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Generalizing TOPSIS for fuzzy multiple-criteria group decision-making

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Abstract

In this paper, we generalize TOPSIS to fuzzy multiple-criteria group decision-making (FMCGDM) in a fuzzy environment. TOPSIS is one of the well-known methods for multiple-criteria decision-making (MCDM). Most of the steps of TOPSIS can be easily generalized to a fuzzy environment, except max and min operations in finding the ideal solution and negative ideal solution. Thus we propose two operators Up and Lo which satisfy the partial ordering relation on fuzzy numbers to the generalization of TOPSIS. In generalized TOPSIS, these two operations (Up and Lo) are employed to find ideal and negative ideal solutions under a fuzzy environment. Then the FMCGDM problem can be solved effectively and efficiently.

Keywords: FMCGDM; Fuzzy TOPSIS; Ideal solution; Negative ideal solution; Up and Lo

1. Introduction

Decision-making is the procedure to find the best alternative among a set of feasible alternatives. Sometimes, decision-making problems considering several criteria are called multi-criteria decision-making (MCDM) problems [1–19]. An MCDM problem with m alternatives and n criteria can be expressed in matrix format as follows:

$$G = \begin{matrix} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} \begin{matrix} \begin{matrix} C_1 & C_2 & \cdots & C_n \end{matrix} \\ \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ G_{m1} & G_{m2} & \cdots & G_{mn} \end{bmatrix} \end{matrix},$$
$$W = [W_1, W_2, \ldots, W_n],$$

where $A_1, A_2, \ldots, A_m$ are feasible alternatives, $C_1, C_2, \ldots, C_n$ are evaluation criteria, G_{ij} is the performance rating of alternative A_i under criterion C_j, and W_j is the weight of criterion C_j.

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The MCDM problems may be divided into two kinds of problem. One is the classical MCDM problems [1–3], among which the ratings and the weights of criteria are measured in crisp numbers. Another is the fuzzy multi-criteria decision-making (FMCDM) problems [4–19], among which the ratings and the weights of criteria evaluated on imprecision, subjective and vagueness are usually expressed by linguistic terms and then set into fuzzy numbers [20–22]. The technique for order preference by similarity to ideal solution (TOPSIS) proposed Hwang and Yoon [1] is one of the well-known methods for classical MCDM. The underlying logic of TOPSIS is to define the ideal solution and negative ideal solution. The ideal solution is the solution that maximizes the benefit criteria and minimizes the cost criteria, whereas the negative ideal solution is the solution that maximizes the cost criteria and minimizes the benefit criteria. In short, the ideal solution consists of all of best values attainable of criteria, whereas the negative ideal solution is composed of all worst values attainable of criteria. The optimal alternative is the one which has the shortest distance from the ideal solution and the farthest distance from the negative ideal solution.

Since TOPSIS is a well-known method for classical MCDM, many researchers have applied TOPSIS to solve FMCDM problems in the past. Some of them [6,17] defuzzify fuzzy ratings and weights into crisp values, whereas the defuzzification will lose some information. Others, such as Chen, Liang, Raj and Kumar [7,14,16], supposed that TOPSIS should be generalized in a fuzzy environment. These methods can decline the loss of fuzzy information, but there are some problems in their works. In Liang's work, he utilized the maximizing set and minimizing set [23] to rank a set of fuzzy evaluated values presented by approximate trapezoidal fuzzy numbers against criteria. However, the distance values from two different alternatives to the ideal solution or negative ideal solution would be indifferent on one criterion, if the intersections of two different evaluated values and the best or worst values on the same criterion are Ø. Raj and Kumar also used the maximizing set and minimizing set to rank alternatives presented by approximate trapezoidal fuzzy numbers. Their process could combat the problem of Liang, but their computation is more difficult and complex than Liang's. In Chen's work, he constructed the normalized values for the ideal solution and negative ideal solution on criteria. The normalized values for the ideal solution and negative ideal solution on criteria are always (1, 1, 1) and (0, 0, 0) respectively. (1, 1, 1) and (0, 0, 0) are extreme values which are possibly far from away true max and min values, so the extreme values could not represent the max and min values of TOPSIS. Beside the disadvantage of extreme values, the weighted ratings on criteria in Chen's work are presented by triangular fuzzy numbers as ratings, and weights are triangular fuzzy numbers. In fact, the multiplication between two triangular fuzzy numbers should be an approximate triangular fuzzy number, not a triangular fuzzy number. Thus, the computation of Chen is very simple, but the weighted ratings could not express approximate triangular fuzzy numbers.

