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Anisotropic brane gravity with a confining potential

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Abstract

We consider an anisotropic brane world with Bianchi type I and V geometries where the mechanism of confining the matter on the brane is through the use of a confining potential. The resulting equations on the anisotropic brane are modified by an extra term that may be interpreted as the x-matter, providing a possible phenomenological explanation for the accelerated expansion of the universe. We obtain the general solution of the field equations in an exact parametric form for both Bianchi type I and V space–times. In the special case of a Bianchi type I the solutions of the field equations are obtained in an exact analytic form. Finally, we study the behavior of the observationally important parameters.

1. Introduction

The type Ia supernovae (SNe Ia) [1] observations provide the first evidence for the accelerating expansion of the present universe. These results, when combined with the observations on the anisotropy spectrum of cosmic microwave background (CMB) [2] and the results on the power spectrum of large scale structure (LSS) [3], strongly suggest that the universe is spatially flat and dominated by a component, though arguably exotic, with large negative pressure, referred to as dark energy [4]. The nature of such dark energy constitutes an open and tantalizing question connecting cosmology and particle physics. Different mechanisms have been suggested over the past few years to accommodate dark energy. The simplest form of dark energy is the vacuum energy (the cosmological constant). A tiny positive cosmological constant which can naturally explain the current acceleration would encounter many theoretical problems such as the fine-tuning problem and the coincidence problem. The former can be stated as the existence of an enormous gap between the vacuum expectation value, in other words the cosmological constant, in particle physics and that observed over cosmic scales. The absence of a fundamental mechanism which sets the cosmological constant to zero or to a very small value is also known as the cosmological constant problem. The latter however relates to the question of the near equality of energy densities of the dark energy and dark matter today.

Another possible form of dark energy is a dynamical, time dependent and spatially inhomogeneous component, called the quintessence [5]. An example of quintessence is the energy associated with a scalar field ϕ slowly evolving down its potential $V(\phi)$ [6,7]. Slow evolution is needed to obtain a negative pressure, $p_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$, so that the kinetic energy density is less than the potential energy density. Yet another phenomenological explanation based on current observational data is given by the x-matter (xCDM) model which is associated with an exotic fluid characterized by an equation of state $p_x = w_x \rho_x$ ($w_x < -\frac{1}{3}$ is the necessary condition to make a universe accelerate), where the parameter w_x can be a constant or more generally a function of time [8].

Over the past few years, we have been witnessing a phenomenal interest in the possibility that our observable four-dimensional (4D) universe may be viewed as a brane hypersurface embedded in a higher-dimensional bulk space. Physical matter fields are confined to this hypersurface, while gravity can propagate in the higher-dimensional space–time as well as on the brane. The most

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popular model in the context of brane world theory is that proposed by Randall and Sundrum (RS). In the so-called RSI model [9], the authors proposed a mechanism to solve the hierarchy problem with two branes, while in the RSII model [10], they considered a single brane with a positive tension, where 4D Newtonian gravity is recovered at low energies even if the extra dimension is not compact. This mechanism provides us with an alternative to compactification of extra dimensions. The cosmological evolution of such a brane universe has been extensively investigated and effects such as a quadratic density term in the Friedmann equations have been found [11–13]. This term arises from the imposition of the Israel junction conditions which is a relationship between the extrinsic curvature and energy–momentum tensor of the brane and results from the singular behavior in the energy–momentum tensor. There has been concerns expressed over applying such junction conditions in that they may not be unique. Indeed, other forms of junction conditions exist, so that different conditions may lead to different physical results [14]. Furthermore, these conditions cannot be used when more than one non-compact extra dimension is involved. To avoid such concerns, an interesting higher-dimensional model was introduced in [15] where particles are trapped on a 4-dimensional hypersurface by the action of a confining potential $\mathcal{V}$. In [16], the dynamics of test particles confined to a brane by the action of such potential at the classical and quantum levels were studied and the effects of small perturbations along the extra dimensions investigated. Within the classical limits, test particles remain stable under small perturbations and the effects of the extra dimensions are not felt by them, making them undetectable in this way. The quantum fluctuations of the brane cause the mass of a test particle to become quantized and, interestingly, the Yang–Mills fields appear as quantum effects. Also, in [17], a braneworld model was studied in which the matter is confined to the brane through the action of such a potential, rendering the use of any junction condition unnecessary and predicting a geometrical explanation for the accelerated expansion of the universe.

In brane theories the covariant Einstein equations are derived by projecting the bulk equations onto the brane. This was first done by Shiromizu, Maeda and Sasaki (SMS) [18] where the Gauss–Codazzi equations together with Israel junction conditions were used to obtain the Einstein field equations on the 3-brane. In a series of recent papers a number of authors [23,24] have presented detailed descriptions of the dynamic of homogeneous and anisotropic brane worlds in the SMS model. The study of anisotropic homogeneous brane world cosmological models has shown an important difference between these models and standard 4D general relativity, namely, that brane universes are born in an isotropic state. For the anisotropic Bianchi type I and V geometries, with a conformally flat bulk (vanishing Weyl tensor), this type of behavior has been found by exactly solving the field equations [25].

In this Letter, we follow [17] and consider an m-dimensional bulk space without imposing the Z_2 symmetry. As mentioned above, to localize the matter on the brane, a confining potential is used rather than a delta-function in the energy–momentum tensor. The resulting equations on the anisotropic brane are modified by an extra term that may be interpreted as the x-matter, providing a possible phenomenological explanation for the accelerated expansion of the universe. The behavior of the observationally important physical quantities such as anisotropy and deceleration parameter is studied in this scenario. We should emphasize here that there is a difference between the model presented in this work and models introduced in [19,20] in that in the latter no mechanism is introduced to account for the confinement of matter to the brane.

