on the control system performance.

3. Distributed genetic algorithm (DGA)

The Genetic Algorithm (GA) has been developed by Holland [35] and documented in his pioneering book. Because of the high capability of GA as a computational method in solving optimization problems [30,31], GA has been used extensively in different fields of engineering applications for solving complicated optimization problems [36,37], especially in the field of structural control systems [38–41]. A version of GA, called Distributed Genetic Algorithm (DGA), has been proposed for solving the optimization problems with large numbers of variables. DGA has shown quicker convergence and higher searching capability, compared to the conventional GA [32,33]. In the DGA, the individuals are divided into smaller size subpopulations, where some individuals can migrate from one subpopulation to others. The number of individuals selected for migration and the interval of migration are determined according to migration rate and migration interval parameters. Joghataie and Mohebbi [23,24] studied convergence behavior and proposed guidelines for selecting parameters of DGA, and used it for designing optimal controllers. Their results have supported the capabilities assumed for DGA. In this paper too, following the proposed procedure in [23,24], DGA has been used to solve the optimization problem of this study. Because of some advantages, such as simple programming, and not needing to convert chromosomes to binary coding, as well as greater freedom to use different genetic operators, the real-valued coding has been used for the representation of variables [42]. Also, the elitist strategy has been used, which allows the best individuals of the current generation to go to the next generation without modification.

4. Optimal design of controllers

Eq. (12) gives the control force as a function of the weighting matrices, $\mathbf{R}$, $\mathbf{Q}_1$, $\mathbf{Q}_2$ and $\mathbf{Q}_3$, as design variables. According to the method proposed by Joghataie and Mohebbi [24], these weights are considered as design variables, and are determined by solving an optimization problem which can be formulated as:

$$\text{Find } \mathbf{Q} = (\mathbf{Q}_1, \mathbf{Q}_2, \mathbf{Q}_3), \qquad (13a)$$

Minimize

$$f = \text{Max.}(\mathbf{f}_k(j) = |\mathbf{u}_k(j)|,\ k = 1, 2, \ldots, k_{\max}) \quad j = 1, 2, \ldots, m, \qquad (13b)$$

$$\text{Subject to } g_1 = \lambda.Y_{\max}/(Y_{\max(\text{uncon.})}) - 1 \le 0.0, \qquad (13c)$$

where m = number of actuators, $\mathbf{u}_k(j)$ = control force applied on the jth story at time step k, and $\mathbf{f}_k(j) = |\mathbf{u}_k(j)|$. Also, $k_{\max}$ = total number of time steps, $\mathbf{Y}_k(i)$ = the relative displacement (drift) of the ith story at time step k, $Y_{\max}$ = maximum drift of structure and λ = response reduction parameter, which is always greater than, or equal to, 1 and is used to define the desired reduction in the response.

Both GA and DGA could be used to solve the optimization problem of Eqs. (13a)–(13c). For better convergence in solving the optimization problem, as shown in [23], it has been decided to use DGA. To solve the problem by DGA, the problem was first reformulated as an unconstrained optimization problem by using the penalty method [43] as:

$$F(\mathbf{Q}) = \mu f + \beta \max\left[0, g_1\right], \qquad (14)$$

where μ and β are two constants that should be specified by the designer [23].

5. Response feedback arrangements

To assess the effect of different combinations of response feedback on the performance of active control systems for nonlinear frames, arrangements A-1 to A-7 have been considered as follows:

(A-1) $\mathbf{Q}_1 \ne 0, \quad \mathbf{Q}_2 = 0, \quad \mathbf{Q}_3 = 0$ (Displacement feedback), (15a)

(A-2) $\mathbf{Q}_1 = 0, \quad \mathbf{Q}_2 \ne 0, \quad \mathbf{Q}_3 = 0$ (Velocity feedback), (15b)

(A-3) $\mathbf{Q}_1 = 0, \quad \mathbf{Q}_2 = 0, \quad \mathbf{Q}_3 \ne 0$ (Acceleration feedback), (15c)

(A-4) $\mathbf{Q}_1 \ne 0, \quad \mathbf{Q}_2 \ne 0, \quad \mathbf{Q}_3 = 0$ (Displacement–velocity feedback), (15d)

(A-5) $\mathbf{Q}_1 \ne 0, \quad \mathbf{Q}_2 = 0, \quad \mathbf{Q}_3 \ne 0$ (Displacement–acceleration feedback), (15e)

