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Higgs phenomenology of supersymmetric economical 3–3–1 model

P.V. Dong ^{*}, D.T. Huong, N.T. Thuy, H.N. Long*Institute of Physics, VAST, PO Box 429, Bo Ho, Hanoi 10000, Viet Nam*

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Abstract

We explore the Higgs sector in the supersymmetric economical 3–3–1 model and find new features in this sector. The charged Higgs sector is revised, i.e., in difference of the previous work, the exact eigenvalues and states are obtained without any approximation. In this model, there are three Higgs bosons having masses equal to that of the gauge bosons—the W and extra X and Y . There is one scalar boson with mass of 91.4 GeV, which is closed to the Z boson mass and in good agreement with present limit: 89.8 GeV at 95% CL. The condition of eliminating for charged scalar tachyon leads to splitting of VEV at the first symmetry breaking, namely, $w \simeq w'$. The interactions among the Standard Model gauge bosons and scalar fields in the framework of the supersymmetric economical 3–3–1 model are presented. From these couplings, at some limit, almost scalar Higgs fields can be recognized in accordance with the Standard Model. The hadronic cross section for production of the bilepton charged Higgs boson at the CERN LHC in the effective vector boson approximation is calculated. Numerical evaluation shows that the cross section can exceed 35.8 fb. © 2007 Elsevier B.V. All rights reserved.

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1. Introduction

Recent neutrino experimental results [1–3] establish the fact that neutrinos have masses and the Standard Model (SM) must be extended. The generation of neutrino masses is thus an impor-

^{*} Corresponding author.

E-mail address: pvdong@iop.vast.ac.vn (P.V. Dong).

tant issue in any realistic extension of the SM. In general, the values of these masses which are of the order of, or less than, 1 eV needed to explain all neutrino oscillation data are not enough to put strong constraints on model building. It means that several models can induce neutrino masses and mixing compatible with experimental data. In such cases it is more useful to consider in any particular model motivation other than that can explain neutrino masses. In addition, although the SM is exceedingly successful in describing charged leptons, quarks and their interactions, it is not considered as the ultimate theory since neither the fundamental parameters, masses and couplings, nor the symmetry pattern are predicted. These elements are merely built into the model. Likewise, the spontaneous electroweak symmetry breaking is simply parametrized by a single Higgs doublet field.

The embedding of the model into a more general framework is therefore expected. If the Higgs boson is light, the SM can naturally be embedded in a grand unified theory. The large energy gap between the low electroweak scale and the high grand unification scale can be stabilized by a supersymmetry (SUSY) transforming bosons into fermions and vice versa [4]. The existence of such a non-trivial extension is highly constrained by theoretical principles and actually provides the link between the experimentally explored interactions at electroweak energy scales and physics at scales close to the Planck scale $M_{\text{pl}} \approx 10^{19}$ GeV where gravity is important. One of the intriguing features of the supersymmetric models is that the Higgs spectrum is quite constrained. This statement is consolidated by our analysis below.

On the other hand, the possibility of a gauge symmetry based on $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3–3–1) [5–7] is particularly interesting, because it explains some fundamental questions that are eluded in the SM. The main motivations to study this kind of model are:

- (1) The family number must be multiple of three;
- (2) It solves the strong CP problem;
- (3) It is the simplest model that includes bileptons of both types: scalar and vectors ones;
- (4) The model has several sources of CP violation;
- (5) The explanation of electric charge quantization [8].

In one of 3–3–1 models [7], the anomaly-free particle content is given by

$$\begin{aligned} L_{aL} &= (v_a, l_a, \nu_a^c)_L^T \sim (1, 3, -1/3), \quad l_{aR} \sim (1, 1, -1), \quad a = 1, 2, 3, \\ Q_{1L} &= (u_1, d_1, u')_L^T \sim (3, 3, 1/3), \\ Q_{\alpha L} &= (d_\alpha, -u_\alpha, d'_\alpha)_L^T \sim (3, 3^*, 0), \quad \alpha = 2, 3, \\ u_{iR} &\sim (3, 1, 2/3), \quad d_{iR} \sim (3, 1, -1/3), \quad i = 1, 2, 3, \\ u'_R &\sim (3, 1, 2/3), \quad d'_{\alpha R} \sim (3, 1, -1/3), \end{aligned}$$

where the values in the parentheses denote quantum numbers based on the $(SU(3)_C, SU(3)_L, U(1)_X)$ symmetry. The exotic quarks u' and d'_α take the same electric charges as of the usual quarks, i.e., $q_{u'} = 2/3$, $q_{d'_\alpha} = -1/3$. The spontaneous symmetry breaking is achieved by two Higgs scalar triplets only

$$\chi = (\chi_1^0, \chi^-, \chi_2^0)^T \sim (1, 3, -1/3), \quad \rho = (\rho_1^+, \rho^0, \rho_2^+)^T \sim (1, 3, 2/3)$$

with all the neutral components χ_1^0 , χ_2^0 and ρ^0 developing the vacuum expectation values (VEVs). Such a scalar sector is minimal, therefore it has been called the economical 3–3–1 model [9,10].

In a series of papers, we have developed and proved that this version is consistent, realistic and very rich in physics. Let us remind some steps in the development: The general Higgs sector is very simple and consists of three physical scalars (two neutral and one charged) and eight Goldstone bosons—the needed number for massive gauge ones [11]. In Refs. [12,13], we have shown that the model under the consideration is realistic, by the mean that, at the one-loop level, all fermions gain consistent masses. It was shown that [11] the economical 3–3–1 model does not furnish any candidate for self-interaction dark matter. This directly relates to the scalar sector in which a significant number of fields and couplings is reduced. With a larger field content in order to provide candidates for dark matter, the supersymmetric version of the economical 3–3–1 model has already been constructed in Ref. [14].

It is well known that the electroweak symmetry breaking in the SM is achieved via the Higgs mechanism. In the Glashow–Weinberg–Salam model there is a single complex Higgs doublet, where the Higgs boson h is the physical neutral Higgs scalar which is the only remaining part of this doublet after spontaneous symmetry breaking. In the extended models there are additional charged and neutral scalar Higgs particles. The prospects for Higgs coupling measurements at the CERN LHC have recently been analyzed in detail in Ref. [15]. The experimental detection of the h will be great triumph of the SM of electroweak interactions and will mark new stage in high energy physics.

In extended Higgs models, which would be deduced in the low energy effective theory of new physics models, additional Higgs bosons like charged and CP-odd scalar bosons are predicted. Unlike the spectrum of squarks, sleptons and gauginos, which are determined by many parameters, the Higgs spectrum is quite constrained. Phenomenology of these extra scalar bosons strongly depends on the characteristics of each new physics model. By measuring their properties like masses, widths, production rates and decay branching ratios, the outline of physics beyond the electroweak scale can be experimentally determined. In the model under consideration, at the tree level, the mass lightest Higgs is the mass of the W boson. This is in agreement with the current experimental limit.

The interesting feature compared with other 3–3–1 models is the Higgs physics. In the 3–3–1 models, the general Higgs sector is very complicated [16,17] and this prevents the models' predictability. Thus, the Higgs sector of the supersymmetric version of the 3–3–1 models are intricate too [18,19]. The Higgs sector of a supersymmetric version of the economical 3–3–1 model is not so complicated and its eigenvalues and states can be found exactly without any approximation. The scalar sector of the supersymmetric economical 3–3–1 model is a subject of the present study. As shown, by couplings of the scalar fields with the ordinary gauge bosons such as the photon, the W and the neutral Z gauge bosons, we are able to identify full content of the Higgs sector in the SM including the neutral h and the Goldstone bosons eaten by their associated massive gauge ones. Almost interactions among Higgs-gauge bosons in the Standard Model are *recovered*.

The aim of this work is to explore more features of the supersymmetric version of the economical 3–3–1 model through the Higgs-gauge boson interactions. In scalar sector of the model, there exists the singly-charged boson ζ_4^\pm , which is a subject of intensive current studies (see, for example, Refs. [20,21]). The trilinear coupling $ZW^\pm\zeta_4^\mp$ which differs, at the tree level, from zero only in the models with Higgs triplets, plays a special role on study phenomenology of these exotic representations. We shall pay particular interest on this boson.

The outline of this paper is as follows. Section 2 is devoted to a brief review of the model. The scalar fields and mass spectrum is revisited in Section 3 and their couplings with the ordinary gauge bosons are given in Section 4. Production of the heavy singly charged Higgs boson ζ_4^\pm

at the CERN LHC are calculated in Section 5. We outline our main results in the last section—Section 6.

2. A review of the model

In this section we first recapitulate the basic elements of the model [14].

2.1. Particle content

The superfield content in this paper is defined in a standard way as follows

$$\hat{F} = (\tilde{F}, F), \quad \hat{S} = (S, \tilde{S}), \quad \hat{V} = (\lambda, V), \quad (1)$$

where the components F , S and V stand for the fermion, scalar and vector fields while their superpartners are denoted as \tilde{F} , \tilde{S} and λ , respectively [4,22].

The superfields for the leptons under the 3–3–1 gauge group transform as

$$\hat{L}_{aL} = (\hat{\nu}_a, \hat{l}_a, \hat{\nu}_a^c)^T \sim (1, 3, -1/3), \quad \hat{l}_{aL}^c \sim (1, 1, 1), \quad (2)$$

where $\hat{\nu}_L^c = (\hat{\nu}_R)^c$ and $a = 1, 2, 3$ is a generation index.

The superfields for the left-handed quarks of the first generation are in triplets

$$\hat{Q}_{1L} = (\hat{u}_1, \hat{d}_1, \hat{u}'_1)^T \sim (3, 3, 1/3), \quad (3)$$

where the right-handed singlet counterparts are given by

$$\hat{u}_{1L}^c, \hat{u}'_{1L}^c \sim (3^*, 1, -2/3), \quad \hat{d}_{1L}^c \sim (3^*, 1, 1/3). \quad (4)$$

Conversely, the superfields for the last two generations transform as antitriplets

$$\hat{Q}_{\alpha L} = (\hat{d}'_{\alpha}, -\hat{u}_{\alpha}, \hat{d}'_{\alpha})^T \sim (3, 3^*, 0), \quad \alpha = 2, 3, \quad (5)$$

where the right-handed counterparts are in singlets

$$\hat{u}_{\alpha L}^c \sim (3^*, 1, -2/3), \quad \hat{d}'_{\alpha L}^c, \hat{d}_{\alpha L}^c \sim (3^*, 1, 1/3). \quad (6)$$

The primes superscript on usual quark types (u' with the electric charge $q_{u'} = 2/3$ and d' with $q_{d'} = -1/3$) indicate that those quarks are exotic ones. The mentioned fermion content, which belongs to that of the 3–3–1 model with right-handed neutrinos [7,10] is, of course, free from anomaly.