To avoid these above problems, we have proposed a fuzzy multiple-criteria group decision-making (FMCGDM) method [19] called fuzzy TOPSIS in a fuzzy environment. In fuzzy TOPSIS, most of the steps of TOPSIS are easily generalized to a fuzzy environment except the min and max operations. The max and min operations are in TOPSIS for finding negative and ideal solutions, whereas the min and max operations are inadequate under a fuzzy environment. We have proposed two operators, MAX and MIN, which satisfy the partial ordering relation on triangular fuzzy numbers. By MAX and MIN operations, we can find the ideal and negative ideal solutions, whereas these fuzzy numbers against criteria to ideal and negative ideal solutions gained by MAX and MIN operations may be not found on these performance ratings of possible alternatives. In this paper, we will propose a new generalized TOPSIS which substitutes Up and Lo operations for MAX and MIN operations. By Up and Lo operations, a set of fuzzy numbers are ranked quickly. Then, we find the ideal and negative ideal solutions easily, and the fuzzy numbers against criteria on ideal and negative ideal solutions can be also found on these possible alternatives.

For the sake of clarity, the related concepts of mathematics are presented in Section 2. The FMCGDM method about generalized TOPSIS is expressed in Section 3. Finally, a numerical example of FMCGDM is illustrated in Section 4.

2. Preliminaries

In this section, we review some basic notions of fuzzy sets [20–22]. These notions of fuzzy sets are expressed as follows.

Definition 2.1. Let U be a universe set. A fuzzy set A of U is defined by a membership function $\mu_A(x) \to [0, 1]$, where $\mu_A(x)$, $\forall x \in U$, indicates the degree of x in A.

Definition 2.2. The α-cut of fuzzy set A is a crisp set $A_\alpha = \{x \mid \mu_A(x) \geq \alpha\}$. The support A is the crisp set $\text{Supp}(A) = \{x \mid \mu_A(x) > 0\}$. A is normal iff $\sup_{x \in U} \mu_A(x) = 1$, where U is the universe set.

Definition 2.3. A fuzzy subset A of universe set U is convex iff $\mu_A(\lambda x + (1-\lambda)y) \geq (\mu_A(x) \wedge \mu_A(y))$, $\forall x, y \in U$, $\forall \lambda \in [0, 1]$, where $\wedge$ denotes the minimum operator.

Definition 2.4. A is a fuzzy number iff A is normal and convex fuzzy set of U.

Definition 2.5. A triangular fuzzy number A is a fuzzy number with piecewise linear membership function μ_A defined by

$$\mu_A = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2, \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3, \\ 0, & \text{otherwise}, \end{cases}$$

which can be denoted as a triplet (a_1, a_2, a_3).

Definition 2.6. Let $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ be two triangular fuzzy numbers. A distance measure function $d(A, B)$ can be defined [7]:

$$d(A, B) = \sqrt{\frac{1}{3}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - a_3)^2]}.$$

Definition 2.7. Let A be a fuzzy number. Then A_α^L and A_α^U are defined as

$$A_\alpha^L = \inf_{\mu_A(z) \geq \alpha}(z)$$

and

$$A_\alpha^U = \sup_{\mu_A(z) \geq \alpha}(z)$$

respectively.

Beside these previous concepts of fuzzy sets, the other related notions are stated as follows.

Definition 2.8. Let $L(S)$ and $U(S)$ be two boundaries for a set of fuzzy numbers $S = \{X_1, X_2, \ldots, X_n\}$, defined as

$$L(S) = \min_{1 \leq j \leq n} \{x_{lj}\}$$

and

$$U(S) = \max_{1 \leq j \leq n} \{x_{rj}\},$$

where X_j is a fuzzy number denoted as the triplet (x_{lj}, x_{mj}, x_{rj}) for $j = 1, 2, \ldots, n$.