(A-6) $\mathbf{Q}_1 = 0, \quad \mathbf{Q}_2 \ne 0, \quad \mathbf{Q}_3 \ne 0$ (Velocity–acceleration feedback), (15f)

(A-7) $\mathbf{Q}_1 \ne 0, \quad \mathbf{Q}_2 \ne 0, \quad \mathbf{Q}_3 \ne 0$ (Full feedback). (15g)

Two different actuator placements have been studied in this paper, where either eight actuators have been placed on the floors, so that an actuator has been placed on each floor [24], or a single actuator has been placed on the ground and applies the

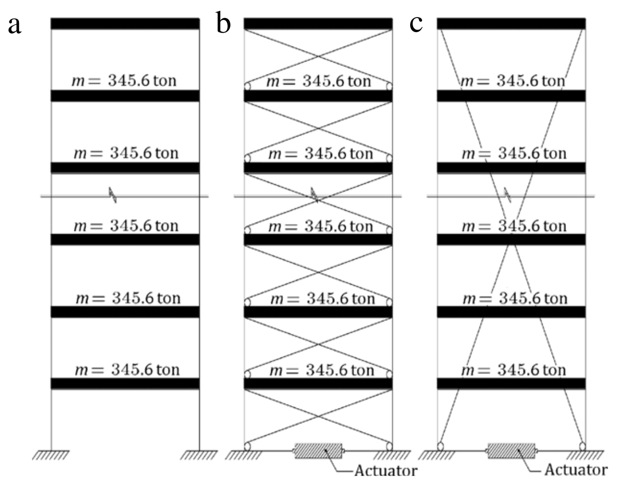

Figure 1: The eight-story shear frame. (a) Uncontrolled; (b) fully controlled; and (c) single actuator.

control force on the eighth floor of the frame. The first actuator placement has been referred to as full controlling, in that it provides the control force on all the degrees of freedom of a shear frame.

Also, a simple arrangement for weighting matrices, $\mathbf{Q}_1$, $\mathbf{Q}_2$ and $\mathbf{Q}_3$, has been considered as follows, which satisfies the necessary conditions of positive semi-definiteness for the weighting matrices.

$$\mathbf{Q}_1 = q_1[I]_{n\times n}, \tag{16a}$$

$$\mathbf{Q}_2 = q_2[I]_{n\times n}, \tag{16b}$$

$$\mathbf{Q}_3 = q_3[I]_{n\times n}, \tag{16c}$$

where $[I]_{n\times n}$ is the unit matrix of size $n \times n$, and there are 3 variables in the optimization problem. For this study, $\mathbf{R}$ has been a diagonal matrix with equal elements, as follows:

$$\mathbf{R} = r[I]_{m\times m}, \tag{17}$$

where m is the number of actuators. $\mathbf{R}$ also satisfies the condition of positive definiteness.

6. Numerical example

The eight-story shear frame [14] shown in Figure 1(a), and mitigation of its vibrations by active controlling have been studied in this section. Two different actuator placements have been considered, as explained in the previous section: either 8 actuators (Figure 1(b)) provide full controlling or a single actuator, which has been placed on the ground and connected to the eighth floor (Figure 1(c)). From previous studies by many authors, it was expected that in a single actuator case, the best results would be obtained when the actuator was connected to the ground and the top of the frame [23,44].

The bilinear hysteretic material behavior with positive post-yielding stiffness and full hysteresis loops, as shown in Figure 2, has been assumed for the structure. It has been assumed that the unloading occurs with the initial stiffness. In Figure 2, the elastic stiffness and post elastic stiffness have been $k_1 = 3.404 \times 10^5$ kN/m and $k_2 = 3.404 \times 10^4$ kN/m, respectively. The floor mass has been 345.6 tons, and the linear viscous damping coefficient, c, is 734.3 kN s/m, which corresponds to the 0.5% damping ratio of the first vibration mode of the structure. The same stiffness has been specified for all floors, assuming a floor would yield when its inter-story drift was $Y_{\text{yielding}} = 2.4$ cm. The uncontrolled structure has been subjected to a white noise, $W(t)$, ground acceleration with Peak Ground Acceleration (PGA) $= 0.4g$, as shown in Figure 3, where the maximum drift and total acceleration of the floors are reported in Table 1. The structure has experienced nonlinear deformation at stories 1, 2 and 3, and the maximum drift has exceeded yielding drift $= Y_{\text{yielding}} = 2.4$ cm. Also, the first story experienced maximum drift amongst all floors, which was $Y_{\max(\text{uncon.})} = 4.75$ cm.