The two superfields $\hat{\chi}$ and $\hat{\rho}$ are at least introduced to span the scalar sector of the economical 3–3–1 model [11]:

$$\hat{\chi} = (\hat{\chi}_1^0, \hat{\chi}^-, \hat{\chi}_2^0)^T \sim (1, 3, -1/3), \quad \hat{\rho} = (\hat{\rho}_1^+, \hat{\rho}^0, \hat{\rho}_2^+)^T \sim (1, 3, 2/3).$$

To cancel the chiral anomalies of Higgsino sector, the two extra superfields $\hat{\chi}'$ and $\hat{\rho}'$ must be added as follows

$$\hat{\chi}' = (\hat{\chi}'_1^0, \hat{\chi}'^+, \hat{\chi}'_2^0)^T \sim (1, 3^*, 1/3), \quad \hat{\rho}' = (\hat{\rho}'_1^-, \hat{\rho}'^0, \hat{\rho}'_2^-)^T \sim (1, 3^*, -2/3). \quad (7)$$

In this model, the $SU(3)_L \otimes U(1)_X$ gauge group is broken via two steps:

$$SU(3)_L \otimes U(1)_X \xrightarrow{w, w'} SU(2)_L \otimes U(1)_Y \xrightarrow{v, v', u, u'} U(1)_Q, \quad (8)$$

where the VEVs are defined by

$$\sqrt{2}\langle\chi\rangle^T = (u, 0, w), \quad \sqrt{2}\langle\chi'\rangle^T = (u', 0, w'), \tag{9}$$

$$\sqrt{2}\langle\rho\rangle^T = (0, v, 0), \quad \sqrt{2}\langle\rho'\rangle^T = (0, v', 0). \tag{10}$$

The VEVs w and w' are responsible for the first step of the symmetry breaking while u, u' and v, v' are for the second one. Therefore, they have to satisfy the constraints:

$$u, u', v, v' \ll w, w'. \tag{11}$$

The vector superfields \hat{V}_c, \hat{V} and \hat{V}' containing the usual gauge bosons are, respectively, associated with the $SU(3)_C, SU(3)_L$ and $U(1)_X$ group factors. The color and flavor vector superfields have expansions in the Gell-Mann matrix bases $T^a = \lambda^a/2$ ($a = 1, 2, \dots, 8$) as follows

$$\begin{aligned} \hat{V}_c \frac{1}{2} \lambda^a \hat{V}_{ca}, \quad \hat{V}_c &= -\frac{1}{2} \lambda^{a*} \hat{V}_{ca}; \\ \hat{V} &= \frac{1}{2} \lambda^a \hat{V}_a, \quad \hat{V} &= -\frac{1}{2} \lambda^{a*} \hat{V}_a, \end{aligned} \tag{12}$$

where an overbar ‘-’ indicates complex conjugation. For the vector superfield associated with $U(1)_X$, we normalize as follows

$$X\hat{V}' = (XT^9)\hat{B}, \quad T^9 \equiv \frac{1}{\sqrt{6}}\text{diag}(1, 1, 1). \tag{13}$$

In the following, we are denoting the gluons by g^a and their respective gluino partners by λ_c^a , with $a = 1, \dots, 8$. In the electroweak sector, V^a and B stand for the $SU(3)_L$ and $U(1)_X$ gauge bosons with their gaugino partners λ_V^a and λ_B , respectively.

The supersymmetric model possesses a full Lagrangian of the form $\mathcal{L}_{\text{susy}} + \mathcal{L}_{\text{soft}}$, where the first term is supersymmetric part, whereas the last term breaks explicitly the supersymmetry. We can find in Ref. [14] for more details on this Lagrangian. In the following, only terms relevant to our calculations are displayed.

2.2. Gauge bosons

The mass Lagrangian for the gauge bosons is given by

$$\begin{aligned} \mathcal{L}_{\text{mass}}^{\text{gauge}} &= (D^\mu\langle\rho\rangle)^+(D_\mu\langle\rho\rangle) + (D^\mu\langle\chi\rangle)^+(D_\mu\langle\chi\rangle) \\ &+ (\bar{D}^\mu\langle\rho'\rangle)^+(\bar{D}_\mu\langle\rho'\rangle) + (\bar{D}^\mu\langle\chi'\rangle)^+(\bar{D}_\mu\langle\chi'\rangle), \end{aligned} \tag{14}$$

where

$$D_\mu = \partial_\mu + igT^a V_{a\mu} + ig'XT^9 B_\mu, \quad \bar{D}_\mu = \partial_\mu - igT^{a*} V_{a\mu} + ig'XT^9 B_\mu. \tag{15}$$

Let us define the charged gauge bosons as follows

$$W_\mu'^{\pm} \equiv \frac{1}{\sqrt{2}}(V_{1\mu} \mp iV_{2\mu}), \quad Y_\mu'^{\pm} \equiv \frac{1}{\sqrt{2}}(V_{6\mu} \pm iV_{7\mu}). \tag{16}$$

The mass matrix of the W'_μ and Y'_μ is obtained then

$$M_{\text{charged}}^2 = \frac{g^2}{4} \begin{pmatrix} V^2 + U^2 & K \\ K & W^2 + V^2 \end{pmatrix}, \tag{17}$$

where

$$\begin{aligned} V^2 &\equiv v^2 + v'^2, & W^2 &\equiv w^2 + w'^2, & U^2 &\equiv u^2 + u'^2 = t_\theta^2 W^2, \\ K &\equiv uw + u'w' = t_\theta W^2, & t &\equiv g'/g. \end{aligned} \tag{18}$$

As in the previous work [14], we have used

$$t_\theta \equiv \frac{u}{w} = \frac{u'}{w'}, \tag{19}$$

and $s_\theta \equiv \sin \theta$, $t_\theta \equiv \tan \theta$, and so forth.

The physical gauge bosons are the SM-like W^\pm and new gauge boson Y^\pm :

$$W_\mu = c_\theta W'_\mu - s_\theta Y'_\mu, \quad Y_\mu = s_\theta W'_\mu + c_\theta Y'_\mu, \tag{20}$$

with the respective masses:

$$m_W^2 = \frac{g^2}{4} V^2, \quad m_Y^2 = \frac{g^2}{4} (V^2 + U^2 + W^2). \tag{21}$$

Therefore, the θ is the mixing angle of $W' - Y'$, which is the same as in the case of non-supersymmetric model [10]. Because of the constraint (11), the mass of W boson is identified with those of the SM, that is

$$\sqrt{v^2 + v'^2} \equiv v_{\text{weak}} = 246 \text{ GeV}. \tag{22}$$

For the remaining gauge vectors (V_3, V_8, B, V_4, V_5), the mass matrix in this basis is given by

$$M_{\text{neutral}}^2 = \begin{pmatrix} M_{\text{mixing}}^2 & 0 \\ 0 & M_{V_5}^2 \end{pmatrix}, \tag{23}$$

where V_5 is decoupled with the mass

$$M_{V_5}^2 \equiv \frac{g^2}{4} (W^2 + U^2), \tag{24}$$

while the mixing part M_{mixing}^2 of (V_3, V_8, B, V_4) is equal to

$$\frac{g^2}{4} \begin{pmatrix} U^2 + V^2 & \frac{1}{\sqrt{3}}(U^2 - V^2) & -\frac{2t}{3\sqrt{6}}(U^2 + 2V^2) & K \\ & \frac{1}{3}(V^2 + U^2 + 4W^2) & \frac{\sqrt{2}t}{9}(2V^2 + 2W^2 - U^2) & -\frac{1}{\sqrt{3}}K \\ & & \frac{2t^2}{27}(4V^2 + U^2 + W^2) & -\frac{4t}{3\sqrt{6}}K \\ & & & U^2 + W^2 \end{pmatrix}. \tag{25}$$

As in the non-supersymmetric version, it can be checked that the matrix (25) contains two *exact* eigenvalues, the photon A_μ and new $V'_{4\mu} \sim V_{4\mu}$, such as

$$M_Y^2 = 0, \quad M_{V'_4}^2 = \frac{g^2}{4} (U^2 + W^2). \tag{26}$$

Due to the fact that V'_4 and V_5 gain the same mass [cf. (26) and (24)], it is worth noting that these boson vectors have to be combined to produce the following physical state [10]

$$X_\mu^0 \equiv \frac{1}{\sqrt{2}}(V'_{4\mu} - iV_{5\mu}), \tag{27}$$

with the mass

$$m_X^2 = \frac{g^2}{4}(U^2 + W^2). \tag{28}$$

Combining Eqs. (21) and (28), as in the non-symmetric version, we get the law of Pythagoras

$$M_Y^2 = M_X^2 + M_W^2. \tag{29}$$

The eigenvectors of (25) are the same in Ref. [10] of the non-supersymmetric version with unique replacement of u, v, w by U, V, W (for details, see [14]). It is worth noting that because of the relation (19), the above diagonalization was eased. For convenience in reading further the mixing matrix of the neutral gauge bosons is given as follows

$$(V_3, V_8, B, V_4)^T = U(A, Z, Z', V_4')^T, \tag{30}$$

where U is given in appendix of Ref. [10].

To finish this section, we mention again that the imaginary part of the non-Hermitian bilepton X^0 is decoupled, while its real part has the mixing among the neutral Hermitian gauge bosons such as, the photon, the neutral Z and the extra Z' .

3. The Higgs sector revisited

The supersymmetric Higgs potential takes the form [14]

$$\begin{aligned} V_{\text{susyeco}} &\equiv V_{\text{scalar}} + V_{\text{soft}} \\ &= \frac{\mu_X^2}{4}(\chi^\dagger \chi + \chi'^\dagger \chi') + \frac{\mu_\rho^2}{4}(\rho^\dagger \rho + \rho'^\dagger \rho') \\ &\quad + \frac{g'^2}{12} \left(-\frac{1}{3} \chi^\dagger \chi + \frac{1}{3} \chi'^\dagger \chi' + \frac{2}{3} \rho^\dagger \rho - \frac{2}{3} \rho'^\dagger \rho' \right)^2 \\ &\quad + \frac{g^2}{8} (\chi_i^\dagger \lambda_{ij}^b \chi_j - \chi_i'^\dagger \lambda_{ij}^{*b} \chi_j' + \rho_i^\dagger \lambda_{ij}^b \rho_j - \rho_i'^\dagger \lambda_{ij}^{*b} \rho_j')^2 \\ &\quad + m_\rho^2 \rho^\dagger \rho + m_X^2 \chi^\dagger \chi + m_{\rho'}^2 \rho'^\dagger \rho' + m_{\chi'}^2 \chi'^\dagger \chi'. \end{aligned} \tag{31}$$

Assuming that the VEVs of neutral components u, u', v, v', w and w' are real, we expand the fields around the VEVs as follows

$$\begin{aligned} \chi^T &= \left(\frac{u + S_1 + iA_1}{\sqrt{2}}, \chi^-, \frac{w + S_2 + iA_2}{\sqrt{2}} \right), & \rho^T &= \left(\rho_1^+, \frac{v + S_5 + iA_5}{\sqrt{2}}, \rho_2^+ \right), \\ \chi'^T &= \left(\frac{u' + S_3 + iA_3}{\sqrt{2}}, \chi'^+, \frac{w' + S_4 + iA_4}{\sqrt{2}} \right), & \rho'^T &= \left(\rho_1'^-, \frac{v' + S_6 + iA_6}{\sqrt{2}}, \rho_2'^- \right). \end{aligned} \tag{32}$$

Requirement of vanishing the linear terms in fields, we get, at the tree-level approximation, the following constraint equations

$$\begin{aligned} \mu_X^2 + 4m_X^2 &= -\frac{g'^2}{54} [w^2 - w'^2 + u^2 - u'^2 + 2(v'^2 - v^2)] \\ &\quad - \frac{g^2}{6} [2(u^2 - u'^2 + w^2 - w'^2) + v'^2 - v^2], \end{aligned}$$

$$\begin{aligned}
 \mu_\rho^2 + 4m_\rho^2 &= -\frac{2g^{2'} + 9g^2}{54} [2(v^2 - v'^2) + w^{2'} - w^2 + u'^2 - u^2], \\
 m_\chi^2 + m_{\chi'}^2 + \mu_\chi^2 &= 0, \quad m_\rho^2 + m_{\rho'}^2 + \mu_\rho^2 = 0, \\
 (w^2 - u^2)u'w' &= (w'^2 - u'^2)uw.
 \end{aligned}
 \tag{33}$$

It is noteworthy that Eq. (33) implies the matching condition previously mentioned in (19). Consequently, the model contains a pair of Higgs triplet χ and antitriplet χ' with the VEVs in top and bottom elements governed by the relation: $u/w = u'/w'$.

