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Minimal seesaw model with tri/bi-maximal mixing and leptogenesis

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Abstract

We examine minimal seesaw mechanism in which the masses of light neutrinos are described with tri/bi-maximal mixing in the basis where the charged-lepton Yukawa matrix and heavy Majorana neutrino mass matrix are diagonal. We search for all possible Dirac mass textures which contain at least one zero entry in 3×2 matrix and evaluate the corresponding lepton asymmetries. We present the baryon asymmetry in terms of a single low energy unknown, a Majorana CP phase to be clued from neutrinoless double beta decay.

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1. Introduction

Thanks to the accumulating data from experiments on the atmospheric and solar neutrinos experiments [1–3], we are now convinced that neutrinos oscillate among three active neutrinos. Interpreting each experiment in terms of two-flavor mixing, the mixing angle for the oscillation of atmospheric neutrinos is understood to be maximal or nearly maximal: $\sin^2 2\theta_{\rm atm} \simeq 1$, whereas the one for the oscillation of solar neutrinos is not maximal but large: $\sin^2\theta_{\rm sol} \simeq 0.3$ [6]. The upper bound for $\theta_{\rm reac}$, $\sin\theta_{\rm reac} \lesssim 0.2$, was obtained from the non-observation of the disappearance of $\bar{\nu}_e$ in the CHOOZ experiment [4] with $\Delta m^2 \leqslant 10^{-3}$ eV^2. The unitary mixing matrix is defined via $\nu_a = \sum_{j=1}^{3} U_{aj}\nu_j$ $(a = e, \mu, \tau)$, where ν_a is a flavor eigenstate and ν_j is a mass eigenstate. Including data from

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SNO [3] and KamLAND [5], the range of the magnitude of the MNS mixing matrix is given by [7–10],

$$|U| = \begin{pmatrix} 0.79\text{–}0.86 & 0.50\text{–}0.61 & 0\text{–}0.16 \\ 0.24\text{–}0.52 & 0.44\text{–}0.69 & 0.63\text{–}0.79 \\ 0.26\text{–}0.52 & 0.47\text{–}0.71 & 0.60\text{–}0.77 \end{pmatrix} \tag{1}$$

at the 90% confidence level. The existing data also show that the neutrino mass squared differences which induce the solar and atmospheric neutrino oscillations are $\Delta m^2_{\text{sol}} \simeq (7^{+10}_{-2}) \times 10^{-5}\ \text{eV}^2$ and $\Delta m^2_{\text{atm}} \simeq (2.5^{+1.4}_{-0.9}) \times 10^{-3}\ \text{eV}^2$, respectively. It can be readily recognized that the central values of elements in the mixing matrix in Eq. (1) are pointing an elegant form, which is called tri/bi-maximal mixing [11],

$$U_{\text{TB}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{2}$$

which consists of $\sin\theta_{\text{sol}} = 1/\sqrt{3}$ and $\sin\theta_{\text{atm}} = 1/\sqrt{2}$. There are some literatures [12] which proposed textures of the mass matrix based on the particular mixing type U_{TB}.

On the other hand, the baryon density of our Universe $\Omega_B h^2 = 0.0224 \pm 0.0009$ implied by WMAP (Wilkinson Microwave Anisotropy Probe) data indicates the observed baryon asymmetry in the Universe [13,14],

$$\eta_B^{\text{CMB}} = \frac{n_B - n_{\bar{B}}}{n_\gamma} = \left(6.5^{+0.4}_{-0.3}\right) \times 10^{-10}, \tag{3}$$

where $n_B, n_{\bar{B}}$ and n_γ are number density of baryon, anti-baryon and photon, respectively. The leptogenesis [15] has become a compelling theory to explain the observed baryon asymmetry in the Universe, due to increasing reliance on the seesaw mechanism from experiments. Theory for lepton asymmetry requires two heavy right-handed neutrinos or more. For that reason, a class of models with two heavy right-handed neutrinos and 3×2 neutrino Dirac mass matrix is called the minimal neutrino seesaw models (MNSMs) which were intensively studied by several authors recently [16,17], especially for simple Dirac mass textures that make prediction compatible with solar and atmospheric neutrino data.

The main framework of our work is seesaw mechanism in bottom-up approach. We launch our analysis by taking U_{TB} for mixing of light neutrinos and then investigate the structure of 3×2 Dirac matrix. That is, our concern remains on the combination of tri/bi-maximal mixing and MNSMs, in order to study the phenomenological implication of the high energy theory based on the low energy theory. One advantage of our framework is that physical observables can be explained in minimal terms of physical parameters. In Section 2, we present the light neutrino mass matrix in terms of the mixing given in Eq. (2) and mass squared differences measured in experiment. The mass matrix reconstructed in that way will constrain Dirac mass matrix. In subsections, depending on the type of mass hierarchy, possible 3×2 Dirac matrices will be examined carefully. In Section 3, leptogenesis will be discussed in details based on the Dirac matrices investigated before. In Section 4, we will present numerical results on leptogenesis in our scheme and a relationship between leptogenesis and neutrinoless double beta decay as well as the lower bound of M_1 will be discussed focusing on the contribution from a single Majorana phase.

