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Interference between diffraction radiation and a scattered plane wave as a possible source of information about properties of a surface

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Abstract

We investigate a possibility of an interference of diffraction radiation with a field, originated from scattering of refracted on a non-uniform surface electromagnetic wave in a case of total internal reflection. An effect of the field from fast charged particle and the refracted field on surface inhomogeneities gives rise to emergence of surface polarization currents. Radiation from these currents can be of use in analysis of surface optical features. The opportunity of studying of surface characteristics by means of diffraction radiation and total internal reflection of the electromagnetic wave is discussed in this paper.

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Keywords: diffraction radiation, total internal reflection, surface inhomogeneities

1. Introduction

When the charged particle moves parallel to the plane surface of homogeneous isotropic media with constant rate, longitudinal (parallel to the particle velocity) momentum is preserved. Therefore, if the conditions of existence of Cherenkov radiation are not fulfilled, photon emission is prohibited by conservation law. However, the presence of

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surface inhomogeneities makes it possible to transfer the longitudinal momentum to the medium and diffraction radiation (DR) appears (see, e.g., Bolotovskii and Voskresenskii (1966), Ryazanov et al. (2004), Ryazanov (2009)).

DR can be useful for an analysis of characteristics of the medium surface as well as for noninvasive diagnostics of charged particle beams. Information obtained via DR can be increased, using an interference of DR with the field refracted on the non-uniform surface, when the total internal reflection on the uniform surface with vanishing refracted wave takes place. Thus both refracted field and DR are raised only because of the presence of surface inhomogeneities. Comparison of the results obtained at various frequencies and intensities of incident waves can increase the scope of useful information.

It is of interest to investigate the interference of DR and refracted wave and also the opportunity of getting more information.

2. Diffraction radiation from inhomogeneous medium surface

In frames of macroscopic electrodynamics the difference of near-surface layer features is usually left out of account as the thickness of the layer is much smaller than wavelength, if one considers boundary conditions on the interface between two mediums. And it is correct if the surface layer features are close to volume features of the medium. Nevertheless, in a number of cases the influence of surface characteristics can be significant (see, e.g., Barrera and Mochan (1986), Ryazanov (1996)). The small thickness of the layer let us take it into account via introduction of the surface polarization current into boundary conditions.

Let us consider DR from the charged particle moving in the vacuum along x axis with constant velocity $\mathbf{v}$ at the distance $z = a$ from the surface $z = 0$ of nonmagnetic lossless medium, which contains one-dimensionally inhomogeneous thin layer with the thickness, smaller than wavelength. The source of DR is the surface polarization current that can be written in a form (Ryazanov (1987)):

$$\mathbf{J}(x, y, \omega) = \zeta(x, \omega)\mathbf{E}^p(x, y, 0, \omega), \tag{1}$$

where $\zeta(x, \omega)$ is the surface conductivity. As the thickness of the near-surface layer is not large, the influence of it is considered as a small perturbation, so the exact field $\mathbf{E}^p(\mathbf{r}, \omega)$, which is the solution of Maxwell's equations, can be expanded to series in terms of the polarization current. Therefore we can neglect the polarization current in Maxwell's equation in a zero-order approximation and the field coincides with the field of the moving in vacuum charge Ze:

$$E_i^{(0)p}(\mathbf{r}, \omega) = \iiint d^3q E_i^p(\mathbf{q})\delta(\omega - \mathbf{qv})e^{i\mathbf{qr}}, \tag{2}$$

$$E_i^{(0)p}(\mathbf{q}) = -\frac{iZe}{2\pi^2}\frac{v_i q_j v_j - q_i c^2}{q^2c^2 - (q_i v_i)^2}e^{-iq_z a}. \tag{3}$$

