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## Quarks and leptons in a hexagonal chain

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### Abstract

The seemingly disparate notions of chiral color and quark–lepton nonuniversality are combined, and shown to be essential to each other as part of an underlying (and unifying) larger symmetry, i.e., supersymmetric  $SU(3)^6$ . Both phenomena are accessible experimentally at the TeV energy scale.

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In the Standard Model of quarks and leptons, the electric charge has two components, i.e.,

$$Q = I_{3L} + \frac{1}{2}Y, \quad (1)$$

where  $I_{3L}$  comes from  $SU(2)_L$  and  $Y$  from  $U(1)_Y$ . If the gauge group is extended to include  $SU(2)_R$ , then there are two possible decompositions of the electric charge. One is based on [1]  $SU(4)_C \rightarrow SU(3)_C \times U(1)_{B-L}$ , i.e.,

$$Q = I_{3L} + I_{3R} + \frac{1}{2}(B - L). \quad (2)$$

The other is based on  $SU(3)_L \rightarrow SU(2)_L \times U(1)_{Y_L}$  and  $SU(3)_R \rightarrow SU(2)_R \times U(1)_{Y_R}$ , i.e.,

$$Q = I_{3L} + I_{3R} - \frac{1}{2}Y_L - \frac{1}{2}Y_R. \quad (3)$$

[The minus signs in the above expression are due to a convention which will become clear later.] Whereas

Eq. (2) is indicative of  $SO(10)$  as the unification group, Eq. (3) is indicative of [2]  $SU(3)_C \times SU(3)_L \times SU(3)_R$ . However, the two are in fact equivalent if considered as subgroups of  $E_6$ .

Using Eq. (2), under  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ , the quarks and leptons transform as

$$\begin{aligned} q &= (u, d)_L \sim \left(2, 1, \frac{1}{6}\right), \\ q^c &= (d^c, u^c)_L \sim \left(1, 2, -\frac{1}{6}\right), \end{aligned} \quad (4)$$

$$\begin{aligned} l &= (v, e)_L \sim \left(2, 1, -\frac{1}{2}\right), \\ l^c &= (e^c, \nu^c)_L \sim \left(1, 2, \frac{1}{2}\right). \end{aligned} \quad (5)$$

They are different in their electric charges because they have different  $B - L$  values. Using Eq. (3), under  $SU(2)_L \times SU(2)_R \times U(1)_{Y_L} \times U(1)_{Y_R}$ ,

$$q \sim \left(2, 1, \frac{1}{6}, 0\right), \quad q^c \sim \left(1, 2, 0, -\frac{1}{6}\right), \quad (6)$$

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$$l \sim \left(2, 1, -\frac{1}{6}, -\frac{1}{3}\right), \quad l^c \sim \left(1, 2, \frac{1}{3}, \frac{1}{6}\right). \quad (7)$$

Here the differences come from the fact that leptons belong to the  $(1, 3, 3^*)$  representation of  $SU(3)_C \times SU(3)_L \times SU(3)_R$ , whereas  $q$  and  $q^c$  belong to the  $(3, 3^*, 1)$  and  $(3^*, 1, 3)$  representations, respectively. Note that  $(Y_L, Y_R) = (-1/3, 0), (0, -1/3), (1/3, 2/3), (-1/3, -2/3)$ , respectively, for  $q, q^c, l, l^c$  in these representations. This explains the minus signs in Eq. (3).

At this point, a curious fact must have already been noticed, i.e., the electric charge has two components in Eq. (1), three in Eq. (2), and four in Eq. (3). How about five or more? Using the idea of a separate  $SU(3)_l$  for leptons [3], which results in a successful nonsupersymmetric  $SU(3)^4$  model [4], the electric charge may indeed have five components, i.e.,

$$Q = I_{3L} + I_{3R} - \frac{1}{2}Y_L - \frac{1}{2}Y_R - \frac{1}{2}Y_l, \quad (8)$$

where  $Y_l$  comes from  $SU(3)_l \rightarrow SU(2)_l \times U(1)_{Y_l}$ . In this case,

$$q \sim \left(2, 1, \frac{1}{6}, 0, 0\right), \quad q^c \sim \left(1, 2, 0, -\frac{1}{6}, 0\right), \quad (9)$$

$$l \sim \left(2, 1, -\frac{1}{6}, 0, -\frac{1}{3}\right), \quad l^c \sim \left(1, 2, 0, \frac{1}{6}, \frac{1}{3}\right). \quad (10)$$