In the following sections, by considering different combinations of response feedback, optimum controllers have been designed for both cases of full controlling and single actuator.

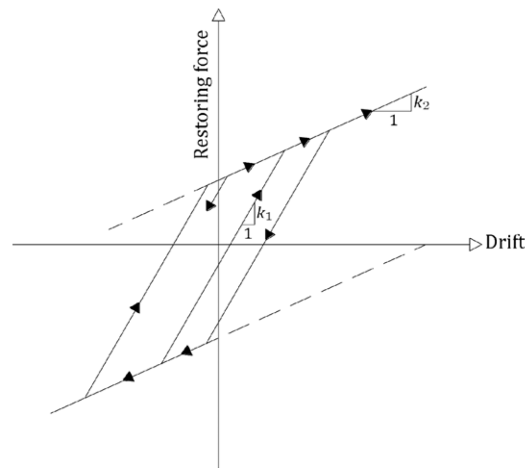

Figure 2: Bilinear elasto–plastic model.

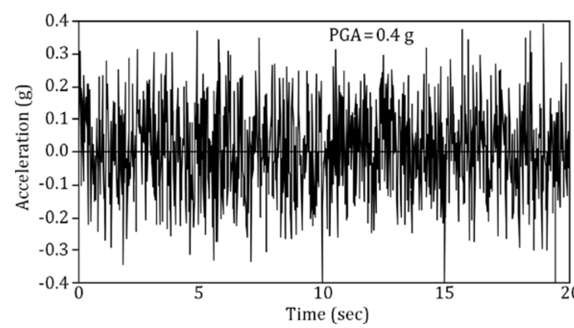

Figure 3: White noise excitation, $W(t)$, with PGA $= 0.4g$.

Table 1: Maximum drift and total acceleration of controlled and uncontrolled nonlinear frames under white noise, $W(t)$, excitation, also maximum required control force for the arrangements A-2 and A-7 when using 8 actuators.

Story no.	Uncontrolled		Controlled					
			Arrangement A-2			Arrangement A-7		
	Drift (cm)	Acc.[a] (cm/s^2)	Drift (cm)	Acc. (cm/s^2)	$u_{\max}(i)$ (kN)	Drift (cm)	Acc. (cm/s^2)	$u_{\max}(i)$ (kN)
1	4.75	573	2.40	591	28	2.40	599	33
2	3.52	724	2.23	645	47	2.19	654	54
3	2.47	815	2.07	713	67	2.04	725	77
4	2.21	852	1.92	769	83	1.80	783	88
5	1.78	859	1.60	796	96	1.49	812	101
6	1.46	908	1.16	846	104	1.07	865	106
7	1.12	911	0.81	817	122	0.83	841	104
8	0.65	951	0.48	784	138	0.49	796	109

[a] Acc. = Acceleration.

6.1. Designing an optimal controller

As a design preference, the objective has been to control the frame to remain linear, hence $\lambda = 2$ in Eq. (13c) has been selected to design the optimum controller, which corresponds to the peak controlled drift of $Y_{\max} = 2.4$ cm and about 50% reduction in the uncontrolled response. By considering the arrangements, A-1 to A-7, for response feedback, optimal controllers have been designed for actuator placement scenarios of either full controlling or using a single controller, as follows.

6.2. Case (a): full controlling, using eight actuators

It has been desired to design the optimum controller to reduce the drift to below yielding level, $Y_{\max} = 2.4$ cm, when a ground white noise acceleration with PGA = $0.4g$ has been applied. The different response feedback arrangements, A-1 to A-7, have been studied. The weighting matrices, $\mathbf{Q}_1$, $\mathbf{Q}_2$ and $\mathbf{Q}_3$, as defined in Eqs. (16a)–(16c), have been considered. Arbitrarily and based on previous experience, $r = 5 \times 10^{-7}$ has been selected in Eq. (17). However, it should be noticed that it is the ratio of $\mathbf{Q}$ to r which is important, and r is in fact a scale factor for the $\mathbf{Q}$s.

For arrangements, A-1 to A-3, which have only one variable (q_1, q_2 or q_3), the variable and the required control force have been determined through trial and error.

Figure 4(a)–(c) show the variation of the maximum required control force versus the different values of q_1, q_2 and q_3, where the maximum required control force to reduce the maximum drift to $Y_{\max} = 2.4$ cm was 819.1 kN, 138.5 kN and 1332.4 kN for displacement, velocity and acceleration feedback, respectively.