Definition 2.9. Let $R_{(S)}(X_j)$ indicate the relation of X_j between $L(S)$ and $U(S)$, where $S = \{X_1, X_2, \ldots, X_n\}$ is a set of fuzzy numbers. Define

$$R_{(S)}(X_j) = \frac{\int_0^1 ((X_j)_\alpha^L - L(S)) \mathrm{d}\alpha}{\int_0^1 ((X_j)_\alpha^L - L(S))\, \mathrm{d}\alpha + \int_0^1 (U(S) - (X_j)_\alpha^U)\, \mathrm{d}\alpha}.$$

Lemma 2.1. *Let $A = (a_1, a_2, a_3)$ be a triangular fuzzy number in S, then*

$$R_{(S)}(A) = \frac{a_1 + a_2 - 2L(S)}{a_1 - a_3 + 2(U(S) - L(S))}.$$

Definition 2.10. Let $\succ$ be a binary relation on fuzzy numbers. Assume A and B to be two fuzzy numbers in S. $A \succ B$ iff $R_{(S)}(A) \geq R_{(S)}(B)$, then A is said to be bigger than or equal to B.

Lemma 2.2. *$\succ$ is a partial ordering relation* [24] *on fuzzy numbers.*

Proof. (1) $\succ$ is reflexive. Since $A \succ A$ iff $R_{(S)}(A) \geq R_{(S)}(A), \forall A \in S$.

(2) $\succ$ is anti-symmetric. If $A \succ B(R_{(S)}(A) \geq R_{(S)}(B))$ and $B \succ A(R_{(S)}(B) \geq R_{(S)}(A))$ then A and B are indifferent $(R_{(S)}(A) = R_{(S)}(B)), \forall A, B \in S$.

(3) $\succ$ is transitive. Assume C to be another fuzzy number for $C \in S$. If $A \succ B$ and $B \succ C$ then $A \succ C$. Since $R_{(S)}(A) \geq R_{(S)}(B)$ and $R_{(S)}(B) \geq R_{(S)}(C)$, we can know that $R_{(S)}(A) \geq R_{(S)}(C), \forall A, B, C \in S$.

It is obvious that $\succ$ is reflexive, anti-symmetric and transitive. Thus $\succ$ is the partial ordering relation on the set of fuzzy numbers.

Definition 2.11. $S = \{X_1, X_2, \ldots, X_n\}$ denotes a set of fuzzy numbers. Define

$$X^+ = \mathrm{Up}(S) = \mathrm{Up}(\{X_1, X_2, \ldots, X_n\}) \text{ to be the fuzzy maximum value in } S$$

and

$$X^- = \mathrm{Lo}(S) = \mathrm{Lo}(\{X_1, X_2, \ldots, X_n\}) \text{ to be the fuzzy minimum value in } S,$$

where

$$X^+ = X_i \quad \text{if } X_i \succ X_t \quad \forall X_t \in S, \quad \text{i.e., } \max_{t=1,2,\ldots,n} \{R_{(S)}(X_t)\} = R_{(S)}(X_i), \quad \text{for } t = 1, 2, \ldots, n$$

and

$$X^- = X_j \quad \text{if } X_t \succ X_j \quad \forall X_t \in S, \quad \text{i.e., } \min_{t=1,2,\ldots,n} \{R_{(S)}(X_t)\} = R_{(S)}(X_j), \quad \text{for } t = 1, 2, \ldots, n.$$

Lemma 2.3. *For $S = \{X_1, X_2, \ldots, X_n\}$, the* Up$(S)$ *and* Lo(S) *operations satisfy the partial ordering relation on S.*

3. The FMCGDM method

Based on the two operations Up and Lo, the FMCGDM method being the generalized TOPSIS in a fuzzy environment is presented as follows. First, performance ratings and weights are evaluated with linguistic terms [25,26]. These linguistic ratings, employed by experts to represent the performances under certain criteria, are very poor (VP), poor (P), medium poor (MP), fair (F), medium good (MG), good (G) and very good (VG). The linguistic weights for presenting the importance of criteria are very low (VL), low (L), medium (M), high (H) and very high (VH). Assume that all linguistic terms can be represented with triangular fuzzy numbers, and that these fuzzy numbers are limited in the interval [0,1]. Thus these performance ratings would be not normalized. Let G_{ijk} be the performance rating given by expert E_k to alternative A_i against criterion C_j, where $G_{ijk} = (g_{1ijk}, g_{2ijk}, g_{3ijk})$ is a triangular fuzzy number, $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$; $k = 1, 2, \ldots, p$. Then G_{ij} is the average performance rating of alternative A_i against criterion C_j. Let $\otimes$ and $\oplus$ be extended multiplication and addition defined by the extension principle; thus

$$G_{ij} = (g_{1ij}, g_{2ij}, g_{3ij}) = (1/p) \otimes (G_{ij1} \oplus G_{ij2} \oplus G_{ij3} \oplus \cdots \oplus G_{ijp}),$$

where $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$.

