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Master formula for twist-3 soft-gluon-pole mechanism to single transverse-spin asymmetry

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Abstract

Perturbative QCD relates the single transverse-spin asymmetries (SSAs) for hard processes at large transverse-momentum of produced particle to partonic matrix elements that describe interference between scattering from a coherent quark–gluon pair and from a single quark, generated through twist-3 quark–gluon correlations inside a hadron. When the coherent gluon is soft at the gluonic poles, its coupling to partonic subprocess can be systematically disentangled, so that the relevant interfering amplitude can be derived entirely from the Born diagrams for the scattering from a single quark. We establish a new formula that represents the exact rules to derive the SSA due to soft-gluon poles from the knowledge of the twist-2 cross-section formula for unpolarized processes. This single master formula is applicable to a range of processes like Drell–Yan and direct-photon production, and semi-inclusive deep inelastic scattering, and is also useful to manifest the gauge invariance of the results.

There have been two systematic frameworks to study single spin asymmetries (SSAs) observed in a variety of high energy semi-inclusive reactions. One is based on the so-called "T-odd" distribution and fragmentation functions with parton's intrinsic transverse momentum [1,2]. It describes SSAs in the region of the small transverse momentum p_T of the final hadrons as a leading twist effect. Its factorization property and the universality of "T-odd" functions have been extensively studied [3]. The other one is the twist-3 mechanism based on the collinear factorization, and is suited to describe SSAs in the large p_T region [4–6]. It relates the SSA to certain quark–gluon correlation functions in the hadrons. Recently it has been shown that these two mechanisms give the identical SSA in the intermediate p_T region for Drell–Yan and semi-inclusive deep inelastic scatterings, i.e., describe the same physics in QCD [7,8].

In the present work, we shall focus on the twist-3 mechanism. A systematic study on this mechanism was first performed in [5] for the direct photon production $p^\uparrow p \to \gamma X$, and the method has been applied to many other processes such as pion production in pp-collision, $p^\uparrow p \to \pi X$ [9–12], hyperon polarization $pp \to \Lambda^\uparrow X$ [13,14], Drell–Yan lepton-pair production $p^\uparrow p \to \ell^+\ell^- X$ [7], pion production in semi-inclusive deep inelastic scattering (SIDIS), $ep^\uparrow \to e\pi X$ [6,8,15]. In our recent work [6], we reexamined the formalism for the twist-3 mechanism and gave a proof for the factorization property and the gauge invariance of the corresponding single-spin-dependent cross sections, which was missing in the previous literature. Through this development, the cross-section formula derived in the previous studies [5–15] have been given a solid theoretical basis.

In the twist-3 mechanism, the strong interaction phase necessary for SSA is provided by the partonic hard scattering: Owing to the insertion of a "coherent gluon" emanating from the twist-3 quark–gluon correlation inside e.g. the polarized nucleon, an internal propagator of the partonic hard part can be on-shell, and its imaginary part (pole contribution) can give rise to the interfering phase that leads eventually to the real single-spin-dependent cross section. Depending on the resulting value of the coherent-gluon's

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momentum fraction at such poles, those poles are classified as soft-gluon-pole (SGP), soft-fermion-pole (SFP) and the hard-pole (HP). In [6], we have given a formula which expresses all these pole contributions in terms of the twist-3 distributions associated with the gluon's field strength tensor (see (1), (3) below). There we have also given a general proof that the SGP contribution appears as both "derivative" and "non-derivative" terms of the twist-3 correlation functions, while the SFP and HP contributions appear only as the non-derivative terms. Since the derivative of the "SGP function", a twist-3 correlation function at the SGP, causes enhancement compared with the non-derivative term in a certain kinematic region, and also one may expect that the soft gluons are more ample in the hadrons, phenomenology based on the SGP contribution only has often been performed in the literature [9–15].

Another peculiar feature of the SGP contribution is that the partonic hard cross section associated with the "derivative term", arising from the twist-3 soft-gluon mechanism, is directly proportional to some twist-2 partonic cross section as noticed for direct photon production [5], Drell–Yan pair production [7], SIDIS [15], $p^\uparrow p \to \pi X$ [9–11] and $pp \to \Lambda^\uparrow X$ [13,14].

In this Letter we propose a new systematic approach to treat the SGPs, which allows us to reveal the novel structure behind the soft-gluon mechanism to the SSA. We disentangle the coupling of the coherent gluon and the associated pole structure from the partonic subprocess, by reorganizing the relevant diagrams using Ward identities and certain decomposition identities for the interacting parton propagators. We find that the many Feynman diagrams can be eventually united into certain derivative of the Born diagrams without the coherent-gluon insertion, which shows that the entire contributions from the SGPs, not only the derivative term but also the non-derivative term, can be derived from the knowledge of the twist-2 cross section formula for the unpolarized process. We establish the corresponding "master formula" that is applicable to a range of processes like SIDIS, Drell–Yan and direct-photon production.

To illustrate our approach, we consider the SIDIS, $e(\ell) + p(p, S_\perp) \to e(\ell') + \pi(P_h) + X$, following the convention of our recent work [6]: We use the kinematic variables, $S_{ep} = (p+\ell)^2$, $q = \ell - \ell'$, $Q^2 = -q^2$, $x_{bj} = Q^2/(2p\cdot q)$, and $z_f = p\cdot P_h/p\cdot q$. All momenta p, P_h, ℓ, and ℓ' of the particles in the initial and the final states can be regarded as light-like in the twist-3 accuracy, $p^2 = P_h^2 = \ell^2 = \ell'^2 = 0$. As usual, we define another light-like vector as $n^\mu = (q^\mu + x_{bj} p^\mu)/p\cdot q$, and the projector onto the transverse direction as $g_\perp^{\mu\nu} = g^{\mu\nu} - p^\mu n^\nu - p^\nu n^\mu$. We also define a space-like vector, $q_t^\mu = q^\mu - (P_h\cdot q/p\cdot P_h)p^\mu - (p\cdot q/p\cdot P_h)P_h^\mu$, which is orthogonal to both p and P_h, and its magnitude as $q_T = \sqrt{-q_t^2}$. Then, in a frame where the 3-momenta $\vec{q}$ and $\vec{p}$ of the virtual photon and the initial nucleon are collinear along the z-axis, like the so-called hadron frame [16], the magnitude of the transverse momentum of the pion is given by $\sqrt{-P_{h\perp}^2} = z_f q_T$.

In the present study we are interested in the SSA for the large-$P_{h\perp}$ pion production, in particular, the contribution from the twist-3 quark–gluon correlation functions for the transversely polarized nucleon, which are defined as [5,17]

$$M_{Fij}^\beta(x_1,x_2) = \int \frac{d\lambda}{2\pi}\int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle pS_\perp|\bar{\psi}_j(0)[0,\mu n] g F^{\beta\rho}(\mu n) n_\rho [\mu n, \lambda n]\psi_i(\lambda n)|pS_\perp\rangle$$
$$= \frac{M_N}{4}(\not{p})_{ij}\epsilon^{\beta p n S_\perp} G_F(x_1,x_2) + i\frac{M_N}{4}(\gamma_5\not{p})_{ij} S_\perp^\beta \tilde{G}_F(x_1,x_2) + \cdots, \tag{1}$$

where the spinor indices i and j associated with the quark field ψ are shown explicitly, $F^{\beta\rho}$ is the gluon field strength tensor, and the spin vector for the transversely polarized nucleon satisfies $S_\perp^\alpha = g_\perp^{\alpha\beta} S_{\perp\beta}$, $S_\perp^2 = -1$. The path-ordered gauge-link, $[\mu n, \lambda n] = P\exp[ig\int_\lambda^\mu dt\, n\cdot A(tn)]$, guarantees gauge invariance of the nonlocal lightcone operator. The second line of (1) defines two dimensionless functions $G_F(x_1,x_2)$ and $\tilde{G}_F(x_1,x_2)$ through the Lorentz decomposition of the matrix element; here M_N is the nucleon mass representing typical mass scale generated by nonperturbative effects, and "$\cdots$" denotes Lorentz structures of twist higher than three. These two functions $G_F(x_1,x_2)$ and $\tilde{G}_F(x_1,x_2)$ constitute a complete set of the twist-3 quark–gluon correlation functions for the transversely polarized nucleon [15,17].