2. Dirac mass matrices in minimal seesaw

In general, a unitary mixing matrix for 3 generations of neutrinos is given by

$$\tilde{U} = R(\theta_{23}) R(\theta_{13}, \delta) R(\theta_{12}) P(\varphi, \varphi'), \tag{4}$$

where R's are rotations with three angles and a Dirac phase δ and the $P = \text{Diag}(1, e^{i\varphi/2}, e^{i\varphi'/2})$ with Majorana phases φ and φ' is a diagonal phase transformation. The mass matrix of light neutrinos is given by

$M_\nu = \tilde{U}\,\mathrm{Diag}(m_1, m_2, m_3)\tilde{U}^{\mathrm{T}}$, where $m_1,\ m_2,\ m_3$ are real positive masses of light neutrinos. Or the Majorana phases can be embedded in the diagonal mass matrix such that

$$M_\nu = U\,\mathrm{Diag}(m_1, \check{m}_2, \check{m}_3)U^{\mathrm{T}}, \tag{5}$$

where $U \equiv \tilde{U}P^{-1}$ and $\check{m}_2 \equiv m_2 e^{i\varphi}$ and $\check{m}_3 \equiv m_3 e^{i\varphi'}$.

If the U_{TB} in Eq. (2) is adopted for the U in Eq. (5), the light neutrino mass is

$$M_\nu = m_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{\check{m}_2 - m_1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{\check{m}_3 - m_1}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \tag{6}$$

which orients toward a minimal model of neutrino sector by removing an angle and the Dirac phase. With SNO and KamLAND, data have narrowed down the possible mass spectrum of neutrinos into two types, Normal Hierarchy (NH), $m_1 \lesssim m_2 < m_3$, and Inverse Hierarchy (IH), $m_3 < m_1 \lesssim m_2$ for MSW LMA. Those two types include the possibility of zero mass for a neutrino, which is necessarily followed by relegating one of the Majorana phases to the unphysical. In other words, the minimal model with the physical observables which the present experimental data guarantee can be obtained by U_{TB} and dictating zero mass to one generation of neutrinos, where the non-zero physical parameters in the model consist of 2 masses, 2 angles, one Majorana phase.

When only two of three neutrinos are massive, by accommodating the experimental results $\Delta m^2_{\mathrm{sol}} = m_2^2 - m_1^2$ and $\Delta m^2_{\mathrm{atm}} = |m_3^2 - m_2^2|$ to the two types of mass hierarchies, one can obtain the following expressions for mass eigenvalues:

$$m_1 = 0, \qquad m_2 = \sqrt{\Delta m^2_{\mathrm{sol}}}, \qquad m_3 = \sqrt{\Delta m^2_{\mathrm{sol}} + \Delta m^2_{\mathrm{atm}}}, \quad \text{for NH}, \tag{7}$$

$$m_1 = \sqrt{\Delta m^2_{\mathrm{atm}} - \Delta m^2_{\mathrm{sol}}}, \qquad m_2 = \sqrt{\Delta m^2_{\mathrm{atm}}}, \qquad m_3 = 0, \quad \text{for IH}. \tag{8}$$

Phase transformation $P = \mathrm{Diag}(1, e^{i\varphi/2}, 1)$ now can replace the phase transformation in Eq. (4) without loss of generality, whether NH or IH, so that one can single $\check{m}_2$ out in order to investigate the CP-violating contribution of the Majorana phase.

Effective neutrino mass models with one zero mass eigenvalue involved in three active neutrinos can be generated naturally from MNSMs. In the basis the mass matrix M_R of right-handed neutrinos $N_R = (N_1, N_2)$ is diagonal, the model is given

$$\mathcal{L} = -\bar{l}_L M_L l_R - \bar{\nu}_L m_D N_R + \frac{1}{2}\bar{N}_R^c M_R N_R + \text{h.c.}, \tag{9}$$

from which the light masses are derived through the seesaw mechanism, $M_\nu = -m_D M_R^{-1} m_D^{\mathrm{T}}$ in top-down approach. On the other hand, the matrix m_D is found as the solution to the seesaw mechanism in bottom-up approach once one launches the analysis with the light neutrino masses M_ν. Let M_1 and M_2 be the masses of right-handed neutrinos and M_{ij} the elements of the matrix M_ν. The Dirac matrix,