2. The model

In this section we present a brief review of the model proposed in [16]. Consider the background manifold $\bar{V}_4$ isometrically embedded in a pseudo-Riemannian manifold V_m by the map $\mathcal{Y}: \bar{V}_4 \to V_m$ such that

$$\mathcal{G}_{AB}\mathcal{Y}^A_{,\mu}\mathcal{Y}^B_{,\nu} = \bar{g}_{\mu\nu}, \qquad \mathcal{G}_{AB}\mathcal{Y}^A_{,\mu}\mathcal{N}^B_a = 0, \qquad \mathcal{G}_{AB}\mathcal{N}^A_a\mathcal{N}^B_b = g_{ab} = \pm 1, \tag{1}$$

where $\mathcal{G}_{AB}(\bar{g}_{\mu\nu})$ is the metric of the bulk (brane) space $V_m(\bar{V}_4)$ in arbitrary coordinates, $\{\mathcal{Y}^A\}(\{x^\mu\})$ is the basis of the bulk (brane) and $\mathcal{N}^A_a$ are $(m-4)$ normal unit vectors, orthogonal to the brane. Perturbation of $\bar{V}_4$ in a sufficiently small neighborhood of the brane along an arbitrary transverse direction ξ is given by

$$\mathcal{Z}^A\left(x^\mu, \xi^a\right) = \mathcal{Y}^A + (\mathcal{L}_\xi \mathcal{Y})^A, \tag{2}$$

where $\mathcal{L}$ represents the Lie derivative and ξ^a $(a = 1, 2, \ldots, m-4)$ is a small parameter along $\mathcal{N}^A_a$ that parameterizes the extra noncompact dimensions. By choosing ξ orthogonal to the brane, we ensure gauge independence [16] and have perturbations of the embedding along a single orthogonal extra direction $\bar{\mathcal{N}}_a$ giving local coordinates of the perturbed brane as

$$\mathcal{Z}^A_{,\mu}\left(x^\nu, \xi^a\right) = \mathcal{Y}^A_{,\mu} + \xi^a \bar{\mathcal{N}}^A_{a,\mu}\left(x^\nu\right). \tag{3}$$

In a similar manner, one can find that since the vectors $\bar{\mathcal{N}}^A$ depend only on the local coordinates x^μ, they do not propagate along the extra dimensions. The above assumptions lead to the embedding equations of the perturbed geometry

$$\mathcal{G}_{\mu\nu} = \mathcal{G}_{AB}\mathcal{Z}^A{}_{,\mu}\mathcal{Z}^B{}_{,\nu}, \qquad \mathcal{G}_{\mu a} = \mathcal{G}_{AB}\mathcal{Z}^A{}_{,\mu}\mathcal{N}^B{}_a, \qquad \mathcal{G}_{AB}\mathcal{N}^A{}_a\mathcal{N}^B{}_b = g_{ab}. \tag{4}$$

If we set $\mathcal{N}^A{}_a = \delta^A_a$, the metric of the bulk space can be written in the following matrix form

$$\mathcal{G}_{AB} = \begin{pmatrix} g_{\mu\nu} + A_{\mu c}A^c{}_\nu & A_{\mu a} \\ A_{\nu b} & g_{ab} \end{pmatrix}, \tag{5}$$

where

$$g_{\mu\nu} = \bar{g}_{\mu\nu} - 2\xi^a \bar{K}_{\mu\nu a} + \xi^a \xi^b \bar{g}^{\alpha\beta} \bar{K}_{\mu\alpha a} \bar{K}_{\nu\beta b}, \tag{6}$$

is the metric of the perturbed brane, so that

$$\bar{K}_{\mu\nu a} = -\mathcal{G}_{AB} \mathcal{Y}^A{}_{,\mu} \mathcal{N}^B{}_{a;\nu}, \tag{7}$$

represents the extrinsic curvature of the original brane (second fundamental form). We use the notation $A_{\mu c} = \xi^d A_{\mu cd}$, where

$$A_{\mu cd} = \mathcal{G}_{AB} \mathcal{N}^A{}_{d;\mu} \mathcal{N}^B{}_c = \bar{A}_{\mu cd}, \tag{8}$$

represents the twisting vector fields (the normal fundamental form). Any fixed ξ^a signifies a new perturbed geometry, enabling us to define an extrinsic curvature similar to the original one by

$$\tilde{K}_{\mu\nu a} = -\mathcal{G}_{AB} \mathcal{Z}^A{}_{,\mu} \mathcal{N}^B{}_{a;\nu} = \bar{K}_{\mu\nu a} - \xi^b \big(\bar{K}_{\mu\gamma a} \bar{K}^\gamma{}_{\nu b} + A_{\mu ca} A^c{}_{b\nu}\big). \tag{9}$$

Note that definitions (5) and (9) require

$$\tilde{K}_{\mu\nu a} = -\frac{1}{2} \frac{\partial \mathcal{G}_{\mu\nu}}{\partial \xi^a}. \tag{10}$$

In geometric language, the presence of gauge fields $A_{\mu a}$ tilts the embedded family of sub-manifolds with respect to the normal vector $\mathcal{N}^A$. According to our construction, the original brane is orthogonal to the normal vector $\mathcal{N}^A$. However, Eq. (4) shows that this is not true for the deformed geometry. Let us change the embedding coordinates and set

$$\mathcal{X}^A_{,\mu} = \mathcal{Z}^A_{,\mu} - g^{ab} \mathcal{N}^A_a A_{b\mu}. \tag{11}$$

The coordinates $\mathcal{X}^A$ describe a new family of embedded manifolds whose members are always orthogonal to $\mathcal{N}^A$. In this coordinates the embedding equations of the perturbed brane is similar to the original one, described by Eq. (1), so that $\mathcal{Y}^A$ is replaced by $\mathcal{X}^A$. This new embedding of the local coordinates is suitable for obtaining induced Einstein field equations on the brane. The extrinsic curvature of a perturbed brane then becomes

$$K_{\mu\nu a} = -\mathcal{G}_{AB} \mathcal{X}^A_{,\mu} \mathcal{N}^B_{a;\nu} = \bar{K}_{\mu\nu a} - \xi^b \bar{K}_{\mu\gamma a} \bar{K}^\gamma{}_{\nu b} = -\frac{1}{2} \frac{\partial g_{\mu\nu}}{\partial \xi^a}, \tag{12}$$