Going back to Eq. (3), it is also clear that quarks and leptons may belong to *different*  $SU(3)_L$ 's and  $SU(3)_R$ 's, so that the electric charge has eight components, i.e.,

$$Q = (I_3)_{qL} + (I_3)_{qR} - \frac{1}{2}Y_{qL} - \frac{1}{2}Y_{qR} + (I_3)_{lL} + (I_3)_{lR} - \frac{1}{2}Y_{lL} - \frac{1}{2}Y_{lR}. \quad (11)$$

Combining this notion of quark–lepton nonuniversality [5–7] with that of chiral color [8], the group  $SU(3)^6$  is then obtained. Note that this is very different from the previously proposed [9] nonsupersymmetric  $SU(4)^6$  model which predicted a value of  $\sin^2 \theta_W$  very far away from the present data. A good way of displaying the structure of this symmetry is again a hexagonal “moose” diagram [10] (Fig. 1) with the assignments

$$q \sim (3, 3^*, 1, 1, 1, 1), \quad (12)$$

$$x \sim (1, 3, 3^*, 1, 1, 1), \quad (13)$$

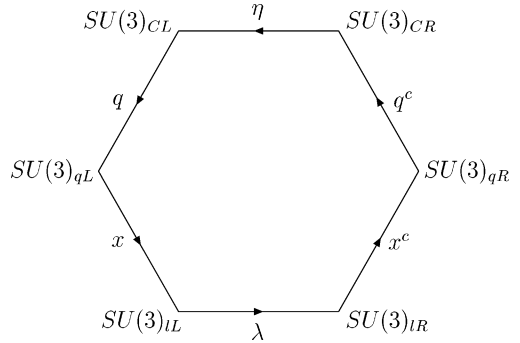


Fig. 1. Moose diagram of quarks and leptons in  $[SU(3)]^6$ .

$$\lambda \sim (1, 1, 3, 3^*, 1, 1), \quad (14)$$

$$x^c \sim (1, 1, 1, 3, 3^*, 1), \quad (15)$$

$$q^c \sim (1, 1, 1, 1, 3, 3^*), \quad (16)$$

$$\eta \sim (3^*, 1, 1, 1, 1, 3), \quad (17)$$

under  $SU(3)_{CL} \times SU(3)_{qL} \times SU(3)_{lL} \times SU(3)_{lR} \times SU(3)_{qR} \times SU(3)_{CR}$ . It reduces to the well-known  $SU(3)_C \times SU(3)_L \times SU(3)_R$  model if the  $x, x^c, \eta$  links are all contracted. It is also the natural *anomaly-free* extension of chiral color (the  $\eta$  link) and quark–lepton nonuniversality (the  $x, x^c$  links). [Anomaly cancellation in an  $SU(n)_1 \times SU(n)_2 \times \dots \times SU(n)_k$  gauge theory is guaranteed [10] whenever the fermions form a moose chain, because each  $n$  representation is accompanied by its conjugate  $n^*$  representation with the same multiplicity, i.e.,  $n$ .]

The particle content of this model is given by

$$q = \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix}, \quad q^c = \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix},$$

$$\lambda = \begin{pmatrix} N & E^c & \nu \\ E & N^c & e \\ \nu^c & e^c & S \end{pmatrix}, \quad (18)$$

where the rows denote  $(I_3, Y) = (1/2, 1/3), (-1/2, 1/3), (0, -2/3)$ , and the columns denote  $(I_3, Y) = (-1/2, -1/3), (1/2, -1/3), (0, 2/3)$ , with  $x$  and  $x^c$  having the same electric charge assignments as  $\lambda$ , and all the components of  $\eta$  are neutral. The doubling of  $SU(3)^3$  to  $SU(3)^6$  also allows the six gauge couplings to unify with  $\sin^2 \theta_W$  equal to the canonical  $3/8$  at the unification scale. To check this, consider the

contributions of  $q, x, \lambda, x^c, q^c, \eta$  to  $\sum I_{3L}^2$  and  $\sum Q^2$ :

$$\begin{aligned} \sin^2 \theta_W &= \frac{\sum I_{3L}^2}{\sum Q^2} = \frac{\frac{3}{2} + 3 + \frac{3}{2} + 0 + 0 + 0}{2 + 4 + 4 + 4 + 2 + 0} \\ &= \frac{6}{16} = \frac{3}{8}, \end{aligned} \quad (19)$$

as expected. Note that in the  $SU(3)^4$  model [4] of leptonic color,  $\sin^2 \theta_W = 1/3$  at the unification scale.