$$m_D = \begin{pmatrix} \sqrt{M_1}a_1 & \sqrt{M_2}b_1 \\ \sqrt{M_1}a_2 & \sqrt{M_2}b_2 \\ \sqrt{M_1}a_3 & \sqrt{M_2}b_3 \end{pmatrix}, \tag{10}$$

is resulted in with

$$\begin{aligned} a_1 &= \sqrt{M_{11} - b_1^2}, \qquad b_1 = \sqrt{M_{11} - a_1^2}, \\ a_i &= \frac{1}{M_{11}}\Big[a_1 M_{1i} - \sigma_i b_1 \sqrt{M_{11}M_{ii} - M_{1i}^2}\Big], \qquad b_i = \frac{1}{M_{11}}\Big[b_1 M_{1i} + \sigma_i a_1 \sqrt{M_{11}M_{ii} - M_{1i}^2}\Big], \\ M_{11}M_{23} &= \Big[M_{12}M_{13} + \sigma_2\sigma_3\sqrt{\big(M_{11}M_{22} - M_{12}^2\big)\big(M_{11}M_{33} - M_{13}^2\big)}\Big], \end{aligned} \tag{11}$$

where the i is 2 or 3, the σ_i is a sign $\pm$, and the sign of a_1 is fixed as positive. The solution in Eq. (12) was first derived and formulated in Ref. [17]. It is clear that only 5 parameters out of 6; a_1, b_1, a_i, b_i, can be specified in terms of the elements of M_ν. There are various ways to decrease the number of parameters in Dirac matrix, posing one or more zeros or posing equalities between elements for the matrix texture. It is known that texture zeros or equalities among matrix entries can be generated by imposing additional symmetries to the theory.

In this Letter, we focus on posing one or more zeros in Dirac matrix, and show that only one-zero textures are allowed for NH and only one-zero and two-zero textures are allowed for IH, accompanied with the low energy mixing U_{TB}. On the other hand, from Eq. (11), one can recognize that, if there exists a kind of symmetry between entries in M_ν, the Dirac matrix also has a symmetry in certain entries inherited from the symmetry of the M_ν. So, there are a number of patterns with equalities among the entries in Dirac matrices obtained with one or two zeros, as a consequence of maximal mixing.

2.1. Normal hierarchy

With $m_1 = 0$, the neutrino mass M_ν in Eq. (6) reduces to

$$M_\nu = \frac{\check{m}_2}{3}\begin{pmatrix} 1 & 1 & 1 \\ & 1+d & 1-d \\ & & 1+d \end{pmatrix}, \tag{12}$$

where $d = 3m_3/2\check{m}_2$, which, using Eq. (11), gives rise to Dirac matrix with the following entries:

$$\begin{aligned} &a_1 = \sqrt{\check{m}_2/3 - b_1^2}, \qquad b_1 = \sqrt{\check{m}_2/3 - a_1^2}, \\ &a_i = a_1 - \sigma_i b_1 \sqrt{d}, \qquad b_i = b_1 + \sigma_i a_1 \sqrt{d}, \quad i = 2, 3, \end{aligned} \tag{13}$$

where $\sigma_2\sigma_3 = -1$. Depending on the position of texture zero, the types of Dirac matrices can be classified as follows:

- NH 1-a: $b_1 = 0, \quad a_1 = \sqrt{\check{m}_2/3}, \quad a_1 = a_2 = a_3, \quad b_2 = -b_3 = \sigma_2\sqrt{m_3/2};$
- NH 1-b: $a_1 = 0, \quad a_1 \leftrightarrow b_1, \quad a_i \leftrightarrow b_i$ in NH 1-a;
- NH 2-a: $b_2 = 0, \quad a_2 = \sqrt{\check{m}_2/3 + m_3/2},$
  $a_1 = \frac{a_2}{1+d}, \quad a_3 = \frac{a_2(1-d)}{1+d}, \quad \frac{b_1}{a_1} = -\sigma_2\sqrt{d}, \quad b_3 = 2b_1;$
- NH 2-b: $a_2 = 0, \quad a_1 \leftrightarrow b_1, \quad a_i \leftrightarrow b_i$ in NH 2-a.

The matrix in Eq. (12) features the equalities between M_{22} and M_{33} and between M_{12} and M_{13} as a consequence of the maximal mixing of atmospheric neutrinos. In the case with $b_2 = 0$, it can be recognized that the ratios of a_1 to a_2 and a_3 to a_2 inherit those of M_{12} to M_{22} and M_{23} to M_{22}, respectively. $b_3 = 0$ or $a_3 = 0$ cases will not be presented as an independent case, since it can be made by exchanging b_3 with b_2 and a_3 with a_2 from NH 2-a and 2-b.