The relevant contributions to the hadronic tensor $W_{\mu\nu}$ arise from the process where the partons from the nucleon in the initial state undergoes the hard interaction with the virtual photon, followed by the fragmentation into the final state with π + anything, as illustrated in Fig. 1. The twist-3 distribution functions contribute to $ep^\uparrow \to e\pi X$ in combination with the twist-2 fragmentation function for the pion, which is immediately factorized from the hadronic tensor as

$$W_{\mu\nu}(p,q,P_h) = \sum_{j=q,g}\int \frac{dz}{z^2} D_j(z) w_{\mu\nu}^j\left(p,q,\frac{P_h}{z}\right), \tag{2}$$

where $D_j(z)$ $(j=q,g)$ is the quark and gluon fragmentation functions for the pion, with z being the momentum fraction. We consider the case for the quark fragmentation in detail and omit the index j from $w_{\mu\nu}^j$ below. Modifications necessary for the gluon-fragmentation case will be discussed later. The lower blobs in Fig. 1 can be written as Fourier transform of the correlation functions for the nucleon, i.e., schematically, $M^{(0)}(k) \sim \langle pS_\perp|\bar{\psi}\psi|pS_\perp\rangle$ and $M^{(1)\sigma}(k_1,k_2) \sim \langle pS_\perp|\bar{\psi}A^\sigma\psi|pS_\perp\rangle$, with the upper indices (0) and (1) representing the number of gluon lines connecting the middle and lower blobs. (See Eqs. (23), (24) of [6] for the explicit definitions.) The momenta of the partons, k and $k_{1,2}$, are assigned as in Fig. 1.

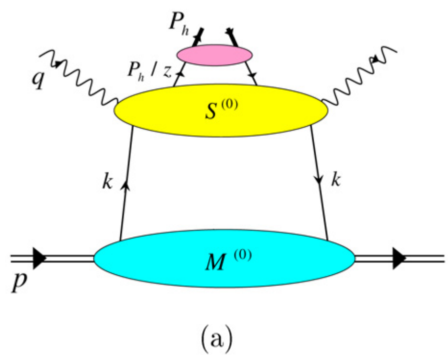

Fig. 1. Generic diagrams for the hadronic tensor of $ep^\uparrow \to e\pi X$, decomposed into the three blobs as nucleon matrix element (lower), pion matrix element (upper), and partonic hard scattering by the virtual photon (middle). The first two terms, (a) and (b), in the expansion by the number of partons connecting the middle and lower blobs are relevant to the twist-3 effect induced by the nucleon.

In the leading-order perturbation theory for the partonic hard scattering, we have shown [6] that the twist-3 contribution to $w_{\mu\nu}$ arises solely from Fig. 1(b), and the entire twist-3 contribution can be written as

$$w_{\mu\nu}\left(p,q,\frac{P_h}{z}\right)=\int dx_1\int dx_2\,\mathrm{Tr}\left[ig_{\perp\beta}^{\alpha}M_F^{\beta}(x_1,x_2)\frac{\partial S^{(1)}(k_1,k_2,P_h/z)}{\partial k_{2\perp}^{\alpha}}\bigg|_{k_i=x_ip}\right],\tag{3}$$

where $M_F^{\beta}(x_1,x_2)$ is defined in (1), and $S^{(1)}(k_1,k_2,P_h/z)\equiv S_\sigma^{(1)}(k_1,k_2,P_h/z)p^\sigma$ represents the middle blob in Fig. 1(b), which denotes the hard scattering function for the partons off the virtual photon, corresponding to the nucleon matrix element $M^{(1)\sigma}(k_1,k_2)$. For simplicity, we suppress momentum q and the Lorentz indices μ, ν for the virtual photon in $S_\sigma^{(1)}(k_1,k_2,P_h/z)$. In (3), $S^{(1)}(k_1,k_2,P_h/z)$ and $M_F^{\beta}(x_1,x_2)$ are matrices in spinor space, and $\mathrm{Tr}[\cdots]$ indicates the trace over Dirac-spinor indices while the color trace is implicit. Here we recall from [6] that, although the straightforward collinear expansion to the twist-3 accuracy produces many other complicated terms, it has been proved that the hard parts associated with those terms vanish in the leading order in QCD perturbation theory, using Ward identities for color gauge invariance. Note that (3) guarantees the factorization property for the twist-3 single-spin-dependent cross section in manifestly gauge-invariant form, which was assumed in the previous literature [8,15] (see also [5,7]). We should also mention that, for the HP and SFP contributions, one can calculate the partonic hard scattering function via $S_\alpha^{(1)}(x_1p,x_2p,P_h/z)$ by using the relation $\partial S^{(1)}(k_1,k_2,P_h/z)/\partial k_{2\perp}^{\alpha}|_{k_i=x_ip}=S_\alpha^{(1)}(x_1p,x_2p,P_h/z)/(x_1-x_2)$ obtained from the Ward identity, while, for the SGP contribution, one has to calculate the "derivative", $\partial S^{(1)}(k_1,k_2,P_h/z)/\partial k_{2\perp}^{\alpha}|_{k_i=x_ip}$, as shown in (3) [6].

Based on (3), our task is to identify the SGP contribution to $S^{(1)}(k_1,k_2,P_h/z)$ and compute its deviation arising linearly in the quark's transverse momentum $k_{2\perp}$ from the value in the collinear limit $k_{1,2}\to x_{1,2}p$. For this purpose, we work in the Feynman gauge and with $k_{i\perp}\ll x_ip$ in $k_i=x_ip+(k_i\cdot p)n+k_{i\perp}$ $(i=1,2)$. In the leading order in QCD perturbation theory, $S^{(1)}(k_1,k_2,P_h/z)$ stands for a set of cut Feynman diagrams which are obtained by attaching the additional gluon to the 2-to-2 partonic Born subprocess, where the large transverse momentum $P_{h\perp}/z$ of the "fragmenting quark" is provided by the recoil from the emission of a hard gluon into the final state. When the coherent gluon couples to an on-shell parton line, the parton propagator adjacent to the coherent gluon produces a pole for the vanishing gluon momentum, $k_2-k_1\to 0$. Only those arising from the diagrams in Fig. 2, where the coherent gluon couples to the final-state quark line fragmenting into π + anything, survive as the SGP contributions [6,8,15], while the other pole contributions cancel out combined with those from the "mirror" diagrams. We denote the contributions of the diagrams in Fig. 2, where the coherent gluon is attached to the LHS of the cut, as $S^{(1)L}(k_1,k_2,P_h/z)$, and those of the mirror diagrams as $S^{(1)R}(k_1,k_2,P_h/z)$, so that $S^{(1)}(k_1,k_2,P_h/z)=S^{(1)L}(k_1,k_2,P_h/z)+S^{(1)R}(k_1,k_2,P_h/z)$. Explicit form of $S^{(1)L}(k_1,k_2,P_h/z)$ is given as