There is another reason for choosing  $SU(3)^6$ . If chiral color is invoked without separate  $SU(2)_L$  gauge groups for quarks and leptons, then an  $SU(3)^4$  model with  $\sin^2 \theta_W = 3/8$  is possible. However, because

$$\frac{1}{\alpha_s} = \frac{1}{\alpha_{CL}} + \frac{1}{\alpha_{CR}}, \quad (20)$$

$\alpha_s$  would be wrong by at least a factor of two. As it is,  $SU(3)^6$  allows the intriguing possibility that *both* chiral color and quark–lepton nonuniversality may exist at experimentally accessible energies, as shown below.

The spontaneous symmetry breaking of  $SU(3)^6$  is achieved through the scalar components of the supermultiplets  $x^c, x$ , and  $\lambda$  of Eqs. (15), (13), and (14). Specifically, the ones corresponding to  $x_{11}^c, x_{22}^c, x_{13}^c, x_{31}^c, x_{33}^c, x_{33}$ , and  $\lambda_{33}$  are assumed to acquire large vacuum expectation values, in which case  $SU(3)^6$  is broken down to

$$\begin{aligned} &SU(3)_{CL} \times SU(3)_{CR} \times SU(2)_{qL} \\ &\times SU(2)_{lL} \times U(1)_Y. \end{aligned} \quad (21)$$

This symmetry embodies both the notions of chiral color and quark–lepton nonuniversality and is assumed to be valid down to  $M_S$ , the supersymmetry breaking scale. The particles which remain massless between  $M_S$  and  $M_U$ , the unification scale, are assumed to be three copies of  $q, q^c$ , and  $\eta$ , three copies of all the components of  $x$  except  $x_{33}$ , three copies of  $(\nu, e)$  and  $(e^c, \nu^c)$  in  $\lambda$ , but only one copy of the  $(N, E; E^c, N^c)$  bidoublet in  $\lambda$ . This particle content generalizes that of the MSSM (Minimal Supersymmetric Standard Model), where there are three families of quarks and leptons, but only one pair of Higgs doublets. The transformation properties of these particles with respect to this symmetry are then given by

$$\begin{aligned} (u, d) &\sim (3, 1, 2, 1, 1/6), & h &\sim (3, 1, 1, 1, -1/3), \\ \eta &\sim (3^*, 3, 1, 1, 0), \end{aligned} \quad (22)$$

$$\begin{aligned} d^c &\sim (1, 3^*, 1, 1, 1/3), & u^c &\sim (1, 3^*, 1, 1, -2/3), \\ h^c &\sim (1, 3^*, 1, 1, 1/3), \end{aligned} \quad (23)$$

$$\begin{aligned} (\nu, e) &\sim (1, 1, 1, 2, -1/2), & e^c &\sim (1, 1, 1, 1, 1), \\ \nu^c &\sim (1, 1, 1, 1, 0), \end{aligned} \quad (24)$$

$$\begin{aligned} (\nu_x, e_x) &\sim (1, 1, 2, 1, -1/2), \\ (e_x^c, \nu_x^c) &\sim (1, 1, 1, 2, 1/2), \end{aligned} \quad (25)$$

$$(N_x, E_x; E_x^c, N_x^c) \sim (1, 1, 2, 2, 0), \quad (26)$$

$$\begin{aligned} (N, E) &\sim (1, 1, 1, 2, -1/2), \\ (E^c, N^c) &\sim (1, 1, 1, 2, 1/2). \end{aligned} \quad (27)$$

Above  $M_S$ , the theory is supersymmetric and all these supermultiplets contribute to the running of the five gauge couplings of Eq. (21). The one-loop renormalization-group equations are given by

$$\frac{1}{\alpha_i(M_S)} - \frac{1}{\alpha_i(M_U)} = \frac{b_i}{2\pi} \ln \frac{M_U}{M_S}, \quad (28)$$

where

$$SU(3)_{CL}: \quad b_{CL} = -9 + 3N_f = 0, \quad (29)$$

$$SU(3)_{CR}: \quad b_{CR} = -9 + 3N_f = 0, \quad (30)$$

$$SU(2)_{qL}: \quad b_{qL} = -6 + 3N_f = 3, \quad (31)$$

$$SU(2)_{lL}: \quad b_{lL} = -6 + 2N_f + 1 = 1, \quad (32)$$

$$U(1)_Y: \quad b_Y = 5N_f + 1 = 16, \quad (33)$$

where  $N_f = 3$  is the number of families.