2.2. Inverse hierarchy

With $m_3 = 0$,

$$M_\nu = \frac{m_1}{3}\begin{pmatrix} x+2 & x-1 & x-1 \\ & x+1/2 & x+1/2 \\ & & x+1/2 \end{pmatrix}, \tag{14}$$

where $x \equiv \check{m}_2/m_1$, which, using Eq. (11), gives rise to Dirac matrix with the following entries:

$$a_1 = \sqrt{m_1(x+2)/3 - b_1^2}, \qquad b_1 = \sqrt{m_1(x+2)/3 - a_1^2},$$
$$a_i = \left((x-1)a_1 - 3\sigma b_1\sqrt{x/2}\right)/(x+2), \qquad b_i = \left((x-1)b_1 + 3\sigma a_1\sqrt{x/2}\right)/(x+2), \quad i = 2,3, \tag{15}$$

where $\sigma \equiv \sigma_i$ and $\sigma_2\sigma_3 = 1$. The equality $M_{22} = M_{23} = M_{33}$ in Eq. (14), which is again a consequence of the maximal mixing of atmospheric neutrinos, constrains the elements of the Dirac matrix such that $a_2 = a_3$, $b_2 = b_3$. Hence, texture with a single zero included appears only if $a_1 = 0$ or $b_1 = 0$, while texture with two zeros appears if $a_2 = a_3 = 0$ or $b_2 = b_3 = 0$.

- IH 1-a: $b_1 = 0, \quad a_1 = \sqrt{\check{m}_2/3 + 2m_1/3}, \quad a_i = \dfrac{a_1(x-1)}{x+2}, \quad \dfrac{b_i}{a_i} = \dfrac{3\sigma\sqrt{x/2}}{x-1};$
- IH 1-b: $a_1 = 0, \quad a_1 \leftrightarrow b_1, \quad a_i \leftrightarrow b_i$ in IH 1-a;
- IH 2-a: $b_2 = b_3 = 0, \quad a_2 = a_3 = \sqrt{\check{m}_2/3 + m_1/6}, \quad a_1 = \dfrac{2a_2(x-1)}{2x+1}, \quad \dfrac{b_1}{a_1} = -\dfrac{3\sigma\sqrt{x/2}}{x-1};$
- IH 2-b: $a_2 = a_3 = 0, \quad a_1 \leftrightarrow b_1, \quad a_i \leftrightarrow b_i$ in IH 2-a.

Listed are all the cases with one or more texture zeros in Dirac matrix derivable from the light neutrino mass with U_{TB}, whether NH or IH. In the following, the eligibility of each case to generate the CP asymmetry for leptogenesis will be examined.

3. Leptogenesis

The baryon asymmetry Eq. (3) can be rephrased

$$Y_B = \frac{n_B - n_{\bar{B}}}{s} \simeq (8.8\text{–}9.8) \times 10^{-11}. \tag{16}$$

The n_γ is the photon number density and the s is entropy density so that the number density with respect to a co-moving volume element is taken into account. The baryon asymmetry produced through sphaleron process is related to the lepton asymmetry [18,19] by

$$Y_B = aY_{B-L} = \frac{a}{a-1} Y_L, \tag{17}$$

where $a \equiv (8N_F + 4N_H)/(22N_F + 13N_H)$, for example, $a = 28/79$ for the Standard Model (SM) with three generations of fermions and a single Higgs doublet, $N_F = 3$, $N_H = 1$. The purpose of this work is to estimate whether the Yukawa interaction which produces the light neutrinos with the mixing Eq. (2) through the seesaw mechanism can also generate a sufficient lepton asymmetry for the observed baryon asymmetry. The generation of a lepton asymmetry requires the CP-asymmetry and out-of-equilibrium condition. The Y_L is explicitly parameterized by two factors, ε, the size of CP asymmetry, and κ, the dilution factor from washout process.

$$Y_L = \frac{n_L - n_{\bar{L}}}{s} = \kappa \frac{\varepsilon_i}{g^*}, \tag{18}$$

where $g^* \simeq 110$ is the number of relativistic degree of freedom. The ε_i is the magnitude of CP asymmetry in decays of heavy Majorana neutrinos [20,21],

$$\varepsilon_i = \frac{\Gamma(N_i \to \ell H) - \Gamma(N_i \to \bar{\ell} H^*)}{\Gamma(N_i \to \ell H) + \Gamma(N_i \to \bar{\ell} H^*)}, \tag{19}$$

where i denotes a generation. When one of two generations of right neutrinos has a mass far below that for the other generation, i.e., $M_1 < M_2$, the ε_i in Eq. (19) is obtained from the decay of M_1 [22–24],

$$\varepsilon_1 = \frac{1}{8\pi v^2} \frac{\mathrm{Im}\left[\left(m_D^\dagger m_D\right)_{12}^2\right]}{\left(m_D^\dagger m_D\right)_{11}} f\left(\frac{M_2}{M_1}\right), \tag{20}$$

where $v = 174$ GeV and $f(M_2/M_1)$ represents loop contribution to the decay width from vertex and self energy and is given by

$$f(x) = x\left[1 - \left(1 + x^2\right) \ln \frac{1 + x^2}{x^2} + \frac{1}{1 - x^2}\right] \tag{21}$$

for the Standard Model. For large value of x, the leading order of $f(x)$ is $(-3/2)x^{-1}$.

