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Effects of curvature-Higgs coupling on electroweak fine-tuning

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ABSTRACT

It is shown that nonminimal coupling between the Standard Model (SM) Higgs field and spacetime curvature, present already at the renormalizable level, can be fine-tuned to stabilize the electroweak scale against power-law ultraviolet divergences. The nonminimal coupling acts as an extrinsic stabilizer with no effect on the loop structure of the SM, if gravity is classical. This novel fine-tuning scheme, which could also be interpreted within Sakharov's induced gravity approach, works neatly in extensions of the SM involving additional Higgs fields or singlet scalars.

The discovery of a fundamental scalar [1] by ATLAS and CMS experiments, and compatibility of this scalar with the SM Higgs boson [1,2] prioritized the disastrous UV sensitivity of the Higgs boson mass [3,4] as the foremost problem [5] to be resolved. This is because, in the LHC searches reaching out to energies fairly above the electroweak scale [6], the Higgs boson seems to lack any companion which would stabilize its mass. This means that the electroweak scale, set by the Higgs vacuum expectation value (VEV)

$$v^2 = \frac{-m_H^2}{\lambda_H} \tag{1}$$

that minimizes the Higgs potential

$$V(H) = V_0 + m_H^2 H^\dagger H + \lambda_H \left(H^\dagger H\right)^2 \tag{2}$$

for $m_H^2 < 0$ and $\lambda_H > 0$, is completely destabilized by the additive power-law quantum corrections $\delta m_H^2 \propto \Lambda_{UV}^2$ [4], where $\Lambda_{UV} \gg v$ is the UV scale which can be as high as M_{Pl} if the SM is valid all the way up to the gravitational scale.

The present paper will point out an exception to this inevitable destabilization by noting that the Higgs field, being a doublet of fundamental scalars, necessarily develops the nonminimal Higgs-curvature interaction [7]

$$\Delta V(H, R) = \zeta R H^\dagger H \tag{3}$$

with which the Higgs VEV in (1) changes to

$$v^2 = \frac{-m_H^2 - \frac{4\zeta V_0}{M_{Pl}^2}}{\lambda_H + \frac{\zeta m_H^2}{M_{Pl}^2}} \tag{4}$$

and this new VEV can be stabilized by fine-tuning ζ to counterbalance the quadratic divergences $\delta m_H^2 \propto \Lambda_{UV}^2$ with the quartic divergences $\delta V_0 \propto \Lambda_{UV}^4$. Quantum corrections to the SM parameters are independent of ζ if gravity is classical, and thus ζ acts as a gyroscope that stabilizes the electroweak scale against violent UV contributions. This novel fine-tuning scheme is in accord with Sakharov's induced gravity approach, and continues to hold also in extensions of the SM involving extra Higgs fields (additional Higgs doublets or singlet scalars or scalar multiplets belonging to larger gauge groups).

Below, we verify these observations by studying effects of the curvature-Higgs interaction (3) on the electroweak breaking, analyzing how fine-tuning of ζ leads to stabilization of the electroweak scale, and determining implications of the mechanism for physics beyond the SM.

In general, Higgs VEV is determined by the fields which can develop nontrivial backgrounds. Thus, only the Higgs VEV v and the corresponding curvature scalar $R(v)$ matter at the tree level whilst all the fields coupling to the Higgs doublet count at the loop level. Explicating only its Higgs and curvature sections, the tree-level action is given by

$$S \supset \int d^4x \sqrt{-g} \left\{ \frac{1}{2} M_{Pl}^2 R - g^{\mu\nu} (\mathcal{D}_\mu H)^\dagger (\mathcal{D}_\nu H) - V(H) - \Delta V(H, R) - [h_f \overline{F_L} H f_R + \text{H.C.}] \right\} \tag{5}$$

where $F_L \sim SU(2)_L \otimes U(1)_Y$ and $f_R \sim U(1)_Y$ are quark and lepton fields, $\mathcal{D}_\mu$ is gauge-covariant derivative, and

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\varphi^+ \\ v + h + i\varphi^0 \end{pmatrix} \tag{6}$$

