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On the Generative Capacity of Conditional Grammars

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A conditional grammar is a Chomsky grammar with languages associated to its rules such that each rule is applicable only to words in the corresponding language. In this paper the generative capacity of type 0, 1, 2, 2 — λ, 3 grammars with associated type 0, 1, 2, 3 languages will be characterized in terms of the Chomsky hierarchy. We shall prove that the generative capacity of context-free and of regular grammars is increased in this way, while for type-0 and type-1 grammars the generative capacity is not modified. Two other variants of these grammars are shown to be equivalent with them.

In recent years, many generalizations of the Chomsky grammars have been introduced that consider various control devices of the use of the rewriting rules. In this way we obtain intermediate families of languages, starting generally from context-free grammars. [Most often the generative capacity of type-0, type-1 and of regular grammars is not modified (see Salomaa, 1973).]

Such a control device is one that associates a language to each rule of a grammar and allows the use of a rule only to rewrite a word in the language associated to it. We call these *conditional grammar*. Conditional grammars with regular languages associated to rules were introduced by Friś (1968). Salomaa (1973) has proved that any type-0 language can be generated by a context-free grammar with regular restrictions, whereas if only λ-free context-free grammars are used, we obtain the family of context-sensitive languages.

In this paper we investigate the generative capacity of conditional grammars of any type in the Chomsky hierarchy. Twenty families of languages are obtained in this way. The generative capacity of type-0 and type-1 grammars is not modified by such a restriction, while that of context-free and regular grammars is increased. Finally we prove that the conditional grammars of Navratil (1970) and of Král (1970) are equivalent as to generative capacity to the conditional grammars discussed here.

CONDITIONAL GRAMMARS

In this paper we use the terminology and results of Salomaa (1973). We denote a Chomsky grammar by $G = (V_N, V_T, S, P)$, where V_N is the nonterminal

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vocabulary, V_T is the terminal vocabulary, $S \in V_N$ is the start symbol of the grammar, and P is the set of rewriting rules. According to the form of its rules, a grammar is said to be of type 0, 1, 2, $2 - \lambda$, 3 (type-0, length-increasing, context-free, λ-free context-free, and regular, respectively). We use the notation $0 < 1 < 2 < 2 - \lambda < 3$. The four families of the Chomsky hierarchy are denoted by $\mathscr{L}_i$, $i = 0, 1, 2, 3$.

DEFINITION 1. A conditional grammar of type (i, j), $i \in \{0, 1, 2, 2 - \lambda, 3\}$, $j \in \{0, 1, 2, 3\}$, is a pair (G, ρ), where $G = (V_N, V_T, S, P)$ is a type-i grammar and ρ is a mapping of P into the family of type-j languages over $V_G = V_N \cup V_T$. For a vocabulary V we denote by V^* the free monoid generated by V under the operation of concatenation and the null element λ. For $x, y \in V_G^*$ we write $x \Rightarrow y$ iff $x = x_1x_2x_3$, $y = x_1x_4x_3$, $x_2 \rightarrow x_4 \in P$, and $x \in \rho(x_2 \rightarrow x_4)$. If $\overset{*}{\Rightarrow}$ is the reflexive transitive closure of $\Rightarrow$, then the language generated by (G, ρ) is

$$L(G, \rho) = \{x \in V_T^* \mid S \overset{*}{\Rightarrow} x\}.$$

We denote by $\mathscr{C}(i, j)$ the family of languages generated by conditional grammars of type (i, j), $i \in \{0, 1, 2, 2 - \lambda, 3\}$, $j \in \{0, 1, 2, 3\}$.

