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Characterizations of k-copwin graphs

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ABSTRACT

We give two characterizations of the graphs on which k cops have a winning strategy in the game of Cops and Robber. One of these is in terms of an order relation, and one is in terms of a vertex ordering. Both generalize characterizations known for the case $k = 1$.

1. Introduction

The perfect-information pursuit game "Cops and Robber" was introduced independently by Quilliot [14] in 1978, and Nowakowski and Winkler in 1983 [13]. It is played on a reflexive graph by two sides: a set of $k > 0$ cops and a single robber. The cops begin the game by each choosing a vertex to occupy, and the robber then chooses a vertex. The two sides move alternately. A move for the cops consists of each cop traversing an edge to a neighbouring vertex. A move for the robber is defined analogously. Since the graph is reflexive, a loop can be used by a player to pass. The cops win if any cop occupies the same vertex as the robber after a finite number of moves (some cop *catches* the robber), and otherwise the robber wins.

Graphs on which the k cops have a winning strategy are called *k-copwin graphs*. The minimum number of cops that suffice to win on a graph G is the *copnumber* of G. The 1-copwin graphs were characterized by Quilliot [14], and Nowakowski and Winkler [13]. They also arise in the study of convexity in graphs [1,5]. No characterization of k-copwin graphs, for any $k > 1$, has appeared. In this paper we provide several characterizations of these graphs. Most of our results hold for variations of the game and some of them hold for infinite graphs. These extensions are discussed in Section Four where we make particular note of the elimination order characterization of *tandem-win* finite reflexive graphs [11]. Our main contributions are extending the relational characterization due to Nowkowski and Winkler to the k-cop case, and using it to obtain a characterization of k-copwin graphs in terms of an ordering of the vertices of the $(k + 1)$-fold categorical product of such a graph with itself. The relational characterization also implies simple polynomial-time algorithms to decide if, for fixed k, a given graph is k-copwin. These improve previous algorithms due to Berarducci and Intriglia [2], and Hahn and MacGillivray [12], and slightly improve the one found by Bonato et al. [3].

We now introduce the basic definitions and results, starting with the structural characterization of finite reflexive 1-copwin graphs [13,14] (see also [4,5]): *A finite, reflexive graph G is 1-copwin if and only if there exists an enumeration $(v_1, v_2, \ldots, v_n)$ of the vertices of G such that, for $i = 1, 2, \ldots, n-1$, there exists $j_i > i$ such that $(N_G(v_i) \cap \{v_i, v_{i+1}, \ldots, v_n\}) \subseteq N_G(v_{j_i})$.* This enumeration of the vertices of G has come to be known as a *copwin ordering* or *dismantling ordering* of G. A graph G is called *dismantlable* if it has such an ordering.

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Although a graph with a copwin ordering is 1-copwin, the cop's strategy may not be immediately clear. A description of a cop strategy can be obtained by using retractions.

Let G be a graph and H be a fixed subgraph of G. A *retraction of G to H* is a homomorphism of G to H that maps H identically to itself. Formally, it is a function $f : V(G) \to V(H)$ such that: (1) if $xy \in E(G)$ then $f(x)f(y) \in E(H)$; and (2) $f(h) = h$, for all vertices h of H. If there exists a retraction of G to H, then H is called a *retract* of G. A *one-point retraction of G* is a retraction of G to $G - x$, for some vertex x. Let G be a reflexive graph. Notice that if $N_G(x) \subseteq N_G(y)$, then the mapping that sends x to y and every vertex of $G - x$ to itself is a one-point retraction of G (to $G - x$).

Let G be a finite, reflexive 1-copwin graph with copwin ordering $(v_1, v_2, \ldots, v_n)$. For $i = 1, 2, \ldots, n-1$, there is a one-point retraction f_i of the graph $G - \{v_1, v_2, \ldots, v_{i-1}\}$ to $G - \{v_1, v_2, \ldots, v_i\}$ that maps v_i to a vertex v_{j_i} such that $(N_G(v_i) - \{v_1, v_2, \ldots, v_i\}) \subseteq N_G(v_{j_i})$. (If there is more than one candidate for v_{j_i}, it does not matter which one is chosen.) On his ith move, the cop plays on $f_{n-i} \circ \cdots \circ f_2 \circ f_1(G) = G - \{v_1, v_2, \ldots, v_{n-i}\}$. No matter where the robber is located on G, his *shadow* (i.e. image under the mapping $f_{n-i} \circ \cdots \circ f_2 \circ f_1$) is located on one of the vertices of this graph. Since the cop is "on" the robber's shadow on his first move, and the one-point retractions allow him to stay on it in each subsequent move, the cop is guaranteed to catch the robber after at most n moves [6,9]. A particular sequence of one-point retractions can be summarized by a *copwin spanning tree* T with $V(T) = V(G)$ and $v_iv_j \in E(T)$ if $i < j$ and $f_i(v_i) = v_j$ [7,8]. We observe that each induced subgraph $G - \{v_1, v_2, \ldots, v_i\}$ is 1-copwin, and $T - \{v_1, v_2, \ldots, v_i\}$ is a copwin spanning tree of this graph.