According to Figure 4(a)–(c), some important results are:

1. For this case study, generally, maximum drift decreased monotonically when q_2 was increased.
2. By increasing q_1 and q_3, the decrease in maximum drift has not been monotonic, and for some values of q_1 and q_3, the increasing of q_1 and q_3 led to an increase in both control force and maximum drift. Hence, more numerical computation is required to find the optimum values of the elements of the weighting matrices.
3. Comparing arrangements, A-1 to A-3, shows that the velocity feedback (arrangement A-2) is more effective in designing the optimum control system, while the acceleration feedback (arrangement A-3) is the least effective. For arrangement A-2, Table 1 shows the maximum drift and total acceleration of the controlled structure, as well as the maximum required control force applied to each story.

Another strategy for defining the response feedback could be to use a combinational feedback of response, so arrangements A-4 to A-7 have also been investigated.

There are 2 variables in the weighting matrices in arrangements A-4 to A-6, and 3 variables in the weighting matrices in arrangement A-7, to be determined. The determination of the optimal values of these variables, by using the traditional optimization methods, requires extensive numerical trial and error, though there is no guarantee that the optimal values for these variables could be obtained. Hence, it has been decided to use a powerful optimization algorithm to determine these variables automatically. Both the genetic algorithm and its improved version, Distributed Genetic Algorithm (DGA), could be used. In this paper, DGA has been utilized because of its better convergence.

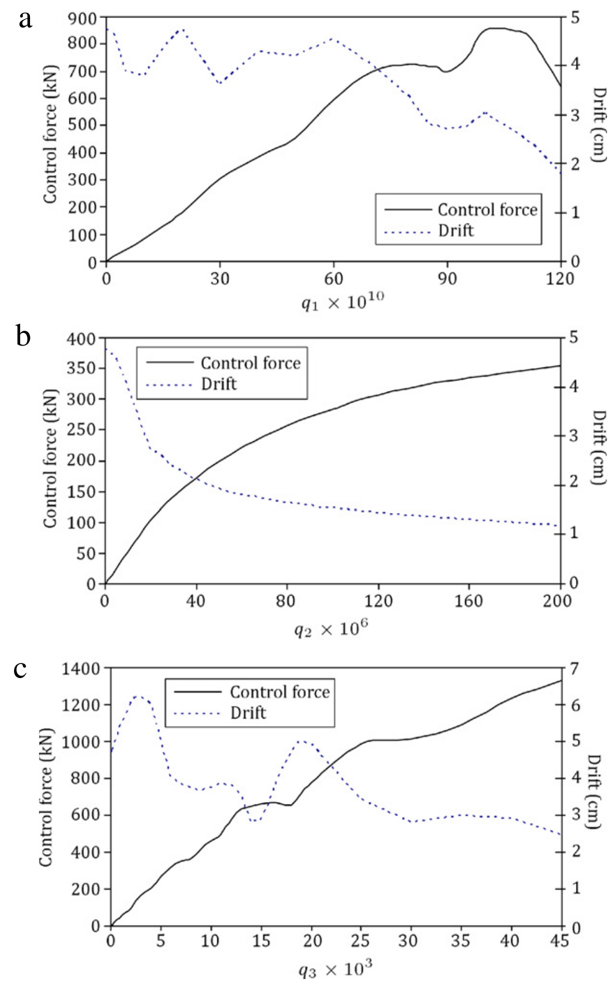

Figure 4: Maximum required control force versus different values of elements of weighting matrices for (a) q_1, (b) q_2, and (c) q_3.

As sample computations, the procedure of DGA for arrangement A-7, which has 3 variables, has been reported here. The optimization problem of Eqs. (13a)–(13c) has been solved to find q_1, q_2 and q_3 to reduce maximum drift to $Y_{\max} = 2.4$ cm. Based on their experience from previous studies [23,24], the authors have selected the following parameters for DGA:

N_{sub} = number of subpopulations = 2,
N_{ind} = number of individuals in each subpopulation = 25,
N_{elites} = number of elites = 5,
N_{new} = number of newborns = 25,
N_{ins} = number of inserted individuals = 20,
m_r = mutation rate = 0.04,
migration interval = 20,
migration rate = 0.20.