THE GENERATIVE CAPACITY OF CONDITIONAL GRAMMARS

Theorem 7.3, page 190 in Salomaa (1973) shows that $\mathscr{C}(1, 2) \subseteq \mathscr{L}_1$ and $\mathscr{C}(2 - \lambda, 3) = \mathscr{L}_1$. (We use $\subseteq$ for inclusion and $\subset$ for strict inclusion.) Moreover, $\mathscr{C}(2, 3) = \mathscr{L}_0$. From Church's thesis it follows that for any i, j we have $\mathscr{C}(i, j) \subseteq \mathscr{L}_0$. On the other hand, since any type-i grammar can be considered a conditional grammar with $\rho(r) = V_G^*$ for each rule r, we have $\mathscr{L}_i \subseteq \mathscr{C}(i, 3)$, for any $i \in \{0, 1, 2, 3\}$. Moreover, the following inclusions are obvious

$$\begin{aligned} &\mathscr{C}(i, j) \subseteq \mathscr{C}(i', j), &&\text{for } i > i' \text{ but } (i, i') \neq (2, 1), \\ &\mathscr{C}(i, j) \subseteq \mathscr{C}(i, j'), &&\text{for } j > j'. \end{aligned}$$

Therefore we have

$$\begin{aligned} \mathscr{L}_0 &= \mathscr{C}(0, 0) = \mathscr{C}(0, 1) = \mathscr{C}(0, 2) = \mathscr{C}(0, 3) = \mathscr{C}(2, 3) = \mathscr{C}(2, 2) \\ &= \mathscr{C}(2, 1) = \mathscr{C}(2, 0), \\ \mathscr{L}_1 &= \mathscr{C}(2 - \lambda, 3) = \mathscr{C}(1, 3). \end{aligned}$$

LEMMA 1. *If $L_1, L_2 \in \mathscr{L}_i$, then $L_1 \cap L_2 \in \mathscr{C}(3, i)$ for all $i \in \{0, 1, 2, 3\}$.*

Proof. Let $V = \{a_1, \ldots, a_n\}$ be such that $L_1, L_2 \subseteq V^*$, $L_1, L_2 \in \mathscr{L}_i$, $i \in \{0, 1, 2, 3\}$. We construct the conditional grammar (G, ρ), $G = (\{S\} \cup$

$\{T_i \mid i = 1, 2,..., n\}$, V, S, P), where P contains the following rules (for each rule $r \in P$ we give the language $\rho(r)$ too):

$$r_i: S \to a_i S,\ \rho(r_i) = V^*\{S\}, \qquad i = 1, 2,..., n, \tag{1}$$

$$r_{ij}: S \to a_i T_j,\quad \rho(r_{ij}) = \partial^r_{a_i a_j}(L_1)\{S\}, \qquad i, j = 1,..., n, \tag{2}$$

($\partial_x{}^r(L)$ denotes the right derivative of L with respect to the string x, that is the set $\{y \in V^* \mid yx \in L\}$).

$$r_i': T_i \to a_i,\ \rho(r_i') = \partial^r_{a_i}(L_2)\{T_i\}, \qquad i = 1, 2,..., n, \tag{3}$$

$$r_i'': S \to a_i,\ \rho(r_i'') = \partial^r_{a_i}(L_1 \cap L_2 \cap V)\{S\}, \qquad i = 1,..., n, \tag{4}$$

$$r_0: S \to \lambda,\ \rho(r_0) = (L_1 \cap L_2 \cap \{\lambda\})\{S\}. \tag{5}$$

Clearly, G is a regular grammar and $\rho(r) \in \mathscr{L}_i$ for any $r \in P$. (All the families $\mathscr{L}_i$, $i = 0, 1, 2, 3$, are closed under right derivative.) It is easy to see that $L(G, \rho) = L_1 \cap L_2$ and the lemma is proved.

The above grammar G does not depend on the languages L_1, L_2 but only on the vocabulary V.

THEOREM 1. *For any* $i \in \{0, 1, 2, 3\}$ *we have* $\mathscr{L}_i \subseteq \mathscr{C}(3, i)$. (*The assertion follows by taking* $L_1 = V^*$, $L_2 \in \mathscr{L}_i$ *in the above lemma.*)

COROLLARY. $\mathscr{L}_0 = \mathscr{C}(3, 0) = \mathscr{C}(2 - \lambda, 0) = \mathscr{C}(2, 0) = \mathscr{C}(1, 0)$.

THEOREM 2. *For any* $L \in \mathscr{L}_0$ *there are a homomorphism* h *and a language* $L' \in \mathscr{C}(3, 2)$ *such that* $L = h(L')$.

Proof. Any type-0 language L can be written in the form $L = h(L_1 \cap L_2)$, where L_1, L_2 are context-free languages and h is a homomorphism (Salomaa, 1973). Using Lemma 1 we obtain the theorem.