Field of the first-order approximation satisfies the boundary conditions with the given surface current, determined by the field of zero-order approximation:

$$\mathbf{J}^{(0)p}(q_x, q_y, \omega) = \frac{iZe}{2\pi F}e^{-aF}\left(\frac{\omega c^2}{v\gamma^2}\mathbf{e}_x + q_y c^2 \mathbf{e}_y\right)\zeta\left(q_x - \frac{\omega}{v}, \omega\right), \tag{4}$$

where $F = \sqrt{q_y^2 + \gamma^{-2}\omega^2/v^2}$, $\gamma^{-2} = 1 - v^2/c^2$ and

$$\zeta\left(q_x - \frac{\omega}{v}, \omega\right) = \frac{1}{2\pi}\int dx \zeta(x, \omega)e^{i\frac{\omega}{v}x - iq_x x}. \tag{5}$$

Using Maxwell's equations and boundary conditions one can obtain z -component of the electric and magnetic fields, produced by the polarization current (4) and propagating out from the interface both into the vacuum

$\left(\mathbf{E}_1^{(1)p}, \mathbf{H}_1^{(1)p}\right)$ and into the medium $\left(\mathbf{E}_2^{(1)p}, \mathbf{H}_2^{(1)p}\right)$:

$$E_{1(2)z}^{(1)p}\left(q_x, q_y, z, \omega\right) = \mp \frac{4\pi}{\omega} \frac{\kappa_{2(1)}\left(q_x J_x^{(0)p}\left(q_x, q_y, \omega\right) + q_y J_y^{(0)p}\left(q_x, q_y, \omega\right)\right)}{\kappa_2 + \varepsilon(\omega)\kappa_1} e^{\pm i\kappa_{1(2)}z}, \tag{6}$$

$$H_{1(2)z}^{(1)p}\left(q_x, q_y, z, \omega\right) = -\frac{4\pi}{c} \frac{q_x J_y\left(q_x, q_y, \omega\right) - q_y J_x\left(q_x, q_y, \omega\right)}{\kappa_2 + \kappa_1} e^{\pm i\kappa_{1(2)}z}. \tag{7}$$

Disposing a detector in the vacuum at the point of observation $\mathbf{r}$ on the plane $z = z_0$, which is far from the interface (in the wave zone), we can measure the energy flux, passing through this plane along the unit vector $\mathbf{n} = \mathbf{r}/r$:

$$W^p = \frac{c}{4\pi} \iiint dxdydt\left(\mathbf{n}\left[\mathbf{E}_1^{(1)p}\left(x, y, z_0, t\right)\mathbf{H}_2^{(1)p}\left(x, y, z_0, t\right)\right]\right). \tag{8}$$

Let us introduce polar angle $\theta = \mathbf{e}_z \wedge \mathbf{n}$ and rewrite (8):

$$W^p = \frac{c}{4\pi}(\cos\theta)^{-1} \iiint dxdydt\left(\mathbf{n}\left[\mathbf{E}_1^{(1)p}\left(x, y, z_0, t\right)\mathbf{H}_2^{(1)p}\left(x, y, z_0, t\right)\right]\right). \tag{9}$$

It is known that all components of electric and magnetic field vectors can be expressed in terms of normal components using Maxwell's equations. Therefore, applying coordinate and time Fourier transform, relation (9) can be expressed as

$$W^p(\mathbf{n}) = 2\pi^2 \iiint dq_x dq_y d\omega \frac{\kappa_1 \omega}{q^2 \cos\theta}\left\{\left|E_{1z}^{(1)p}\right|^2 + \left|H_{1z}^{(1)p}\right|^2\right\}, \tag{10}$$

or, making a substitution $q = (\omega/c)\sin\theta$, one can obtain distribution of energy of DR over the frequency $d\omega$ and solid angle $d\Omega = \sin\theta d\theta d\varphi$:

$$\begin{gathered}\frac{d^2W^p(\mathbf{n},\omega)}{d\Omega d\omega} = 8\pi^2 \frac{\omega^4}{c} \frac{Z^2e^2\, ctg\theta}{F} e^{-2aF}\left|\zeta\left(q_x - \frac{\omega}{v}, \omega\right)\right|^2 \times \\ \times\left\{\begin{aligned}&\frac{\varepsilon - \sin^2\theta}{\left(\sqrt{\varepsilon - \sin^2\theta} + \varepsilon\cos\theta\right)^2}\left(\frac{\cos\varphi}{\sqrt{\gamma^2\left(\gamma^2 - 1\right)}} + \sin\varphi\sin\theta\right)^2 + \\ &+ \frac{1}{\left(\sqrt{\varepsilon - \sin^2\theta} + \cos\theta\right)^2}\left(\sin\theta\sin\varphi\cos\varphi - \frac{\sin\varphi}{\sqrt{\gamma^2\left(\gamma^2 - 1\right)}}\right)^2\end{aligned}\right\}\end{gathered} \tag{11}$$

It is seen that the spectral-angular distribution of DR depends on the variation of dielectric properties in near-surface layer that is described by the surface conductivity function ζ. Thus, if there is no inhomogeneities on the medium surface, i.e. $\zeta(x) = const$, wave vector $\boldsymbol{\kappa}_1$ becomes imaginary so the fields (6), (7) attenuate and DR does not arise.

3. Discussion

Relation (11) allows us to obtain the absolute value of the function ζ. However, the near-surface layer can also contain absorbing impurity atoms and it becomes necessary to describe surface properties by complex surface conductivity $\zeta = \zeta' + i\zeta''$. To get information about the real (ζ') and imaginary (ζ'') parts separately, let us introduce an electromagnetic wave $\mathbf{E}_{in}^{(0)w}(\mathbf{r},t) = \mathbf{E}_{in}^{(0)w}\exp\left(ik_{0x}x + ik_{0z}z - i\omega_0 t\right)$, propagating in the medium towards its interface $z = 0$, linearly polarized in the plane, which makes an angle α with *xz* incident plane. At incident angle β_0 that obeys the condition $\sin\beta_0 \geq \{\varepsilon(\omega_0)\}^{-1/2}$, total internal reflection does not occur due to the existence of inhomogeneous surface layer, where a refracted wave is scattering. The scattered fields can also be found using the perturbation theory and therefore can be written as Eqs. (6) and (7) but with the different surface current:

$$\mathbf{J}^{(0)w}\left(q_x, q_y, \omega\right) = \mathbf{E}_{tr}^{(0)w}\zeta\left(q_x - k_{0x}, \omega\right)\delta\left(\omega - \omega_0\right)\delta\left(q_y\right), \tag{12}$$

where $k_{0x} = \omega/c$ in case of total internal reflection, $\mathbf{E}_{tr}^{(0)w}$ is an amplitude of the refracted wave, connected with $\mathbf{E}_{in}^{(0)w}$ by well-known Fresnel's relations (Landau and Lifshitz (1960)) and

$$\zeta\left(q_x - k_{0x}, \omega\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dx\,\zeta\left(x, \omega\right)e^{ik_{0x}x - iq_x x}. \tag{13}$$

In presence of this electromagnetic wave the energy flux, registered by the detector, alters:

$$W = W^p + W^w + W^{pw}.$$

The spectral-angular distribution of energy, produced by the refracted field, can be found in the same manner using Eq. (10):

$$\begin{gathered}\frac{d^2W^w(\mathbf{n},\omega)}{d\Omega d\omega} = 8\pi^2 TL\frac{\omega}{c^2}ctg\theta\left|\zeta\left(q_x - k_{0x}, \omega\right)\right|^2\delta\left(\sin\varphi\right)\delta\left(\omega - \omega_0\right)\times \\ \times\left\{\begin{aligned}&\frac{\varepsilon(\omega) - \sin^2\theta}{\left(\sqrt{\varepsilon(\omega) - \sin^2\theta} + \varepsilon(\omega)\cos\theta\right)^2}\left|E_x\cos\varphi + E_y\sin\varphi\right|^2 + \\ &+\frac{1}{\left(\sqrt{\varepsilon(\omega_0) - \sin^2\theta} + \cos\theta\right)^2}\left|E_y\cos\varphi - E_x\sin\varphi\right|^2\end{aligned}\right\}\end{gathered} \tag{14}$$