$$S^{(1)L}\left(k_1,k_2,\frac{P_h}{z}\right)=\frac{-1}{2N_c}\bar{\Gamma}_\alpha\left(k_2,\frac{P_h}{z}\right)\frac{\not{P}_h}{z}i\gamma_\sigma p^\sigma\frac{i}{\frac{\not{P}_h}{z}+\not{k}_1-\not{k}_2+i\varepsilon}\Gamma_\beta\left(k_1,\frac{P_h}{z}+k_1-k_2\right)\mathcal{D}_+^{\alpha\beta}\left(k_2+q-\frac{P_h}{z}\right),\tag{4}$$

where $\bar{\Gamma}_\alpha(k_2,P_h/z)\equiv\gamma^0\Gamma_\alpha^\dagger(k_2,P_h/z)\gamma^0$ denotes the photon–quark–gluon vertex function of Fig. 3 which appears in the RHS of the cut in the diagrams of Fig. 2. In $\bar{\Gamma}_\alpha(k_2,P_h/z)$, the factors for the external lines are amputated. With this Γ_α, the photon–quark–gluon vertex function appearing in the LHS of the cut in Fig. 2 is given by $\Gamma_\beta(k_1,P_h/z+k_1-k_2)$. The structure $\not{P}_h/z$ projects the final-state quark onto the twist-2 fragmentation process to produce the pion (see (2)), and $i\gamma_\sigma p^\sigma$ is the quark–coherent-gluon vertex. $\mathcal{D}_+^{\alpha\beta}(k)=2\pi\delta(k^2)\theta(k^0)\sum_\lambda\epsilon_{(\lambda)}^{\alpha}(k)\epsilon_{(\lambda)}^{*\beta}(k)$ is the cut propagator for the final-state hard gluon with the polarization vector $\epsilon_{(\lambda)}^{\alpha}(k)$. Note that we have already worked out the color structure associated with (3), simplifying the color matrices contained in Fig. 2 as $t^at^bt^a=(-1/2N_c)t^b$ and performing the color trace to obtain the gauge-invariant matrix element (1). We also note the relation $S^{(1)R}(k_1,k_2,P_h/z)=\bar{S}^{(1)L}(k_2,k_1,P_h/z)\equiv\gamma^0S^{(1)L\dagger}(k_2,k_1,P_h/z)\gamma^0$.

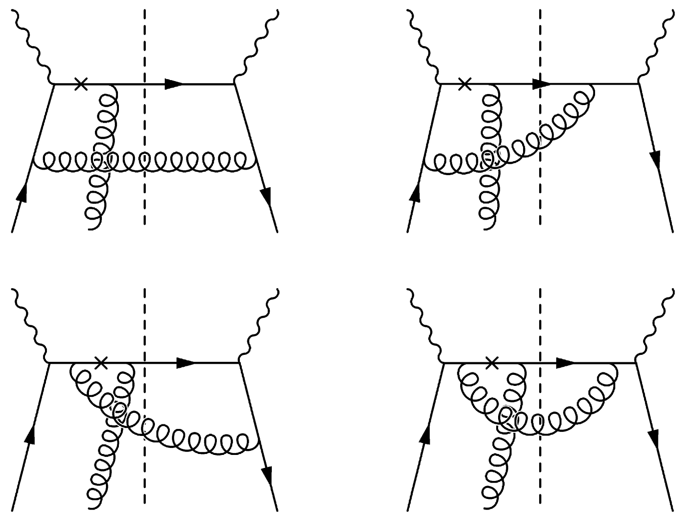

Fig. 2. Feynman diagrams which give rise to the SGP contributions in the quark fragmentation channel, where the hard quark fragments into the final-state with pion and the hard gluon goes into unobserved final state. The cross × denotes the quark propagator which gives the SGP contribution. Mirror diagrams also contribute.

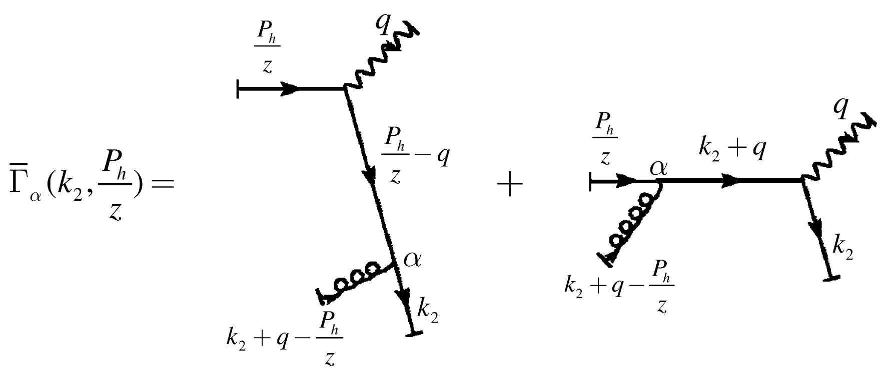

Fig. 3. The definition of the photon–quark–gluon vertex function $\bar{\Gamma}_\alpha(k_2, P_h/z)$, as the sum of two types of diagrams which appear in the RHS of the cut in Fig. 2. The amputated lines are identified by bars at their ends.

To proceed further, one may employ direct evaluation of each diagram of Fig. 2, substituting the explicit form corresponding to Fig. 3 into the vertex functions $\bar{\Gamma}_\alpha(k_2, P_h/z)$ and $\Gamma_\beta(k_1, P_h/z + k_1 - k_2)$ of (4), and working out the necessary Dirac algebra. Such calculation has been done in [6,8,15]. Here we employ another approach to disentangle the coherent-gluon vertex and the corresponding SGP structure from the diagrams of Fig. 2. An earlier attempt in the same spirit investigated decoupling property of the soft-gluon vertex in the collinear limit, $k_{1\perp} = k_{2\perp} = 0$, for the SGP contribution in hadron–hadron scattering, using the helicity basis technique [20]; but fully consistent evaluation of the SGP contribution following [6] requires to treat the deviation of the hard part due to nonzero $k_{2\perp}$, as noted above, and in this case straightforward application of the helicity basis technique would be useless. Our new systematic approach uses Ward identities and decomposition identities for the parton propagators interacting with the coherent gluon, which allows us to disentangle the coherent-gluon vertex from the derivative $\partial S^{(1)}(k_1, k_2, P_h/z)/\partial k_{2\perp}^\alpha|_{k_i = x_i p}$ in (3). A key idea is to reorganize the terms that contribute to this derivative by rewriting p^σ contracted with the quark–gluon vertex in (4) as

$$p^\sigma = \frac{1}{x_2 - x_1 - i\varepsilon}\left(k_2^\sigma - k_1^\sigma\right) - \frac{1}{x_2 - x_1 - i\varepsilon}\left(k_{2\perp}^\sigma - k_{1\perp}^\sigma\right), \tag{5}$$

up to the irrelevant $O((k_{2\perp} - k_{1\perp})^2)$ correction. Correspondingly, $S^{(1)L}(k_1, k_2, P_h/z)$ can be decomposed as

$$S^{(1)L}\left(k_1, k_2, \frac{P_h}{z}\right) = S^{(1\cdot)L}\left(k_1, k_2, \frac{P_h}{z}\right) + S^{(1\perp)L}\left(k_1, k_2, \frac{P_h}{z}\right), \tag{6}$$

where the first and second terms in the RHS correspond to those in (5), respectively. In (5), "$-i\varepsilon$" in the denominator is chosen such that each term in (6) does not produce pinch singularity at $x_1 = x_2$.