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is the SM Higgs field encoding the Higgs boson h and Goldstone bosons $\varphi^{+,-,0}$. Its VEV v is determined to read as in (4) after a self-consistent solution of the curvature scalar

$$R(v) = \frac{1}{M_{Pl}^2 - \zeta v^2}\left(4V_0 + 2m_H^2 v^2 + \lambda_H v^4\right) \tag{7}$$

and the Higgs equation of motion

$$m_H^2 + \lambda_H v^2 + \zeta R(v) = 0 \tag{8}$$

in the constant v and $R(v)$ backgrounds. On physical grounds, $V_0 \neq 0$ as there exists no symmetry that dictates it. Quite expectedly, the Higgs VEV in (4) tends to the usual Higgs VEV in (1) as $\zeta \to 0$. It is through Eqs. (7) and (8) that the denominator in (4) reads $\lambda_H + (\zeta m_H^2)/M_{Pl}^2$ not just λ_H. Irrespective of if the Higgs field couples minimally or nonminimally, the Higgs VEV induces the Higgs boson mass $m_h^2 = 2\lambda_H v^2$ properly if $m_H^2 + \frac{4\zeta V_0}{M_{Pl}^2} < 0$, and ensures strict masslessness of the Goldstone bosons on its equation of motion (8). (The spacetime curvature can influence symmetry breaking [8].)

In general, ζ is a free parameter that can be assigned to appropriate values depending on the physical process under consideration. For instance, it is known to affect the LHC Higgs boson candidate [9] and weak boson scattering [10] for $\zeta \simeq 10^{15}$, and facilitate successful inflation for $\zeta \simeq 10^4$ [11].

The Higgs VEV, which sets the electroweak scale and generates the masses of the SM particles, is the germinal physical observable. Its scale value is crucial for phenomenological success of the electroweak theory, and hence, its stability against quantum fluctuations is a vital issue by itself. Concerning the computation of quantum corrections to Higgs VEV, it is natural to construct the effective action [12,16] corresponding to the tree-level action (5) by incorporating into it the effects of the quantum fluctuations whose frequencies range from $\Lambda_{IR} \gtrsim v$ up to Λ_{UV}. Taking gravity classical to avoid nonrenormalizable quantum gravitational effects [18], the action (5) is found to form a renormalizable setup if one sticks to constant-curvature backgrounds [17] enabling direct comparison with the tree-level geometry in (7). Then, one-loop quantum corrections to the parameters in Higgs potential (2) are given by

$$\delta V_0 = \frac{1}{(4\pi)^2}\left[\frac{1}{4}(n_F - n_B)\Lambda_{UV}^4 + 2m_H^2\Lambda_{UV}^2 + m_H^4 \log\frac{\Lambda_{IR}^2}{\Lambda_{UV}^2}\right] \tag{9}$$

$$\delta m_H^2 = \frac{3}{(4\pi)^2}\left[\left(2\lambda_H + \frac{g_Y^2}{4} + \frac{3g_2^2}{4} - 2h_t^2\right)\Lambda_{UV}^2 + 2\lambda_H m_H^2 \log\frac{\Lambda_{IR}^2}{\Lambda_{UV}^2}\right] \tag{10}$$

$$\delta\lambda_H = \frac{3}{(4\pi)^2}\left[4\lambda_H^2 + \frac{g_Y^4}{16} + \frac{g_2^2 g_Y^2}{8} + \frac{3g_2^4}{16} + h_t^4\right]\log\frac{\Lambda_{IR}^2}{\Lambda_{UV}^2} \tag{11}$$

where h_t is top quark Yukawa coupling, g_Y (g_2) is the hypercharge (isospin) gauge coupling, and n_F (n_B) is the total number of fermions (bosons) in the SM. Unlike δV_0, δm_H^2 and $\delta\lambda_H$, all of which are independent of ζ, quantum corrections to gravity sector parameters

$$\delta M_{Pl}^2 = -\frac{1}{6(4\pi)^2}\left[(1-24\zeta)\Lambda_{UV}^2 + 4m_H^2(1-6\zeta)\log\frac{\Lambda_{IR}^2}{\Lambda_{UV}^2}\right] \tag{12}$$

$$\delta\zeta = -\frac{1}{(4\pi)^2}\left[\lambda_H(1-6\zeta) - \frac{3g_Y^2}{8} - \frac{9g_2^2}{8} + \frac{h_t^2}{12}\right]\log\frac{\Lambda_{IR}^2}{\Lambda_{UV}^2} \tag{13}$$

explicitly involve ζ. Despite its quadratic divergence, δM_{Pl}^2 is a tiny quantum correction because $\Lambda_{UV} \lesssim M_{Pl}$.