The interaction between DR and the refracted field leads to emergence of interference term:

$$\begin{gathered}\frac{d^2W^{pw}(\mathbf{n},\omega)}{d\Omega d\omega} = 32\pi^3\frac{\omega^2}{c^2}\frac{Ze\,ctg\theta}{F}e^{-aF}\delta\left(\sin\varphi\right)\delta\left(\omega - \omega_0\right)\times \\ \times\mathrm{Im}\left\{\zeta^*\left(q_x - \frac{\omega}{v}, \omega\right)\zeta\left(q_x - k_{0x}, \omega\right)\right\}\times \\ \times\left\{\begin{aligned}&\frac{\varepsilon(\omega) - \sin^2\theta}{\left(\sqrt{\varepsilon(\omega) - \sin^2\theta} + \varepsilon(\omega)\cos\theta\right)^2}\left((\gamma^2\beta)^{-1}\cos\varphi + \sin\theta\sin^2\varphi\right)\left(E_x\cos\varphi + E_y\sin\varphi\right) + \\ &+\frac{1}{\left(\sqrt{\varepsilon(\omega) - \sin^2\theta} + \cos\theta\right)^2}\left(\sin\theta\sin\varphi\cos\varphi - (\gamma^2\beta)^{-1}\sin\varphi\right)\left(E_y\cos\varphi - E_x\sin\varphi\right)\end{aligned}\right\}\end{gathered} \tag{15}$$

Thus, having measured the spectral-angular distribution of energy both in case of moving charge in absence of the electromagnetic wave and in case of existence of the electromagnetic wave only and then in the simultaneous presence of moving particle and incident wave, we can get a set of equations:

$$\zeta'^2\left(q_x - \omega/v\right) + \zeta''^2\left(q_x - \omega/v\right) = f\left(\Omega, \omega\right), \tag{16}$$

$$\zeta'^2\left(q_x-\omega/c\right)+\zeta''^2\left(q_x-\omega/c\right)=g\left(\Omega,\omega\right), \tag{17}$$

$$\zeta'\left(q_x-\omega/v\right)\zeta''\left(q_x-\omega/c\right)-\zeta'\left(q_x-\omega/c\right)\zeta''\left(q_x-\omega/v\right)=h\left(\Omega,\omega\right), \tag{18}$$

where f, g, h can be taken from Eqs. (11), (14) and (15). If the particle velocity is close to the light velocity, function ζ can be rewritten as:

$$\zeta\left(q_x-\omega/v\right)=\zeta\left(q_x-\omega/c-\omega/v\left(1-v/c\right)\right)\simeq\zeta\left(q_x-\omega/c\right)-\frac{\omega}{2v\gamma^2}\frac{d\zeta\left(q_x-\omega/c\right)}{dq_x}. \tag{19}$$

Therefore, using Eqs. (16)-(19) and neglecting the quantity $\left(d\zeta/dq_x\right)^2$, one can obtain following differential equation

$$\frac{d\zeta'\left(q_x-\omega/c\right)}{dq_x}=-\frac{\left(f-g\right)\zeta'\left(q_x-\omega/c\right)+2h\sqrt{g-\zeta'^2\left(q_x-\omega/c\right)}}{g\omega/v\gamma^2}, \tag{20}$$

solution of which can be found in two presumable approximations:

$$\zeta'\left(q_x-\omega/c\right)=\begin{cases}\exp\left[-\dfrac{v\gamma^2}{\omega}\displaystyle\int\frac{f-g+2ih}{g}dq_x\right], & g<<\zeta'^2\\ \displaystyle\int\frac{2h\sqrt{g}}{g\omega/v\gamma^2}\exp\left[\int\frac{f-g}{g\omega/v\gamma^2}dq_x\right]dq_x, & g>>\zeta'^2\end{cases} \tag{21}$$

or by means of numerical methods. Thus the interference between DR and scattered field of electromagnetic wave can give us information about the real (imaginary) part of the surface conductivity function.

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