which is the generalized York's relation and shows how the extrinsic curvature propagates as a result of the propagation of the metric in the direction of extra dimensions. The components of the Riemann tensor of the bulk written in the embedding vielbein $\{\mathcal{X}^A_{,\alpha}, \mathcal{N}^A_a\}$, lead to the Gauss–Codazzi equations [22]

$$R_{\alpha\beta\gamma\delta} = 2g^{ab} K_{\alpha[\gamma a} K_{\delta]\beta b} + \mathcal{R}_{ABCD} \mathcal{X}^A_{,\alpha} \mathcal{X}^B_{,\beta} \mathcal{X}^C_{,\gamma} \mathcal{X}^D_{,\delta}, \tag{13}$$

$$2K_{\alpha[\gamma c;\delta]} = 2g^{ab} A_{[\gamma ac} K_{\delta]\alpha b} + \mathcal{R}_{ABCD} \mathcal{X}^A_{,\alpha} \mathcal{N}^B_c \mathcal{X}^C_{,\gamma} \mathcal{X}^D_{,\delta}, \tag{14}$$

where $\mathcal{R}_{ABCD}$ and $R_{\alpha\beta\gamma\delta}$ are the Riemann tensors for the bulk and the perturbed brane respectively. Contracting the Gauss equation (13) on α and γ, we find

$$R_{\mu\nu} = \big(K_{\mu\alpha c} K_\nu{}^{\alpha c} - K_c K_{\mu\nu}{}^c\big) + \mathcal{R}_{AB} \mathcal{X}^A_{,\mu} \mathcal{X}^B_{,\nu} - g^{ab} \mathcal{R}_{ABCD} \mathcal{N}^A_a \mathcal{X}^B_{,\mu} \mathcal{X}^C_{,\nu} \mathcal{N}^D_b, \tag{15}$$

and the Einstein tensor of the brane is given by

$$G_{\mu\nu} = G_{AB} \mathcal{X}^A_{,\mu} \mathcal{X}^B_{,\nu} + Q_{\mu\nu} + g^{ab} \mathcal{R}_{AB} \mathcal{N}^A_a \mathcal{N}^B_b g_{\mu\nu} - g^{ab} \mathcal{R}_{ABCD} \mathcal{N}^A_a \mathcal{X}^B_{,\mu} \mathcal{X}^C_{,\nu} \mathcal{N}^D_b, \tag{16}$$

where

$$Q_{\mu\nu} = -g^{ab}\big(K^\gamma_{\mu a} K_{\gamma\nu b} - K_a K_{\mu\nu b}\big) + \frac{1}{2}\big(K_{\alpha\beta a} K^{\alpha\beta a} - K_a K^a\big) g_{\mu\nu}. \tag{17}$$

As can be seen from the definition of $Q_{\mu\nu}$, it is independently a conserved quantity, that is $Q^{\mu\nu}{}_{;\nu} = 0$ [19]. Using the decomposition of the Riemann tensor into the Weyl curvature, the Ricci tensor and the scalar curvature

$$\mathcal{R}_{ABCD} = C_{ABCD} - \frac{2}{(m-2)} (\mathcal{G}_{B[D} \mathcal{R}_{C]A} - \mathcal{G}_{A[D} \mathcal{R}_{C]B}) - \frac{2}{(m-1)(m-2)} \mathcal{R}(\mathcal{G}_{A[D} \mathcal{G}_{C]B}), \tag{18}$$

we obtain the 4D Einstein equations as

$$\begin{aligned} G_{\mu\nu} = {} & G_{AB} \mathcal{X}^A_{,\mu} \mathcal{X}^B_{,\nu} + Q_{\mu\nu} - \mathcal{E}_{\mu\nu} + \frac{m-3}{(m-2)} g^{ab} \mathcal{R}_{AB} \mathcal{N}^A_a \mathcal{N}^B_b g_{\mu\nu} \\ & - \frac{m-4}{(m-2)} \mathcal{R}_{AB} \mathcal{X}^A_{,\mu} \mathcal{X}^B_{,\nu} + \frac{m-4}{(m-1)(m-2)} \mathcal{R} g_{\mu\nu}, \end{aligned} \tag{19}$$

where

$$\mathcal{E}_{\mu\nu} = g^{ab} C_{ABCD} \mathcal{N}_a^A \mathcal{X}^B_{,\mu} \mathcal{N}_b^D \mathcal{X}^C_{,\nu}, \tag{20}$$

is the electric part of the Weyl tensor C_{ABCD}. Now, let us write the Einstein equation in the bulk space as

$$G^{(b)}_{AB} + \Lambda^{(b)} \mathcal{G}_{AB} = \alpha^* S_{AB}, \tag{21}$$

where

$$S_{AB} = T_{AB} + \frac{1}{2}\mathcal{V}\mathcal{G}_{AB}, \tag{22}$$

here $\alpha^* = \frac{1}{M_*^{m-2}}$ (M_* is the fundamental scale of energy in the bulk space), $\Lambda^{(b)}$ is the cosmological constant of the bulk and T_{AB} is the energy–momentum tensor of the matter confined to the brane through the action of the confining potential $\mathcal{V}$. We require $\mathcal{V}$ to satisfy three general conditions: firstly, it has a deep minimum on the non-perturbed brane, secondly, depends only on extra coordinates and thirdly, the gauge group representing the subgroup of the isometry group of the bulk space is preserved by it [16]. The vielbein components of the energy–momentum tensor are given by

$$S_{\mu\nu} = S_{AB} \mathcal{X}^A_{,\mu} \mathcal{X}^B_{,\nu}, \qquad S_{\mu a} = S_{AB} \mathcal{X}^A_{,\mu} \mathcal{N}^B_a, \qquad S_{ab} = S_{AB} \mathcal{N}^A_a \mathcal{N}^B_b. \tag{23}$$

Use of Eq. (22) then gives

$$\mathcal{R}_{AB} = -\frac{\alpha^*}{(m-2)} \mathcal{G}_{AB} S + \frac{2}{(m-2)} \Lambda^{(b)} \mathcal{G}_{AB} + \alpha^* S_{AB}, \tag{24}$$

and

$$\mathcal{R} = -\frac{2}{m-2}\left(\alpha^* S - m\Lambda^{(b)}\right). \tag{25}$$