Pertaining to a renormalizable theory, the loop-level Higgs VEV is expected to have the same form as the tree-level VEV in (4). Thus, in response to the quantum corrections above, it changes by

$$\delta v^2 \simeq \frac{3Q(\zeta)}{(4\pi)^2\lambda_H}\left(2h_t^2 - \frac{3}{4}g_2^2 - \frac{1}{4}g_Y^2 - 2\lambda_H\right)\Lambda_{UV}^2 \tag{14}$$

as follows from (10) and (9) after neglecting logarithmic UV contributions and dropping minuscule $\mathcal{O}(m_H^2/M_{Pl}^2)$ and $\mathcal{O}(V_0/M_{Pl}^4)$ terms. This correction differs from the well-known Veltman condition [4] by the loop factor

$$Q(\zeta) = 1 - \frac{\zeta(n_F - n_B)}{3(2h_t^2 - \frac{3}{4}g_2^2 - \frac{1}{4}g_Y^2 - 2\lambda_H)}\frac{\Lambda_{UV}^2}{M_{Pl}^2} \tag{15}$$

which is nothing but the ratio of the quartic divergence in V_0 to the quadratic divergence in m_H^2. It is this factor that differentiates between nonminimally- and minimally-coupled Higgs fields. Indeed, as $\zeta \to 0$, $Q(\zeta) \to 1$ and Higgs VEV starts developing quadratic divergence, as expected from the familiar Higgs VEV (1) holding for minimally-coupled Higgs.

A short glance at (14) reveals that, as a new means not possible for minimally-coupled Higgs, one can suppress δv^2 to admissible level by imposing

$$\left|Q(\zeta)\right| \lesssim \frac{v^2}{\Lambda_{UV}^2} \tag{16}$$

which amounts to an extreme fine-tuning of some 30 decimal places after comma. Thus, the nonminimal Higgs-curvature coupling ζ provides the SM with a novel fine-tuning mechanism for stabilizing the electroweak scale against power-law UV effects. This stabilization leads to stabilization of all particle masses, including that of the Higgs boson.

Suppression of $Q(\zeta)$ in (16) is accomplished by finely adjusting ζ in (15). Its duty is to counterbalance the quadratic divergence in m_H^2 with the quartic divergence in V_0. This is evident from the Veltman condition (14) supplemented by (15). The working of the fine-tuning in (16) is best exemplified by the special value of ζ

$$\zeta_0 = \frac{1}{(n_F - n_B)}\left(6h_t^2 - 6\lambda_H - \frac{3g_Y^2}{4} - \frac{9g_2^2}{4}\right)\frac{M_{Pl}^2}{\Lambda_{UV}^2} \tag{17}$$

for which $Q(\zeta_0) = 0$. This specific nonminimal coupling has the numerical value $\zeta_0 \approx 1/15$ for $\Lambda_{UV} \approx M_{Pl}$. It is smaller than the conformal value 1/6 [7] and much much smaller than the Higgs inflation value 10^4 [11]. As a function of Λ_{UV}, ζ_0 completely eradicates the power-law UV contribution (14), and the concealed logarithmic corrections give the usual renormalization properties of the Higgs VEV. Obviously, smaller the Λ_{UV} larger the ζ_0 though there remains lesser and lesser need to fine-tuning if Λ_{UV} gets closer and closer to the Fermi scale.

The nonminimal coupling ζ, as explicated in (13), receives additive logarithmic UV corrections involving the SM gauge and Yukawa couplings. Hence, the tuned value of ζ in (16), as exemplified by (17), receives small logarithmic corrections for which the value of ζ can be adjusted order by order in perturbation theory. The essential point is that it is the tree-level coupling ζ, not any of the SM parameters or momentum cutoffs $\Lambda_{UV/IR}$, which is finely tuned to achieve the suppression in (16).

Quantum corrections to the SM parameters (Higgs, gauge and Yukawa sectors) do not involve ζ. This is already evinced by (9), (10) and (11). This observation is actually an all-loop feature ensured by the classical nature of gravity, and makes certain that the SM maintains all of its IR/UV quantum structures as if ζ does not exist. In other words, all the SM parameters run from scale to scale with no parameter tunings, coarse or fine. In essence, ζ behaves as an external gyroscope that maintains the bound (16) despite various violent UV effects, and it is with this functionality that the matter and forces in the SM find themselves under optimal conditions for weak interactions to occur correctly.