We can exploit an elementary Ward identity in $S^{(1\cdot)L}(k_1, k_2, P_h/z)$ in order to disentangle the scalar-polarized gluon vertex, $\not{k}_2 - \not{k}_1$, as well as the quark propagator adjacent to it, as $(\not{P}_h/z)(\not{k}_2 - \not{k}_1)[1/(\not{P}_h/z + \not{k}_1 - \not{k}_2 + i\varepsilon)] = (\not{P}_h/z)(\not{k}_2 - \not{k}_1 -$

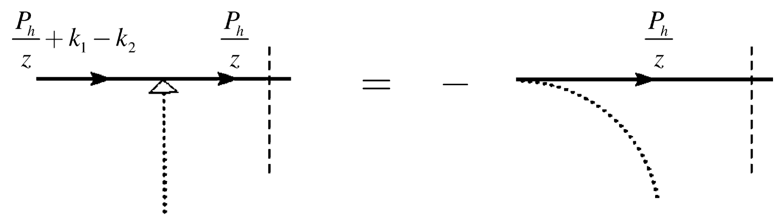

Fig. 4. Diagrammatic representation of Ward identity for the coupling of the scalar-polarized gluon to the final-state quark line.

$\not{P}_h/z)[1/(\not{P}_h/z+\not{k}_1-\not{k}_2+i\varepsilon)]=-\not{P}_h/z$, as illustrated in Fig. 4, and get

$$S^{(1\cdot)L}\left(k_1,k_2,\frac{P_h}{z}\right)=\frac{-1}{2N_c}\frac{1}{x_2-x_1-i\varepsilon}\bar{\Gamma}_\alpha\left(k_2,\frac{P_h}{z}\right)\frac{\not{P}_h}{z}\Gamma_\beta\left(k_1,\frac{P_h}{z}+k_1-k_2\right)\mathcal{D}_+^{\alpha\beta}\left(k_2+q-\frac{P_h}{z}\right). \tag{7}$$

This exhibits the SGP only as the single pole, so that we can put $x_2=x_1$ except for the factor $1/(x_2-x_1-i\varepsilon)$, without affecting the results for the SGP contribution. Combining this with the corresponding result for $S^{(1\cdot)R}(k_1,k_2,P_h/z)=\bar{S}^{(1\cdot)L}(k_2,k_1,P_h/z)$ and Taylor expanding the total result $S^{(1\cdot)}(k_1,k_2,P_h/z)=S^{(1\cdot)L}(k_1,k_2,P_h/z)+S^{(1\cdot)R}(k_1,k_2,P_h/z)$ in terms of $k_{1\perp}$ and $k_{2\perp}$, we get

$$S^{(1\cdot)}\left(k_1,k_2,\frac{P_h}{z}\right)=\frac{1}{2N_c}\frac{k_{2\perp}^\sigma-k_{1\perp}^\sigma}{x_2-x_1-i\varepsilon}\frac{\partial}{\partial r_\perp^\sigma}\bar{\Gamma}_\alpha(x_1p,r)\frac{\not{P}_h}{z}\Gamma_\beta(x_1p,r)\mathcal{D}_+^{\alpha\beta}(x_1p+q-r)\bigg|_{r\to\frac{P_h}{z}}, \tag{8}$$

up to the irrelevant terms of the second or higher order in $k_{1\perp}$, $k_{2\perp}$. Here r denotes a four-vector not restricted to being light-like, $r^2\neq0$.

Next we consider the contribution from $S^{(1\perp)L}(k_1,k_2,P_h/z)$ of the second term of (6). Because the coherent gluon vertex associated with $S^{(1\perp)L}(k_1,k_2,P_h/z)$ is proportional to $k_{2\perp}^\sigma-k_{1\perp}^\sigma$ (see (4), (5)), we can set $k_{1\perp}=k_{2\perp}=0$ in $S^{(1\perp)L}(k_1,k_2,P_h/z)$ except for this vertex factor, up to irrelevant corrections. One thus obtains

$$\begin{aligned}S^{(1\perp)L}\left(k_1,k_2,\frac{P_h}{z}\right)&=\frac{-1}{2N_c}\frac{k_{2\perp}^\sigma-k_{1\perp}^\sigma}{x_2-x_1-i\varepsilon}\bar{\Gamma}_\alpha\left(x_2p,\frac{P_h}{z}\right)\\&\quad\times\frac{\not{P}_h}{z}\gamma_\sigma\frac{1}{\frac{\not{P}_h}{z}+(x_1-x_2)\not{p}+i\varepsilon}\Gamma_\beta\left(x_1p,\frac{P_h}{z}+x_1p-x_2p\right)\mathcal{D}_+^{\alpha\beta}\left(x_2p+q-\frac{P_h}{z}\right).\end{aligned} \tag{9}$$

In this result we decompose the product of the quark–gluon vertex and the quark propagator, adjacent to the fragmentation insertion $\not{P}_h/z$, as

$$\frac{\not{P}_h}{z}\gamma_\sigma\frac{\not{P}_h/z+(x_1-x_2)\not{p}}{(\frac{P_h}{z}+(x_1-x_2)p)^2+i\varepsilon}=\frac{\not{P}_h}{z}\left(-\frac{P_{h\sigma}}{P_h\cdot p}\frac{1}{x_2-x_1-i\varepsilon}+\gamma_\sigma\frac{z\not{p}}{2P_h\cdot p}\right), \tag{10}$$

where the first term in the parentheses is given by the eikonal vertex and eikonal propagator, and the second term is the "contact term". Combining the result obtained from (9), (10) with the corresponding result for $S^{(1\perp)R}(k_1,k_2,P_h/z)=\bar{S}^{(1\perp)L}(k_2,k_1,P_h/z)$, we find that the double pole term in $S^{(1\perp)L}(k_1,k_2,P_h/z)$, which originates from the eikonal propagator in (10) and gives the contribution proportional to $\delta'(x_1-x_2)$, cancels the corresponding double-pole term in $S^{(1\perp)R}(k_1,k_2,P_h/z)$, and the remaining single-pole contributions eventually give, for $S^{(1\perp)}(k_1,k_2,P_h/z)=S^{(1\perp)L}(k_1,k_2,P_h/z)+S^{(1\perp)R}(k_1,k_2,P_h/z)$,

$$\begin{aligned}S^{(1\perp)}\left(k_1,k_2,\frac{P_h}{z}\right)&=\frac{1}{2N_c}\frac{k_{2\perp}^\sigma-k_{1\perp}^\sigma}{x_2-x_1-i\varepsilon}\frac{\partial}{\partial r_\rho}\left\{g_{\sigma\rho}\bar{\Gamma}_\alpha\left(x_1p,\frac{P_h}{z}\right)\not{r}\Gamma_\beta\left(x_1p,\frac{P_h}{z}\right)\mathcal{D}_+^{\alpha\beta}\left(x_1p+q-\frac{P_h}{z}\right)\right.\\&\quad\left.-\frac{P_{h\sigma}p_\rho}{P_h\cdot p}\bar{\Gamma}_\alpha(x_1p,r)\not{r}\Gamma_\beta(x_1p,r)\mathcal{D}_+^{\alpha\beta}(x_1p+q-r)\right\}\bigg|_{r\to\frac{P_h}{z}}.\end{aligned} \tag{11}$$

Combining (8) and (11), the entire SGP contribution for (3) reads[1]

$$\frac{\partial S^{(1)}(k_1,k_2,P_h/z)}{\partial k_{2\perp}^\sigma}\bigg|_{k_i=x_ip}=\frac{1}{2N_cC_F}\frac{z}{x_2-x_1-i\varepsilon}\frac{\partial S^{(0)}(x_1p,\frac{P_h}{z})}{\partial P_{h\perp}^\sigma}, \tag{12}$$

[1] We also obtain $(\partial/\partial k_{1\perp}^\rho+\partial/\partial k_{2\perp}^\rho)S^{(1)}(k_1,k_2,P_h/z)|_{k_i=x_ip}=0$ for the SGP contribution, which has been proved in [6] by the detailed inspection of the diagrams in Fig. 2. This holds for the gluon fragmentation channel, too (see (19)).