By the extension principle, we have

$$g_{1ij} = \sum_{k=1}^{p} g_{1ijk}/p,$$

$$g_{2ij} = \sum_{k=1}^{p} g_{2ijk}/p$$

and

$$g_{3ij} = \sum_{k=1}^{p} g_{3ijk}/p.$$

A decision-making matrix G is composed of the performance ratings of alternative $A_1, A_2, \ldots, A_m$; that is,

$$G = [G_{ij}]_{m \times n}.$$

$[G_{i1}, G_{i2}, \ldots, G_{in}]$ denotes the performance ratings of alternative A_i on all criteria.

Let A^- and A^+ denote the negative ideal solution and ideal solution respectively; thus

$$A^- = [G_1^-, G_2^-, \ldots, G_n^-]$$

and

$$A^+ = [G_1^+, G_2^+, \ldots, G_n^+],$$

where

$$G_j^- = \text{Lo}(\{G_{1j}, G_{2j}, \ldots, G_{mj}\})$$

and

$$G_j^+ = \text{Up}(\{G_{1j}, G_{2j}, \ldots, G_{mj}\}),$$

for $j = 1, 2, \ldots, n$.

By the partial ordering relation, we know

$$G_j^+ \succ G_{ij} \succ G_j^-,$$

where $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$.

By Definition 2.6, we compute the distance from alternatives to the ideal solution (or negative ideal solution). Let d_{ij}^- and d_{ij}^+ be the distance from G_{ij} to G_j^- and G_j^+ respectively; thus

$$d_{ij}^- = d(G_{ij}, G_j^-)$$

and

$$d_{ij}^+ = d(G_{ij}, G_j^+),$$

where $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$.

Let $W_{jk} = (w_{1jk}, w_{2jk}, w_{3jk})$ denote the weight evaluated by expert E_k under criterion C_j, where $j = 1, 2, \ldots, n$; $k = 1, 2, \ldots, p$. Assume W_j to be the average weight on criterion C_j; thus

$$W_j = (w_{1j}, w_{2j}, w_{3j}) = (1/p) \otimes (W_{j1} \oplus W_{j2} \oplus W_{j3} \oplus \cdots \oplus W_{jp}),$$

where $j = 1, 2, \ldots, n$.

By the extension principle, we have

$$w_{1j} = \sum_{k=1}^{p} w_{1jk}/p,$$

$$w_{2j} = \sum_{k=1}^{p} w_{2jk}/p$$

and

$$w_{3j} = \sum_{k=1}^{p} w_{3jk}/p.$$

Table 1
The weighted distance values of alternatives

Alternative	Negative weighted distance	Positive weighted distance
A_1	D_1^-	D_1^+
A_2	D_2^-	D_2^+
$\vdots$	$\vdots$	$\vdots$
A_m	D_m^-	D_m^+

Then D_i^- and D_i^+ express the weighted distance from alternative A_i to negative ideal solution A^- and ideal solution A^+ respectively. Let

$$D_i^- = \sum_{j=1}^{n} W_j \otimes d_{ij}^-$$

and

$$D_i^+ = \sum_{j=1}^{n} W_j \otimes d_{ij}^+,$$

where $i = 1, 2, \dots, m$.

An alternative under any one criterion can be presented by two weighted distance values, i.e., positive weighted distance value and negative weighted distance value. The positive weighted distance is the distance form the alternative to the ideal solution, and the negative weighted distance is that from the alternative to the negative ideal solution. These weighted distance values are presented in Table 1.