It is convenient to consider separately the factor that depends on Dirac matrix in ε_1 in Eq. (20) at this stage.

$$\frac{\mathrm{Im}\left[\left(m_D^\dagger m_D\right)_{12}^2\right]}{\left(m_D^\dagger m_D\right)_{11}} = M_2 \frac{\mathrm{Im}[(a_1^* b_1 + a_2^* b_2 + a_3^* b_3)^2]}{|a_1|^2 + |a_2|^2 + |a_3|^2} \equiv M_2 \Delta_1, \tag{22}$$

where a's and b's are defined in Eq. (11). From a number of types of matrices with a texture zero derived in Eq. (13) and Eq. (15), only 6 different non-zero values of Δ_1's can be evaluated. Those particular Dirac matrices to contribute the imaginary parts are the matrix with $b_2 = 0$ and that with $a_2 = 0$ for NH, and the matrix with $b_1 = 0$, that with $a_1 = 0$, that with $b_2 = b_3 = 0$, and that with $a_2 = a_3 = 0$ for IH. For NH, if $a_1 = 0$, or $b_1 = 0$, the $(m_D^\dagger m_D)_{12}$ vanishes from the trivial relation between entries. Applying Eq. (13) and Eq. (15) for Eq. (22), one can find that each type of Dirac matrix gives rise to Δ_1 as follows:

$$\Delta_1(\text{NH 2-a}) = \frac{6 m_2 m_3 (m_3^2 - m_2^2) \sin\varphi}{(2m_2^2 + 3m_3^2)\sqrt{4m_2^2 + 9m_3^2 + 12 m_2 m_3 \cos\varphi}}, \tag{23}$$

$$\Delta_1(\text{NH 2-b}) = \frac{-6(m_3^2 - m_2^2)\sin\varphi}{5\sqrt{4m_2^2 + 9m_3^2 + 12 m_2 m_3 \cos\varphi}}, \tag{24}$$

where m_2 and m_3 are given in terms of Δm_{sol}^2 and Δm_{atm}^2 in Eq. (7),

$$\Delta_1(\text{IH 1-a}) = \frac{-2 m_1 m_2 (m_2^2 - m_1^2)\sin\varphi}{(2m_1^2 + m_2^2)\sqrt{4m_1^2 + m_2^2 + 4 m_1 m_2 \cos\varphi}}, \tag{25}$$

$$\Delta_1(\text{IH 1-b}) = \frac{2(m_2^2 - m_1^2)\sin\varphi}{3\sqrt{4m_1^2 + m_2^2 + 4 m_1 m_2 \cos\varphi}}, \tag{26}$$

$$\Delta_1(\text{IH 2-a}) = \frac{-2 m_1 m_2 (m_2^2 - m_1^2)\sin\varphi}{(m_1^2 + 2m_2^2)\sqrt{m_1^2 + 4m_2^2 + 4 m_1 m_2 \cos\varphi}}, \tag{27}$$

$$\Delta_1(\text{IH 2-b}) = \frac{2(m_2^2 - m_1^2)\sin\varphi}{3\sqrt{m_1^2 + 4m_2^2 + 4 m_1 m_2 \cos\varphi}}, \tag{28}$$

where m_1 and m_2 are given in terms of Δm_{sol}^2 and Δm_{atm}^2 in Eq. (8). Thus, for $M_2 \gg M_1$ case, the CP asymmetry in Eq. (20) reduces to $\varepsilon_1 \approx 3/(16\pi v^2) M_1 \Delta_1$, which is now parameterized by the lightest mass of heavy neutrino M_1 and Majorana phase φ. The sign of ε_1 depends on the position of a texture zero in a row of Dirac matrix.