The squared-mass matrix derived from (31) can be divided into three 6×6 matrices respective to the charged, scalar and pseudoscalar bosons. Note that there is no mixing among the scalar and pseudoscalar bosons.

3.1. Pseudoscalar sector

- (1) There are two decoupled massless particles: A_5, A_6 .
- (2) Three massless states are mixing of

$$A'_1 = s_\beta A_1 - c_\beta A_3, \quad A'_2 = s_\beta A_2 - c_\beta A_4, \quad \varphi_A = s_\theta A'_3 + c_\theta A'_4,
 \tag{34}$$

where

$$A'_3 = c_\beta A_1 + s_\beta A_3, \quad A'_4 = c_\beta A_2 + s_\beta A_4,
 \tag{35}$$

with

$$t_\beta \equiv \frac{w}{w'}.
 \tag{36}$$

- (3) One massive eigenstate,

$$\phi_A = c_\theta A'_3 - s_\theta A'_4,
 \tag{37}$$

with mass is equal to those of the X bilepton [14]

$$m_{\phi_A}^2 = \frac{g^2}{4} (1 + t_\theta^2) (w^2 + w'^2) = m_X^2.
 \tag{38}$$

Hence, in the pseudoscalar sector, we get five Goldstone bosons: $A_5, A_6, A'_1, A'_2, \varphi_A$ and one massive ϕ_A having the mass equal to those of the bilepton X .

3.2. Scalar sector

In this sector, six particles are mixing in terms of a 6×6 squared-mass matrix. To study physical eigenvalues and eigenstates, we change the basis to such $(S'_1, S'_2, S'_3, S'_4, S'_5, S'_6)$ as

$$\begin{pmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{pmatrix} = \begin{pmatrix} s_\theta & -c_\theta & 0 & 0 & 0 & 0 \\ c_\theta & s_\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & s_\theta & -c_\theta & 0 & 0 \\ 0 & 0 & c_\theta & s_\theta & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{v'}{\sqrt{v^2+v'^2}} & \frac{-v}{\sqrt{v^2+v'^2}} \\ 0 & 0 & 0 & 0 & \frac{v}{\sqrt{v^2+v'^2}} & \frac{v'}{\sqrt{v^2+v'^2}} \end{pmatrix} \begin{pmatrix} S'_1 \\ S'_2 \\ S'_3 \\ S'_4 \\ S'_5 \\ S'_6 \end{pmatrix}.
 \tag{39}$$

For using further, we just introduce the following notation

$$\begin{pmatrix} S'_{1a} \\ S'_{3a} \\ S'_{6a} \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta & 0 \\ -s_\beta & c_\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} S'_1 \\ S'_3 \\ S'_6 \end{pmatrix}. \tag{40}$$

With these combinations, we get the physical fields as follows:

- (1) Three massless fields: S'_5 , S'_{1a} , and

$$\varphi_{S_{24}} = s_\beta S'_2 + c_\beta S'_4. \tag{41}$$

- (2) Three massive fields corresponding to the masses:

$$\phi_{S_{24}} = c_\beta S'_2 - s_\beta S'_4, \quad m_{\phi_{S_{24}}}^2 = \frac{g^2}{4}(1 + t_\theta^2)(w^2 + w'^2) = m_X^2, \tag{42}$$

$$\begin{aligned} \varphi_{S_{a36}} &= s_\alpha S'_3 + c_\alpha S'_6, \\ m_{\varphi_{S_{a36}}}^2 &= \frac{1}{2} [m_{33a}^2 + m_{66a}^2 - \sqrt{(m_{33a}^2 - m_{66a}^2)^2 + 4m_{36a}^4}], \end{aligned} \tag{43}$$

$$\begin{aligned} \phi_{S_{a36}} &= c_\alpha S'_3 - s_\alpha S'_6, \\ m_{\phi_{S_{a36}}}^2 &= \frac{1}{2} [m_{33a}^2 + m_{66a}^2 + \sqrt{(m_{33a}^2 - m_{66a}^2)^2 + 4m_{36a}^4}], \end{aligned}$$

where

$$m_{33a}^2 = \frac{18g^2 + g'^2}{54c_\theta^2}(w^2 + w'^2), \quad m_{66a}^2 = \frac{9g^2 + 2g'^2}{27}(v^2 + v'^2),$$

$$m_{36a}^2 = \frac{(9g^2 + 2g'^2)\sqrt{(v^2 + v'^2)(w^2 + w'^2)}}{54c_\theta}$$

and

$$t_{2\alpha} \equiv \frac{-2m_{36a}^2}{m_{66a}^2 - m_{33a}^2}. \tag{44}$$

From (43), we get

$$m_{\varphi_{S_{a36}}}^2 \simeq \frac{h_1 h_2 - h_3^2}{h_1}(v^2 + v'^2), \tag{45}$$

where

$$h_1 \equiv \frac{18g^2 + g'^2}{54c_\theta^2}, \quad h_2 \equiv \frac{9g^2 + 2g'^2}{27}, \quad h_3 \equiv \frac{9g^2 + 2g'^2}{54c_\theta^2}.$$

Taking into account $\alpha = \frac{e^2}{4\pi} = \frac{1}{128}$, $s_W^2 = 0.2312$, $t = \frac{g'}{g} = \frac{3\sqrt{2}s_W}{\sqrt{4c_W^2 - 1}}$ [23] we have

$$m_{\varphi_{S_{a36}}} \simeq 91.4 \text{ GeV}. \tag{46}$$

This value is very closed to the lower limit of 89.8 GeV (95% CL) given in Ref. [24, p. 32]. It is interesting to note that this mass is also closed to the Z boson mass.

Let us note that ϕ_A and $\phi_{S_{24}}$ have the same mass, which can be combined to become a physical neutral complex field $H_X^0 = (\phi_{S_{24}} + i\phi_A)/\sqrt{2}$ with mass equal to m_X of the neutral non-Hermitian gauge boson X^0 .

3.3. Charged Higgs sector

In Ref. [14], to solve the characteristic equation for charged Higgs sector, we had to use the approximation (11). In this sector we are revising the previous work. Our result below is exact without any approximation.

In the base of $(\chi_a^+, \chi_a'^+, \rho_{1a}^+, \rho_{2a}^+, \rho_{1a}'^+, \rho_{2a}'^+)$, the mass matrix becomes [14]

$$M_{a6}^2{}_{\text{charged}} = \frac{g^2}{4} \begin{pmatrix} m_{a11}^2 & m_{a12}^2 & 0 & m_{a14}^2 & 0 & m_{a16}^2 \\ & m_{a22}^2 & 0 & 0 & 0 & 0 \\ & & v'^2 & 0 & -vv' & 0 \\ & & & m_{a44}^2 & 0 & -vv' \\ & & & & v^2 & 0 \\ & & & & & m_{a66}^2 \end{pmatrix}, \tag{47}$$

where

$$\begin{aligned} m_{a11}^2 &= -c_{2\beta}(\cot_\gamma^2 - 1)v'^2, & m_{a12}^2 &= -s_{2\beta}(\cot_\gamma^2 - 1)v'^2, \\ m_{a22}^2 &= c_{2\beta}(\cot_\gamma^2 - 1)v'^2 + (1 + t_\beta^2)(1 + t_\theta^2)w'^2, \\ m_{a14}^2 &= \sqrt{(1 + t_\beta^2)(1 + t_\theta^2)}w'v, & m_{a16}^2 &= \sqrt{(1 + t_\beta^2)(1 + t_\theta^2)}w'v', \\ m_{a44}^2 &= (t_\beta^2 - 1)(t_\theta^2 + 1)w'^2 + v'^2, & m_{a66}^2 &= -(t_\beta^2 - 1)(t_\theta^2 + 1)w'^2 + v^2, \end{aligned}$$

with

$$\cot_\gamma \equiv \frac{v}{v'}. \tag{48}$$

Since the block intersected by the third, fifth rows and columns is decoupled, it can be diagonalized and this yields two eigenvalues as follows

$$m_{\varrho_2^+}^2 = 0, \quad m_{\varrho_1^+}^2 = \frac{g^2}{4}(v^2 + v'^2) = m_W^2. \tag{49}$$

Here the Goldstone boson ϱ_2^+ and Higgs boson ϱ_1^+ are, respectively, defined by

$$\varrho_1^+ = s_\gamma \rho_{1a}^+ - c_\gamma \rho_{1a}'^+, \quad \varrho_2^+ = c_\gamma \rho_{1a}^+ + s_\gamma \rho_{1a}'^+. \tag{50}$$

Eq. (49) shows that *one charged Higgs boson has the mass equal to those of W boson*, i.e., $m_{\varrho_1^\pm} = m_{W^\pm}$. This result is in agreement with the experimental current limit $m > 79.3$ GeV, CL = 95% [24].

The remaining part of $(\chi_a^+, \chi_a'^+, \rho_{2a}^+, \rho_{2a}'^+)$ is still mixing in terms of a 4×4 submatrix of (47). This matrix can be diagonalized to get eigenvalues as following

$$m_{\xi_1^+}^2 = 0, \tag{51}$$

$$m_{\xi_2^+}^2 = \frac{g^2}{4}[(t_\beta^2 - 1)(u'^2 + w'^2) - (\cot_\gamma^2 - 1)v'^2], \tag{51}$$

$$m_{\xi_3^+}^2 = -m_{\xi_2^+}^2, \tag{52}$$

$$m_{\xi_4^+}^2 = \frac{g^2}{4}(U^2 + V^2 + W^2) = m_Y^2 \tag{53}$$

and the corresponding eigenvalues

$$\begin{aligned} \zeta_1^+ &= \frac{\sqrt{(t_\beta^2 + 1)(u'^2 + w'^2)}}{\sqrt{\frac{4m_{\phi_4^+}^2}{g^2}}} \chi_a^+ - \frac{v}{\sqrt{\frac{4m_{\phi_4^+}^2}{g^2}}} \rho_{2a}^+ - \frac{v'}{\sqrt{\frac{4m_{\phi_4^+}^2}{g^2}}} \rho_{2a}'^+, \\ \zeta_2^+ &= \frac{1}{\sqrt{u'^2 + w'^2 + v^2}} \left[\frac{1}{\sqrt{1 + t_\beta^2}} (v \chi_a^+ + t_\beta v \chi_a'^+) + \sqrt{u'^2 + w'^2} \rho_{2a}^+ \right], \\ \zeta_3^+ &= \frac{1}{\sqrt{v'^2 + w^2 + u^2}} \left[\frac{v'}{\sqrt{1 + t_\beta^2}} (t_\beta \chi_a^+ - \chi_a'^+) + \sqrt{u^2 + w^2} \rho_{2a}'^+ \right], \\ \zeta_4^+ &= \frac{1}{\sqrt{k_1^2 + k_2^2 + k_3^2 + 1}} (k_1 \chi_a^+ + k_2 \chi_a'^+ + k_3 \rho_{2a}^+ + \rho_{2a}'^+), \end{aligned}$$

where

$$\begin{aligned} k_1 &\equiv -\frac{(t_\beta^2 v^2 - v'^2) \sqrt{u'^2 + w'^2}}{v' \sqrt{1 + t_\beta^2} (u'^2 + v^2 + w'^2)}, \\ k_2 &\equiv \frac{\sqrt{u^2 + w^2} [v^2 + v'^2 + (1 + t_\beta^2)(u^{2'} + w^{2'})]}{v' \sqrt{1 + t_\beta^2} (u'^2 + w^{2'} + v^2)}, \\ k_3 &\equiv -\frac{v(v'^2 + u^2 + w^2)}{v'(u'^2 + w'^2 + v^2)}. \end{aligned}$$

As in the gauge boson sector, from (42), (49) and (53), we get again the law of Pythagoras

$$m_{\zeta_4^+}^2 = m_{H_X^0}^2 + m_{\phi_1^+}^2. \tag{54}$$

It is easy to check that the physical field ζ_1^+ is Goldstone bosons and charged Higgs boson ζ_4 has the mass equal to those of Y . This matrix also gives us two physical fields ζ_2^+ and ζ_3^+ with their mass are the same value but opposite sign. Therefore, one of them can be identified with tachyon fields.