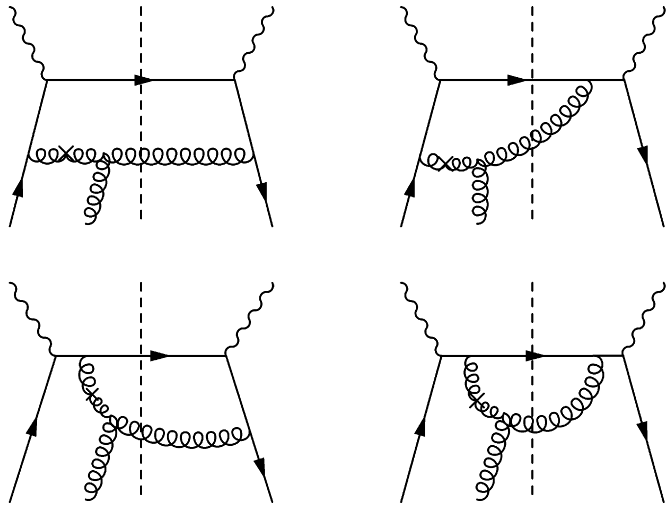

Fig. 5. Same as Fig. 2, but for the gluon fragmentation channel where the hard gluon fragments into the final-state with pion and the hard quark goes into unobserved final state.

where $C_F = (N_c^2 - 1)/(2N_c)$ and

$$S^{(0)}\left(x_1 p, \frac{P_h}{z}\right) = C_F \bar{\Gamma}_\alpha\left(x_1 p, \frac{P_h}{z}\right)\frac{\not{P}_h}{z}\Gamma_\beta\left(x_1 p, \frac{P_h}{z}\right)\mathcal{D}_+^{\alpha\beta}\left(x_1 p + q - \frac{P_h}{z}\right) \tag{13}$$

is exactly the leading-order hard scattering function in the collinear limit $k \to xp$, which is represented as the middle blob in Fig. 1(a). In (12), we have used the relation,

$$\left(g_{\sigma\rho} - \frac{P_{h\sigma} p_\rho}{P_h \cdot p}\right)\frac{\partial \varphi(r)}{\partial r_\rho}\bigg|_{r \to \frac{P_h}{z}} = z\frac{\partial \varphi(P_h/z)}{\partial P_h^\sigma}, \tag{14}$$

for $P_h = -(P_{h\perp}^2/2P_h \cdot p)p + (P_h \cdot p)n + P_{h\perp}$, which holds for an arbitrary function $\varphi(r)$. Note that $P_h \cdot n$ is not an independent variable to perform the derivative in the RHS of (14), corresponding to that the LHS vanishes when contracted by p^σ. Substituting (1) and (12) into (3), and integrating over x_2 to get the imaginary phase from the SGP of (12), one gets for $w_{\mu\nu}$ as

$$w_{\mu\nu} = \frac{1}{2N_c C_F}\left(\frac{-\pi M_N z}{4}\right)\epsilon^{\sigma p n S_\perp}\frac{\partial}{\partial P_{h\perp}^\sigma}\int dx\, G_F(x,x)\,\mathrm{Tr}\left[S^{(0)}\left(xp, \frac{P_h}{z}\right)\not{p}\right] + \cdots, \tag{15}$$

where the ellipses stand for the contribution from the second term in the RHS of (1), which does not contribute to the cross section when contracted with the symmetric leptonic tensor for the unpolarized electron. We recall the similar formula for the leading-order twist-2 hadronic tensor,

$$w_{\mu\nu}^{\text{tw-2}} = \frac{1}{2}\int dx\, f_q(x)\,\mathrm{Tr}\left[S^{(0)}\left(xp, \frac{P_h}{z}\right)\not{p}\right] + \cdots, \tag{16}$$

where $S^{(0)}(xp, P_h/z)$ in (15) appears and $f_q(x)$ is the unpolarized quark distribution.

It is straightforward to extend the above results to the gluon fragmentation case by mostly trivial substitutions, noting that essential part of our above derivation is based on diagrammatic manipulation. The diagrams for the SGP contribution in this case are shown in Fig. 5 [6,15]. The corresponding hard scattering function is given by

$$\begin{aligned} S_g^{(1)L}\left(k_1, k_2, \frac{P_h}{z}\right) = {} & \frac{iN_c}{2}\bar{\Lambda}_\alpha\left(k_2, \frac{P_h}{z}\right)\mathcal{S}_+\left(k_2 + q - \frac{P_h}{z}\right)\Lambda_\beta\left(k_1, \frac{P_h}{z} + k_1 - k_2\right)\left[-\hat{g}_t^{\alpha\eta}(P_h)\right] \\ & \times V_{\sigma\eta\beta}\left(k_2 - k_1, -\frac{P_h}{z}, \frac{P_h}{z} + k_1 - k_2\right)p^\sigma\frac{-i}{(\frac{P_h}{z} + k_1 - k_2)^2 + i\varepsilon}, \end{aligned} \tag{17}$$

where the relevant photon–quark–gluon vertex functions $\bar{\Lambda}_\alpha(k_2, P_h/z)$ and $\Lambda_\beta(k_1, P_h/z + k_1 - k_2)$ can be expressed using that of Fig. 3 by appropriate substitutions of the momenta, $\Lambda_\alpha(k_2, P_h/z) = \Gamma_\alpha(k_2, k_2 + q - P_h/z)$. $V_{\mu_1\mu_2\mu_3}(q_1, q_2, q_3) = (q_1 - q_2)_{\mu_3} g_{\mu_1\mu_2} +$ (cyclic permutation) denotes the ordinary three-gluon vertex except for the color structure that has been used as $f^{cab}t^a t^b = (iN_c/2)t^c$ to obtain the prefactor $iN_c/2$, the structure $-\hat{g}_t^{\alpha\eta}(P_h) = -g^{\alpha\eta} + (P_h^\alpha p^\eta + P_h^\eta p^\alpha)/(P_h \cdot p)$ projects the final-state gluon onto the twist-2 fragmentation process to produce the pion (see (2)), and $\mathcal{S}_+(k) = 2\pi\delta(k^2)\theta(k^0)\not{k}$ is the cut propagator for the final-state hard quark. The mirror diagrams of Fig. 5 gives $S_g^{(1)R}(k_1, k_2, P_h/z) = \bar{S}_g^{(1)L}(k_2, k_1, P_h/z)$. Similarly to (6),