Substituting $\mathcal{R}_{AB}$ and $\mathcal{R}$ from the above into Eq. (19) and using Eq. (23), we obtain

$$G_{\mu\nu} = Q_{\mu\nu} - \mathcal{E}_{\mu\nu} + \frac{(m-3)}{(m-2)} \alpha^* g^{ab} S_{ab} g_{\mu\nu} + \frac{2\alpha^*}{(m-2)} S_{\mu\nu} - \frac{(m-4)(m-3)}{(m-1)(m-2)} \alpha^* S g_{\mu\nu} + \frac{(m-7)}{(m-1)} \Lambda^{(b)} g_{\mu\nu}. \tag{26}$$

On the other hand, again from Eq. (21), the trace of the Codazzi equation (14) gives the “gravi-vector equation”

$$K^\delta_{a\gamma;\delta} - K_{a,\gamma} - A_{ba\gamma} K^b + A_{ba\delta} K^{b\delta} + B_{a\gamma} = \frac{3(m-4)}{m-2} \alpha^* S_{a\gamma}, \tag{27}$$

where

$$B_{a\gamma} = g^{mn} C_{ABCD} \mathcal{N}^A_m \mathcal{N}^B_a \mathcal{X}^C_{,\gamma} \mathcal{N}^D_n. \tag{28}$$

Finally, the “gravi-scalar equation” is obtained from the contraction of (15), (19) and using equation (21)

$$\alpha^* \left[\frac{m-5}{m-1} S - g^{mn} S_{mn}\right] g_{ab} = \frac{m-2}{6}(Q + R + W) g_{ab} - \frac{4}{m-1} \Lambda^{(b)} g_{ab}, \tag{29}$$

where

$$W = g^{ab} g^{mn} C_{ABCD} \mathcal{N}^A_m \mathcal{N}^B_b \mathcal{N}^C_n \mathcal{N}^D_a. \tag{30}$$

Eqs. (26)–(30) represent the projections of the Einstein field equations on the brane–brane, bulk–brane, and bulk–bulk directions.

As was mentioned in the introduction, localization of matter on the brane is realized in this model by the action of a confining potential. Since the potential $\mathcal{V}$ is assumed to have a minimum on the brane, which can be taken as zero, localization of matter may simply be realized by taking Eq. (22) and consider its components on the brane, in which case we may write

$$\alpha\tau_{\mu\nu} = \frac{2\alpha^*}{(m-2)} T_{\mu\nu}, \qquad T_{\mu a} = 0, \qquad T_{ab} = 0, \tag{31}$$

where α is the scale of energy on the brane. Now, the induced Einstein field equations on the original brane can be written as

$$G_{\mu\nu} = \alpha\tau_{\mu\nu} - \frac{(m-4)(m-3)}{2(m-1)} \alpha\tau g_{\mu\nu} - \Lambda g_{\mu\nu} + Q_{\mu\nu} - \mathcal{E}_{\mu\nu}, \tag{32}$$

where $\Lambda = -\frac{(m-7)}{(m-1)}\Lambda^{(b)}$ and $Q_{\mu\nu}$ is a conserved quantity which according to [19] may be considered as an energy–momentum tensor of a dark energy fluid representing the x-matter, the more common phrase being 'x-Cold-Dark Matter' (xCDM). This matter has the most general form of the equation of state which is characterized by the following conditions [21]: violation of the strong energy condition at the present epoch for $\omega_x < -1/3$ where $p_x = \omega_x \rho_x$, local stability i.e. $c_s^2 = \delta p_x/\delta\rho_x \geqslant 0$ and preservation of causality i.e. $c_s \leqslant 1$. Ultimately, we have three different types of 'matter' on the right-hand side of Eq. (32), namely, ordinary confined conserved matter represented by $\tau_{\mu\nu}$, the matter represented by $Q_{\mu\nu}$ which will be discussed later and finally, the Weyl matter represented by $\mathcal{E}_{\mu\nu}$.

3. Field equations on the anisotropic brane

In the following we will investigate the influence of the extrinsic curvature terms on the anisotropic universe described by Bianchi type I and V geometries. We restrict our analysis to a constant curvature bulk, so that $\mathcal{E}_{\mu\nu} = 0$. The constant curvature bulk is characterized by the Riemann tensor

$$\mathcal{R}_{ABCD} = k_*(\mathcal{G}_{AC}\mathcal{G}_{BD} - \mathcal{G}_{AD}\mathcal{G}_{BC}), \tag{33}$$

where $\mathcal{G}_{AB}$ denotes the bulk metric components in arbitrary coordinates and k_* is either zero for the flat bulk, or proportional to a positive or negative bulk cosmological constant respectively, corresponding to two possible signatures (4, 1) for the dS_5 bulk and (3, 2) for the AdS_5 bulk. We take, in the embedding equations, $g^{55} = \varepsilon = \pm 1$. With this assumption the Gauss–Codazzi equations reduce to

$$R_{\alpha\beta\gamma\delta} = \frac{1}{\varepsilon}(K_{\alpha\gamma}K_{\beta\delta} - K_{\alpha\delta}K_{\beta\gamma}) + k_*(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}), \tag{34}$$

$$K_{\alpha[\beta;\gamma]} = 0. \tag{35}$$

The effective equations derived in the previous section with constant curvature bulk can be written as

$$G_{\mu\nu} = \alpha\tau_{\mu\nu} - \lambda g_{\mu\nu} + Q_{\mu\nu}. \tag{36}$$

Here, λ is the effective cosmological constant in four dimensions with $Q_{\mu\nu}$ being a completely geometrical quantity given by

$$Q_{\mu\nu} = \frac{1}{\varepsilon}\left[\left(KK_{\mu\nu} - K_{\rho\alpha}K^{\alpha}_{\nu}\right) + \frac{1}{2}\left(K_{\alpha\beta}K^{\alpha\beta} - K^2\right)g_{\mu\nu}\right], \tag{37}$$

where $K = g^{\mu\nu}K_{\mu\nu}$. To proceed further, the confined source on the brane should now be specified. The most common matter source used in cosmology is that of a perfect fluid which, in co-moving coordinates, is given by

$$\tau_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}, \qquad u_\mu = -\delta^0_\mu, \qquad p = (\gamma - 1)\rho, \quad 1 \leqslant \gamma \leqslant 2, \tag{38}$$

where $\gamma = 2$ represents the stiff cosmological fluid describing the high energy density regime of the early universe.