Denoting the vector of variables by $\mathbf{Q} = (q_1, q_2, q_3)$, two subpopulations, each with 25 randomly generated vectors of control parameters, have been generated as the initial population. The processes of DGA have been continued until convergence has been achieved. Figure 5(a) shows the best fitness value, $F(\mathbf{Q})$, obtained for 4 different runs (Run 1–Run 4) starting from 4 different randomly generated initial populations. All the runs have ended up with the same optimum answer, though with different convergence speeds. The fitness values of the chromosomes at the final generations are shown in

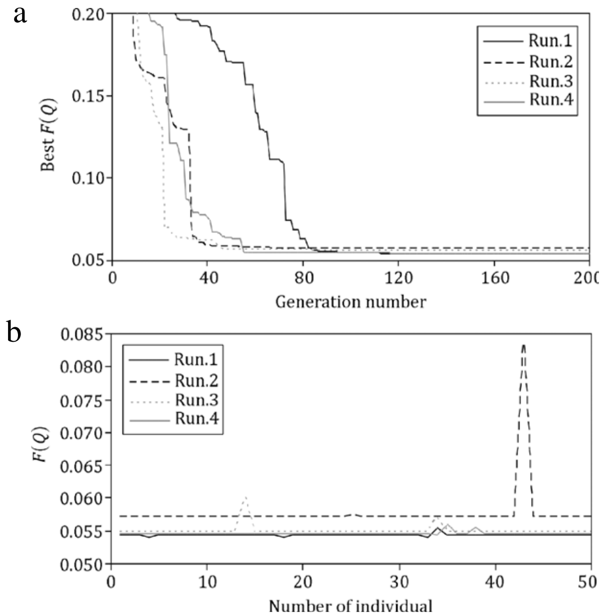

Figure 5: DGA results. (a) The best fitness value of chromosomes in four different runs of DGA; and (b) fitness value of chromosomes at the final generation for four runs.

Figure 5(b) for the 4 runs, which show that most chromosomes have converged to the same value. The optimum controller has been $\mathbf{Q} = (1.0422e11, 2.2643e7, 505.57)$, $u_{\max}$ = maximum control force = 109.0 kN and $Y_{\max}$ = maximum drift = 2.4 cm. Table 1 shows the maximum drift and total acceleration of the controlled structure, as well as the maximum required control force applied to each story.

The time history of drift of the first, second, third and eighth floors of the uncontrolled and controlled frames have been shown in Figure 6. The results show that though the 1st, 2nd and 3rd floors have experienced nonlinear behavior in the uncontrolled frame, they have been controlled successfully to remain within the linear domain, using the active control system. The response corresponding to the other floors has been linear in the uncontrolled frame, and hence as an example, only the time history of the 8th floor, which is the top floor of the frame, has been shown in Figure 6(d). Figure 7 compares the hysteresis loops of the uncontrolled and controlled frames for stories 1, 2 and 3. These results show that the objective of designing an optimum active control system to reduce the maximum drift to $Y_{\max}$ = 2.4 cm, to keep the frame response in its linear domain, has been achieved.

The same procedure has been followed for arrangements A-4 to A-6, which have 2 variables. The peaks of drift, total acceleration, Root-Mean-Square (RMS) of drift and control force corresponding to each arrangement, which have been normalized by division to the peaks of the uncontrolled response and the peak required control force, have been plotted in Figure 8(a).

From Figure 8(a), some important results can be concluded:

1. Arrangement A-7 required the least peak control force amongst all arrangements.
2. The displacement–velocity feedback, A-4, works better to reduce the drift, compared to arrangements A-5 and A-6.
3. It can generally be said that without exception, the inclusion of velocity feedback has improved the performance of the controller.