The κ in Eq. (18) is determined by solving the full Boltzmann equations. The κ can be simply parameterized in terms of K defined as the ratio of Γ_1 the tree-level decay width of N_1 to H the Hubble parameter at temperature M_1, where $K \equiv \Gamma_1/H < 1$ describes processes out of thermal equilibrium and $\kappa < 1$ describes washout effect [19,25],

$$\kappa \simeq \frac{0.3}{K(\ln K)^{0.6}} \quad \text{for } 10 \lesssim K \lesssim 10^6, \tag{29}$$

$$\kappa \sim \frac{1}{2\sqrt{K^2+9}} \quad \text{for } 0 \lesssim K \lesssim 10. \tag{30}$$

The decay width of N_1 by the Yukawa interaction at tree level and Hubble parameter in terms of temperature T and the Planck scale $M_{\rm pl}$ are $\Gamma_1 = (m_D^\dagger m_D)_{11} M_1/(8\pi v^2)$ and $H = 1.66 g_*^{1/2} T^2/M_{\rm pl}$, respectively. At temperature $T = M_1$, the ratio K is

$$K = \frac{M_{\rm pl}}{1.66\sqrt{g^*}(8\pi v^2)} \frac{(m_D^\dagger m_D)_{11}}{M_1}, \tag{31}$$

which reduces to, using the Dirac matrices in Eq. (10),

$$K \approx \frac{1}{10^{-3}\ \text{eV}} \left(|a_1|^2 + |a_2|^2 + |a_3|^2\right), \tag{32}$$

where all fixed numbers are included in a factor of order. As done for the Δ_1's, one can apply Eq. (13) and Eq. (15) for Eq. (32) to find dilution factor κ when the decay width is determined by Yukawa couplings in each type of Dirac matrix. For the six types of Dirac matrices that are eligible for the CP asymmetry as in Eqs. (23)–(28), the ratio K for each case is

$$K(\text{NH 2-a, 2-b}) \approx \frac{(2m_2^2 + 3m_3^2, 5m_2 m_3)}{(10^{-3}\ \text{eV})\sqrt{4m_2^2 + 9m_3^2 + 12 m_2 m_3 \cos\varphi}}, \tag{33}$$

$$K(\text{IH 1-a, 1-b}) \approx \frac{(2m_1^2 + m_2^2, 3m_1 m_2)}{(10^{-3}\ \text{eV})\sqrt{4m_1^2 + m_2^2 + 4 m_1 m_2 \cos\varphi}}, \tag{34}$$

$$K(\text{IH 2-a, 2-b}) \approx \frac{(m_1^2 + 2m_2^2, 3m_1 m_2)}{(10^{-3}\ \text{eV})\sqrt{m_1^2 + 4m_2^2 + 4 m_1 m_2 \cos\varphi}}, \tag{35}$$

which shows that the dilution factor also depends on the phase φ, but it does not significantly affect the order of magnitude. Out of all the types of Dirac matrices examined, there is no such a case that Yukawa couplings originate decays of neutrinos N_1 which satisfy the out-of-equilibrium condition $K < 1$ at $T = M_1$. The washout effect of asymmetry is most suppressed with the Dirac matrix of type NH 2-b, where, depending on φ, the dilution factor ranges from 0.010 to 0.013, the amount of asymmetry survived from washout is at most about 1%. When $T < M_1$, the Boltzmann equations still depict the finite value of κ as M_1/T increases for the Universe evolution [20,21,26].

4. Discussion

Based on the formulation of the leptogenesis derived in the previous section, we numerically analyze baryon asymmetry for each case classified as NH or IH. For the numerical calculation, we take $\Delta m^2_{\rm sol} = 7.0 \times 10^{-5}\ \text{eV}^2$ and $\Delta m^2_{\rm atm} = 2.5 \times 10^{-3}\ \text{eV}^2$ as inputs.

Consider a model with neutrino masses in normal hierarchy. In Fig. 1, we plot the baryon asymmetry Y_B as a function of the Majorana phase φ for NH 2-b. The different curves correspond to $M_1 = 2.0 \times 10^{11}$ to 2.0×10^{13} GeV for fixed $M_2/M_1 = 5$. We note that we can choose any reasonable M_2/M_1 value which can protect

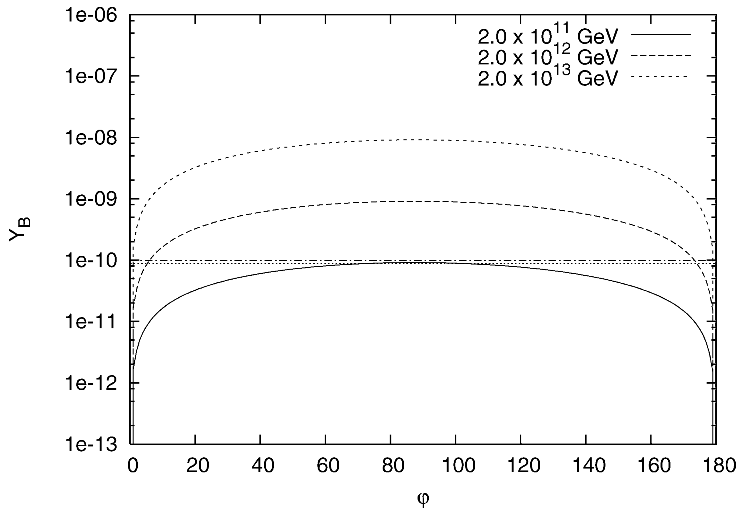

Fig. 1. Y_B as a function of Majorana CP phase for case NH 2-b, with various values of M_1 where $M_2/M_1 = 5$. The horizontal lines are the current cosmological bound of Y_B.