From a formal point of view the Bianchi type I and V geometries are described by the line element

$$ds^2 = -dt^2 + a_1^2(t)\,dx^2 + a_2^2(t)e^{-2\beta x}\,dy^2 + a_3^2(t)e^{-2\beta x}\,dz^2. \tag{39}$$

The metric for the Bianchi type I geometry corresponds to the case $\beta = 0$, while for the Bianchi type V case we have $\beta = 1$. Here $a_i(t)$, $i = 1, 2, 3$, are the expansion factors in different spatial directions. For later convenience, we define the following variables [25]

$$\upsilon = \prod_{i=1}^{3} a_i, \qquad H_i = \frac{\dot{a}_i}{a_i}, \quad i = 1, 2, 3, \qquad 3H = \sum_{i=1}^{3} H_i, \qquad \Delta H_i = H_i - H, \quad i = 1, 2, 3. \tag{40}$$

In Eq. (40), υ is the volume scale factor, H_i, $i = 1, 2, 3$, are the directional Hubble parameters, and H is the mean Hubble parameter. From Eq. (40) we also obtain $H = \frac{\dot{\upsilon}}{3\upsilon}$. The physical quantities of observational importance in cosmology are the expansion scalar Θ, the mean anisotropy parameter A, and the deceleration parameter q, which are defined according to

$$\Theta = 3H, \qquad 3A = \sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \qquad q = \frac{d}{dt}H^{-1} - 1 = -H^{-2}\left(\dot{H} + H^2\right). \tag{41}$$

We note that $A = 0$ for an isotropic expansion. Moreover, the sign of the deceleration parameter indicates how the universe expands. A positive sign for q corresponds to the standard decelerating models whereas a negative sign indicates an accelerating expansion in late times.

Using the York's relation

$$K_{\mu\nu a} = -\frac{1}{2}\frac{\partial g_{\mu\nu}}{\partial\xi^a}, \tag{42}$$

we realize that in a diagonal metric, $K_{\mu\nu a}$ is diagonal. After separating the spatial components, the Codazzi equations reduce to (here $\alpha, \beta, \gamma, \sigma = 1, 2, 3$)

$$K^{\alpha}{}_{\gamma a,\sigma} + K^{\beta}{}_{\gamma a}\Gamma^{\alpha}{}_{\beta\sigma} = K^{\alpha}{}_{\sigma a,\gamma} + K^{\beta}{}_{\sigma a}\Gamma^{\alpha}{}_{\beta\gamma}, \tag{43}$$

$$K^{\alpha}{}_{\gamma a,0} + \frac{\dot{a}_i}{a_i}K^{\alpha}{}_{\gamma a} = \frac{\dot{a}_i}{a_i}K^{0}{}_{0a}, \quad i = 1, 2, 3. \tag{44}$$

The first equation gives $K^{1}{}_{1a,\sigma} = 0$ for $\sigma \neq 1$, since $K^{1}{}_{1a}$ does not depend on the spatial coordinates. Repeating the same procedure for $\alpha, \gamma = i$, $i = 2, 3$, we obtain $K^{2}{}_{2a,\sigma} = 0$ for $\sigma \neq 2$ and $K^{3}{}_{3a,\sigma} = 0$ for $\sigma \neq 3$. Assuming $K^{1}{}_{1a} = K^{2}{}_{2a} = K^{3}{}_{3a} = b_a(t)$, where $b_a(t)$ are arbitrary functions of t, the second equation gives

$$\dot{b}_a + \frac{\dot{a}_i}{a_i}b_a = \frac{\dot{a}_i}{a_i}K^{0}{}_{0a}, \quad i = 1, 2, 3. \tag{45}$$

Summing Eq. (45) we find

$$K_{00a} = -\left(\frac{3\dot{b}_a v}{\dot{v}} + b_a\right). \tag{46}$$

For $\mu, \nu = 1, 2, 3$ we obtain

$$K_{\mu\nu a} = b_a g_{\mu\nu}. \tag{47}$$

Assuming further that the functions b_a are equal and denoting $b_a = b$, $\theta = \frac{\dot{b}}{b}$ and $\Theta = \frac{\dot{v}}{v}$, we find from Eq. (37) that

$$K_{\alpha\beta}K^{\alpha\beta} = b^2\left(9\frac{\theta^2}{\Theta^2} + 6\frac{\theta}{\Theta} + 4\right), \qquad K = b\left(3\frac{\theta}{\Theta} + 4\right), \tag{48}$$

$$Q_{\mu\nu} = -\frac{3b^2}{\varepsilon}\left(\frac{2\theta}{\Theta} + 1\right)g_{\mu\nu}, \quad \mu, \nu = 1, 2, 3, \qquad Q_{00} = \frac{3b^2}{\varepsilon}. \tag{49}$$

Now, using these relations and Eq. (36), the modified Friedmann equations become

$$3\dot{H} + \sum_{i=1}^{3} H_i^2 = \lambda - \frac{\alpha}{2}\rho(3\gamma - 2) + \frac{3b^2}{\varepsilon}\left(\frac{3v\dot{b}}{\dot{v}b} + 1\right), \tag{50}$$

$$\frac{1}{v}\frac{d}{dt}(vH_i) = 2\beta^2 v^{-2/3} + \lambda - \frac{\alpha}{2}\rho(\gamma - 2) + \frac{3b^2}{\varepsilon}\left(\frac{v\dot{b}}{\dot{v}b} + 1\right), \quad i = 1, 2, 3. \tag{51}$$

For $\beta = 0$ we obtain the field equations for Bianchi type I geometry, while $\beta = 1$ gives the Bianchi type V equations on the brane world. These equations are modified with respect to the standard equations by the components of the extrinsic curvature. Such term may be used to offer an explanation for the x-matter. In the next section we discuss the ramifications of this term on the cosmology of our model. We also note the implicit effects of the bulk signature ε on the expansion of the universe.