Nowakowski and Winkler [13] also gave a relational characterization of the 1-copwin reflexive (possibly infinite) graphs. Define, for each ordinal α, a binary relation $\preceq_\alpha$ on $V(G)$ as follows:

(1) $x \preceq_0 x$ for every $x \in V(G)$; and
(2) For $\alpha > 0$, set $x \preceq_\alpha y$ if and only if, for all $u \in N_G(x)$, there exists $v \in N_G(y)$ such that $u \preceq_\rho v$, for some $\rho < \alpha$.

Notice that $\preceq_\alpha$ contains $\preceq_\rho$ for all $\rho < \alpha$. Let β be the least ordinal such that $\preceq_\beta \, = \, \preceq_{\beta+1}$, and set $\preceq \, = \, \preceq_\beta$. The main result is that: *A reflexive graph G is 1-copwin if and only if the relation $\preceq$ is trivial, that is $x \preceq y$ for all $x, y \in V(G)$.* The cop's strategy is apparent from the definition of the relation $\preceq$. On his first move the cop locates himself on any vertex y. On each subsequent move the cop moves so that his position and that of the robber belong to a sequence of relations indexed by a strictly decreasing sequence of ordinals: when the robber moves to u, the cop moves to a vertex $v \in N_G(y)$ such that $u \preceq_\rho v$, for some $\rho < \beta$. We will see in Section 2 that this result follows as a corollary of Theorem 2.1, our main result.

Finally, let G and H be graphs. The *categorical product* of G and H is the graph $G \times H$ with vertex set $V(G) \times V(H)$ and edge set $\{(a, b)(c, d) : ac \in E(G) \text{ and } bd \in E(H)\}$. Cops and Robber is sometimes defined on irreflexive graphs and the rules allow a side to not move on its turn. In that case, the *strong product* of G and H is used. It has the same vertex set as $G \times H$, and the edges $\{(a, b)(c, d) : a = c \text{ and } bd \in E(H), \text{ or } ac \in E(G) \text{ and } b = d\}$ in addition to the edges of $G \times H$. When G and H are reflexive graphs, these two products coincide.

2. A relational characterization

In this section, we give a relational characterization of k-copwin graphs. Our relation is very similar to the one described by Nowakowski and Winkler [13], but is not a relation on $V(G)$. Similar ideas were used by Hahn and MacGillivray [12] to obtain an algorithmic characterization of k-copwin finite graphs and digraphs in terms of a labelling of an auxiliary directed graph. Their labelling algorithm is also strongly similar to the Nowakowski–Winkler relation. The results in this section (and also the next) lead to $O(|V(G)|^{2k+2})$ algorithms to decide if G is k-copwin. These slightly improve other known algorithms [2,3,12].

Let $\mathcal{P} = \mathcal{P}(G)$ be the graph whose vertices are the possible positions of the k cops on the graph G, with $pq \in E(\mathcal{P})$ if and only it is possible for the cops to move from position p to position q, or vice versa. Then $\mathcal{P}$ is the k-fold categorical product of G with itself. (If the cops are regarded as indistinguishable, $\mathcal{P}$ can be replaced by the graph whose vertices are the k-element multi-subsets of $V(G)$.) When $k = 1$, $\mathcal{P} = G$, and what follows is exactly the same as in the paper by Nowakowski and Winkler [13]. The theorem holds for infinite reflexive graphs, but we state it for finite reflexive graphs.

For $i = 0, 1, \ldots$, the relation $\preceq_i$ from $V(G)$ to $V(\mathcal{P})$ is defined inductively as follows:

(1) For all $x \in V(G)$ and $p \in V(\mathcal{P})$, $x \preceq_0 p$ if, in position p, one of the cops is located at vertex x.
(2) For $i > 0$, $x \preceq_i p$ if, for every $y \in N_G(x)$, there exists $q \in N_{\mathcal{P}}(p)$ such that $y \preceq_j q$, for some $j < i$.