Thus the weighted distance of A_i can be expressed by $[D_i^-, D_i^+]$. Let

$$LD^- = \mathrm{Lo}(\{D_1^-, D_2^-, \dots, D_m^-\}),$$
$$UD^- = \mathrm{Up}(\{D_1^-, D_2^-, \dots, D_m^-\}),$$
$$LD^+ = \mathrm{Lo}(\{D_1^+, D_2^+, \dots, D_m^+\})$$

and

$$UD^+ = \mathrm{Up}(\{D_1^+, D_2^+, \dots, D_m^+\}).$$

By the two operations of Lo and Up, we know that the negative ideal solution is $[LD^-, UD^+]$ and the ideal solution is $[UD^-, LD^+]$ for weighted distance values of all alternatives. Let A_i^- denote the distance from $[D_i^-, D_i^+]$ to $[LD^-, UD^+]$, and A_i^+ denote the distance from $[D_i^-, D_i^+]$ to $[UD^-, LD^+]$. Define

$$A_i^- = d(D_i^-, LD^-) + d(D_i^+, UD^+)$$

and

$$A_i^+ = d(D_i^-, UD^-) + d(D_i^+, LD^+),$$

where $i = 1, 2, \dots, m$.

Finally, the closeness coefficient A_i^* of alternative A_i is defined:

$$A_i^* = \frac{A_i^-}{A_i^- + A_i^+},$$

where $i = 1, 2, \dots, m$.

Obviously, $0 \le A_i^* \le 1$, where $i = 1, 2, \dots, m$. If $A_i^* = 0$, alternative A_i would be the negative ideal solution. In contrast, $A_i^* = 1$ denotes A_i to be ideal solution. An alternative A_i is closer to the negative ideal solution and farther from the ideal solution as A_i^* approaches 0, whereas alternative A_i is closer to the ideal solution and farther from

Table 2
The linguistic performance ratings of the three airports

		A_1	A_2	A_3
(E_1, E_2, E_3, E_4)	C_1	(MG, G, G, VG)	(VG, G, MG, MG)	(MG, F, MG, F)
	C_2	(MG, VG, G, MG)	(G, G, VG, G)	(G, VG, G, G)
	C_3	(F, F, F, MG)	(VG, G, MG, G)	(VG, VG, G, G)
	C_4	(VG, G, VG, VG)	(F, MG, MG, MG)	(MG, MG, G, MG)
	C_5	(G, MG, F, G)	(MG, G, F, G)	(F, VG, G, MG)
	C_6	(VG, G, VG, VG)	(MG, VG, G, G)	(G, F, MG, G)
	C_7	(F, GMG, G)	(G, MG, VG, MG)	(VG, MG, VG, G)
	C_8	(MG, VG, MG, G)	(VG, F, VG, G)	(G, G, VG, MG)
	C_9	(VG, G, G, VG)	(MG, G, G, VG)	(VG, G, VG, VG)
	C_{10}	(G, G, G, F)	(G, MG, G, G)	(G, VG, G, MG)
	C_{11}	(MG, VG, MG, MG)	(VG, MG, G, MG)	(VG, MG, G, G)
	C_{12}	(G, VG, G, MG)	(VG, G, VG, G)	(G, G, VG, MG)
	C_{13}	(F, MG, MG, G)	(F, MG, F, MG)	(G, G, VG, VG)
	C_{14}	(VG, MG, MG, VG)	(MG, MG, G, VG)	(F, MG, G, MG)
	C_{15}	(G, VG, F, G)	(MG, F, VG, G)	(F, F, F, F)

the negative ideal solution as A_i^* approaches 1. Therefore, we can determine the ranking order of a set of alternatives according to their closeness coefficients, and then the best alternative is found from the set of alternatives.