For the sake of completeness, let us compare the model presented in this work to the usual brane worlds models where the Israel junction condition is used to calculate the extrinsic curvature in terms of the energy–momentum tensor on the brane and its trace, that is

$$K_{\mu\nu} = -\frac{1}{2}\alpha^{*2}\left(\tau_{\mu\nu} - \frac{1}{3}\tau g_{\mu\nu}\right), \tag{52}$$

where α^* is proportional to the gravitational constant in the bulk. If we did that, we would obtain $b(t) = -\frac{1}{6}\alpha^{*2}\rho$, which, upon its substitution in Eq. (50), gives

$$3\dot{H} + \sum_{i=1}^{3} H_i^2 = \lambda + \frac{\alpha}{2}\rho(2 - 3\gamma) + \frac{\alpha^{*4}}{12\varepsilon}\rho^2(1 - 3\gamma), \tag{53}$$

which is same as Eq. (16) in [25]. Therefore, the emergence of a ρ^2 term in the Friedmann equations is a consequence of the discontinuity in the bulk and the brane system. The existence of this term either does not agree with observations or requires extra parameters and fine tuning.

4. Dark energy and role of extrinsic curvature

As we noted before, $Q_{\mu\nu}$ is an independently conserved quantity, suggesting that an analogy with the energy momentum of an uncoupled non-conventional energy source would be in order. To evaluate the compatibility of such geometrical model with the present experimental data, we identify $Q_{\mu\nu}$ with x-matter [21] by defining $Q_{\mu\nu}$ as a perfect fluid and write

$$Q_{\mu\nu} = \frac{1}{\alpha}\big[(\rho_x + p_x)u_\mu u_\nu + p_x g_{\mu\nu}\big], \quad p_x = (\gamma_x - 1)\rho_x. \tag{54}$$

Comparing $Q_{\mu\nu}$, $\mu, \nu = 1, 2, 3$ and Q_{00} from Eq. (54) with the components of $Q_{\mu\nu}$ and Q_{00} given by Eq. (49), we obtain

$$p_x = -\frac{3b^2}{\alpha\varepsilon}\left(\frac{2\theta}{\Theta} + 1\right), \qquad \rho_x = \frac{3b^2}{\alpha\varepsilon}. \tag{55}$$

Eq. (54) was chosen in accordance with the weak-energy condition corresponding to the positive energy density and negative pressure with $\varepsilon = +1$. Use of the above equations leads to an equation for $b(t)$

$$\frac{\dot b}{b} = -\frac{\gamma_x}{2}\left(\frac{\dot v}{v}\right). \tag{56}$$

If γ_x is taken as a constant, the solution for $b(t)$ is

$$b(t) = b_0 v^{-\gamma_x/2}. \tag{57}$$

Using Eq. (55) and this solution, the energy density of xCDM becomes

$$\rho_x = \frac{3b_0^2}{\alpha} v^{-\gamma_x}. \tag{58}$$

A brief discussion on the energy–momentum conservation on the brane would be appropriate at this point. The contracted Bianchi identities in the bulk space, $G^{AB}{}_{;A} = 0$, using Eq. (21), imply

$$\left(T^{AB} + \frac{1}{2}\mathcal{V}\mathcal{G}^{AB}\right)_{;A} = 0. \tag{59}$$

Since the potential $\mathcal{V}$ has a minimum on the brane, the above conservation equation reduces to

$$\tau^{\mu\nu}{}_{;\mu} = 0, \tag{60}$$

and gives

$$\dot\rho + \frac{\dot v}{v}\gamma\rho = 0. \tag{61}$$

Thus, the time evolution of the energy density of the matter is given by

$$\rho = \rho_0 v^{-\gamma}. \tag{62}$$

Using the geometrical energy density for $Q_{\mu\nu}$ and the evolution law of the matter energy density, the field equations (50)–(51) become

$$3\dot H + \sum_{i=1}^{3} H_i^2 = \lambda + \frac{\alpha}{2}\rho_0(2 - 3\gamma)v^{-\gamma} + \frac{3}{2}b_0^2(2 - 3\gamma_x)v^{-\gamma_x}, \tag{63}$$

$$\frac{1}{v}\frac{d}{dt}(vH_i) = 2\beta^2 v^{-2/3} + \lambda + \frac{\alpha}{2}\rho_0(2 - \gamma)v^{-\gamma} + \frac{3}{2}b_0^2(2 - \gamma_x)v^{-\gamma_x}, \quad i = 1, 2, 3. \tag{64}$$

Summing Eq. (64) we find

$$\frac{1}{v}\frac{d}{dt}(vH) = 2\beta^2 v^{-2/3} + \lambda + \frac{\alpha}{2}\rho_0(2 - \gamma)v^{-\gamma} + \frac{3}{2}b_0^2(2 - \gamma_x)v^{-\gamma_x}. \tag{65}$$

Now, substituting back Eq. (65) into Eq. (64) we obtain

$$H_i = H + \frac{h_i}{v}, \quad i = 1, 2, 3, \tag{66}$$

with h_i, $i = 1, 2, 3$, being constants of integration satisfying the consistency condition $\sum_{i=1}^{3} h_i = 0$. The basic equation describing the dynamics of the anisotropic brane world with a constant curvature bulk can be written as

$$\ddot v = 6\beta^2 v^{1/3} + 3\lambda v + \frac{3\alpha}{2}\rho_0(2 - \gamma)v^{1-\gamma} + \frac{9}{2}b_0^2(2 - \gamma_x)v^{1-\gamma_x}. \tag{67}$$