Note that $\preceq_i$ contains $\preceq_j$ for all $j < i$. Since G and $\mathcal{P}$ are both finite, there exists an integer M such that $\preceq_M \, = \, \preceq_t$, for all $t \geq M$. Let $\preceq \, = \, \preceq_M$.

Theorem 2.1. *A finite reflexive graph G is k-copwin if and only if there exists $p \in V(\mathcal{P})$ such that $x \preceq p$, for every $x \in V(G)$.*

Proof. ($\Leftarrow$) We first show by induction on i that if $x \preceq_i p$ for $i \leq M$ then cops in position p can catch a robber located on x in at most i moves.

The statement is clearly true if $i = 0$. Assume it is true for $i = t$, for some $0 \leq t < M$, and suppose $x \preceq_{t+1} p$. If the robber moves from x to $y \in N_G(x)$ (possibly $y = x$), then the cops move to position $q \in N_{\mathcal{P}}(p)$ such that $y \preceq_t q$ (note that $\preceq_j \subseteq \preceq_t$, for all $j < t$). The result now follows from the induction hypothesis.

($\Rightarrow$) Suppose that, for every $p \in V(\mathcal{P})$, there exists $x \in V(G)$ such that $x \not\preceq p$. Any such x must have a neighbour y such that $y \not\preceq q$, for any $q \in N_{\mathcal{P}}(p)$. In particular, neither position has a cop located at the same vertex as the robber.

Consider the game when the cops start in position p. The robber then chooses x such that $x \not\preceq p$. If the cops move to position q, then the robber moves to $y \in N_G(x)$ such that $y \not\preceq q$. Continuing in this way, the robber is never caught. $\square$

For a finite reflexive graph G, define a bipartite graph $B = B(G)$ with bipartition $(V(G), V(\mathcal{P}))$ and $x \in V(G)$ adjacent to $p \in V(\mathcal{P})$ when $x \preceq p$.

Corollary 2.2. *A finite reflexive graph G is k-copwin if and only if there is a vertex p in $V(\mathcal{P})$ that is adjacent in B to every vertex of $V(G)$.*

Corollary 2.3. *A connected finite reflexive graph G is k-copwin if and only if $x \preceq p$ for every $x \in V(G)$ and $p \in V(\mathcal{P})$.*

Proof. If $x \preceq p$ for every $x \in V(G)$ and $p \in V(\mathcal{P})$, then G is k-copwin by Theorem 2.1. Conversely, if G is k-copwin then, by Theorem 2.1, there exists $p \in V(\mathcal{P})$ and a least integer $i \geq 0$ such that $x \preceq_i p$ for every $x \in V(G)$. Let $q \in N_{\mathcal{P}}(p)$. By definition of $\preceq_{i+1}$ we have $x \preceq_{i+1} q$ for every $x \in V(G)$. Since G is connected and finite, the result follows by applying this argument repeatedly. $\square$

Corollary 2.4. *A connected finite reflexive graph G is k-copwin if and only if $B(G)$ is complete bipartite.*

Both the relation $\preceq$ and the bipartite graph B can be computed in polynomial time for fixed k. The results in this section imply $O(|V(G)|^{2k+2})$ algorithms to decide whether a given graph G is k-copwin: Each iteration adds at least one pair to the relation, and hence $O(|V(G)|^{k+1})$ iterations are required. Determining if a pair (p, x) can be added to the relation $\preceq_i$ requires examining $O(|V(\mathcal{P})| \cdot |V(G)|)$ pairs (y, q), where $y \in N_G(x)$ and $q \in N_{\mathcal{P}}(p)$. Determining whether each such pair belongs to $\preceq_{i-1}$ can be done in constant time if the relation is stored as a $|V(G)| \times |V(\mathcal{P})|$ matrix.

3. A vertex elimination order characterization

In this section, we obtain a vertex elimination order characterization of k-copwin graphs. Instead of the elimination order being of the vertices of G as in the one-cop case, it is an ordering of the vertices of the $(k+1)$-fold categorical product of G with itself. In the one-cop case we relate this ordering to copwin orderings. We note that the vertex labelling of the auxiliary digraph of Hahn and MacGillivray [12] also gives an elimination ordering of its vertices.

Let G be a finite reflexive graph. For $y \in V(G)$, let $\mathcal{J}_y = \{(q, y) \in V(\mathcal{P} \times G)\}$ and, for $p \in V(\mathcal{P})$, let $\mathcal{I}_p = \{(p, z) \in V(\mathcal{P} \times G)\}$. We will refer to $\mathcal{J}_y$ as the *row of* y and $\mathcal{I}_p$ as the *column of* p. Since G is a reflexive graph, every vertex (p, x) of $\mathcal{P} \times G$ has a neighbour in $\mathcal{J}_y$ for every $y \in N_G(w)$, namely (p, y).