To discuss further, we state that ζ fine-tuning can have a variety of implications for model building and phenomenology. Below we highlight some of them briefly:

- Our setup of classical gravity plus quantized matter can be consistently interpreted within Sakharov's induced gravity [19,16]. In this framework, gravity is induced by matter loops as a long-distance effective theory, and this typically requires additional matter multiplets to rightly induce the Planck scale M_{Pl} [19,20]. This means that fine-tuning of ζ in (16) might be deduced from symmetries of the non-SM matter multiplets. Interestingly, the non-SM matter here does not have to conform to supersymmetry or other UV-safe extensions of the SM [21].
- The matter sector does not have to be precisely the SM. The fine-tuning mechanism here works also in extensions of the SM which include extra scalar fields provided that each scalar assumes a nonminimal coupling to curvature as in (3). The scalar fields can be additional Higgs doublets, singlet scalars or multiplets of scalars belonging to larger gauge groups. The VEV of each scalar is of the form in (4), and can be fine-tuned individually without interfering with the VEVs of the remaining scalars. These extended Higgs sectors can be probed at the LHC and other colliders [22]. The singlet scalars, in particular, can explain the cold Dark Matter [23] in Universe and enhance the invisible width of the Higgs boson [24].
- The classical gravity assumption in the present work can be lifted to include quantum gravitational effects. In this case, nonminimal coupling spreads into the SM parameters through graviton loops. Moreover, this quantum gravitational setup is inherently nonrenormalizable [18]. These factors can obscure the process of fine-tuning ζ.
- There have been various attempts [25] to nullify the quadratic divergence in Higgs VEV by introducing singlet scalars. This is now known to be not possible at all, even when vector-like fermions are included [26]. Nevertheless, nonminimal coupling between curvature scalar and some scalar fields can help stabilize both electroweak and hidden scales as in (14), and then masses of the particles in the SM and hidden sector get automatically stabilized.
- Throughout the discussions, cosmological constant problem [27] is left aside as in supersymmetry and other UV-safe extensions of the SM. The assumption is that it is a separate, independent naturalness problem pertaining to deep IR rather than electroweak or higher energy scales. The alleged mechanism that solves the cosmological constant problem must degravitate or dilute the vacuum energy at large distances. This can be accomplished presumably via modifications of gravity in the deep IR. In the present work, using (9) in (7) one finds $R(v) \sim \mathcal{O}(\Lambda_{UV}^2)$ which is some 120 orders of magnitude larger than its observational value of $R(exp) \simeq 10^{-47}$ eV^2 [28], and modified gravitational dynamics becomes essential for diluting this curvature at large distances. In this connection, one notes the empirical modifications of gravity which degravitate the vacuum energy [29] or canalize vacuum energy to gravitational constant instead of cosmological constant [30].

To conclude, we reiterate that the nonminimal curvature-Higgs coupling ζ plays a crucial role in stabilizing the electroweak scale. If Higgs field were minimally-coupled, quadratic divergences in m_H^2 would induce the same divergences in v^2, simply because the latter is proportional to the former. Nevertheless, nonminimal curvature-Higgs interaction disrupts this proportionality by bringing V_0 into the game. Essentially, ζ causes Higgs VEV to involve not only the Higgs mass parameter m_H^2 but also the vacuum energy V_0, and the quadratic divergence of the former can be counterbalanced with the quartic divergence of the latter if ζ is finely adjusted. Then, ζ acts as an external stabilizer that sets the electroweak scale without intervening with the quantum structure of the SM.

The various investigation directions commented above give an idea of how widespread the implications of the ζ fine-tuning scheme could be. It would be an important advancement to relate the fine-tuning constraints on ζ at low-energies to the symmetries and spectra of the non-SM matter multiplets needed for inducing the Planck mass (matter multiplicity and Λ_{UV} set M_{Pl}). On the other side, the LHC phenomenology of the extra Higgs fields and analysis of the singlet scalars in regard to electroweak stability and Dark Matter phenomenology would be another important direction to explore. Last but not least, a fundamental understanding of the modified gravity models that render the vacuum weightless would be a crucial step towards completing the fine-tuning scheme presented in the present work.

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