4. Numerical example

A numerical example is illustrated for presenting generalized TOPSIS to evaluate airport operation performance with group decision-making. Assume that three airports A_1, A_2 and A_3 are evaluated by four experts E_1, E_2, E_3 and E_4 under a fuzzy environment for operation performance [3,6,17,27–33] against 15 criteria, $C_1, C_2, \ldots, C_{15}$. These criteria are:
return on operation profit to capital (C_1),
comfort and cleanness of airport terminal (C_2),
trolleys approach travelers (C_3),
signal and direction (C_4),
aerodrome control (C_5),
security measures (C_6),
check-in and check-out time (C_7),
aircraft take-off and loading time (C_8),
traffic connecting city or out-bound (C_9),
courtesy of crew (C_{10}),
parking lots (C_{11}),
airport scale (C_{12}),
navigation equipment (C_{13}),
noise pollution control (C_{14}), and
flight safety control (C_{15}),
where $W_1, W_2, \ldots, W_{15}$ are related weights of the criteria $C_1, C_2, \ldots, C_{15}$ respectively. The elements of the linguistic performance rating set, {VP, P, MP, F, MG, G, VG}, are used to present seven situations of performance ratings, and then set into the following fuzzy numbers, where $\mathrm{VP} = (0, 0, 0.2)$, $\mathrm{P} = (0, 0.2, 0.4)$, $\mathrm{MP} = (0.2, 0.4, 0.5)$, $\mathrm{F} = (0.4, 0.5, 0.6)$, $\mathrm{MG} = (0.5, 0.6, 0.8)$, $\mathrm{G} = (0.6, 0.8, 1)$ and $\mathrm{VG} = (0.8, 1, 1)$. The elements of the linguistic weight set, {VL, L, M, H, VH}, are used to describe five states for weights, and then set into the following numbers, where $\mathrm{VL} = (0, 0, 0.3)$, $\mathrm{L} = (0, 0.3, 0.5)$, $\mathrm{M} = (0.3, 0.5, 0.7)$, $\mathrm{H} = (0.5, 0.7, 1)$ and $\mathrm{VH} = (0.7, 1, 1)$. The linguistic ratings and weights of operation performance employed by four experts under 15 criteria for the three airports are presented in Table 2. The fuzzy average ratings of the three airports computed from Table 2 are shown in Table 3.

From Table 3, we know that the ideal and negative ideal solutions are

$$A^+ = [G_1^+, G_2^+, \ldots, G_{15}^+]$$

Table 3
The average ratings of the three airport on 15 criteria

G_{ij}	A_1	A_2	A_3
C_1	(0.625, 0.8, 0.95)	(0.6, 0.75, 0.9)	(0.45, 0.55, 0.7)
C_2	(0.6, 0.75, 0.9)	(0.65, 0.85, 1)	(0.65, 0.85, 1)
C_3	(0.425, 0.525, 0.65)	(0.625, 0.8, 0.95)	(0.7, 0.9, 1)
C_4	(0.75, 0.95, 1)	(0.475, 0.575, 0.75)	(0.525, 0.65, 0.85)
C_5	(0.4, 0.5, 0.6)	(0.525, 0.675, 0.85)	(0.575, 0.725, 0.85)
C_6	(0.75, 0.95, 1)	(0.625, 0.8, 0.95)	(0.525, 0.675, 0.85)
C_7	(0.525, 0.675, 0.85)	(0.6, 0.75, 0.9)	(0.675, 0.85, 0.95)
C_8	(0.6, 0.75, 0.9)	(0.65, 0.825, 0.9)	(0.625, 0.8, 0.95)
C_9	(0.7, 0.9, 1)	(0.625, 0.8, 0.95)	(0.75, 0.95, 1)
C_{10}	(0.55, 0.725, 0.9)	(0.575, 0.75, 0.95)	(0.625, 0.8, 0.95)
C_{11}	(0.575, 0.7, 0.85)	(0.6, 0.75, 0.9)	(0.625, 0.8, 0.95)
C_{12}	(0.625, 0.8, 0.95)	(0.7, 0.9, 1)	(0.625, 0.8, 0.95)
C_{13}	(0.5, 0.625, 0.8)	(0.45, 0.55, 0.7)	(0.7, 0.9, 1)
C_{14}	(0.65, 0.8, 0.9)	(0.6, 0.75, 0.9)	(0.5, 0.625, 0.8)
C_{15}	(0.6, 0.775, 0.9)	(0.575, 0.725, 085)	(0.4, 0.5, 0.6)