L-violating processes with N_1 from the wash-out when $T < M_2$. As expected from Eq. (20), the value of Y_B for a fixed φ increases with M_1. The horizontal line in Fig. 1 presents the current cosmological observation of Y_B given in Eq. (16). From the analysis, we see that the current observation on Y_B constrains the lower bound of M_1, which turns out to be $M_1 \gtrsim 2.0 \times 10^{11}$ GeV. It is clear that the CP asymmetry in high energy is almost proportional to the imaginary part of Majorana CP contribution in low energy from Eqs. (22)–(28). Thus, the plots show that the lower bound of M_1 to generate the observed baryon asymmetry should be raised if the imaginary contribution of low energy phase is decreased as the φ approaches 0 or π. In all aspects of the prediction of Y_B, NH 2-a and NH 2-b are quite similar to each other except an overall factor. The Y_B for NH 2-b is enhanced from both the enhancement of CP asymmetry, $\Delta_1(b)/\Delta_1(a) \simeq 3.6$, and the suppression of wash-out effect, $\kappa(b)/\kappa(a) \simeq 4.5$. The lower bound of M_1 with $\varphi = \pi/2$ is pulled down to 2.0×10^{11} GeV for NH 2-b, whereas that for NH 2-a is 3.2×10^{12} GeV. Suppressing a certain Yukawa coupling by putting a texture zero can vary the amount of the asymmetry by order of magnitude.

In Fig. 2, we plot Y_B as a function of the Majorana phase φ for IH 1-a. The different curves correspond to $M_1 = 5.5 \times 10^{13}$ to 5.5×10^{15} GeV for fixed $M_2/M_1 = 5$. As in NH, we obtain a lower bound on $M_1 \gtrsim 5.5 \times 10^{12}$ GeV for IH. The prediction of Y_B for IH with the same value of M_1 is smaller than that for NH because Δ_1 for IH is proportional to $m_2^2 - m_1^2$ which corresponds to the solar mass squared difference, while Δ_1 for NH is proportional to $m_3^2 - m_2^2$ which corresponds to the atmospheric mass squared difference. We expect from Eqs. (25)–(28) that the predictions of Y_B's for other cases of IH are almost the same as that for IH 1-a because $m_1 m_2/(2m_1^2 + m_2^2) \sim 1/3$.

Although the Majorana phase is not detectable through neutrino oscillations, it may affect the amplitude of neutrinoless double beta decay. Thus, one can anticipate that there may exist a correlation between leptogenesis and neutrinoless double beta decay in our scenario where the heavy Majorana neutrino mass matrix and the charged-lepton Yukawa matrix are both diagonal. The neutrinoless double beta decay amplitude is proportional to the effective Majorana mass $|\langle m_{ee}\rangle|$ which can be written in the form:

$$\left|\langle m_{ee}\rangle\right| = \left|\sum_{i=1}^{3} U_{ei}^2 m_i e^{i\varphi_i}\right| = \begin{cases} m_2/3, & \text{NH}, \\ \frac{1}{3}(4m_1^2 + m_2^2 + 4m_1 m_2 \cos\varphi)^{1/2}, & \text{IH}, \end{cases} \tag{36}$$

where φ_i are Majorana CP-violating phases. The $|\langle m_{ee}\rangle|$ depends on the CP phase φ only with inverted hierarchy, so that one can draw a simple correlation between leptogenesis and neutrinoless double beta decay only for the

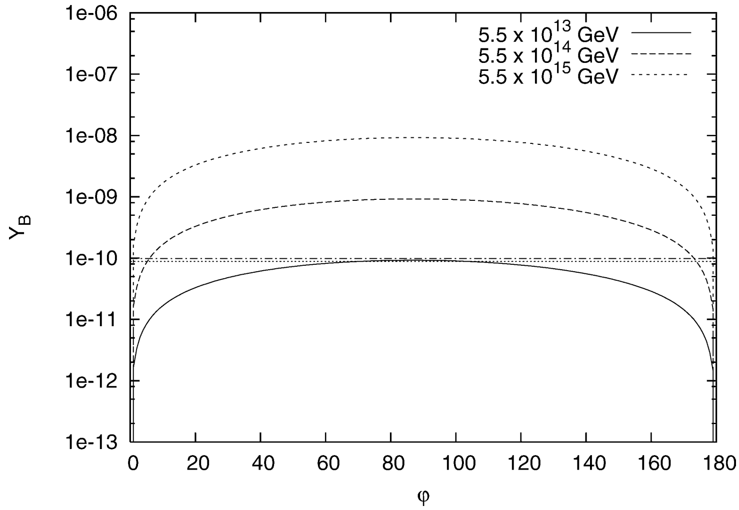

Fig. 2. Y_B as a function of Majorana CP phase for case IH 1-a, with various values of M_1 where $M_2/M_1 = 5$.