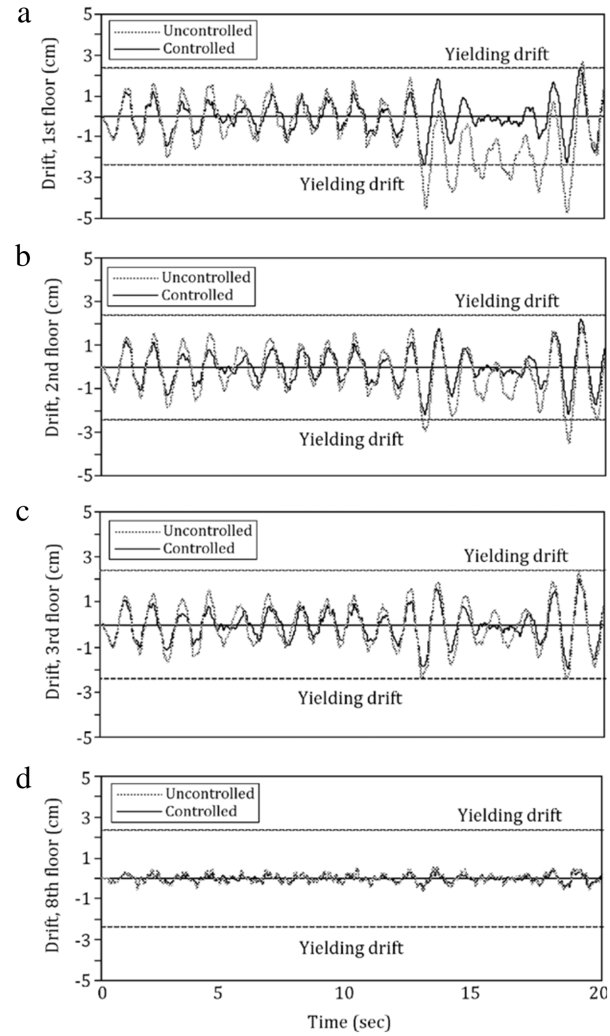

Figure 6: 20 s of uncontrolled and controlled drifts by eight actuators when using arrangement A-7 shown for (a) first floor, (b) second floor, (c) third floor and (d) eighth floor.

4. Using the full feedback of response has decreased maximum control force by about 12%, 61% and 21%, in comparison with arrangements A-4 to A-6, hence it works approximately as well as the displacement–velocity feedback.

Figure 9(a) compares the required control force applied on the eighth floor for arrangements A-2, as the optimum case of using only one response as feedback, and A-7, as the optimum case of combinational arrangements. Results show about 22% reduction in maximum control force using full response as feedback.

6.3. Case (b): control, using a single actuator

Joghataie and Mohebbi [24] studied the optimum actuator placement and concluded that when using a single actuator, it is optimal to place the control force on the eighth floor. So in this paper too, it has been decided to place the actuator on the ground and apply the control force on the eighth floor, as shown in Figure 1(c). Following the same procedure explained for the case of full controlling, for arrangements A-1 to A-7, the optimal controllers have been designed to reduce the maximum drift to $Y_{\max}$ = 2.4 cm under white noise excitation, $W(t)$. Figure 8(b)

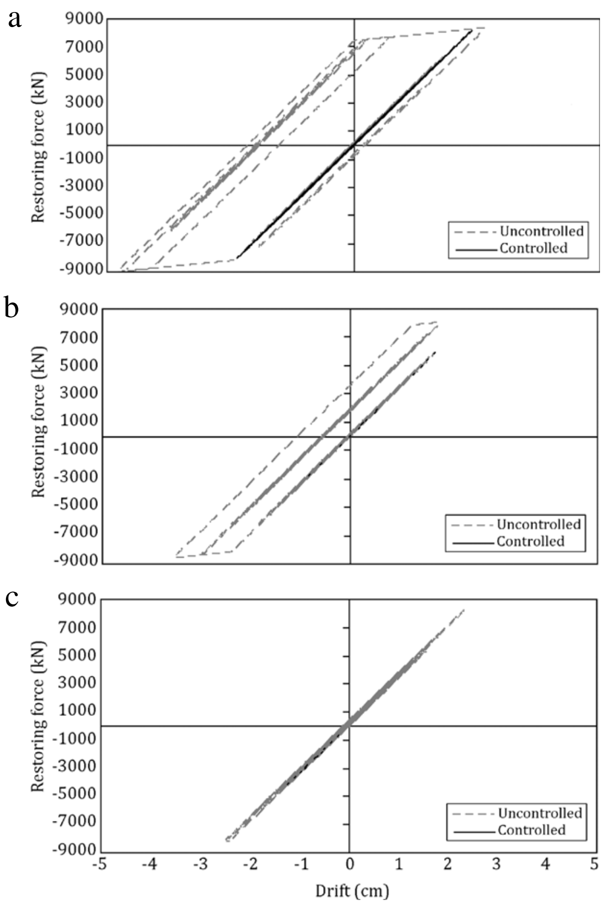

Figure 7: Hysteresis loops of the uncontrolled and controlled frame by eight actuators when using arrangement A-7, shown for (a) first floor, (b) second floor, and (c) third floor.