$S_g^{(1)L}$ can be decomposed as

$$S_g^{(1)L}\left(k_1,k_2,\frac{P_h}{z}\right)=S_g^{(1\cdot)L}\left(k_1,k_2,\frac{P_h}{z}\right)+S_g^{(1\perp)L}\left(k_1,k_2,\frac{P_h}{z}\right),\tag{18}$$

by using (5). The three-gluon vertex coupling to the scalar-polarized coherent gluon in $S_g^{(1\cdot)L}(k_1,k_2,P_h/z)$ can be disentangled using Ward identity similarly to (7), but this time we also obtain some additional ghost-like gauge terms. It is indeed not difficult to show that those gauge terms drop in the final result applying Ward identities further. Alternatively, one may employ background field gauge [21]: In the Feynman gauge under the background coherent-gluon field, the three-gluon vertex in (17) is replaced as $V_{\sigma\eta\beta}(k_2-k_1,-P_h/z,P_h/z+k_1-k_2)\to V^{BG}_{\sigma\eta\beta}(k_2-k_1,-P_h/z,P_h/z+k_1-k_2)\equiv V_{\sigma\eta\beta}(k_2-k_1,-P_h/z,P_h/z+k_1-k_2)-(P_h/z)_\eta g_{\sigma\beta}-(P_h/z+k_1-k_2)_\beta g_{\sigma\eta}$, and the resulting $S_g^{(1\cdot)L}(k_1,k_2,P_h/z)$ obeys simple Ward identity without unwanted gauge terms, which is analogous to Fig. 4 for the quark–gluon vertex. As a result, we get for $S_g^{(1\cdot)}(k_1,k_2,P_h/z)=S_g^{(1\cdot)L}(k_1,k_2,P_h/z)+S_g^{(1\cdot)R}(k_1,k_2,P_h/z)$

$$S_g^{(1\cdot)}\left(k_1,k_2,\frac{P_h}{z}\right)=\frac{N_c}{2}\frac{k^\sigma_{2\perp}-k^\sigma_{1\perp}}{x_2-x_1-i\varepsilon}\frac{\partial}{\partial r^\sigma_\perp}\bar{\Lambda}_\alpha(x_1p,r)\mathcal{S}_+(x_1p+q-r)\Lambda_\beta(x_1p,r)\hat{g}_t^{\alpha\beta}(P_h)\bigg|_{r\to\frac{P_h}{z}},\tag{19}$$

similarly to (8), up to the irrelevant higher order terms.

The contribution $S_g^{(1\perp)L}(k_1,k_2,P_h/z)$ in (18) can be also reduced similarly to (9): We can use the decomposition of the product of the three-gluon vertex and the gluon propagator, adjacent to the fragmentation insertion $\hat{g}_t^{\alpha\eta}(P_h)$, into the eikonal propagator term and the contact term as

$$\begin{aligned}&\hat{g}_t^{\alpha\eta}(P_h)V^{BG}_{\sigma\eta\beta}\left(x_2p-x_1p,-\frac{P_h}{z},\frac{P_h}{z}+x_1p-x_2p\right)\frac{-1}{(\frac{P_h}{z}+(x_1-x_2)p)^2+i\varepsilon}\\&\quad=\hat{g}_t^{\alpha\eta}(P_h)\left(-\frac{P_{h\sigma}g_{\eta\beta}}{P_h\cdot p}\frac{1}{x_2-x_1-i\varepsilon}+g_{\sigma\eta}p_\beta\frac{z}{P_h\cdot p}\right),\end{aligned}\tag{20}$$

for $\sigma=\perp$. This simple formula analogous to (10) holds in the background field gauge mentioned above. Again, only the single pole terms eventually survive in the total contribution, $S_g^{(1\perp)}(k_1,k_2,P_h/z)=S_g^{(1\perp)L}(k_1,k_2,P_h/z)+S_g^{(1\perp)R}(k_1,k_2,P_h/z)$, and the result is given by the RHS of (11) with the substitutions necessary for "translating" (8) into (19). Combining this result with (19) and taking the similar steps as those in (12)–(15), we find that $w_{\mu\nu}$ for the gluon fragmentation channel is given by (15) with the replacement $1/N_c\to -N_c$ and $S^{(0)}(xp,P_h/z)\to S_g^{(0)}(xp,P_h/z)$, where

$$S_g^{(0)}\left(xp,\frac{P_h}{z}\right)=C_F\bar{\Lambda}_\alpha\left(xp,\frac{P_h}{z}\right)\mathcal{S}_+\left(xp+q-\frac{P_h}{z}\right)\Lambda_\beta\left(xp,\frac{P_h}{z}\right)\left[-\hat{g}_t^{\alpha\beta}(P_h)\right].\tag{21}$$

This $S_g^{(0)}(xp,P_h/z)$ is the hard scattering function for the 2-to-2 partonic Born subprocess leading to the gluon fragmentation, and participates in the twist-2 contribution for the unpolarized SIDIS as (16) with $S^{(0)}(xp,P_h/z)\to S_g^{(0)}(xp,P_h/z)$.

Substituting (13)–(16) and (21) into (2), we contract the result with the leptonic tensor for the unpolarized electron, $L_{\mu\nu}(\ell,\ell')=2(\ell_\mu\ell'_\nu+\ell_\nu\ell'_\mu)-g_{\mu\nu}Q^2$. Working out the phase space factor corresponding to the differential elements, $[d\omega]=dx_{bj}\,dQ^2\,dz_f\,dq_T^2\,d\phi$, with the kinematical variables introduced above (1) and the azimuthal angle ϕ for the observed final-state pion (see [6,15]), we immediately obtain the formula for the single-spin-dependent cross section in SIDIS, $ep^\uparrow\to e\pi X$, associated with the soft-gluon mechanism induced by the twist-3 effects inside the nucleon, as

$$\frac{d\sigma^{\rm SGP}_{\text{tw-3}}}{[d\omega]}=\frac{\pi M_N}{2C_F}\epsilon^{\sigma pnS_\perp}\sum_{j=q,g}\mathcal{C}_j\int\frac{dz}{z}\int\frac{dx}{x}D_j(z)\frac{\partial H_{jq}(x,z,q_T^2)}{\partial(P^\sigma_{h\perp}/z)}G_F^q(x,x),\tag{22}$$

where the sum over all quark and antiquark flavors $q=u,\bar{u},d,\bar{d},\ldots$ is implicit for the index q, and $G_F^q(x,x)$ denotes the "soft-gluon-pole function" from (1) for the flavor q. The color factors are introduced as $\mathcal{C}_q=-1/(2N_c)$ and $1/(2N_c)$ for quark and antiquark flavors, respectively,[2] and $\mathcal{C}_g=N_c/2$. $H_{jq}(x,z,q_T^2)$ for $j=q$ and g are, respectively, equal to $\mathrm{Tr}[S^{(0)}(xp,P_h/z)\not{p}]$ and $\mathrm{Tr}[S_g^{(0)}(xp,P_h/z)\not{p}]$, up to the kinematical factor. They are exactly the partonic hard scattering cross sections which participate in the twist-2 factorization formula of the unpolarized cross section for $ep\to e\pi X$ as

$$\frac{d\sigma^{\rm unpol}_{\text{tw-2}}}{[d\omega]}=\sum_{j=q,g}\int\frac{dz}{z}\int\frac{dx}{x}D_j(z)H_{jq}\big(x,z,q_T^2\big)f_q(x).\tag{23}$$

[2] The relative minus sign in $\mathcal{C}_q$ between quark and antiquark is due to that of the color charge between them, to which the coherent gluon couples.

Our results (22) and (23) represent the SGP contribution as the response of 2-to-2 partonic Born subprocess to the change of the transverse momentum carried by the "fragmenting parton". It is worth noting that our results (22) and (23) hold in any Lorentz frame with $p_\perp = 0$, as is seen from the above derivation. In particular, we recall that the derivative with respect to the transverse momentum in (22) was introduced merely as a formal recipe via the relation (14), so that one can freely move to any frame even with $P_{h\perp} = 0$ after performing the derivative.