From (51) and (52), to cancel the tachyon field, we have to put the condition

$$(t_\beta^2 - 1)(u'^2 + w'^2) - (\cot_\gamma^2 - 1)v'^2 = 0. \tag{55}$$

This yields

$$1 + \frac{u'^2}{w'^2} = \frac{v^2 - v'^2}{w^2 - w'^2}. \tag{56}$$

This means that in the limit $w' \gg u'$, we have the splitting formula

$$w^2 - w'^2 = v^2 - v'^2 \leq 246^2 \text{ GeV}^2. \tag{57}$$

It is noteworthy that the relation (57) is very good addition to (29). In particular notice that, the condition of eliminating tachyon fields (55) does not violate minimum of Higgs potential.

The above condition of tachyon eliminating means that the minimum is *non-trivial*. The same conclusion has been happened for positivity of $m_{H^\pm}^2$ in the MSSM [25].

Finally, let us summarize the physical fields of the scalar sector in the model. There are eight neutral massless particles: five pseudoscalars $A_5, A_6, A'_1, A'_2, \varphi_A$, and three scalars $S'_5, \varphi_{S_{24}}, S'_{1a}$. There is one complex neutral Higgs H_X^0 with mass equal to those of the bilepton m_X , and two massive scalars $\varphi_{S_{a36}}, \phi_{S_{a36}}$. There are four charged massless scalar fields $\varrho_2^\pm, \zeta_1^\pm, \zeta_2^\pm$ and ϱ_3^\pm , and two massive charged bosons ϱ_1^\pm and ζ_4^\pm with masses equal to that of the W boson and the bilepton Y , respectively: $m_{\varrho_1^\pm} = m_W, m_{\zeta_4^\pm} = m_Y$.

The masses of Higgs bosons given in this section is the tree level ones. At one-loop level, besides contributions from the non-supersymmetric version, we have additional loops with sfermions, Higgsino-gaugino, fermion-Higgsino and tadpoles of sfermions. We do hope that, LHC will discovery long-waiting Higgs bosons and then we can fix more free parameters. Radiative corrections to masses of the main Higgs bosons will be our next study.

4. Higgs-gauge boson couplings

With above content of Higgs sector, we can now calculate the Higgs-gauge boson interactions. These interactions exist in part from

$$\mathcal{L}_{\text{kinetic}} = (D^\mu \chi)^+ D_\mu \chi + (D^\mu \rho)^+ D_\mu \rho + (\bar{D}^\mu \chi')^+ \bar{D}_\mu \chi' + (\bar{D}^\mu \rho')^+ \bar{D}_\mu \rho'. \quad (58)$$

In this paper, the gauge bosons are limited to be the gauge bosons of Glashow–Weinberg–Salam model, i.e., photon, W and Z bosons. Using mixing matrices given in Appendix A, we are able to get interactions of the physical fields.

Despite mixing, electromagnetic interactions are unchanged

$$ieA^\mu H^- \overleftrightarrow{\partial}_\mu H^+, \quad H^- = \varrho_1^-, \varrho_2^-, \zeta_1^-, \zeta_2^-, \zeta_3^-, \zeta_4^-. \quad (59)$$

Formula (59) shows that the scalar Higgs physical states and mixings given in Section 3 are correct.

For the W boson, we get couplings of pair W^+W^- with neutral Higgs bosons presented in Table 1.

The interactions of single W boson with two Higgs bosons are presented in Table 2, where $m_{x,y}, x, y = 1, 2, 3, 4$, are given in Appendix A. Other vertices are

$$\begin{aligned} \mathcal{V}(W^- \varrho_1^+ A_5) &= -\mathcal{V}(W^- \varrho_2^+ A_6) = \frac{gv'}{2\sqrt{v^2 + v'^2}}, \\ \mathcal{V}(W^- \varrho_2^+ A_5) &= \mathcal{V}(W^- \varrho_1^+ A_6) = \frac{gv}{2\sqrt{v^2 + v'^2}}, \\ \mathcal{V}(W^- \varrho_2^+ S'_5) &= \frac{-1}{c_\alpha} \mathcal{V}(W^- \varrho_1^+ \varphi_{S_{a36}}) = \frac{1}{s_\alpha} \mathcal{V}(W^- \varrho_1^+ \phi_{S_{a36}}) = -\frac{igvv'}{v^2 + v'^2}, \\ \mathcal{V}(W^- \varrho_1^+ S'_5) &= \frac{1}{c_\alpha} \mathcal{V}(W^- \varrho_2^+ \varphi_{S_{a36}}) = \frac{-1}{s_\alpha} \mathcal{V}(W^- \varrho_2^+ \phi_{S_{a36}}) = \frac{ig(v^2 - v'^2)}{v^2 + v'^2}. \end{aligned}$$

Non-zero quartic couplings of pair W^+W^- with two Higgs bosons are given in Table 3. Addition to this table, we have also five interactions

Table 1
Trilinear couplings of W^+W^- with neutral Higgs bosons

| Vertex | Coupling |
|---------------------------|-----------------------------------------------------------|
| $W^+W^-S'_5$ | $\frac{g^2vv'}{\sqrt{v^2+v'^2}}$ |
| $W^+W^-\varphi_{S_{a36}}$ | $\frac{g^2c_\alpha}{2} \frac{v'^2-v^2}{\sqrt{v'^2+v^2}}$ |
| $W^+W^-\phi_{S_{a36}}$ | $-\frac{g^2s_\alpha}{2} \frac{v'^2-v^2}{\sqrt{v'^2+v^2}}$ |

Table 2
Trilinear coupling constants of W^- with two Higgs bosons

| Vertex | Coupling |
|-----------------------------------------------------------------------|--------------------------------------------------------------------------------------|
| $W^{\mu-}\zeta_x^+\overleftrightarrow{\partial}_\mu A'_1$ | $-\frac{gw(2m_{2x}uu'+m_{1x}(u^2-u'^2))}{2(u^2+u'^2)\sqrt{u^2+w^2}}, x = 1, 2, 3, 4$ |
| $W^{\mu-}\zeta_x^+\overleftrightarrow{\partial}_\mu \varphi_A$ | $-\frac{gu^2w(m_{2x}u-m_{1x}u')}{2(u^2+u'^2)(u^2+w^2)}, x = 1, 2, 3, 4$ |
| $W^{\mu-}\zeta_x^+\overleftrightarrow{\partial}_\mu A'_2$ | $\frac{gu(2m_{2x}uu'+m_{1x}(u^2-u'^2))}{2(u^2+u'^2)\sqrt{u^2+w^2}}, x = 1, 2, 3, 4$ |
| $W^{\mu-}\zeta_x^+\overleftrightarrow{\partial}_\mu H_X^0$ | $-\frac{ig(-2m_{1x}uu'+m_{2x}(u^2-u'^2))}{\sqrt{2}(u^2+u'^2)}, x = 1, 2, 3, 4$ |
| $W^{\mu-}\zeta_x^+\overleftrightarrow{\partial}_\mu \varphi_{S_{24}}$ | $\frac{ig(2m_{2x}uu'+m_{1x}(u^2-u'^2))}{2(u^2+u'^2)}, x = 1, 2, 3, 4$ |

Table 3
Non-zero quartic coupling constants of W^+W^- with Higgs bosons

| Vertex | Coupling | Vertex | Coupling |
|--------------------------------------|-----------------------------|--------------------------------------------|---------------------------------------------------------------|
| $W^+W^-A'_1A'_2$ | $-\frac{g^2uw}{2(u^2+w^2)}$ | $W^+W^-\varphi_{S_{a36}}\phi_{S_{a36}}$ | $-\frac{g^2s_{2\alpha}}{4}$ |
| $W^+W^-A'_1A'_1$ | $\frac{g^2w^2}{2(u^2+w^2)}$ | $W^+W^-A'_2A'_2$ | $\frac{g^2u^2}{2(u^2+w^2)}$ |
| $W^+W^-A_5A_5$ | $\frac{g^2}{2}$ | $W^+W^-\varphi_{S_{a36}}\varphi_{S_{a36}}$ | $\frac{g^2c_\alpha^2}{2}$ |
| $W^+W^-\phi_{S_{a36}}\phi_{S_{a36}}$ | $\frac{g^2s_\alpha^2}{2}$ | $W^+W^-\zeta_x^+\zeta_y^-$ | $\frac{g^2(m_{1x}m_{1y}+m_{2x}m_{2y})}{2}, x, y = 1, 2, 3, 4$ |

$$\begin{aligned} \mathcal{V}(W^+W^-A_5A_5) &= \mathcal{V}(W^+W^-A_6A_6) = \mathcal{V}(W^+W^-S'_5S'_5) \\ &= \mathcal{V}(W^+W^-H_X^0H_X^{0*}) = \mathcal{V}(W^+W^-e_1^+e_1^-) = \mathcal{V}(W^+W^-e_2^+e_2^-). \end{aligned}$$

For the neutral Z boson, the triple coupling of single Z with two charged Higgs bosons are presented in Table 4, where U_{22}, \dots are elements in the mixing matrix U of the neutral gauge bosons. We have also

$$\mathcal{V}(Ze_1^-e_1^+) = \mathcal{V}(Ze_2^-e_2^+).$$

The notations are given by

$$\begin{aligned} f_1 &= u \left(3U_{12} + \sqrt{3} \left(U_{22} - 2s_W \sqrt{\frac{1}{3-4s_W^2}} U_{32} \right) \right) + 3wU_{42}, \\ f_2 &= 3U_{12} - \sqrt{3} \left(U_{22} + 4s_W \sqrt{\frac{1}{3-4s_W^2}} U_{32} \right), \end{aligned}$$