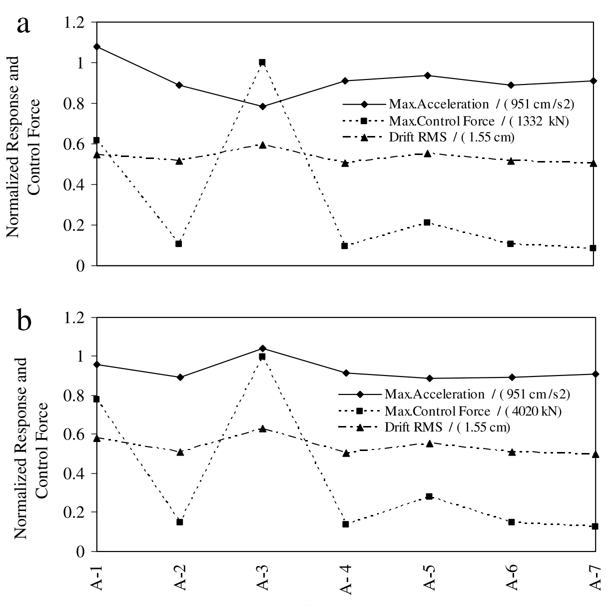

Figure 8: Normalized response and control force for different arrangements when using (a) eight actuators (full controlling), and (b) single actuator.

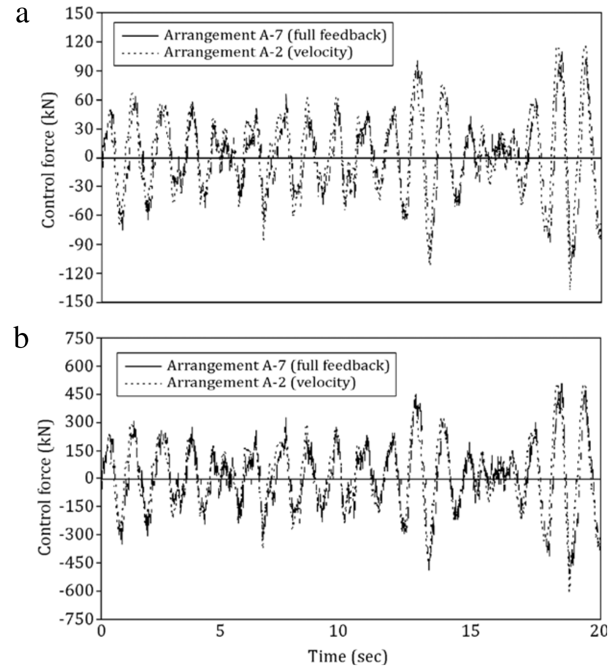

Figure 9: 20 s of control force applied on the 8th floor for arrangements A-2 (velocity feedback) and A-7 (full feedback) when using (a) eight actuators (full controlling), and (b) single actuator.

summarizes the results including the normalized maximum drift, total acceleration, RMS of drift and maximum required control force corresponding to each arrangement. From the results it can be concluded that similar to the case of full controlling, in the case of a single actuator too, the performance of the control system has been dependent on the response feedback, where using the full feedback of response in the control law has reduced the maximum control force by about 8%, 54% and 16%, in comparison with arrangements A-4 to A-6. Therefore, in this case, too, the displacement–velocity feedback (A-4) works as well as the full feedback arrangement (A-7). In Figure 9(b), the required control forces for arrangements A-2 and A-7 have been compared, which shows about 16% reduction in maximum control force using A-7.

6.4. Comparing arrangements regarding time delay and actuator capacity

For practical implementation of active control systems, it is important to use the least number of response measurements in order to reduce the time of online measurements. On the other hand, to improve the performance of the control system, it is desired to use the arrangement which requires minimum peak control force. For the case study of this paper, under $W(t)$ excitation, among arrangements A-1 to A-3, which use 8 response measurements to feedback in the control law, velocity feedback (A-2) has emerged as the optimal arrangement, while for arrangements A-4 to A-6, which require 16 numbers of response measurements, displacement–velocity feedback (A-4) has been the most effective. Under white noise excitation, arrangement A-7 (full feedback) uses the most number of responses (24 numbers) in the control law, though in comparison with velocity and displacement–velocity feedback, it has about 22% and 12% reduction in the maximum control force for full controlling, and 16% and 8% reduction