and

$$A^- = [G_1^-, G_2^-, \ldots, G_{15}^-],$$

where

$$\begin{aligned}
&G_1^+ = (0.625, 0.8, 0.95), \quad G_1^- = (0.45, 0.55, 0.7),\\
&G_2^+ = (0.65, 0.85, 1), \quad G_2^- = (0.6, 0.75, 0.9),\\
&G_3^+ = (0.7, 0.9, 1), \quad G_3^- = (0.425, 0.525, 0.65),\\
&G_4^+ = (0.75, 0.95, 1), \quad G_4^- = (0.475, 0.575, 0.75),\\
&G_5^+ = (0.575, 0.725, 0.85), \quad G_5^- = (0.4, 0.5, 0.6),\\
&G_6^+ = (0.75, 0.95, 1), \quad G_6^- = (0.525, 0.675, 0.85),\\
&G_7^+ = (0.675, 0.85, 0.95), \quad G_7^- = (0.525, 0.675, 0.85),\\
&G_8^+ = (0.65, 0.825, 0.9), \quad G_8^- = (0.6, 0.75, 0.9),\\
&G_9^+ = (0.75, 0.95, 1), \quad G_9^- = (0.625, 0.8, 0.95),\\
&G_{10}^+ = (0.625, 0.8, 0.95), \quad G_{10}^- = (0.55, 0.725, 0.9),\\
&G_{11}^+ = (0.625, 0.8, 0.95), \quad G_{11}^- = (0.575, 0.7, 0.85),\\
&G_{12}^+ = (0.7, 0.9, 1), \quad G_{12}^- = (0.625, 0.8, 0.95),\\
&G_{13}^+ = (0.7, 0.9, 1), \quad G_{13}^- = (0.45, 0.55, 0.7),\\
&G_{14}^+ = (0.65, 0.8, 0.9), \quad G_{14}^- = (0.5, 0.625, 0.8),\\
&G_{15}^+ = (0.6, 0.775, 0.9), \quad G_{15}^- = (0.4, 0.5, 0.6).
\end{aligned}$$

Then we can calculate the distance values $d(G_{ij}, G_j^+)$ and $d(G_{ij}, G_j^-)$ from $G_{ij} (i = 1, 2, 3;\ j = 1, 2, \ldots, 15)$ to G_j^+ and G_j^- respectively. Table 4 lists these distance values.

Table 5 shows the related weights of the 15 criteria employed by the experts.

From the Table 5, the average weights against the 15 criteria are calculated:

$$\begin{aligned}
&W_1 = (0.45, 0.675, 0.85),\\
&W_2 = (0.5, 0.725, 925),\\
&W_3 = (0.35, 0.55, 0.775),\\
&W_4 = (0.325, 0.575, 0.725),
\end{aligned}$$

Table 4
The distance values for the three airports on 15 criteria

	A_1		A_2		A_3	
	$d(G_{1j}, G_j^+)$	$d(G_{1j}, G_j^-)$	$d(G_{2j}, G_j^+)$	$d(G_{2j}, G_j^-)$	$d(G_{3j}, G_j^+)$	$d(G_{3j}, G_j^-)$
C_1	0	0.2278	0.0433	0.1848	0.2278	0
C_2	0.0866	0	0	0.0866	0	0.0866
C_3	0.336	0	0.0777	0.2618	0	0.336
C_4	0	0.3048	0.3048	0	0.2332	0.0777
C_5	0.2189	0	0.0408	0.1904	0	0.2189
C_6	0	0.2227	0.1164	0.109	0.2227	0
C_7	0.1451	0	0.0777	0.0677	0	0.1451
C_8	0.0520	0	0	0.052	0.0354	0.0433
C_9	0.0408	0.0777	0.1164	0	0	0.1164
C_{10}	0.0677	0	0.0408	0.0354	0	0.0677
C_{11}	0.0866	0	0.0433	0.0433	0	0.0866
C_{12}	0.0777	0	0	0.0777	0.0777	0
C_{13}	0.2278	0.0777	0.3028	0	0	0.3028
C_{14}	0	0.1451	0.0408	0.109	0.1451	0
C_{15}	0	0.2618	0.0433	0.2189	0.2618	0