In the hadron frame mentioned above (1), P_h^σ is parameterized as $\sqrt{-P_{h\perp}^2} = z_f q_T$, $P_h^- = z_f Q/\sqrt{2}$ and $P_h^+ = -P_{h\perp}^2/2P_h^- = z_f q_T^2/\sqrt{2}Q$, and thus the derivative on the Lorentz-scalar functions $H_{jq}(x, z, q_T^2)$ with respect to $P_{h\perp}^\sigma$ can be performed through q_T that is indicated explicitly in their argument. Therefore the results (22) and (23) can be expressed as

$$\frac{d\sigma_{\text{tw-3}}^{\text{SGP}}}{[d\omega]} = \frac{\pi M_N}{C_F z_f^2} \epsilon^{pnS_\perp P_{h\perp}} \frac{\partial}{\partial q_T^2} \frac{d\sigma_{\text{tw-2}}^{\text{unpol}}}{[d\omega]} \bigg|_{f_q(x) \to G_F^q(x,x),\ D_j(z) \to \mathcal{C}_j z D_j(z)}. \tag{24}$$

The partonic Born cross section in (23) was derived as [16,18,19]

$$H_{jq}\big(x, z, q_T^2\big) = \frac{\alpha_{em}^2 \alpha_s e_q^2}{8\pi x_{bj}^2 S_{ep}^2 Q^2} \sum_{k=1}^{4} \mathcal{A}_k \hat{\sigma}_k^{jq} \delta\left(\frac{q_T^2}{Q^2} - \left(\frac{1}{\hat{x}} - 1\right)\left(\frac{1}{\hat{z}} - 1\right)\right), \tag{25}$$

where $\mathcal{A}_1 = 1 + \cosh^2\psi$, $\mathcal{A}_2 = -2$, $\mathcal{A}_3 = -\cos\phi \sinh 2\psi$, and $\mathcal{A}_4 = \cos 2\phi \sinh^2\psi$, with $\cosh\psi = 2x_{bj} S_{ep}/Q^2 - 1$, parameterize different azimuthal dependence, and we introduced auxiliary variables $\hat{x} = x_{bj}/x$ and $\hat{z} = z_f/z$. $\hat{\sigma}_k^{jq}$ are the functions of $\hat{x}, \hat{z}, q_T^2$, and Q^2, whose explicit form can be found in Eqs. (57) and (59) of [6]. Therefore, the derivative $\partial/\partial q_T^2$ of (24) can act on either $\hat{\sigma}_k^{jq}$ or the delta-function $\delta(q_T^2/Q^2 - (1/\hat{x} - 1)(1/\hat{z} - 1))$, and the latter contribution produces the "derivative term" proportional to $dG_F^q(x,x)/dx$ as well as the "non-derivative term" with $G_F^q(x,x)$, after the partial integration with respect to x. We get

$$\begin{aligned}\frac{d\sigma_{\text{tw-3}}^{\text{SGP}}}{dx_{bj}\, dQ^2\, dz_f\, dq_T^2\, d\phi} &= \frac{\alpha_{em}^2 \alpha_s e_q^2}{8\pi x_{bj}^2 S_{ep}^2 Q^2} \frac{\pi M_N}{C_F z_f Q^2} \epsilon^{pnS_\perp P_{h\perp}} \sum_{k=1}^{4} \mathcal{A}_k \sum_{j=q,g} \mathcal{C}_j \\ &\quad \times \int \frac{dz}{z} \int \frac{dx}{x} D_j(z) \left\{ \frac{\hat{x}}{1-\hat{z}} \hat{\sigma}_k^{jq} x \frac{dG_F^q(x,x)}{dx} + \left[\frac{1}{\hat{z}} Q^2 \frac{\partial \hat{\sigma}_k^{jq}}{\partial q_T^2} - \frac{\hat{x}}{1-\hat{z}} \frac{\partial(\hat{x}\hat{\sigma}_k^{jq})}{\partial \hat{x}} \right] G_F^q(x,x) \right\} \\ &\quad \times \delta\left(\frac{q_T^2}{Q^2} - \left(\frac{1}{\hat{x}} - 1\right)\left(\frac{1}{\hat{z}} - 1\right)\right).\end{aligned} \tag{26}$$

Substituting the explicit formulae for $\hat{\sigma}_k^{jq}$, it is straightforward to see that this result completely coincides with that obtained recently in [6] by direct evaluation of each Feynman diagram in Figs. 2 and 5, for all azimuthal dependence, where $\mathcal{A}_{1,2}$ give the same azimuthal dependence as the Sivers effect [1] while $\mathcal{A}_3$ and $\mathcal{A}_4$ give additional terms beyond the Sivers effect. Our result reveals that not only the derivative term of the SGP cross section but the whole partonic hard scattering functions for the SGP contributions are completely determined from $\hat{\sigma}_k^{jq}$.

We emphasize that our result (24), and its immediate consequence (26), show extremely nontrivial structure behind the SGP contribution, which would not be unveiled without using our new approach discussed above. Indeed it is completely hopeless to infer from the complicated formulae (82) and (87) of [6] for the non-derivative term as a function of $\hat{x}, \hat{z}, q_T^2$, and Q^2 that the non-derivative term can be expressed in terms of $\hat{\sigma}_k^{jq}$ as in (26) with a form common to all channel ($j = q, g$; $k = 1, \ldots, 4$). We also note an important point which is most clearly indicated by the compact form (24): Apparently the cross section (25) for the 2-to-2 partonic Born subprocess is gauge-independent, and so is its derivative with respect to a kinematical variable q_T. Combined with (24), this fact guarantees the gauge invariance of the hard-scattering function for the twist-3 SGP contribution. In this connection we recall that a straightforward check of gauge invariance is hampered for the SGP contribution, in contrast to the case for the SFP and HP contributions, because Ward identities are useless to the coupling of the zero-momentum coherent gluon.

The above results (22) and (23) that lead to (24), (26) have been derived using only the properties satisfied by the partonic subprocess in QCD perturbation theory. Therefore, by analytically continuing the external momenta, the same formula represents the exact relations associated with other hard processes. In fact, the corresponding formula for the Drell–Yan process can be obtained by switching the final-state "fragmenting parton" into the initial-state parton, and the initial-state photon with space-like momentum into the final-state photon with time-like momentum. This crossing transformation, as illustrated in Fig. 6, is accomplished by the corresponding substitutions: $P_h \to -p'$, $1/z \to x'$, $D_q(z) \to f_{\bar{q}}(x')$, $D_g(z) \to f_g(x')$, and $q^\mu \to -q^\mu$ in (22) and (23), where $f_{\bar{q}}(x')$ and $f_g(x')$ denote the twist-2 parton distributions for the unpolarized initial hadron with momentum p', and the new q^μ denotes the momentum of the virtual photon which is produced by the hard interaction of the partons and goes into the lepton pair $\ell^+\ell^-$ with the invariant mass squared, $q^2 \equiv Q^2$, in the final state. With this replacement, our "master formula" (22) also describes the single-spin-dependent cross section for the Drell–Yan process $p^\uparrow p \to \ell^+\ell^- X$, relating it to (23) for the spin-averaged case

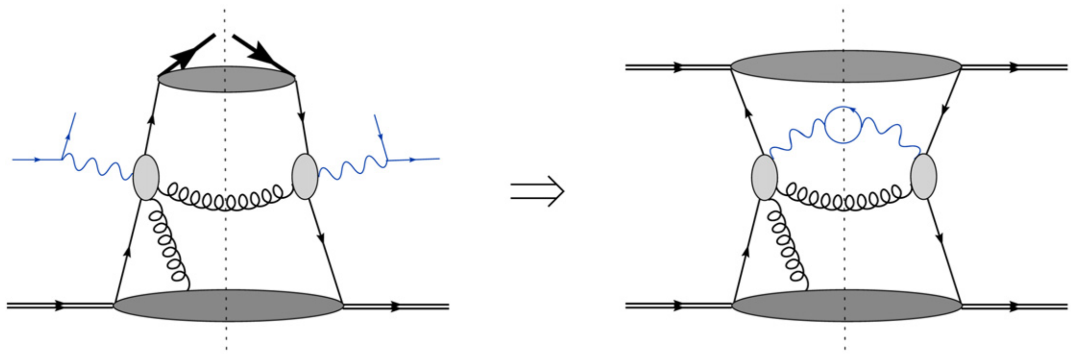

Fig. 6. Diagrammatic representation of the crossing transformation of SIDIS into Drell–Yan process.