In view of Theorems 3 and 5 presented below, this is a stronger result than Theorem 9.10 on page 90 in Salomaa (1973), which says that any type-0 language is the homomorphic image of a type-1 language.

THEOREM 3. $\mathscr{L}_1 = \mathscr{C}(1, 1)$.

Proof. The inclusion $\mathscr{L}_1 \subseteq \mathscr{C}(1, 1)$ is obvious.

Let us consider a length-increasing grammar $G = (V_N, V_T, S, P)$ and let $\rho: P \to \mathscr{P}(V_G^*)$ be such that $\rho(r) \in \mathscr{L}_1$ for any $r \in P$. Assume that $P = \{r_1,..., r_m\}$ and let r_i be of the form $r_i: z_1 \cdots z_{n_i} \to z_1' \cdots z_{t_i}'$, whereas $\rho(r_i) = L_i = L(G_i)$, with $G_i = (V_N{}^i, V_G, S^i, P^i)$, $i = 1,..., m$. We suppose that V_N, $V_N{}^i$ are pairwise disjoint vocabularies and we construct the grammar $G' = (V_N', V_T, S', P')$, where

$$V_N = (V_N \cup V_T) \times (W \cup \{b\}) \cup \{S, B, X, Y, Z\} \cup \{Y_i, Y_i' \mid i = 1,..., m\}$$
$$\cup \{Z_{i,j} \mid j = 1,..., n_i, i = 1,..., m\}$$

with $W = V_T \cup \bigcup_{i=1}^{m} V_N{}^i$, ($b$, S, B, X, Y, Z, X_i, Y_i, $Z_{i,j}$ new symbols) and P' contains the following rules (each group of rules is followed by informal explanations):

$$S' \to BX(S, b)B. \tag{1}$$

(The derivation begins by introducing the end markers B and the nonterminals X and (S, b).)

$$X(z, b) \to Y_i(z, S^i),\ z \in V_N \cup V_T, \qquad i = 1,..., m.$$

(The nonterminal Y_i was introduced in order to determine a derivation in the grammar G_i of the second components of the symbols in the current sequence.)

$$(z_1, b)(z_2, z) \to (z_1, z)(z_2, b), \tag{3}$$
$$(z_1, z)(z_2, b) \to (z_1, b)(z_2, z), \qquad z_1, z_2 \in V_N \cup V_T, z \in W.$$

(The "blank" symbol b is moved to the right or to the left.)

$$(x_1, b) \cdots (x_t, b)(x_{t+1}, z_1) \cdots (t_{t+r}, z_r) \to (x_1, z_1') \cdots (x_{t+r}, z_{t+r}'), \tag{4}$$

for each rule of G_i of the form

$$z_1 \cdots z_r \to z_1' \cdots z_{t+r}', \qquad i = 1,..., m.$$

(A rule of G_i is simulated on the second components of the symbols in the current sequence.)

$$Y_i(z, z) \to (z, b)Y_i,\ z \in V_N \cup V_T, \qquad i = 1,..., m. \tag{5}$$

(The nonterminal Y_i checks whether the first components' string is or is not identical to the second components' string.)

$$Y_iB \to Y_i'B, \qquad i = 1,..., m. \tag{6}$$

(When the two strings are equal, the nonterminal Y_i is replaced by Y_i'.)

$$(z, b)Y_i' \to Y_i'(z, b),\ z \in V_N \cup V_T, \qquad i = 1,..., m. \tag{7}$$

(The nonterminal Y_i' freely goes to the left.)

$$(z_{n_i}, b)Y_i' \to Z_{i,n_i}, \tag{8}$$

$$(z_k, b)Z_{i,k+1} \to Z_{i,k}, \qquad k = 1,\ldots, n_i - 1,$$

$$Z_{i,1} \to Y(z_1', b) \cdots (z_{t_i}', b) \qquad \text{for each rule } r_i : z_1 \cdots z_{n_i},$$

$$\to z_1' \cdots z_{t_i}', r_i \in P, \qquad i = 1,\ldots, m.$$

(The rule r_i is used to derive the first components of the string.)

$$\begin{aligned} &(z, b)Y \to Y(z, b), \qquad z \in V_N \cup V_T, \\ &BY \to BX. \end{aligned} \tag{9}$$

(The nonterminal Y is replaced by X in order to begin a new derivation.)

$$BX \to Z. \tag{10}$$

(The symbol Z will determine the end of the derivation.)

$$Z(a, b) \to aZ, a \in V_T. \tag{11}$$

(Moving to the right, the nonterminal Z transforms the nonterminals from $V_T \times \{b\}$ into terminals.)

$$ZB \to \lambda. \tag{12}$$

(The derivation ends.)