Table 5
The linguistic weights for 15 criteria

	E_1	E_2	E_3	E_4
C_1	M	VH	M	H
C_2	H	H	M	VH
C_3	M	M	H	M
C_4	L	M	VH	M
C_5	VH	VH	VH	VH
C_6	VH	H	VH	VH
C_7	H	VH	M	H
C_8	M	H	VH	M
C_9	M	M	H	M
C_{10}	L	M	H	VH
C_{11}	VH	H	VH	M
C_{12}	H	H	M	L
C_{13}	H	M	H	H
C_{14}	M	H	M	H
C_{15}	H	VH	H	VH

$$W_5 = (0.7, 1, 1),$$
$$W_6 = (0.65, 0.925, 1),$$
$$W_7 = (0.5, 0.725, 0.925),$$
$$W_8 = (0.45, 0.675, 0.85),$$
$$W_9 = (0.35, 0.55, 0.775),$$
$$W_{10} = (0.375, 0.625, 0.8),$$
$$W_{11} = (0.55, 0.8, 0.925),$$
$$W_{12} = (0.325, 0.55, 0.8),$$
$$W_{13} = (0.45, 0.65, 0.925),$$
$$W_{14} = (0.4, 0.6, 0.85)$$

and

$$W_{15} = (0.6, 0.85, 1).$$

The weighted distance values of three airports are presented as follows.

$$D_1^+ = (0.6251, 0.9316, 1.1766),$$
$$D_2^+ = (0.5472, 0.8343, 1.0680),$$
$$D_3^+ = (0.5794, 0.8701, 1.0628),$$
$$D_1^- = (0.6236, 0.9379, 1.1545),$$
$$D_2^- = (0.7168, 1.0579, 1.2884),$$
$$D_3^- = (0.6814, 1.0180, 1.2913);$$

thus

$$UD^+ = (0.6251, 0.9316, 1.1766),$$
$$LD^+ = (0.5472, 0.8343, 1.0680),$$
$$UD^- = (0.7168, 1.0579, 1.2884),$$
$$LD^- = (0.6236, 0.9379, 1.1545),$$

and

$$d(D_1^+, UD^+) = 0, \qquad d(D_1^+, LD^+) = 0.0954,$$
$$d(D_2^+, UD^+) = 0.0954, \qquad d(D_2^+, LD^+) = 0,$$
$$d(D_3^+, UD^+) = 0.0792, \qquad d(D_3^+, LD^+) = 0.0280,$$
$$d(D_1^-, UD^-) = 0.1169, \qquad d(D_1^-, LD^-) = 0,$$
$$d(D_2^-, UD^-) = 0, \qquad d(D_2^-, LD^-) = 0.1169,$$
$$d(D_3^-, UD^-) = 0.0308, \qquad d(D_3^-, LD^-) = 0.0974.$$

From these previous distance values, A_i^+ and A_i^- $(i = 1, 2, 3)$ can be calculated:

$$A_1^+ = d(D_1^+, LD^+) + d(D_1^-, UD^-) = 0.0954 + 0.1169 = 0.2123,$$
$$A_2^+ = d(D_2^+, LD^+) + d(D_2^-, UD^-) = 0 + 0 = 0,$$
$$A_3^+ = d(D_3^+, LD^+) + d(D_3^-, UD^-) = 0.0280 + 0.0308 = 0.0588,$$
$$A_1^- = d(D_1^+, UD^+) + d(D_1^-, LD^-) = 0 + 0 = 0,$$
$$A_2^- = d(D_2^+, UD^+) + d(D_2^-, LD^-) = 0.0954 + 0.1169 = 0.2123$$

and

$$A_3^- = d(D_3^+, UD^+) + d(D_3^-, LD^-) = 0.0792 + 0.0974 = 0.1766.$$

Finally, the evaluated results about the operation performance of the three airports are presented as follows.

$$A_1^* = \frac{0}{0 + 0.2123} = 0,$$
$$A_2^* = \frac{0.2123}{0.2123 + 0} = 1$$

and

$$A_3^* = \frac{0.1766}{01766 + 0.0588} = 0.7502.$$

Clearly, the ranking order is A_2, A_3 and A_1 by comparing their closeness coefficients. Therefore, the best performance is in A_2.

5. Conclusion

In this paper, we present a method for FMCGDM. With our method, TOPSIS is generalized under a fuzzy environment to solve FMCGDM problems. In the generalized TOPSIS, finding the ideal solution and negative ideal solution is easy, because we propose Up and Lo operations on fuzzy numbers to find the ideal solution and negative ideal solution. The Up and Lo operators, satisfying the partial ordering relation on fuzzy numbers, can rank a set of fuzzy numbers quickly. By Up and Lo operations, TOPSIS can be easily generalized in a fuzzy environment and then FMCGDM problems are solved effectively and efficiently.

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