At  $M_S$ , in addition to the breaking of supersymmetry, assume as well that  $SU(3)_{CL} \times SU(3)_{CR}$  is broken to  $SU(3)_C$  through the vacuum expectation values of the diagonal elements of  $\eta$ , and  $SU(2)_{qL} \times SU(2)_{lL}$  is broken to  $SU(2)_L$  through the vacuum expectation values of  $N_x$  and  $N_x^c$ . The boundary conditions at  $M_S$  are

$$\frac{1}{\alpha_s(M_S)} = \frac{1}{\alpha_{CL}(M_S)} + \frac{1}{\alpha_{CR}(M_S)}, \quad (34)$$

$$\frac{1}{\alpha_2(M_S)} = \frac{1}{\alpha_{qL}(M_S)} + \frac{1}{\alpha_{lL}(M_S)}. \quad (35)$$

Below  $M_S$ , the particle content becomes that of the Standard Model, but with two Higgs doublets, i.e.,

$$SU(3)_C: \quad b_s = -11 + (4/3)N_f = -7, \quad (36)$$

$$SU(2)_L: \quad b_2 = -22/3 + (4/3)N_f + 1/3 = -3, \quad (37)$$

$$U(1)_Y: \quad b_Y = (20/9)N_f + 1/3 = 7. \quad (38)$$

At  $M_U$ , all six gauge couplings are assumed equal. Using  $\sin^2 \theta_W(M_U) = 3/8$ , this means that

$$\frac{3}{5\alpha_Y(M_U)} = \frac{1}{\alpha_2(M_U)} = \frac{2}{\alpha_U}. \quad (39)$$

Putting all these together, the constraints on the gauge couplings of this model are then given by

$$\frac{1}{\alpha_s(M_Z)} = -\frac{7}{2\pi} \ln \frac{M_S}{M_Z} + \frac{2}{\alpha_U}, \quad (40)$$

$$\frac{1}{\alpha_2(M_Z)} = -\frac{3}{2\pi} \ln \frac{M_S}{M_Z} + \frac{2}{\pi} \ln \frac{M_U}{M_S} + \frac{2}{\alpha_U}, \quad (41)$$

$$\frac{3}{5\alpha_Y(M_Z)} = \frac{21}{10\pi} \ln \frac{M_S}{M_Z} + \frac{24}{5\pi} \ln \frac{M_U}{M_S} + \frac{2}{\alpha_U}. \quad (42)$$

These equations are easily solved for  $\alpha_s(M_Z)$  and  $M_U/M_Z$  in terms of  $\alpha_2(M_Z)$ ,  $\alpha_Y(M_Z)$ , and  $M_S/M_Z$ , i.e.,

$$\frac{1}{\alpha_s(M_Z)} = \frac{3}{7} \left( \frac{4}{\alpha_2(M_Z)} - \frac{1}{\alpha_Y(M_Z)} \right) + \frac{4}{7\pi} \ln \frac{M_S}{M_Z}, \quad (43)$$

$$\ln \frac{M_U}{M_Z} = \frac{\pi}{14} \left( \frac{3}{\alpha_Y(M_Z)} - \frac{5}{\alpha_2(M_Z)} \right) - \frac{2}{7} \ln \frac{M_S}{M_Z}. \quad (44)$$

Using the input [11]

$$\alpha_2(M_Z) = (\sqrt{2}/\pi) G_F M_W^2 = 0.0340, \quad (45)$$

$$\alpha_Y(M_Z) = \alpha_2(M_Z) \tan^2 \theta_W = 0.0102, \quad (46)$$

the value of  $\alpha_s(M_Z)$  is predicted to be in the range 0.119 to 0.115 for  $M_S/M_Z$  in the range 1 to 5, as shown in Table 1, compared to the experimental world average [11] of  $0.117 \pm 0.002$ . The value of  $M_U$  is of order  $10^{16}$  GeV, in good agreement with the usual theoretical expectation. Thus a *new* and remarkably *successful* model of grand unification is obtained. It is also experimentally *verifiable* because it predicts specific new particles *necessarily* below the TeV energy scale.