Therefore, before applying a rule of G to the first components of the symbols, we have to check whether the string belongs to the language associated to this rule; only when the answer is positive is the derivation allowed. Consequently, we have $L(G, \rho) = L(G')$.

On the other hand, we have $\mathrm{WS}(x) = |x| + 3$, for any $x \in L(G')$. (WS denotes the work-space and $|x|$ is the length of x.) According to the work-space theorem of Salomaa (1973) it follows that $L(G') \in \mathscr{L}_1$ and the theorem is proved.

COROLLARY. $\mathscr{L}_1 = \mathscr{C}(1, 1) = \mathscr{C}(1, 2) = \mathscr{C}(1, 3) = \mathscr{C}(2 - \lambda, 1) = \mathscr{C}(2 - \lambda, 2) = \mathscr{C}(2 - \lambda, 3) = \mathscr{C}(3, 1)$.

We still have to investigate the families $\mathscr{C}(3, 2)$ and $\mathscr{C}(3, 3)$.

THEOREM 4. $\mathscr{L}_3 = \mathscr{C}(3, 3)$.

Proof. The inclusion $\subseteq$ is trivial.

Let (G, ρ) be a conditional grammar with $G = (V_N, V_T, S, P)$, $\rho(r) \in \mathscr{L}_3$ for each $r \in P$ and $P = \{r_1, ..., r_n\}$. For a rule r_i of the form $A \to z$ we denote $\text{Left}(r_i) = A$.

Taking $\rho'(r_i) = \rho(r_i) \cap V_T^* \text{ Left}(r_i)$ for each $r_i \in P$ we obtain $\rho'(r_i) \in \mathscr{L}_3$ and $L(G, \rho) = L(G, \rho')$. Let $W = \{N_1, ..., N_n\}$, where N_i is a new symbol associated to the rule r_i. We define the finite substitution s: $V_T^* \to \mathscr{P}((V_T \cup V_N \cup W)^*)$, by $s(a) = \{\text{Left}(r_i)N_i a \mid i = 1, ..., n\}$, for any $a \in V_T$.

Let us now consider the grammar $G' = (V_N', V_T', S', P')$ with

$$V_N' = V_N \times W \cup \{S'\}, \qquad S' \text{ is a new symbol,}$$

$$V_T' = V_N \cup V_T \cup W,$$

$$\begin{aligned} P' &= \{(X, N_i) \to XN_i a(Y, N_j) \mid r_i: X \to aY \in P, \\ Y &= \text{Left}(r_j),\ i, j \in \{1, 2, ..., n\}\} \cup \{(X, N_i) \to XN_i a \mid r_i: X \to a \in P, \\ i &= 1, ..., n\} \cup \{(S' \to (S, N_i) \mid \text{Left}(r_i) = S\}. \end{aligned}$$

Clearly, each string in $L(G')$ is of the form

$$SN_{i_1}a_{i_1}X_{i_2}N_{i_2}a_{i_2} \cdots a_{i_k}X_{i_{k+1}}N_{i_{k+1}}a_{i_{k+1}},$$

with

$$a_{i_1} \cdots a_{i_{k+1}} \in L(G) \qquad \text{and} \qquad r_{i_1}: S \to a_{i_1}X_{i_2} \in P,\ r_{i_j}: X_{i_j} \to a_{i_j}X_{i_{j+1}} \in P,$$

$$j = 2, ..., k,\ r_{i_{k+1}}: X_{i_{k+1}} \to a_{i_{k+1}} \in P.$$

For any $r_i \in P$, r_i: $X_i \to a_i\alpha$, $\alpha \in V_N \cup \{\lambda\}$, we consider the language

$$M_i = (V_T'^* - s(\partial_{X_i}^r(\rho'(r_i))))\{X_i N_i a_i\}\, s(V_T^*) \cap L(G').$$

In plain words, M_i contains all the strings in $L(G')$ having a prefix xX_iN_i such that $x \notin s(\partial_{X_i}^r(\rho'(r_i)))$. As x is a prefix of a string in $L(G')$, it follows that the string of symbols in V_T which occur in x does not belong to $\partial_{X_i}^r(\rho'(r_i))$.