$pp \to \ell^+\ell^- X$. The partonic hard scattering cross sections are now expressed by the variables, $s = (p + p')^2$, $\hat{s} = (xp + x'p')^2$, $\hat{t} = (xp - q)^2$, and $\hat{u} = (x'p' - q)^2$, as (see [7])

$$H_{jq}(\hat{s},\hat{t},\hat{u}) = \frac{\alpha_{em}^2 \alpha_s e_q^2}{3\pi N_c s Q^2} \hat{\sigma}_{jq}^{\mathrm{DY}}(\hat{s},\hat{t},\hat{u}) \delta\big(\hat{s}+\hat{t}+\hat{u}-Q^2\big), \tag{27}$$

with ($T_R = 1/2$)

$$\hat{\sigma}_{\bar{q}q}^{\mathrm{DY}}(\hat{s},\hat{t},\hat{u}) = 2C_F\left(\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + \frac{2Q^2\hat{s}}{\hat{u}\hat{t}}\right), \qquad \hat{\sigma}_{gq}^{\mathrm{DY}}(\hat{s},\hat{t},\hat{u}) = 2T_R\left(\frac{\hat{s}}{-\hat{t}} + \frac{-\hat{t}}{\hat{s}} - \frac{2Q^2\hat{u}}{\hat{s}\hat{t}}\right). \tag{28}$$

Thus the derivative in (22), which is now with respect to $-x'p'^{\sigma}_{\perp}$ instead of $P^{\sigma}_{h\perp}/z$,[3] can be performed through that for $\hat{u}$ (see the discussion below (23)). After performing the derivative, one can go over to a frame where the two colliding nucleons are collinear along the z-axis, and the produced virtual photon has large transverse momentum $q_\perp$, which is provided by the recoil from the hard parton going into the unobserved final state. We thus obtains, similarly to (24) and (26),

$$\frac{d\sigma_{\text{tw-3}}^{\mathrm{SGP,DY}}}{dQ^2\,dy\,d^2q_\perp} = \frac{\alpha_{em}^2\alpha_s e_q^2}{3\pi N_c s Q^2}\frac{\pi M_N}{C_F}\epsilon^{pnS_\perp q_\perp}\sum_{j=\bar{q},g}\mathcal{C}_{\bar{j}}\int\frac{dx'}{x'}\int\frac{dx}{x}\delta\big(\hat{s}+\hat{t}+\hat{u}-Q^2\big)f_j(x')$$
$$\times\left\{\frac{\hat{\sigma}_{jq}^{\mathrm{DY}}}{-\hat{u}}x\frac{dG_F^q(x,x)}{dx} + \left[\frac{\hat{\sigma}_{jq}^{\mathrm{DY}}}{\hat{u}} - \frac{\partial\hat{\sigma}_{jq}^{\mathrm{DY}}}{\partial\hat{u}} - \frac{\hat{s}}{\hat{u}}\frac{\partial\hat{\sigma}_{jq}^{\mathrm{DY}}}{\partial\hat{s}} - \frac{\hat{t}-Q^2}{\hat{u}}\frac{\partial\hat{\sigma}_{jq}^{\mathrm{DY}}}{\partial\hat{t}}\right]G_F^q(x,x)\right\}, \tag{29}$$

where y is the rapidity of the virtual photon, and $\mathcal{C}_{\bar{g}} \equiv \mathcal{C}_g$. This obeys exactly the same pattern as (26) for both derivative and non-derivative terms. Substituting (28), this formula completely coincides with the result of [7] which was obtained by direct evaluation of the Feynman diagrams.

We can also derive the single-spin-dependent cross section for the direct-photon production, $p^\uparrow p \to \gamma X$, immediately: We make the formal replacement $\alpha_{em}/(3\pi Q^2) \to \delta(Q^2)$ in (29), corresponding to the real photon in the final state, and define $\hat{\sigma}_{jq}^{\mathrm{DP}}$ as the $Q^2 \to 0$ limit of $\hat{\sigma}_{jq}^{\mathrm{DY}}$ of (28). Because $\hat{\sigma}_{jq}^{\mathrm{DP}}$ are invariant under the scale transformation of the partonic variables $\hat{s},\hat{t}$ and $\hat{u}$, we have $(\hat{u}\partial/\partial\hat{u} + \hat{s}\partial/\partial\hat{s} + \hat{t}\partial/\partial\hat{t})\hat{\sigma}_{jq}^{\mathrm{DP}} = 0$. Accordingly, the single-spin-dependent cross section for the direct-photon (DP) production reads

$$E_q\frac{d\sigma_{\text{tw-3}}^{\mathrm{SGP,DP}}}{d^3q} = \frac{\alpha_{em}\alpha_s e_q^2}{N_c s}\frac{\pi M_N}{C_F}\epsilon^{pnS_\perp q_\perp}\sum_{j=\bar{q},g}\mathcal{C}_{\bar{j}}\int\frac{dx'}{x'}\int\frac{dx}{x}\delta(\hat{s}+\hat{t}+\hat{u})f_j(x')\frac{\hat{\sigma}_{jq}^{\mathrm{DP}}}{-\hat{u}}\left(x\frac{dG_F^q(x,x)}{dx} - G_F^q(x,x)\right), \tag{30}$$

with $E_q = |\vec{q}|$. Note the same twist-2 unpolarized hard cross section appears both for derivative and non-derivative terms, as the coefficient for the combination, $x\,dG_F^q(x,x)/dx - G_F^q(x,x)$. This remarkably compact result does not agree with that of [5],[4] but is reminiscent of the recent result for $p^\uparrow p \to \pi X$ [12], where a similar compact formula was obtained for the SGP cross section. One can extend the present approach to other processes such as $p^\uparrow p \to \pi X$ and show that the compact result found in [12] is also a consequence of the simplification due to the scale invariance of the 2-to-2 Born subprocess among massless partons, similarly to the present case (30) [22].

To summarize, we have studied the SGP contribution in the twist-3 mechanism for the SSA. We have developed a new approach that allows systematic reduction of the coupling of the soft coherent-gluon and the associated pole contribution, using Ward identities and decomposition identities for the interacting parton propagator, and derived the master formula which gives the twist-3

[3] This corresponds to the fact that the SGP in Drell–Yan comes from the initial-state interaction, while that in SIDIS is from the final-state interaction.

[4] This is because our result (29) for the Drell–Yan process agrees with that in [7] which is different from the result in [5] in the $Q^2 \to 0$ limit.

SGP contributions to the SSA entirely in terms of the knowledge of the twist-2 factorization formula for the unpolarized cross section. We find that this novel result is also useful for establishing the gauge invariance of the hard-scattering function for the SGP contribution. So far this master formula was derived for Drell–Yan process, direct γ production and SIDIS. Since our diagrammatic manipulation technique uses only elementary identities which hold for any diagram with the gluon insertion to cause the SGP, it is applicable to other processes such as $p^{\uparrow}p \to \pi X$ and $pp \to \Lambda^{\uparrow}X$, etc., and may lead to a similar relation between the twist-3 SGP contribution and the twist-2 cross section, which allows us to reveal the corresponding gauge-invariant structure.

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