Table 4
Trilinear coupling constants of Z^μ with two charged Higgs bosons

| Vertex | Coupling |
|-------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $Z^\mu \varrho_1^- \overleftrightarrow{\partial}_\mu \varrho_1^+$ | $-\frac{ig}{6(u^2+w^2)}\{-2\sqrt{3}u^2(U_{22}-2s_W\sqrt{\frac{1}{3-4s_W^2}}U_{32})-6uU_{42}w$ $+ [3U_{12} + \sqrt{3}(U_{22} + 4s_W\sqrt{\frac{1}{3-4s_W^2}}U_{32})]w^2\}$ |
| $Z^\mu \xi_x^- \overleftrightarrow{\partial}_\mu \varrho_1^+$ | $-\frac{ig}{2\sqrt{v^2+v'^2}(u^2+w^2)}(m_{4x}v - m_{3x}v')[-u^2U_{42} + u(U_{12} + \sqrt{3}U_{22})w + U_{42}w^2]$, $x = 1, 2, 3, 4$ |
| $Z^\mu \xi_x^- \overleftrightarrow{\partial}_\mu \varrho_2^+$ | $\frac{ig}{2\sqrt{v^2+v'^2}(u^2+w^2)}(m_{3x}v + m_{4x}v')[-u^2U_{42} + u(U_{12} + \sqrt{3}U_{22})w + U_{42}w^2]$, $x = 1, 2, 3, 4$ |
| $Z^\mu \xi_x^- \overleftrightarrow{\partial}_\mu \xi_y^+$ | $-\frac{ig}{6s_W(u^2+w^2)}\{(m_{1y}m_{1x} + m_{2y}m_{2x})[3U_{12} - \sqrt{3}(U_{22} - 2s_W\sqrt{\frac{1}{3-4s_W^2}}U_{32})]$ $\times (u^2 + w^2) + (m_{3y}m_{3x} + m_{4y}m_{4x})(u^2[3U_{12} + \sqrt{3}(U_{22} + 4s_W\sqrt{\frac{1}{3-4s_W^2}}U_{32})]$ $+ 6uwU_{42} - 2\sqrt{3}w^2(U_{22} - 2s_W\sqrt{\frac{1}{3-4s_W^2}}U_{32}))\}$, $x, y = 1, 2, 3, 4$ |

$$\begin{aligned}
 f_3 &= -3U_{12}^2 - 2\sqrt{3}U_{12}\left(U_{22} - 2s_W\sqrt{\frac{1}{3-4s_W^2}}U_{32}\right) \\
 &\quad + 3U_{22}\left(U_{22} + 4s_W\sqrt{\frac{1}{3-4s_W^2}}U_{32}\right), \\
 f_4 &= 3U_{12}^2 + U_{22}^2 + \frac{8s_W U_{22}U_{32}}{\sqrt{3-4s_W^2}} - \frac{16s_W^2 U_{32}^2}{-3+4s_W^2} - 2\sqrt{3}U_{12}\left(U_{22} + \frac{4s_W^2}{\sqrt{3-4s_W^2}}U_{42}\right), \\
 f_5 &= 3U_{12}^2 + U_{22}^2 - \frac{4s_W U_{22}U_{32}}{\sqrt{3-4s_W^2}} - \frac{4s_W^2 U_{32}^2}{-3+4s_W^2} - 2\sqrt{3}U_{12}\left(-U_{22} + \frac{2s_W U_{32}}{\sqrt{3-4s_W^2}}\right) + 3U_{42}^2, \\
 f_6 &= 4U_{22}^2 + \frac{8s_W U_{22}U_{32}}{\sqrt{3-4s_W^2}} + 3U_{42}^2 - \frac{4s_W^2}{-3+4s_W^2}U_{32}^2, \\
 f_7 &= 3uU_{42} - 2\sqrt{3}w\left(U_{22} + s_W\sqrt{\frac{1}{3-4s_W^2}}U_{32}\right), \\
 f_8 &= u^2\left[3U_{12} + \sqrt{3}\left(U_{22} - 2s_W\sqrt{\frac{1}{3-4s_W^2}}U_{32}\right)\right] + 6uwU_{42} \\
 &\quad - 2\sqrt{3}w^2\left(U_{22} + s_W\sqrt{\frac{1}{3-4s_W^2}}U_{32}\right).
 \end{aligned}$$

Similarly, trilinear coupling of the single Z with two neutral Higgs bosons are given in Table 5. The triple couplings of pair ZZ with one scalar Higgs boson are given in Table 6, where

$$\begin{aligned}
 a_1 &= (a_{11}^2 + U_{42}^2)u + (a_{11} + a_{33})U_{42}w, & a_2 &= (a_{33}^2 + U_{42}^2)w + (a_{11} + a_{33})U_{42}u, \\
 a_3 &= (a_{11}^2 + U_{42}^2)u' - (a_{11} + a_{33})U_{42}w', & a_4 &= (a_{33}^2 + U_{42}^2)w' - (a_{11} + a_{33})U_{42}u', \\
 a_{11} &= U_{12} + \frac{1}{\sqrt{3}}U_{22} - \frac{t}{\sqrt{3}}\sqrt{\frac{2}{3}}U_{32}, & a_{33} &= -\frac{2}{\sqrt{3}}U_{22} - \frac{t}{\sqrt{3}}\sqrt{\frac{2}{3}}U_{32}.
 \end{aligned}$$

Table 5
Trilinear couplings of Z_μ with two neutral Higgs bosons

| Vertex | Coupling | Vertex | Coupling |
|------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------|-------------------------------------------------------------|
| $Z^\mu A'_1 \overset{\leftrightarrow}{\partial}_\mu S'_{1a}$ | $-\frac{guu'}{3(u^2+u'^2)\sqrt{u^2+w^2}} f_1$ | $Z^\mu A'_2 \overset{\leftrightarrow}{\partial}_\mu S'_{1a}$ | $-\frac{guu'}{3(u^2+u'^2)\sqrt{u^2+w^2}} f_7$ |
| $Z^\mu \varphi_A \overset{\leftrightarrow}{\partial}_\mu S'_{1a}$ | $\frac{g(u^2-u'^2)}{6(u^2+u'^2)(u^2+w^2)} f_8$ | $Z^\mu A_5 \overset{\leftrightarrow}{\partial}_\mu \varphi_{S_{a36}}$ | $-\frac{gv'c_\alpha}{6\sqrt{v^2+v'^2}} f_2$ |
| $Z^\mu A_6 \overset{\leftrightarrow}{\partial}_\mu \varphi_{S_{a36}}$ | $-\frac{gv'c_\alpha}{6\sqrt{v^2+v'^2}} f_2$ | $Z^\mu A'_2 \overset{\leftrightarrow}{\partial}_\mu \varphi_{S_{a36}}$ | $\frac{gs_\alpha(u^2-u'^2)}{6(u^2+u'^2)\sqrt{u^2+w^2}} f_7$ |
| $Z^\mu A'_1 \overset{\leftrightarrow}{\partial}_\mu \varphi_{S_{a36}}$ | $\frac{gs_\alpha(u^2-u'^2)}{6(u^2+u'^2)\sqrt{u^2+w^2}} f_1$ | $Z^\mu \varphi_A \overset{\leftrightarrow}{\partial}_\mu \varphi_{S_{a36}}$ | $\frac{gs_\alpha uu'}{3(u^2+u'^2)(u^2+w^2)} f_8$ |
| $Z^\mu A_5 \overset{\leftrightarrow}{\partial}_\mu S'_5$ | $\frac{gv'}{6\sqrt{v^2+v'^2}} f_2$ | $Z^\mu A_6 \overset{\leftrightarrow}{\partial}_\mu S'_5$ | $-\frac{gv}{6\sqrt{v^2+v'^2}} f_2$ |
| $Z^\mu A_5 \overset{\leftrightarrow}{\partial}_\mu \phi_{S_{a36}}$ | $\frac{gvs_\alpha}{6\sqrt{v^2+v'^2}} f_2$ | $Z^\mu A_6 \overset{\leftrightarrow}{\partial}_\mu \phi_{S_{a36}}$ | $\frac{gv's_\alpha}{6\sqrt{v^2+v'^2}} f_2$ |
| $Z^\mu A'_1 \overset{\leftrightarrow}{\partial}_\mu \phi_{S_{a36}}$ | $\frac{gc_\alpha(u^2-u'^2)}{6(u^2+u'^2)\sqrt{u^2+w^2}} f_1$ | $Z^\mu A'_2 \overset{\leftrightarrow}{\partial}_\mu \phi_{S_{a36}}$ | $\frac{gc_\alpha(u^2-u'^2)}{6(u^2+u'^2)\sqrt{u^2+w^2}} f_7$ |
| $Z^\mu A'_1 \overset{\leftrightarrow}{\partial}_\mu \varphi_{S_{24}}$ | $\frac{g}{6\sqrt{u^2+w^2}} [-3uU_{42} + (3U_{12} + \sqrt{3}(U_{22} - 2s_W \sqrt{\frac{1}{3-4s_W^2}} U_{32}))w]$ | $Z^\mu \varphi_A \overset{\leftrightarrow}{\partial}_\mu \phi_{S_{a36}}$ | $\frac{gc_\alpha uu'}{3(u^2+u'^2)(u^2+w^2)} f_8$ |
| $Z^\mu A'_2 \overset{\leftrightarrow}{\partial}_\mu \varphi_{S_{24}}$ | $\frac{g}{6\sqrt{u^2+w^2}} [2\sqrt{3}u(U_{22} + s_W \sqrt{\frac{1}{3-4s_W^2}} U_{32}) + 3wU_{42}]$ | | |

Table 6
Trilinear coupling constants of ZZ with one scalar bosons

| Vertex | Coupling |
|-----------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|
| ZZS'_5 | $\frac{g^2 v v'}{3\sqrt{v^2+v'^2}} f_4$ |
| ZZS'_{1a} | $\frac{g^2 u'}{3\sqrt{u^2+u'^2}\sqrt{u^2+w^2}} (u^2 f_5 + 2uwU_{42} + w^2 f_6)$ |
| $ZZ\varphi_{S_{24}}$ | $\frac{g^2 \sqrt{u^2+u'^2}}{6u\sqrt{u^2+w^2}} [(u^2 - w^2)U_{42} f_2 + u w f_3]$ |
| $ZZ\varphi_{S_{a36}}$ | $\frac{g^2}{4} \{[-(a_1 s_\theta + a_2 c_\theta) s_\beta + (a_3 s_\theta + a_4 c_\theta) c_\beta] s_\alpha + \frac{v'-v}{\sqrt{v^2+v'^2}} c_\alpha\}$ |
| $ZZ\phi_{S_{a36}}$ | $\frac{g^2}{4} \{[-(a_1 s_\theta + a_2 c_\theta) s_\beta + (a_3 s_\theta + a_4 c_\theta) c_\beta] c_\alpha - \frac{v'-v}{\sqrt{v^2+v'^2}} s_\alpha\}$ |

Non-zero quartic couplings of pair ZZ with two scalar Higgs bosons are given in Table 7.

Non-zero quartic of pair ZZ with two charged Higgs bosons are presented in Table 8.