Now, let us consider the language

$$M = \bigcap_{k=1}^{n} ((V_T'^* - M_k) \cap L(G')).$$

Let $z \in M$. As $z \in (V_T'^* - M_k) \cap L(G')$ for any k, it follows that any prefix of z of the form $x \text{ Left}(r_k)N_k$ is in $s(\partial_{\text{Left}(r_k)}^r(\rho'(r_k)) \text{ Left}(r_k)N_k$. Therefore, the derivation of z in the grammar G' corresponds to a correct derivation in G according to ρ'. Let h be the homomorphism which erases all the symbols in $V_N \cup W$. We have $h(z) \in L(G, \rho')$, hence $h(M) \subseteq L(G, \rho')$. As $\lambda \notin h(M)$, we have $h(M) \subseteq L(G, \rho') - \{\lambda\}$.

Conversely, let $x \in L(G, \rho') - \{\lambda\}$ and let us consider a derivation of x in (G, ρ'). We can introduce the symbols occuring on the left-hand sides of the rules together with the symbols N_i associated to these rules in such a way that a string $\alpha \in L(G')$ is obtained, corresponding to x. Clearly, $\alpha \in (V_T'^* - M_k)$ for any k, hence $\alpha \in M$. As $h(\alpha) = x$, we have $L(G, \rho') - \{\lambda\} \subseteq h(M)$.

In conclusion, $L(G, \rho') - \{\lambda\} = h(M)$. But, $L(G')$, V_T^*, and $\rho'(r_k)$ are regular languages and $\mathscr{L}_3$ is closed under concatenation, intersection, complementation, arbitrary homomorphisms, right derivative, and substitution (Salomaa, 1973). Therefore, $M \in \mathscr{L}_3$ so the inclusion $\mathscr{C}(3, 3) \subseteq \mathscr{L}_3$ is proved too.

THEOREM 5. *The following strict inclusions hold*

$$\mathscr{L}_2 \subset \mathscr{C}(3, 2) \subset \mathscr{L}_1 .$$

Proof. The inclusions follow from the above considerations. To see that $\mathscr{L}_2$ is properly included in $\mathscr{C}(3, 2)$, let us consider the grammar

$$G = (\{S\}, \{a, b, c\}, S, \{S \to aS,\ S \to bS,\ S \to cS,\ S \to c\})$$

and let us define the mapping ρ by

$$\begin{aligned}
\rho(S \to aS) &= \{a^iS \mid i \geqslant 0\} = L_1 ,\\
\rho(S \to bS) &= \{a^ib^jS \mid i > j \geqslant 0\} = L_2 ,\\
\rho(S \to cS) &= \{a^ib^ic^jS \mid i \geqslant 0, j \geqslant 0\} = L_3 ,\\
\rho(S \to c) &= \{a^ib^jc^{j-1}S \mid i \geqslant 0, j \geqslant 2\} = L_4 .
\end{aligned}$$

Obviously, the languages L_1, L_2, L_3, L_4 are context-free, hence $L(G, \rho) \in \mathscr{C}(3, 2)$. But, note that $L(G, \rho) = \{a^nb^nc^n \mid n \geqslant 2\}$ is not a context-free language.

To prove the proper inclusion $\mathscr{C}(3, 2) \subset \mathscr{L}_1$ we shall use the following lemma, which is a generalization of the similar result known for context-free languages (Salomaa, 1973).