We first look at the construction of the sequence of relations $\preceq_i$ as determining a "painting" of the vertices of $\mathcal{P} \times G$:

(1) For all $x \in V(G)$ and $p \in V(\mathcal{P})$, paint (p, x) red if, in position p, one of the cops is located at vertex x.
(2) For $i > 0$, paint (p, x) red if it is adjacent to a red vertex in row $\mathcal{J}_y$, for every $y \in N_G(x)$.

The vertices $(p, x) \in V(\mathcal{P} \times G)$ painted red in the ith step correspond in a natural way with the pairs $x \in V(G)$ and $p \in V(\mathcal{P})$ such that $x \preceq_i p$. By Theorem 2.1, a finite reflexive graph G is k-copwin if and only if, at the end of the process, there is a column $\mathcal{I}_p$ in which every vertex is painted red. By Corollary 2.3, a connected finite reflexive graph G is k-copwin if and only if this procedure results in every vertex of $\mathcal{P} \times G$ being painted red. The sequence in which this happens gives an elimination ordering of the vertices of $\mathcal{P} \times G$.

A vertex (p, x) of $\mathcal{P} \times G$ is called *removable with respect to* $S \subseteq V(\mathcal{P} \times G)$ if either

(1) in position p, one of the cops is located at vertex x; or
(2) $N_{\mathcal{P}\times G}((p, x)) \cap \mathcal{J}_y \cap S \neq \emptyset$, for every $y \in N_G(x)$.

Theorem 3.1. *A finite reflexive graph G is k-copwin if and only if there is a sequence $S = (p_1, x_1), (p_2, x_2), \ldots, (p_t, x_t)$ of vertices of $\mathcal{P} \times G$ such that*

(1) $t \leq |V(\mathcal{P} \times G)|$;
(2) *for* $1 \leq i \leq t$, *the vertex* (p_i, x_i) *is removable with respect to* $\{(p_j, x_j) : j < i\}$; *and*
(3) (p_t, x) *belongs to* S, *for every* $x \in V(G)$.

Proof. Suppose G is k-copwin. By Theorem 2.1 and the definition of $\preceq$, listing the vertices in $\preceq_0$ and, for $i \geq 0$, all those in $\preceq_{i+1}$ following those in $\preceq_i$ and stopping once the desired pair (p_t, x) appears is such an enumeration.

Suppose the vertices of $\mathcal{P} \times G$ can be enumerated as in the statement. We will show by induction on i that cops in position p_i can catch a robber located at x_i in at most i moves. The statement holds for $i = 1$ since (p_1, x_1) is removable with respect to $\emptyset$ which implies that in position p_1, one of the cops is located at vertex x_1. Suppose it holds for $i = 1, 2, \ldots, \ell$, for some ℓ with $t > \ell \geq 1$.

Consider $(p_{\ell+1}, x_{\ell+1})$. Then, in position $p_{\ell+1}$, either one of the cops is located at vertex $x_{\ell+1}$, or $N_{\mathcal{P}\times G}((p_{\ell+1}, x_{\ell+1})) \cap \mathcal{J}_y \cap S \neq \emptyset$ for every $y \in N_G(x_{\ell+1})$. In the former case, the cops have caught the robber. In the latter case, since $(p_{\ell+1}, x_{\ell+1})$ is removable with respect to $\{(p_j, x_j) : j < \ell + 1\}$, for every $y \in N_G(x_{\ell+1})$, there exists $q \in N_{\mathcal{P}}(p_{\ell+1})$ such that $(q, y) \in \{(p_j, x_j) : j < i\}$. The result now follows by induction. $\square$

The following is obtained similarly to the way that Corollary 2.3 is derived from Theorem 2.1.

Corollary 3.2. *A connected finite reflexive graph G is k-copwin if and only if the vertices of* $\mathcal{P} \times G$ *can be enumerated* $(p_1, x_1), (p_2, x_2), \ldots$ *such that, for* $1 \le i \le |V(\mathcal{P} \times G)|$*, the vertex* (p_i, x_i) *is removable with respect to* $\{(p_j, x_j) : j < i\}$.

For fixed k, it can be determined in polynomial time whether $\mathcal{P} \times G$ has a removable vertex ordering, or an ordering of the type in Theorem 3.1 (note that $\mathcal{P}$ has $|V(G)|^k$ vertices). Thus Theorem 3.1 and Corollary 3.2 also imply polynomial time algorithms that decide if a given finite reflexive graph G is k-copwin. These require $O(|V(\mathcal{P} \times G)|)$ iterations of a procedure (to check if (p, x) is removable) that requires $O(|V(\mathcal{P}) \times V(G)|)$ steps (for each $y \in N_G(x)$, check if (p, x) is adjacent to a "red" vertex in row y).