Another interaction is

$$\mathcal{V}(ZZ\rho_1\rho_1) = \mathcal{V}(ZZ\rho_2\rho_2).$$

In the special limit

$$u = u', \quad w = w', \quad v' = 0, \quad w, w' \gg u, u', v, v', \tag{60}$$

the effective couplings are summarized in Table 9. From (50) and Appendix C we get the following limit for physical fields

$$S_5 \rightarrow -\varphi_{S_{a36}}, \quad \rho_1^\dagger \rightarrow -\varrho_2^\dagger. \tag{61}$$

Table 7
Non-zero quartic coupling constants of ZZ with two neutral scalar bosons

| Vertex | Coupling |
|------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $ZZA'_1A'_2$ | $\frac{g^2}{6}U_{42}f_2$ |
| $ZZA'_1A'_1$ | $\frac{g^2}{6}f_5$ |
| $ZZA'_2A'_2$ | $\frac{g^2}{6}f_6$ |
| $ZZHH$ | $\frac{g^2}{6}f_4, H = S'_5, A_5, A_6$ |
| $ZZS'_{1a}\varphi S_{24}$ | $\frac{g^2uu'}{3(u^2+u'^2)+(u^2+w^2)}[(u^2-w^2)U_{42}f_2+uwf_3]$ |
| $ZZ\varphi S_{24}\phi S_{a36}$ | $-\frac{g^2c_\alpha(u^2-u'^2)}{6(u^2+u'^2)+(u^2+w^2)}[(u^2-w^2)U_{42}f_2+uwf_3]$ |
| $ZZ\varphi S_{24}\varphi S_{a36}$ | $-\frac{g^2s_\alpha(u^2-u'^2)}{6(u^2+u'^2)+(u^2+w^2)}[(u^2-w^2)U_{42}f_2+uwf_3]$ |
| $ZZS'_{1a}S'_{1a}$ | $\frac{g^2}{6(u^2+w^2)}(u^2f_5+2uwU_{42}f_2+w^2f_6)$ |
| $ZZ\varphi S_{24}\varphi S_{24}$ | $\frac{g^2}{6(u^2+w^2)}(u^2f_6-2uwU_{42}f_2+w^2f_5)$ |
| $ZZH_X^0H_X^{0*}$ | $\frac{g^2}{6(u^2+w^2)}(u^2f_6-2uwU_{42}f_2+w^2f_5)$ |
| $ZZ\varphi S_{a36}\varphi S_{a36}$ | $\frac{g^2}{6(u^2+w^2)}[c_\alpha^2(u^2+w^2)f_4+s_\alpha^2(u^2f_5+2uwU_{42}f_2+w^2f_6)]$ |
| $ZZ\varphi_A\varphi_A$ | $\frac{g^2}{6(u^2+w^2)}(u^2f_5+2uwU_{42}f_2+w^2f_6)$ |
| $ZZ\phi S_{a36}\phi S_{a36}$ | $\frac{g^2}{6(u^2+w^2)}[s_\alpha^2(u^2+w^2)f_4+c_\alpha^2(u^2f_5+2uwU_{42}f_2+w^2f_6)]$ |
| $ZZ\varphi S_{a36}\phi S_{a36}$ | $\frac{g^2s_{2\alpha}}{12(u^2+w^2)}\{u^2[4\sqrt{3}U_{12}(U_{22}+s_W\sqrt{\frac{1}{3-4s_W^2}}U_{32})+3U_{42}^2-\frac{12s_WU_{22}U_{32}}{\sqrt{3-4s_W^2}}+\frac{12s_W^2U_{32}^2}{-3+4s_W^2}]-2uwU_{42}[-3U_{12}+\sqrt{3}(U_{22}+4s_W\sqrt{\frac{1}{3-4s_W^2}}U_{32})]+[-3U_{12}^2+2\sqrt{3}U_{12}(U_{22}+4s_W\sqrt{\frac{1}{3-4s_W^2}}U_{32})+3(U_{22}^2+U_{42}^2+\frac{4s_W^2U_{32}^2}{-3+4s_W^2})]w^2\}$ |

Therefore, the Higgs triplet responsible for the second step of symmetry breaking ρ can be represented as

$$\rho \Rightarrow \begin{pmatrix} -\varrho_2^+ \\ \frac{v-\varphi_{S_{a36}}+iA_5}{\sqrt{2}} \\ \xi_2^+ \end{pmatrix}. \quad (62)$$

Remind that both ϱ_2^+ and A_5 are massless. By Table 9, we can identify them as Goldstone bosons for the W and Z ones (neglecting the minus sign), respectively. This yields

$$\rho \Rightarrow \begin{pmatrix} \mathbf{G}_{W^+} \\ \frac{v+h+iG_Z}{\sqrt{2}} \\ \xi_2^+ \end{pmatrix}. \quad (63)$$

Hence, all the effective couplings of the gauge bosons with scalar fields of the SM can be recovered, which most of them are presented in Table 10.

In principle, we cannot put $v' = 0$. The above analysis just shows that our calculations are correct.

Table 8
Non-zero quartic coupling constants of ZZ with two charged bosons

| Vertex | Coupling |
|-----------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $ZZ\varrho_1^- \varrho_1^+$ | $\frac{g^2}{6(u^2+w^2)}\{u^2[4U_{22}^2 - \frac{16s_W U_{22}U_{32}}{\sqrt{3-4s_W^2}} - \frac{16s_W^2 U_{32}^2}{-3+4s_W^2} + 3U_{42}^2] - 2uw[3U_{12} - \sqrt{3}(U_{22} - \frac{8s_W U_{32}}{\sqrt{3-4s_W^2}})]U_{42} + w^2[3U_{12}^2 + U_{22}^2 + \frac{8s_W U_{22}U_{32}}{\sqrt{3-4s_W^2}} - \frac{16s_W^2 U_{32}^2}{-3+4s_W^2} + 2\sqrt{3}U_{12}(U_{22} + \frac{4s_W U_{32}}{\sqrt{3-4s_W^2}}) + 3U_{42}^2]\}$ |
| $ZZ\varrho_1^- \zeta_x^+$ | $-\frac{g^2(m_{4x}v-m_{3x}v')}{6(u^2+w^2)\sqrt{v^2+v'^2}}\{(u^2-w^2)[3U_{12} - \sqrt{3}(U_{22} - \frac{8s_W U_{32}}{\sqrt{3-4s_W^2}})]U_{42} - uw[3U_{12}^2 - 3U_{22}(U_{22} - \frac{8s_W U_{32}}{\sqrt{3-4s_W^2}}) + 2\sqrt{3}U_{12}(U_{22} + \frac{4s_W U_{32}}{\sqrt{3-4s_W^2}})]\}, x = 1, 2, 3, 4$ |
| $ZZ\varrho_2^- \zeta_x^+$ | $\frac{g^2(m_{3x}v+m_{4x}v')}{6(u^2+w^2)\sqrt{v^2+v'^2}}\{(u^2-w^2)[3U_{12} - \sqrt{3}(U_{22} - \frac{8s_W U_{32}}{\sqrt{3-4s_W^2}})]U_{42} - uw[3U_{12}^2 - 3U_{22}(U_{22} - \frac{8s_W U_{32}}{\sqrt{3-4s_W^2}}) + 2\sqrt{3}U_{12}(U_{22} + \frac{4s_W U_{32}}{\sqrt{3-4s_W^2}})]\}, x = 1, 2, 3, 4$ |
| $ZZ\zeta_y^- \zeta_x^+$ | $\frac{g^2}{6(u^2+w^2)}\{(m_{1y}m_{1x} + m_{2y}m_{2x})(u^2 + w^2)[3U_{12}^2 + U_{22}^2 - \frac{4s_W U_{22}U_{32}}{\sqrt{3-4s_W^2}} - \frac{4s_W^2 U_{32}^2}{-3+4s_W^2}] - 2\sqrt{3}U_{12}(U_{22} - \frac{2s_W U_{32}}{\sqrt{3-4s_W^2}})] + (m_{3y}m_{3x} + m_{4y}m_{4x})[u^2(3U_{12}^2 + U_{22}^2 + \frac{8s_W U_{22}U_{32}}{\sqrt{3-4s_W^2}} - \frac{16s_W^2 U_{32}^2}{-3+4s_W^2}) + 2\sqrt{3}U_{12}(U_{22} + \frac{4s_W U_{32}}{\sqrt{3-4s_W^2}}) + 3U_{42}^2] + 2uw(3U_{12} - \sqrt{3}(U_{22} - \frac{8s_W U_{32}}{\sqrt{3-4s_W^2}}))U_{42} + w^2(4U_{22}^2 - \frac{16s_W U_{22}U_{32}}{\sqrt{3-4s_W^2}} - \frac{16s_W^2 U_{32}^2}{-3+4s_W^2} + 3U_{42}^2)]\}$ |

5. Production of charged ζ_4^\pm via WZ fusion at LHC

The possibility to detect the neutral Higgs boson in the minimal version at e^+e^- colliders was considered in [26] and production of the SM-like neutral Higgs boson in the 3–3–1 model with right-handed neutrinos at the CERN LHC was considered in Ref. [27]. The decay and production at the CERN LHC of the bilepton charged Higgs in the non-supersymmetric version of the considering was given in Ref. [13]. This section is devoted to the decay modes and production of the charged ζ_4^\pm at the CERN LHC.

Let us first discuss on the mass of this Higgs boson. Eq. (53) gives us a connection between its mass and those of the singly-charged bilepton Y . The bilepton mass limit can be obtained from the “wrong” muon decay $\mu^- \rightarrow e^- \nu_e \tilde{\nu}_\mu$ mediated, at the tree level, by both the W and the Y boson. Taking into account of the famous experimental data [24]

$$R_{\text{muon}} \equiv \frac{\Gamma(\mu^- \rightarrow e^- \nu_e \tilde{\nu}_\mu)}{\Gamma(\mu^- \rightarrow e^- \tilde{\nu}_e \nu_\mu)} < 1.2\% \quad 90\% \text{ CL} \tag{64}$$

we get the constraint: $R_{\text{muon}} \simeq \frac{M_W^4}{M_Y^4}$. Therefore, it follows that $M_Y \geq 230$ GeV. This bound is consistent with that followed from the oblique consideration in Ref. [28]. However, the stronger bilepton mass bound has been derived from consideration of experimental limit on lepton-number violating charged lepton decays [29] of 440 GeV.

Table 9
The non-zero coupling constants in the effective limit

| Vertex | Coupling | Vertex | Coupling |
|-----------------------------------|--------------------------------------|-------------------------|------------------------------------------------------------------------------------------------------------|
| $W^+W^-S'_5$ | g^2vs_γ | $ZW^+\zeta_4^-$ | $\frac{g^2u}{2\sqrt{2}c_W}$ |
| $W^+W^-\varphi_{S_{a36}}$ | $-\frac{g^2vc_{2\gamma}}{2c_\gamma}$ | $ZA_5\varphi_{S_{a36}}$ | $-\frac{gc_\gamma}{2c_W}$ |
| $W^-\varrho_1^+\varphi_{S_{a36}}$ | $\frac{igs_{2\gamma}}{2}$ | $ZA_6\varphi_{S_{a36}}$ | $-\frac{gs_\gamma}{2c_W}$ |
| $W^-\varrho_1^+S'_5$ | $\frac{igc_{2\gamma}}{2}$ | $ZA_5S'_5$ | $\frac{gs_\gamma}{2c_W}$ |
| $W^-\varrho_1^+A_5$ | $\frac{gs_\gamma}{2}$ | $ZA_6S'_5$ | $-\frac{gc_\gamma}{2c_W}$ |
| $W^-\varrho_1^+A_6$ | $\frac{gc_\gamma}{2}$ | $ZA'_1\varphi_{S_{24}}$ | $\frac{g}{2c_W}$ |
| $W^-\varrho_2^+\varphi_{S_{a36}}$ | $\frac{igc_{2\gamma}}{2}$ | $ZZ\varphi_{S_{24}}$ | $-\frac{g^2u}{2\sqrt{2}c_W^2}$ |
| $W^-\varrho_2^+S'_5$ | $-\frac{igs_{2\gamma}}{2}$ | $ZZ\varphi_{S_{a36}}$ | $-\frac{g^2vc_{2\gamma}}{2c_W^2c_\gamma}$ |
| $W^-\varrho_2^+A_6$ | $-\frac{gs_\gamma}{2}$ | ZZS'_5 | $\frac{g^2vs_\gamma}{c_W^2}$ |
| $W^-\varrho_2^+A_5$ | $\frac{gc_\gamma}{2}$ | $Z\Psi^+\Psi^-$ | $-\frac{igs_W^2}{2c_W}, \Psi = \zeta_2, \zeta_3$ |
| $W^-\zeta_1^+H_X^0$ | $\frac{ig}{\sqrt{2}}$ | $Z\Psi^+\Psi^-$ | $\frac{igc_{2W}}{2c_W}, \Psi = \varrho_1, \varrho_2, \zeta_1, \zeta_4$ |
| $W^-\zeta_4^+\varphi_{S_{24}}$ | $\frac{ig}{2}$ | $ZZ\Psi^-\Psi^+$ | $\frac{2g^2s_W^4}{c_W^2}, \Psi = \zeta_2, \zeta_3$ |
| $W^-\zeta_4^+A'_1$ | $-\frac{g}{2}$ | $ZZ\Psi^-\Psi^+$ | $\frac{g^2c_W^2}{2c_W^2}, \Psi = \varrho_1, \varrho_2, \zeta_1, \zeta_4$ |
| $AW^+\varrho_2^-$ | $-\frac{e^2v}{2s_Wc_\gamma}$ | $WWHH$ | $\frac{g^2}{2}, H = A'_1, \varphi_{S_{a36}}, S'_5, A_6, H_X^0, \zeta_1, \zeta_4, \varrho_1, \varrho_2$ |
| $ZW^+\varrho_2^-$ | $\frac{e^2v}{2c_Wc_\gamma}$ | $ZZHH$ | $\frac{g^2}{2c_W^2}, H = S'_5, \varphi_{S_{24}}, H_X^0, \varphi_{S_{a36}}, \phi_{S_{a36}}, A'_1, A_5, A_6$ |