LEMMA 2. *Any one-letter language in $\mathscr{C}(3, 2)$ is regular.*

Proof. Let us consider a conditional grammar (G, ρ) with $G = (V_N, \{a\}, S, P)$ and ρ: $P \to \mathscr{P}((V_N \cup \{a\})^*) \cap \mathscr{L}_2$. Taking a mapping ρ' defined by $\rho'(r) = \rho(r) \cap \{a\}^* V_N$ we obtain $L(G, \rho) = L(G, \rho')$ Clearly, $\rho'(r) \in \mathscr{L}_3$ for any $r \in P$, hence $L(G, \rho) \in \mathscr{C}(3, 3)$. From the above theorem it follows that $L(G, \rho) \in \mathscr{L}_3$.

There are context-sensitive languages in the vocabulary with only one element which are not regular; hence the inclusion $\mathscr{C}(3, 2) \subset \mathscr{L}_1$ is proper.

Table I summarizes the above results (in the ith row and jth column we have the family $\mathscr{C}(i, j)$).

Conjecture. It seems to us that any language in $\mathscr{C}(3, 2)$ can be written as a finite intersection of context-free languages. If this conjecture is proved, then Theorem 5 and Lemma 2 will be obtained as direct consequences. Moreover,

TABLE I

i	j: 0	1	2	3
0	$\mathscr{L}_0$	$\mathscr{L}_0$	$\mathscr{L}_0$	$\mathscr{L}_0$
1	$\mathscr{L}_0$	$\mathscr{L}_0$	$\mathscr{L}_1$	$\mathscr{L}_1$
2	$\mathscr{L}_0$	$\mathscr{L}_1$	$\mathscr{L}_0$	$\mathscr{L}_0$
$2-\lambda$	$\mathscr{L}_0$	$\mathscr{L}_1$	$\mathscr{L}_1$	$\mathscr{L}_1$
3	$\mathscr{L}_0$	$\mathscr{L}_1$	$\mathscr{L}_2 \subset \mathscr{C}(3, 2) \subset \mathscr{L}_1$	$\mathscr{L}_3$

it will follow that there are matrix languages (even equal matrix languages (Siromoney, 1969)) which are not in $\mathscr{C}(3, 2)$. One such language will be $\{wcw \mid w \in \{a, b\}^*\}$. (This language cannot be written as a finite intersection of context-free languages (Păun, 1978).)

Two Variants of Conditional Grammars

Navratil (1970) introduced another kind of conditional grammars, namely,

Definition 2. A *2-conditional grammar* of type (i, j), $i \in \{0, 1, 2, 2-\lambda, 3\}$, $j \in \{0, 1, 2, 3\}$, is a triple (G, ρ_1, ρ_2), where $G = (V_N, V_T, S, P)$ is a type-i grammar and ρ_1, ρ_2 are mappings of P into the family of type-j languages over V_G.

For $x, y \in V_G^*$ we write $x \Rightarrow y$ iff $x = x_1x_2x_3$, $y = x_1x_4x_3$, $x_2 \to x_4 \in P$, and $x_1 \in \rho_1(x_2 \to x_4)$, $x_3 \in \rho_2(x_2 \to x_4)$. The language $L(G, \rho_1, \rho_2)$ is defined in the usual way. Let us denote by $\mathscr{C}_2(i, j)$ the family of languages generated by 2-conditional grammars of type (i, j).

Navratil (1970) proved that $\mathscr{L}_0 = \mathscr{C}_2(2, 3)$ and $\mathscr{L}_1 = \mathscr{C}_2(2-\lambda, 3)$. It follows that $\mathscr{L}_0 = \mathscr{C}(2, i) = \mathscr{C}(0, i) = \mathscr{C}_2(0, i) = \mathscr{C}_2(2, i)$, $i = 0, 1, 2, 3$, and $\mathscr{L}_1 = \mathscr{C}(2-\lambda, i) = \mathscr{C}_2(2-\lambda, i)$, $i = 1, 2, 3$. As in Theorem 3 we can prove that $\mathscr{C}_2(1, 1) = \mathscr{L}_1$, hence $\mathscr{L}_1 = \mathscr{C}(1, i) = \mathscr{C}_2(1, i)$ too, $i = 1, 2, 3$.