The expected new particles and their supersymmetric partners are as follows. (1) Three copies of the exotic  $h(h^c)$  quarks as in  $SU(3)^3$  or  $E_6$ . (2) Eight axigluons [8] corresponding to the breaking of  $SU(3)_{CL} \times SU(3)_{CR}$  to  $SU(3)_C$ . (3) Three sets of neutral (3, 3\*)  $\eta$  particles which are reorganized into octets and singlets under  $SU(3)_C$ . (4) Three vector gauge bosons corresponding to the breaking of  $SU(2)_{qL} \times SU(2)_{lL}$

Table 1  
Values of  $\alpha_s(M_Z)$  and  $M_U/M_Z$  as functions of  $M_S/M_Z$

$M_S/M_Z$	$\alpha_s(M_Z)$	$M_U/M_Z$
1.0	0.119	$2.1 \times 10^{14}$
1.5	0.118	$1.9 \times 10^{14}$
2.2	0.117	$1.7 \times 10^{14}$
3.3	0.116	$1.5 \times 10^{14}$
5.0	0.115	$1.4 \times 10^{14}$
7.6	0.114	$1.2 \times 10^{14}$

to  $SU(2)_L$ . (5) Three (2, 2) bidoublets which are reorganized into triplets and singlets under  $SU(2)_L$ . (6) Three sets of  $SU(2)_L$  doublets ( $\nu_x, e_x$ ) and ( $e_x^c, \nu_x^c$ ).

The presence of  $SU(2)_L$  nonsinglets is a potential phenomenological problem with respect to the precision electroweak measurements. However, all such new particles, i.e., those listed under (4), (5), and (6) in the above, have masses which are invariant under  $SU(2)_L$ . Hence their contributions to the electroweak oblique parameters are all very much suppressed. They are however necessary because they render the  $SU(2)_{qL} \times SU(2)_{lL}$  gauge extension [7] anomaly-free. This is also the purpose of the new  $SU(3)_C$  octets and singlets (the  $\eta$  particles) with respect to  $SU(3)_{CL} \times SU(3)_{CR}$ . Thus this model has very unique predictions which are verifiable experimentally.

If the scale at which  $SU(3)_{CL} \times SU(3)_{CR}$  reduces to the canonical  $SU(3)_C$  is  $M_A$  instead of  $M_S$  with  $M_A > M_S$ , then Eqs. (43) and (44) are modified to read

$$\frac{1}{\alpha_s(M_Z)} = \frac{3}{7} \left( \frac{4}{\alpha_2(M_Z)} - \frac{1}{\alpha_Y(M_Z)} \right) + \frac{1}{14\pi} \left( 35 \ln \frac{M_S}{M_Z} - 27 \ln \frac{M_A}{M_Z} \right), \quad (47)$$

$$\ln \frac{M_U}{M_Z} = \frac{\pi}{14} \left( \frac{3}{\alpha_Y(M_Z)} - \frac{5}{\alpha_2(M_Z)} \right) - \frac{1}{14} \left( 7 \ln \frac{M_S}{M_Z} - 3 \ln \frac{M_A}{M_Z} \right). \quad (48)$$

This would allow  $M_S$  to be somewhat larger, say of order 1 TeV.

In conclusion, a new model of grand unification has been proposed based on  $SU(3)^6$ , which is a natural extension of two seemingly disparate notions, i.e., chiral color and quark–lepton nonuniversality. In the context of supersymmetry, it has been shown that the five gauge couplings corresponding to  $SU(3)_{CL} \times$

$SU(3)_{CR} \times SU(2)_{qL} \times SU(2)_{lL} \times U(1)_Y$  naturally converge to a single value at  $M_U$  of order  $10^{16}$  GeV, and the scale at which this symmetry reduces to the standard  $SU(3)_C \times SU(2)_L \times U(1)_Y$  is not more than a few times that of electroweak symmetry breaking. This is a natural generalization of the MSSM and may be easily distinguished from it experimentally because specific new particles are predicted to exist below the TeV energy scale, and should be accessible at accelerators in the near future.

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