Here, we note that for a stiff fluid ($\gamma = 2$), the dynamics of the matter on the brane is solely determined by the geometrical matter (xCDM). The general solution of Eq. (67) becomes

$$t - t_0 = \int \left(9\beta^2 v^{4/3} + 3\lambda v^2 + 3\alpha\rho_0 v^{2-\gamma} + 9b_0^2 v^{2-\gamma_x} + C\right)^{-1/2} dv, \tag{68}$$

where C is a constant of integration. The time variation of the physically important parameters described above in the exact parametric form, with v taken as a parameter, is given by

$$\Theta = 3H = \frac{(9\beta^2 v^{4/3} + 3\lambda v^2 + 3\alpha\rho_0 v^{2-\gamma} + 9b_0^2 v^{2-\gamma_x} + C)^{1/2}}{v}, \tag{69}$$

$$a_i = a_{0i} v^{1/3} \exp\left[h_i \int v^{-1}\left(9\beta^2 v^{4/3} + 3\lambda v^2 + 3\alpha\rho_0 v^{2-\gamma} + 9b_0^2 v^{2-\gamma_x} + C\right)^{-1/2} dv\right], \quad i = 1, 2, 3, \tag{70}$$

$$A = 3h^2\left(9\beta^2 v^{4/3} + 3\lambda v^2 + 3\alpha\rho_0 v^{2-\gamma} + 9b_0^2 v^{2-\gamma_x} + C\right)^{-1}, \tag{71}$$

$$q = \frac{9\beta^2 v^{4/3} + \frac{9\alpha\gamma}{2}\rho_0 v^{2-\gamma} + \frac{27\gamma_x}{2} b_0^2 v^{2-\gamma_x} + 3C}{(9\beta^2 v^{4/3} + 3\lambda v^2 + 3\alpha\rho_0 v^{2-\gamma} + 9b_0^2 v^{2-\gamma_x} + C)} - 1, \tag{72}$$

where $h^2 = \sum_{i=1}^{3} h_i^2$. In addition, the integration constants h_i and C must satisfy the consistency condition $h^2 = \frac{2}{3}C$. For $\beta = 0$ we obtain the general solutions for Bianchi type I geometry, while $\beta = 1$ gives the Bianchi type V solutions on the anisotropic brane world.

In a matter dominated Bianchi type I universe, $\gamma = 1$ with $\gamma_x = 0$, Eq. (68) becomes

$$t - t_0 = \int \frac{dv}{\sqrt{3\alpha\rho_0 v + 9b_0^2 v^2 + C}}, \tag{73}$$

where for later convenience, we take $C = \frac{3\alpha^2\varrho_0^2}{12b_0^2}$. The time dependence of the volume scale factor of the Bianchi type I universe is given by

$$v(t) = e^{3b_0(t-t_0)} - \frac{\alpha\rho_0}{6b_0^2}, \tag{74}$$

which for $t = t_0 + \frac{1}{3b_0}\ln(\frac{\alpha\rho_0}{6b_0^2})$ becomes zero. By reparameterizing the initial value of the cosmological time according to $e^{-3b_0 t_0} = \frac{\alpha\rho_0}{6b_0^2}$, the evolution of the anisotropic brane universe starts at $t = 0$ from a singular state $v(t = 0) = 0$. Therefore the expansion scalar, scale factors, mean anisotropy, and decelerating parameter are given by

$$\Theta(t) = \frac{3b_0 e^{3b_0(t-t_0)}}{e^{3b_0(t-t_0)} - \frac{\alpha\rho_0}{6b_0^2}}, \tag{75}$$

$$a_i = a_{0i}\left[e^{3b_0(t-t_0)} - \frac{\alpha\rho_0}{6b_0^2}\right]^{1/3}\left[1 - \frac{\alpha\rho_0}{6b_0^2} e^{-3b_0(t-t_0)}\right]^{\frac{2b_0 h_i}{\alpha\rho_0}}, \quad i = 1, 2, 3, \tag{76}$$

$$A(t) = \frac{h^2}{3b_0^2 e^{6b_0(t-t_0)}}, \tag{77}$$

$$q(t) = \frac{\alpha\rho_0}{2b_0^2} e^{-3b_0(t-t_0)} - 1. \tag{78}$$

We consider $\lambda = 0$ and show that, within the context of the present model, the extrinsic curvature can be used to account for the accelerated expansion of the universe. In Fig. 1 we have plotted the behavior of the deceleration parameter for different values of γ. The behavior of this parameter shows that when the geometrical energy density is positive and the pressure is negative the AdS_5 bulk is not compatible with the expansion of the universe. Also, this behavior is much dependent on the range of the values that γ_x can take. The use of the de Sitter bulk with $\rho_x > 0$ allows us to use the wealth of available data from the recent measurements to determine limits on the values of γ_x in our geometric model. For having an accelerating universe we distinguish $\gamma_x < \frac{2}{3}$. As mentioned before, $q(t) > 0$ corresponds to the standard decelerating models whereas $q(t) < 0$ indicates an accelerating expansion at late times. Therefore, the universe undergoes an accelerated expansion at late times in the absence of a positive cosmological constant. As has been noted in [20], it should be emphasized here too that the geometrical approach considered here is based on three basic postulates, namely, the confinement of the standard gauge interactions, the existence of quantum gravity in the bulk and finally, the embedding of the brane world. All other model dependent properties such as warped metric, mirror symmetries, radion

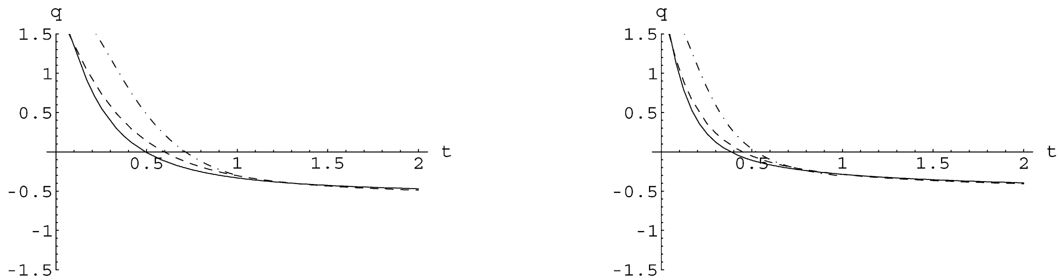

Fig. 1. Left, the deceleration parameter of the Bianchi type I universe and right, the same parameter in the Bianchi type V universe for the de Sitter bulk with $\gamma = 1$ (solid line), $\gamma = 4/3$ (dashed line), $\gamma = 2$ (dot-dashed line), $\gamma_x = 0.3$ and $\lambda = 0$.