Consider now a conditional grammar (G, ρ) with G regular. We construct the 2-conditional grammar (G, ρ_1, ρ_2) with $\rho_1(r) = \partial_X{}^r(\rho(r))$, $\rho_2(r) = \{\lambda\}$ for any $r: X \to z$ in P. Clearly, $L(G, \rho) = L(G, \rho_1, \rho_2)$, hence $\mathscr{C}(3, i) \subseteq \mathscr{C}_2(3, i)$, $i = 0, 1, 2, 3$. On the other hand, if (G, ρ_1, ρ_2) is a 2-conditional grammar with G regular, we have $L(G, \rho_1, \rho_2) = L(G, \rho_1')$, where for any rule $r: X \to z$ we put $\rho_1'(r) = \rho_1(r)\{X\}$. Consequently, we have

Theorem 6. *$\mathscr{C}(i, j) = \mathscr{C}_2(i, j)$ for any $i = 0, 1, 2, 2-\lambda, 3$, $j = 0, 1, 2, 3$.*

Král (1970) considered another class of conditional grammars, namely, DEFINITION 3. A *weakly conditional grammar* of type (i, j) is a pair (G, M), where G is a type-i grammar and M is a type-j language over V_G, $i = 0, 1, 2, 2 - \lambda, 3, j = 0, 1, 2, 3$.

For $x, y \in V_G^*$ we define the relation $x \Rightarrow y$ iff $x \Rightarrow y$ in the grammar G and $x \in M$. (In plain words, a weakly conditional grammar is a conditional grammar with $\rho(r) = \rho(r')$ for any two rules r, r'.) The language $L(G, M)$ is defined in the usual way and the family of weakly conditional languages of type (i, j) is denoted by $\mathscr{C}'(i, j)$.

Clearly, $\mathscr{C}'(i, j) \subseteq \mathscr{C}(i, j)$ for any i, j. In fact, we have

THEOREM 7. *$\mathscr{C}'(i, j) = \mathscr{C}(i, j)$ for any $i = 0, 1, 2, 2 - \lambda, 3, j = 0, 1, 2, 3$.*

Proof. Král (1970) proved that $\mathscr{C}'(2 - \lambda, 3) = \mathscr{L}_1$. In the same way we can prove that $\mathscr{C}'(2, 3) = \mathscr{L}_0$. It follows that $\mathscr{L}_0 = \mathscr{C}'(2, i) = \mathscr{C}'(0, i) = \mathscr{C}(0, i) = \mathscr{C}(2, i)$, $i = 0, 1, 2, 3$. As $\mathscr{C}'(i, j) \subseteq \mathscr{C}(i, j)$, it also follows that $\mathscr{C}'(1, i) = \mathscr{C}'(2 - \lambda, i) = \mathscr{C}(1, i) = \mathscr{C}(2 - \lambda, i) = \mathscr{L}_1$, $i = 1, 2, 3$.

Moreover, we have $\mathscr{C}'(3, i) = \mathscr{C}(3, i)$, $i = 0, 1, 2, 3$. Indeed, let (G, ρ) be a conditional grammar with $G = (V_N, V_T, S, P)$ regular and $\rho(r) \in \mathscr{L}_i$ for any $r \in P$. We shall modify the rules in P in the following way: If $r: X \to aY$, $q: Y \to z$ are in P, then we introduce the rule $r(q)$: $(X, r) \to a(Y, q)$. If $r: X \to a$ is in P, then we introduce r': $(X, r) \to a$. We also introduce a rule $S' \to x$ for each rule $(S, r) \to x$ obtained as above. Let $G' = (V_N', V_T, S', P')$ be the grammar obtained in this way. ($V_N' = \{S'\} \cup \{(X, r) \mid r \in P, X \in V_N\}$). Moreover, if $r: X \to z$, then we replace the occurrences of X in the strings of $\rho(r)$ by (X, r); let $M(r)$ be the new language. There is a one-to-one correspondence between the nonterminals in $V_N' - \{S'\}$ and the languages $M(r)$, therefore, taking $M = \bigcup_{r \in P} M(r) \cup \{S'\}$, we obviously have the equation $L(G, \rho) = L(G, M)$. As $M \in \mathscr{L}_i$ for $\rho(r) \in \mathscr{L}_i$, $i = 0, 1, 2, 3$, the theorem is proved.

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