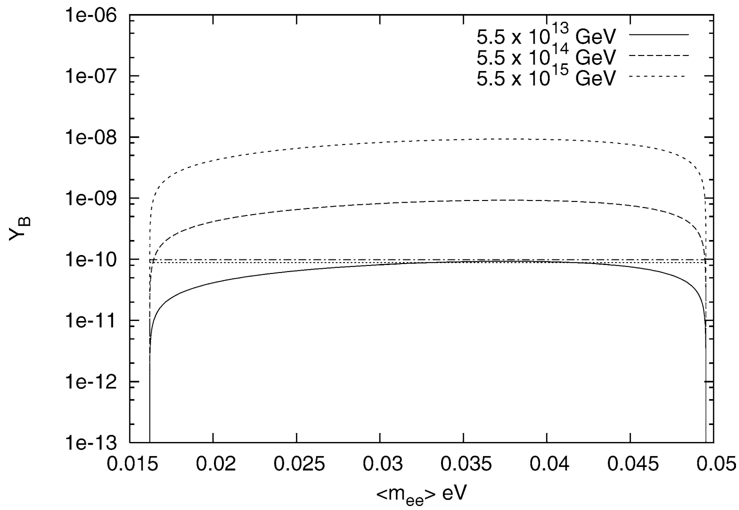

Fig. 3. Y_B as a function of $|\langle m_{ee}\rangle|$ for case IH 1-a, with various values of M_1 where $M_2/M_1 = 5$.

particular case. In Fig. 3, we present a correlation between Y_B and $|\langle m_{ee}\rangle|$ for IH 1-a. The inputs are taken to be the same as in Fig. 2. As the value of $|\langle m_{ee}\rangle|$ approaches to that with $\varphi = \pi/2$, the asymmetry is enhanced and the bound of M_1 becomes lower. The lower bound of M_1 as a function of Majorana phase or that of effective Majorana mass is obtained from the current cosmological observation of Y_B. In Fig. 4, we present a correlation between the lower bound of M_1 and $|\langle m_{ee}\rangle|$.

We examined the minimal seesaw mechanism of 3×2 Dirac matrix by starting our analysis with the masses of light neutrinos with tri/bi-maximal mixing in the basis where the charged-lepton Yukawa matrix and heavy Majorana neutrino mass matrix are diagonal. We found all possible Dirac mass textures which contain one zero entry or two in the matrix and evaluated the corresponding lepton asymmetries. The baryon asymmetry can be presented in terms of low energy observables, where only one Majorana CP phase among them remains yet unknown. The

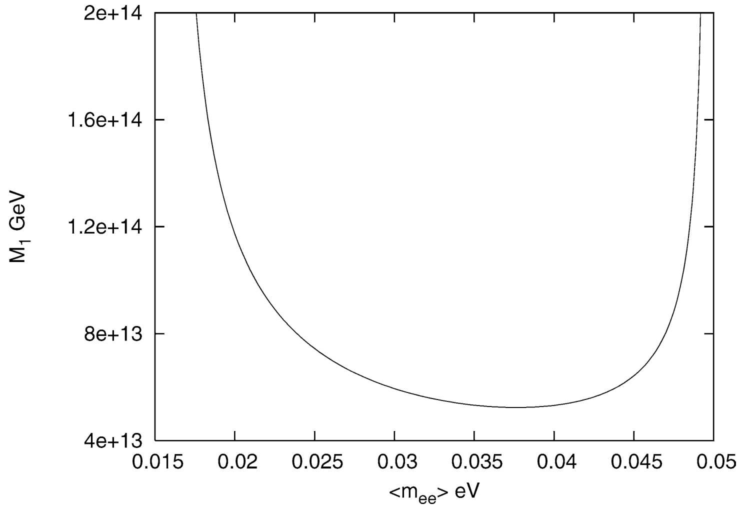

Fig. 4. The lower bound of M_1 as a function of $|\langle m_{ee}\rangle|$ for case IH 1-a.

numerical work exhibits the dependence of both the size of baryon asymmetry and the lower bound of M_1 upon the low energy CP phase to be clued from neutrinoless double beta decay.

Note added

After completing this work, we have been noticed that similar analysis for the hierarchical case in supersymmetric seesaw model appeared in Ref. [27].

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