Table 10
The SM coupling constants in the effective limit

| Vertex | Coupling | Vertex | Coupling |
|-----------|----------------------|------------|-----------------------------|
| $WWhh$ | $\frac{g^2}{2}$ | G_WG_WA | ie |
| WWh | $\frac{g^2}{2}v$ | WWG_ZG_Z | $\frac{g^2}{2}$ |
| WG_Wh | $-\frac{ig}{2}$ | WWG_WG_W | $\frac{g^2}{2}$ |
| WG_WG_Z | $\frac{g}{2}$ | ZZh | $\frac{g^2}{2c_W^2}v$ |
| $ZZhh$ | $\frac{g^2}{2c_W^2}$ | ZZG_ZG_Z | $\frac{g^2}{2c_W^2}$ |
| AWG_W | $\frac{g^2}{2}vs_W$ | ZWG_W | $-\frac{g^2}{2}vs_Wt_W$ |
| ZG_Zh | $-\frac{g}{2c_W}$ | ZG_WG_W | $\frac{ig}{2c_W}(1-2s_W^2)$ |

Taking into account that, in the effective approximation, ζ_4^- is the bilepton, we get the dominant decay channels as follows

$$\zeta_4^- \rightarrow \begin{cases} l\nu_l, U^c d, u^c D, \\ ZW^-, \tilde{H}^0 \tilde{W}^-. \end{cases} \quad (65)$$

Table 11

Trilinear coupling constants of W^+ with neutral gauge boson and the charged scalar boson

| Vertex | Coupling |
|-------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $ZW^+\varrho_2^-$ | $\frac{g^2\sqrt{v^2+v'^2}}{12(u^2+w^2)}\{u^2[3U_{12}+\sqrt{3}(U_{22}-\frac{8s_W U_{32}}{\sqrt{3-4s_W^2}})]+6uwU_{42}-2\sqrt{3}w^2(U_{22}+\frac{4s_W U_{32}}{\sqrt{3-4s_W^2}})\}$ |
| $ZW^+\zeta_x^-$ | $-\frac{g^2}{4u\sqrt{u^2+u'^2}(u^2+w^2)^{\frac{3}{2}}}\{[(u^2-w^2)U_{42}-uw(U_{12}+\sqrt{3}U_{22})][m_{1x}(u^2-u'^2)(u^2+w^2)+u(\sqrt{u^2+u'^2}\sqrt{u^2+w^2}(m_{3x}v+m_{4x}v')+2m_{2x}u'(u^2+w^2))]\}$, $x = 1, 2, 3, 4$ |
| $AW^+\varrho_2^-$ | $\frac{g^2\sqrt{v^2+v'^2}}{12(u^2+w^2)}\{u^2[3U_{11}+\sqrt{3}(U_{21}-\frac{8s_W U_{31}}{\sqrt{3-4s_W^2}})]+6uwU_{41}-2\sqrt{3}w^2(U_{21}+\frac{4s_W U_{31}}{\sqrt{3-4s_W^2}})\}$ |
| $AW^+\zeta_x^-$ | $-\frac{g^2}{4u\sqrt{u^2+u'^2}(u^2+w^2)^{\frac{3}{2}}}\{[(u^2-w^2)U_{41}-uw(U_{11}+\sqrt{3}U_{21})][m_{1x}(u^2-u'^2)(u^2+w^2)+u(\sqrt{u^2+u'^2}\sqrt{u^2+w^2}(m_{3x}v+m_{4x}v')+2m_{2x}u'(u^2+w^2))]\}$, $x = 1, 2, 3, 4$ |

Assuming that masses of the exotic quarks (U, D_α) and both gaugino and Higgsino are larger than $M_{\zeta_4^\pm}$, we come to the fact that the hadron and sparticle modes are absent in the decay of the charged Higgs boson. Because the Yukawa couplings of $\zeta_4^\pm l^\mp \nu$ are very small, the coupling of a singly-charged Higgs boson (ζ_4^\pm) with the weak gauge bosons, $\zeta_4^\pm W^\mp Z$, can dominate. Note that the charged Higgs bosons in doublet models such as the two-Higgs doublet model or the minimal supersymmetric Standard Model, have both hadronic and leptonic modes [21]. This is a specific feature of the model under consideration. It is of particular importance for the electroweak symmetry breaking. Its magnitude is directly related to the structure of the extended Higgs sector under global symmetries [30]. This coupling can appear at the tree level in models with scalar triplets, while it is induced at the loop level in multi scalar doublet models. The coupling, in our model, differs from zero at the tree level due to the fact that the ζ_4^\pm belongs to a triplet.

Thus, for the charged Higgs boson ζ_4^\pm , it is important to study the couplings given by the interaction Lagrangian

$$\mathcal{L}_{\text{int}} = f_{ZW\zeta_4} \zeta_4^\pm W_\mu^\mp Z^\mu, \tag{66}$$

where $f_{ZW\zeta_4}$, at tree level, is given in Table 11. The same as in [20], the dominant rate is due to the diagram connected with the W and Z bosons. Putting necessary matrix elements in Table 11, we get

$$f_{ZW\zeta_4} = \frac{g^2 w^2 t_\theta}{2(1+t_\beta^2)} \frac{v^2(2t_\gamma^2 - t_\beta^2 + 1) + \frac{1+t_\beta^2}{t_\beta^2}(u^2+w^2)}{X} \times \frac{s_\varphi \sqrt{(4c_W^2 - 1)(1 + 4t_{2\theta}^2)} - c_\varphi}{\sqrt{c_W^2 + t_{2\theta}^2(4c_W^2 - 1)} \sqrt{1 + 4t_{2\theta}^2}},$$

where

$$X^2 = v^4 t_\gamma^2 V^2 + \frac{u^2+w^2}{t_\beta^2} v^4 (t_\beta^2 + t_\gamma^4 + 2t_\gamma^2 t_\beta^2 + 2t_\gamma^2) + \frac{(u^2+w^2)^2}{t_\beta^4} v^2 (t_\beta^4 + t_\gamma^2 + 2t_\gamma^2 t_\beta^2 + 2t_\beta^2) + \frac{(u^2+w^2)^3}{t_\beta^6} (1 + t_\beta^2).$$

Thus, the form factor, at the tree-level, is obtained by

$$\begin{aligned}
 F &\equiv \frac{f_Z w_{\zeta_4}}{g M_W} \\
 &= \frac{w^2 t_\theta}{V(1+t_\beta^2)} \frac{v^2(2t_\gamma^2 - t_\beta^2 + 1) + \frac{1+t_\beta^2}{t_\beta^2}(u^2 + w^2)}{X} \frac{s_\varphi \sqrt{(4c_W^2 - 1)(1 + 4t_{2\theta}^2) - c_\varphi}}{\sqrt{c_W^2 + t_{2\theta}^2(4c_W^2 - 1)}\sqrt{1 + 4t_{2\theta}^2}}. \quad (67)
 \end{aligned}$$

The decay width of $\zeta_4^\pm \rightarrow W_i^\pm Z_i$, where $i = L, T$ represent respectively the longitudinal and transverse polarizations, is given by [20]

$$\Gamma(\zeta_4^\pm \rightarrow W_i^\pm Z_i) = M_{\zeta_4^\pm} \frac{\lambda^{1/2}(1, w, z)}{16\pi} |M_{ii}|^2,$$

where $\lambda(1, w, z) = (1 - w - z)^2 - 4wz$, $w = M_W^2/M_{\zeta_4^\pm}^2$ and $z = M_Z^2/M_{\zeta_4^\pm}^2$. The longitudinal and transverse contributions are given in terms of F by

$$|M_{LL}|^2 = \frac{g^2}{4z}(1 - w - z)^2 |F|^2, \quad |M_{TT}|^2 = 2g^2 w |F|^2.$$

For the case of $M_{\zeta_4^\pm} \gg M_Z$, we have $|M_{TT}|^2/|M_{LL}|^2 \sim 8M_W^2 M_Z^2/M_{\zeta_4^\pm}^4$ which implies that the decay into a longitudinally polarized weak boson pair dominates that into a transversely polarized one.

Next, let us study the impact of the $\zeta_4^\pm W^\mp Z$ vertex on the production cross section of $pp \rightarrow W^{\pm*} Z^* X \rightarrow \zeta_4^\pm X$ which is a pure electroweak process with high p_T jets going into the forward and backward directions from the decay of the produced scalar boson without color flow in the central region. The hadronic cross section for $pp \rightarrow \zeta_4^\pm X$ via $W^\pm Z$ fusion is expressed in the effective vector boson approximation [31] by

$$\sigma_{\text{eff}}(s, M_{\zeta_4^\pm}^2) \simeq \frac{16\pi^2}{\lambda(1, w, z) M_{\zeta_4^\pm}^3} \sum_{\lambda=T,L} \Gamma(\zeta_4^\pm \rightarrow W_\lambda^\pm Z_\lambda) \tau \frac{d\mathcal{L}}{d\tau} \Big|_{pp/W_\lambda^\pm Z_\lambda},$$

where $\tau = M_{\zeta_4^\pm}^2/s$, and

$$\frac{d\mathcal{L}}{d\tau} \Big|_{pp/W_\lambda^\pm Z_\lambda} = \sum_{ij} \int_\tau^1 \frac{d\tau'}{\tau'} \int_{\tau'}^1 \frac{dx}{x} f_i(x) f_j(\tau'/x) \frac{d\mathcal{L}}{d\xi} \Big|_{q_i q_j / W_\lambda^\pm Z_\lambda},$$

with $\tau' = \hat{s}/s$ and $\xi = \tau/\tau'$. Here $f_i(x)$ is the parton structure function for the i th quark, and

$$\begin{aligned}
 \frac{d\mathcal{L}}{d\xi} \Big|_{q_i q_j / W_T^\pm Z_T} &= \frac{c}{64\pi^4} \frac{1}{\xi} \ln\left(\frac{\hat{s}}{M_W^2}\right) \ln\left(\frac{\hat{s}}{M_Z^2}\right) \times [(2 + \xi)^2 \ln(1/\xi) - 2(1 - \xi)(3 + \xi)], \\
 \frac{d\mathcal{L}}{d\xi} \Big|_{q_i q_j / W_L^\pm Z_L} &= \frac{c}{16\pi^4} \frac{1}{\xi} [(1 + \xi) \ln(1/\xi) + 2(\xi - 1)],
 \end{aligned}$$

where $c = \frac{g^4 c_\theta^2}{16c_W^2} [g_{1V}^2(q_j) + g_{1A}^2(q_j)]$ with $g_{1V}(q_j)$, $g_{1A}(q_j)$ for quark q_j are given in Table I of Ref. [10].