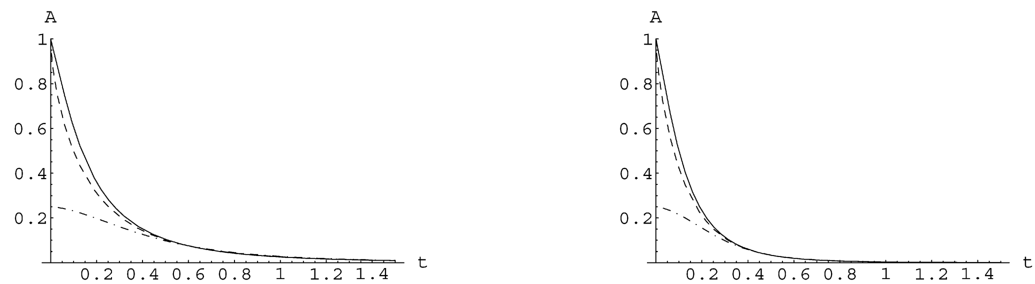

Fig. 2. Left, the anisotropy parameter of the Bianchi type I universe and right, the same parameter in the Bianchi type V universe for the de Sitter bulk with $\gamma = 1$ (solid line), $\gamma = 4/3$ (dashed line), $\gamma = 2$ (dot-dashed line), $\gamma_x = 0.3$ and $\lambda = 0$.

or extra scalar fields, fine tuning parameters like the tension of the brane and the choice of a junction condition are left out as much as possible in our calculations.

To understanding the behavior of the mean anisotropy parameter in our model, let us consider it as a function of the volume scale factor

$$A(v) = \frac{3h^2}{9\beta^2 v^{4/3} + 3\lambda v^2 + 3\alpha\rho_0 v^{2-\gamma} + 9b_0^2 v^{2-\gamma_x} + C}. \tag{79}$$

The behavior of the anisotropy parameter at the initial state depends on the values of γ and γ_x. For an accelerating universe we obtain $\gamma_x < \frac{2}{3}$. From Eq. (79), in the limit $v \to 0$ (singular state) and taking $\gamma_x < \frac{2}{3}$, we find

$$\lim_{v\to 0} A(v) = \frac{3h^2}{C}, \quad 1 \leqslant \gamma \leqslant 2. \tag{80}$$

Therefore for a brane world scenario with a confining potential, the initial state is always anisotropic. In our model the behavior of the anisotropy parameter coincides with the standard 4D cosmology and is different from the brane world models where a delta-function in the energy–momentum tensor is used [26] to confine the mater on the brane. The behavior of the mean anisotropy parameter of the Bianchi type I and V geometries are illustrated, for $\gamma_x = 0.3$ and different values of γ, in Fig. 2. The behavior of this parameter shows that the universe starts from a singular state with maximum anisotropy and ends up in an isotropic de Sitter inflationary phase at late times. In Fig. 3 we have plotted the deceleration parameter and the anisotropy parameter of the Bianchi type I geometry for $\gamma = 1$ and different values of $\gamma_x = 0, 0.3, 0.5$.

At this point it would be appropriate to compare our model with other forms of dark energy such as the 4D quintessence. One may consider a 4D effective Lagrangian whose variation with respect to $g_{\mu\nu}$ would result in the dynamical equations (36) compatible with the embedding and with the confined matter hypotheses [20]. In contrast to the standard model, our model corresponds to an Einstein–Hilbert Lagrangian which is modified by extrinsic curvature terms. The resulting Einstein equations are thus modified by the term $Q_{\mu\nu}$. Since, as was mentioned before, this quantity is independently conserved, there is no exchange of energy between this geometrical correction and the confined matter source. Such an aspect has one important consequence however; if $Q_{\mu\nu}$ is to be related to dark energy, as we did in this Letter, it does not exchange energy with ordinary matter, like the coupled quintessence models [27]. The coupled scalar field models may avoid the cosmic coincidence problem with the available data being used to fix the corresponding dynamics and, consequently, the scalar field potential responsible for the present accelerating phase of the universe.

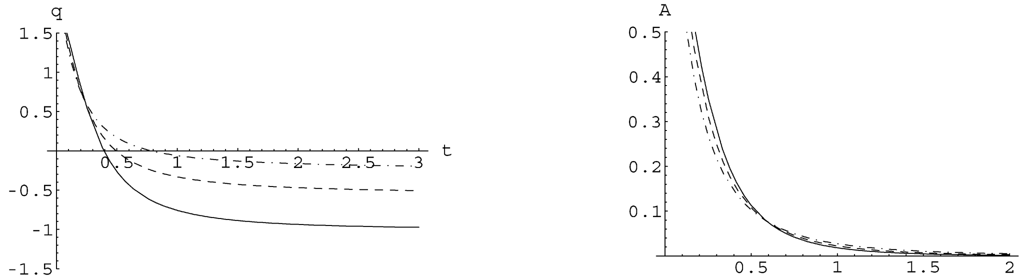

Fig. 3. Left, the deceleration parameter of the Bianchi type I geometry and right, the anisotropy parameter in the Bianchi type I geometry for the de Sitter bulk with $\gamma_x = 0$ (solid line), $\gamma_x = 0.3$ (dashed line), $\gamma_x = 0.5$ (dot-dashed line), $\gamma = 1$ and $\lambda = 0$.

5. Conclusions

In this Letter, we have studied an anisotropic brane world model in which the matter is confined to the brane through the action of a confining potential, rendering the use of any junction condition redundant. We have shown that in an anisotropic brane world embedded in a constant curvature dS_5 bulk the accelerating expansion of the universe can be a consequence of the extrinsic curvature and thus a purely geometrical effect. The study of the behavior of the anisotropy parameter shows that in our model the universe starts as a singular state with maximum anisotropy and reaches, for both Bianchi type I and V space–times, an isotropic state in the late time limit. The study of this scenario in an inhomogeneous brane will be the subject of a future investigation.

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