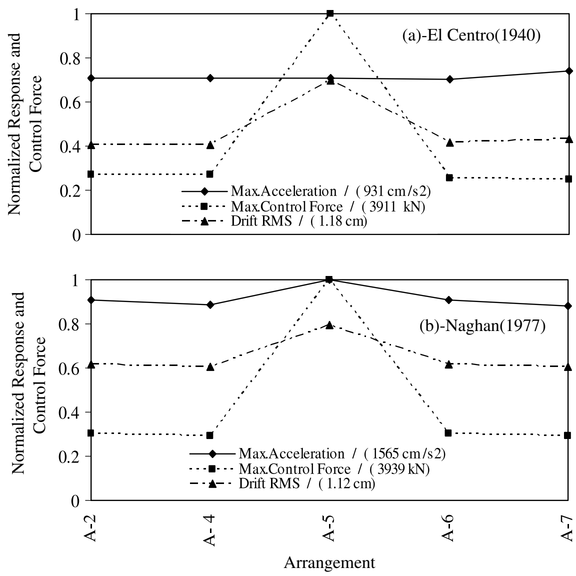

Figure 10: Normalized response and control force for different arrangements when using single actuator connected to eighth floor under (a) El Centro (1940), and (b) Naghan (1977) earthquakes.

for a single actuator. A discussion of the best combination for response feedback, regarding the capacity of actuators and the number of sensors, requires cost analysis and structural design considerations, which have been planned for later study by the authors.

6.5. Designing optimal controllers for real earthquakes

In previous sections, the controllers have been designed using white noise, $W(t)$, excitation. To assess the effect of input excitation in designing optimal controllers for nonlinear frames, other earthquakes have been used to design controllers for arrangements A-1 to A-7, in order to reduce maximum drift to $Y_{\max} = 2.4$ cm. The control force has been assumed to be applied by a single actuator, which has been connected to the 8th floor of the frame. The earthquakes included El Centro (PGA $= 0.34g$, 1940) and Naghan (PGA $= 0.72g$, 1977), which are of different intensity and frequency content. The peak nonlinear response values of the controlled frame, divided by the corresponding maximum response values of the uncontrolled frame, as well as the normalized maximum required control force, have been shown in Figure 10. Arrangements A-1 and A-3 have been unable to keep the structure in the linear domain under both El Centro (1940) and Naghan (1977) earthquakes, and hence their results have not been included in Figure 10. The other arrangements have been able to control the structure to remain within linearity. Arrangements A-2, A-4 and A-6, which have received velocity feedback, have worked very similarly to arrangement A-7 with full feedback. However, A-5 which has not received velocity feedback, and works with displacement-acceleration feedback only, has required larger control forces in comparison with A-7 (with full feedback), where the maximum control force has been about 300% and 240% under El Centro (1940) and Naghan (1977) earthquakes, respectively.

7. Conclusions

In this paper, the effect of the different combinations of response measurements, which are fed back to the active controller on the performance of the control system for nonlinear frames, has been studied. For the purpose of active controlling of a nonlinear frame, a nonlinear instantaneous optimal control algorithm has been used based on the Newmark integration method and Distributed Genetic Algorithms (DGA). Different combinations of response feedback have been defined for the performance index. The Distributed Genetic Algorithms (DGA) have been used to determine the parameters of the weighting matrices for each arrangement of the response feedback. For each combination, an optimal controller has been designed to mitigate the response to below the yielding level (about 50% reduction in maximum drift), while using the minimum required control force.

For verification, an eight-story shear frame with bilinear nonlinearity and hysteretic behavior under white noise excitation has been studied, where the active tendon mechanism was assumed to be capable of applying either a control force on each floor, using 8 actuators or a single actuator placed on the ground, which could apply a control force on the eighth floor. Optimal controllers for each arrangement of response feedback in both cases of full controlling and using a single actuator have been designed. The obtained results have shown that the performance of the active control system significantly depends on the type of response feedback combination included in the control law. From numerical simulation and analysis, it has been concluded that defining a performance index as a function of full feedback of response, including acceleration, velocity and displacement at each and every degree of freedom, can lead to the design of a controller that requires minimum control force. However, it also requires the use of further numbers of response feedback, which is costly but without any significant advantage, in comparison with displacement–velocity and velocity feedbacks. Also, it has been found that the combinations which use velocity feedback specify a smaller peak control force. Also, when the real earthquakes of El Centro (1940) and Naghan (1977) were used in designing the controllers instead of white noise, the obtained results were the same as when white noise had been used. From the viewpoint of implementation of the active control system, and in order to minimize the number of online measurements, it is preferable to use the controller which uses only the velocity feedback. Another result of this study is in relation to the use of the distributed genetic algorithm, which has proven to be a strong and capable engine for optimization problems in designing active controllers.

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