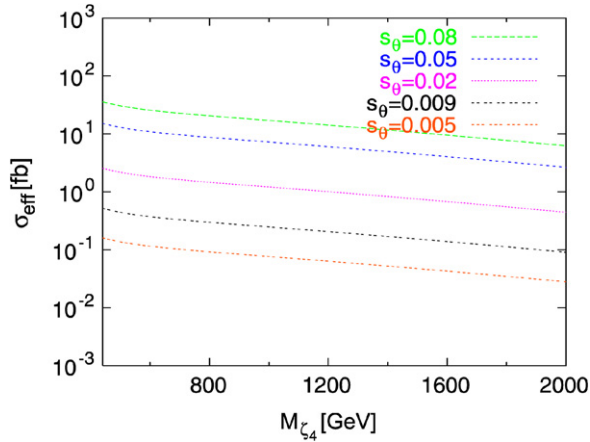


Fig. 1. Hadronic cross section for production of charged ζ_4^\pm via WZ fusion as a function of the charged Higgs boson mass for five cases of $\sin\theta$.

Using CTEQ6L [32], in Fig. 1, we plot $\sigma_{\text{eff}}(s, M_{\zeta_4^\pm}^2)$ at $\sqrt{s} = 14$ TeV as a function of M_{ζ_4} in range of 440–2000 GeV, where the parameters in the F factor are set as follows $v' = 0$, $v = 246$ GeV, $t_\beta = 1$, and t_φ obtained from Ref. [10]. If the mass of the charged Higgs boson is in range of 440 GeV and $s_\theta = 0.08$, the cross section can exceed 35.8 fb: i.e., 10 740 of ζ_4^\pm can be produced at the integrated LHC luminosity of 300 fb^{-1} .

6. Conclusions

In this paper we have explored the Higgs sector of the supersymmetric economical 3–3–1 model and found more new interesting features in this sector. We have revised the charged Higgs sector, i.e., the exact eigenvalues and states of the charged Higgs fields were obtained without any approximation. In this model, there are three Higgs bosons having masses equal to that of the gauge bosons and one neutral complex Higgs boson with mass of the neutral non-Hermitian bilepton X_0 . Therefore, as in the gauge sector, we get the law of Pythagoras among Higgs boson masses: $m_{\zeta_4^+}^2 = m_{H_X^0}^2 + m_{\varrho_1^+}^2$.

There is one scalar boson with mass of 91.4 GeV, which is closed to the Z boson mass and in good agreement with present limit: 89.8 GeV at 95% CL.

The mass matrix of charged Higgs bosons gives two physical fields ζ_2^+ and ζ_3^+ with their square mass are the same value but opposite sign. To solve this problem, we have got very interesting relation which leads to $w \simeq w'$, $u \simeq u'$ in high mass limit.

In the model under consideration, at the tree level, the lightest Higgs boson is the charged with the mass equal to those of the W boson. This is in agreement with the current experimental limit: 79.3 GeV at 95% CL.

It is worth mentioning that the Higgs sector in this model is very constrained. At the tree level, we cannot fix *only* one heavy scalar Higgs boson $\phi_{S_{a36}}$ with mass, while all remaining fields gain masses of the gauge bosons in the model. This is nice feature of the supersymmetric version.

The interactions among the Standard Model gauge bosons and scalar fields in the framework of the supersymmetric economical 3–3–1 model are also presented. From these couplings, all

scalar fields including the neutral scalar h and the Goldstone bosons can be identified and their couplings with the usual gauge bosons such as the photon, the charged W^\pm and the neutral Z , without any additional condition, are recovered.

Despite the mixing among the photon with the non-Hermitian neutral bilepton X^0 as well as with the Z and the Z' gauge bosons, the electromagnetic couplings remain unchanged.

After all we focused attention to the singly-charged Higgs boson ζ_4^\pm with mass equal to the bilepton mass M_Y . Mass of the ζ_4^\pm is estimated to be larger than 440 GeV. This boson, in difference with those arisen in the Higgs doublet models, does not have the hadronic and leptonic decay modes. The trilinear coupling $ZW^\pm\zeta_4^\mp$ which differs, at the tree level, while the similar coupling of the photon $\gamma W^\pm\zeta_4^\mp$ as expected, vanishes. If the mass of the above mentioned Higgs boson is in range of 440 GeV, however, the cross section can exceed 35.8 fb: i.e., 10 740 of ζ_4^\pm can be produced at the CERN LHC for the luminosity of 300 fb^{-1} . By measuring this process we can obtain useful information to determine the structure of the Higgs sector.

LEP II placed the problem of Higgs physics at the forefront of supersymmetry phenomenology. While earlier one might have viewed the Higgs fields as just one of many features of low energy supersymmetric models, the constraints on the Higgs mass are now problematic. In the model under consideration, the Higgs bosons gain masses equal to that of the gauge bosons. This feature deserves further studies.

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Appendix A. Mixing matrix for neutral scalars

$$\begin{pmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{pmatrix} = \begin{pmatrix} c_\beta s_\theta & -s_\beta c_\theta & -c_\beta c_\theta & -s_\alpha s_\beta s_\theta & -c_\alpha s_\beta s_\theta & 0 \\ c_\beta c_\theta & s_\beta s_\theta & c_\beta s_\theta & -s_\alpha s_\beta c_\theta & -c_\alpha s_\beta c_\theta & 0 \\ s_\beta s_\theta & -c_\beta c_\theta & s_\beta c_\theta & s_\alpha c_\beta s_\theta & c_\alpha c_\beta s_\theta & 0 \\ s_\beta c_\theta & c_\beta s_\theta & -s_\beta s_\theta & s_\alpha c_\beta c_\theta & c_\alpha c_\beta c_\theta & 0 \\ 0 & 0 & 0 & -c_\alpha c_\gamma & s_\alpha c_\gamma & s_\gamma \\ 0 & 0 & 0 & c_\alpha s_\gamma & -s_\alpha s_\gamma & c_\gamma \end{pmatrix} \begin{pmatrix} S'_{1a} \\ \varphi_{S_{24}} \\ \phi_{S_{24}} \\ \varphi_{S_{a36}} \\ \phi_{S_{a36}} \\ S'_5 \end{pmatrix}. \quad (\text{A.1})$$

Appendix B. Mixing matrix for neutral pseudoscalars

$$\begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} = \begin{pmatrix} s_\beta & 0 & c_\beta s_\theta & c_\beta c_\theta \\ 0 & s_\beta & c_\beta c_\theta & -c_\beta s_\theta \\ -c_\beta & 0 & s_\beta s_\theta & s_\beta c_\theta \\ 0 & -c_\beta & s_\beta c_\theta & -s_\beta s_\theta \end{pmatrix} \begin{pmatrix} A'_1 \\ A'_2 \\ \varphi_A \\ \phi_A \end{pmatrix}. \quad (\text{B.1})$$

Two massless physical fields are A_5 and A_6 .

Appendix C. Mixing matrix for charged scalars

$$\begin{pmatrix} \chi \\ \chi' \\ \rho_1 \\ \rho_2 \\ \rho'_1 \\ \rho'_2 \end{pmatrix} = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} & \mathcal{M}_{13} & \mathcal{M}_{14} & 0 & 0 \\ \mathcal{M}_{21} & \mathcal{M}_{22} & \mathcal{M}_{23} & \mathcal{M}_{24} & 0 & 0 \\ m_{31}s_\theta & m_{32}s_\theta & m_{33}s_\theta & m_{34}s_\theta & -c_\theta s_\gamma & -c_\theta c_\gamma \\ m_{31}c_\theta & m_{32}c_\theta & m_{33}c_\theta & m_{34}c_\theta & s_\theta s_\gamma & s_\theta c_\gamma \\ m_{41}s_\theta & m_{42}s_\theta & m_{43}s_\theta & m_{44}s_\theta & c_\theta c_\gamma & -c_\theta s_\gamma \\ m_{41}c_\theta & m_{42}c_\theta & m_{43}c_\theta & m_{44}c_\theta & -s_\theta c_\gamma & s_\theta s_\gamma \end{pmatrix} \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \\ \varrho_1 \\ \varrho_2 \end{pmatrix}, \tag{C.1}$$

where

$$\begin{aligned} \mathcal{M}_{11} &= m_{11}s_\beta + m_{21}c_\beta, & \mathcal{M}_{12} &= m_{12}s_\beta + m_{22}c_\beta, \\ \mathcal{M}_{13} &= m_{13}s_\beta + m_{23}c_\beta, & \mathcal{M}_{14} &= m_{14}s_\beta + m_{24}c_\beta, \\ \mathcal{M}_{21} &= m_{21}s_\beta - m_{11}c_\beta, & \mathcal{M}_{22} &= m_{22}s_\beta - m_{12}c_\beta, \\ \mathcal{M}_{23} &= m_{23}s_\beta - m_{13}c_\beta, & \mathcal{M}_{24} &= m_{24}s_\beta - m_{14}c_\beta \end{aligned}$$

and

$$\begin{aligned} m_{11} &= \frac{g\sqrt{(t_\beta^2 + 1)(u'^2 + w'^2)}}{2\sqrt{m_{\zeta_4}^2}}, & m_{12} &= \frac{1}{\sqrt{u'^2 + w'^2 + v^2}} \frac{v}{\sqrt{1 + t_\beta^2}}, \\ m_{13} &= \frac{1}{\sqrt{u^2 + v'^2 + w^2}} \frac{v't_\beta}{\sqrt{1 + t_\beta^2}}, & m_{14} &= \frac{k_1}{\sqrt{k_1^2 + k_2^2 + k_3^2 + 1}}, \\ m_{21} &= 0 = m_{33} = m_{42}, \\ m_{22} &= \frac{1}{\sqrt{u'^2 + w'^2 + v^2}} \frac{v'}{\sqrt{1 + t_\beta^2}}, & m_{23} &= \frac{1}{\sqrt{u^2 + v'^2 + w^2}} \frac{-v'}{\sqrt{1 + t_\beta^2}}, \\ m_{24} &= \frac{k_2}{\sqrt{k_1^2 + k_2^2 + k_3^2 + 1}}, & m_{31} &= \frac{-gv}{2\sqrt{m_{\zeta_4}^2}}, \\ m_{32} &= \frac{\sqrt{u'^2 + w'^2}}{\sqrt{u'^2 + w'^2 + v^2}}, & m_{34} &= \frac{k_3}{\sqrt{k_1^2 + k_2^2 + k_3^2 + 1}}, \\ m_{41} &= \frac{-gv'}{2\sqrt{m_{\zeta_4}^2}}, & m_{43} &= \frac{\sqrt{u^2 + w^2}}{\sqrt{v'^2 + w^2 + u^2}}, & m_{44} &= \frac{1}{\sqrt{k_1^2 + k_2^2 + k_3^2 + 1}}. \end{aligned}$$

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