We now consider finite reflexive 1-copwin graphs G and show how to use copwin orderings to obtain removable vertex orderings (i.e. the orderings in Corollary 3.2). In this situation, $\mathcal{P} = G$ and the graph G is connected. Fix a copwin ordering $(v_1, v_2, \ldots, v_n)$ and let T be a copwin spanning tree arising from this ordering. Root T at v_n. Our removable vertex ordering S begins $(v_1, v_1), (v_2, v_2), \ldots, (v_n, v_n)$. These pairs are all removable by definition. Let a be a vertex of T and suppose that all pairs (a, d), where d is a descendant of a in T, belong to S. If $a \neq v_n$, let w be the parent of a in T, that is, the unique neighbour of w on the (v_n, a)-path in T. We claim that all pairs (w, x), where x belongs to the subtree rooted at a, are removable. By definition of the sequence of one-point retractions, for any $y \in N_G(x)$, either $y \in N_G(w)$ or y is a descendant of a in T. In the former case, (w, x) is adjacent in $G \times G$ to the vertex (y, y) in S. In the latter case, (w, x) is adjacent in $G \times G$ to the vertex (a, y) in S. This proves the claim. Therefore, there is a sequence of removable pairs containing all pairs (v_n, y), where $y \in V(G)$. Using the same argument as in the proof of Corollary 2.3, a removable vertex ordering is obtained.

Conversely, it is possible to obtain a copwin ordering of an n vertex reflexive graph G from a removable vertex ordering S of $G \times G$. Consider the first pair (x_1, v_1) in S with $x_1 \neq v_1$. The definition of removable immediately gives $N_G(v_1) \subseteq N_G(x_1)$. Let S_1 be the sequence obtained from S by

(1) deleting all pairs (u, v_1), where $u \in V(G)$;
(2) replacing each pair (v_1, y), where $y \in V(G)$, by (x_1, y); and
(3) deleting the second instance of all repeated pairs.

We claim that S_1 is a removable vertex ordering of $G - v_1$. The claim follows from the observation that, since $N_G(v_1) \subseteq N_G(x_1)$, if (v_1, y) is removable, then so is (x, y). Repeating this argument $n - 1$ times to define $v_1, v_2, \ldots, v_{n-1}$ and then letting v_n be vertex of G not belonging to this sequence gives a copwin ordering of G.

Since removable vertex orderings can be obtained from $\preceq$, a similar connection can be made between copwin orderings and the relation $\preceq$.

4. Generalizations

We note that our results hold for several variations of the game. For finite graphs, all that is required for Theorem 2.1 and Corollary 2.2 is that the *cops' position graph* $\mathcal{P}$ and the *robber's move graph* G both exist and be reflexive. The remaining results rely on connectivity of $\mathcal{P}$. In the usual game, connectivity of $\mathcal{P}$ follows from connectivity of G.

As a step towards achieving a characterization of 2-copwin graphs, Clarke and Nowakowski [10,11] (see also [7]) defined *tandem-win graphs*. In the tandem version of the Cops and Robber game, the two cops are constrained to positions in which they are located on adjacent vertices. Here, the cops' position graph $\mathcal{P}$ is the graph $L(G)^+$ obtained from the line graph of G by adding the edges $(ab)(cd)$ and $(bc)(ad)$, if necessary, for every four-cycle a, b, c, d, a in G. Corollary 3.2 then becomes the following:

Corollary 4.1. *A connected finite reflexive graph G is tandem-win if and only if there is a sequence* $(p_1, x_1), (p_2, x_2), \ldots$ *of vertices of* $L(G)^+ \times G$ *such that, for* $1 \le i \le |V(L(G)^+ \times G)|$*, the vertex* (p_i, x_i) *is removable with respect to* $\{(p_j, x_j) : j < i\}$.

Similar comments as above apply to finite reflexive digraphs. Neighbourhoods must be replaced by out-neighbourhoods, and the connectivity assumption must be replaced by the existence of a directed path from any given vertex of $\mathcal{P}$ to a vertex $p \in V(\mathcal{P})$ such that $x \preceq p$, for every $x \in V(G)$. In particular, the results hold for finite reflexive strongly connected digraphs. Theorem 3.1, however, gives a characterization that applies to all finite reflexive digraphs.

As in the paper of Nowakowski and Winkler [13], the results in Section 2 hold for infinite graphs.

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