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# Engineering an achromatic Bessel beam using a phase-only spatial light modulator and an iterative Fourier transformation algorithm

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ABSTRACT

Bessel illumination is an established method in optical imaging and manipulation to achieve an extended depth of field without compromising the lateral resolution. When broadband or multicolour imaging is required, wavelength-dependent changes in the radial profile of the Bessel illumination can complicate further image processing and analysis.

We present a solution for engineering a multicolour Bessel beam that is easy to implement and promises to be particularly useful for broadband imaging applications. A phase-only spatial light modulator (SLM) in the image plane and an iterative Fourier Transformation algorithm (IFTA) are used to create an annular light distribution in the back focal plane of a lens. The 2D Fourier transformation of such a light ring yields a Bessel beam with a constant radial profile for different wavelength.



## 1. Introduction

Bessel beams were first presented by Durnin et al. [1]. They have the same intensity distribution (proportional to a zero-order Bessel function of the first kind) in every plane normal to the optical axis [2]. The radially symmetric beam profile exhibits a central maximum surrounded by many concentric side lobes. Theoretically, these side lobes extend indefinitely and carry infinite amount of energy. Since real Bessel beam realizations are always aperture-limited, they are only approximations with an unchanged intensity profile over an extended, but finite region. For simplicity, we will refer to such zero order quasi-Bessel beam realizations as *Bessel beams* throughout this paper.

The two-dimensional Fourier transformation of a Bessel beam amplitude distribution yields a ring spectrum in $k$-space. For an ideal Bessel, this spectrum only consists of an off-axis delta function at constant $|k|$. In an experimental realization the approximated Bessel beam is composed of a set of plane waves under a small range of angles and the interference maxima of their superposition form the Bessel beam. Hence, the interference at any position $z_0$ on the optical axis is independent of previous $z$ and, unlike a Gaussian beam, the Bessel beam's central maximum can reappear behind a partial obstruction on the optical axis [4], which is interesting for Optical Tweezers [5] and Extended Focus microscopy [6].

There are several established methods of creating Bessel beams. The refractive axicon [7,8] generally uses light efficiently, but can be difficult to align and has little design flexibility. Furthermore, the tip of the axicon and other imperfections can cause strong axial intensity fluctuations of the Bessel beam that need to be appropriately compensated [9]. In analogy to the refractive axicon, a diffractive "axicon" can be implemented with a spatial light modulator (SLM) [10].

Ring illumination of the back focal plane of a lens (with the help of a ring-shaped aperture) will also generate a Bessel beam [11]. This method wastes a lot of the incoming light if simply a ring aperture is used to truncate any undesired light in the back focal plane (BFP), but the loss can be greatly reduced in combination with diffractive elements that fill the aperture, such as holograms [12] or SLMs [13].

Botcherby et al. [11] used a fixed binary phase-only grating in the plane conjugate to the image plane to produce the intermediate ring distribution and a narrow annular aperture in the Fourier plane to truncate undesired areas of the field and thereby suppress an undesired strong second ring close by. Instead of the conjugate plane, it is also possible to alter the phase distribution of the incoming light in the Fourier plane of the desired extended beam as demonstrated by Čižmár & Dholakia with an SLM as a hologram [14].

Arbitrary order Bessel beams can be generated efficiently with the so-called Bessel beam kinoforms, whose phase modulation coincides with the phase modulation of the Bessel beam itself, and a narrow annular spatial filter [15,16].

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McCutchen showed that the axial intensity is related to the pupil function by a one-dimensional Fourier transformation and that size and range of Bessel beams change with the wavelength $\lambda$ for a given pupil function [3]. This approach can be utilized to design desired depth-of-focus distributions with an SLM in the Fourier plane [17].

The aim of our work was to produce a Bessel beam illumination for multicolour and broad-spectrum imaging applications. It is however not straightforward to apply any of the aforementioned approaches for working with a broad wavelength range or multicolour illumination: with a diffractive element in the image plane, the radius of the ring pupil function will change with $\lambda$ due to wavelength-dependent diffraction, thus a narrow annular aperture as a spatial filter would not be suited. Furthermore the different diffraction orders for different $\lambda$ can overlap.

Leach et al. displayed phase holograms with an additional phase ramp on the SLM to introduce a phase deviation between the diffraction orders [18]. This allowed them to spatially filter the first diffraction order in the Fourier plane of the SLM. To compensate for the resulting angular dispersion they imaged the SLM plane to a dispersive component (prism or grating). This technique is also suitable for achromatic higher-order Bessel beams. However, the compensation plane is conjugated with the Bessel beam in free space. For imaging applications, a dispersive component (such as a prism) in the image plane can lead to problems further along the imaging path such as a tilted image plane on the camera. A positive axicon was used for generating the Bessel beam, therefore the achievable axial extend is effectively limited to half the size compared to the approach presented here.

Nevertheless it is desirable to use the conjugate as opposed to the Fourier plane to maintain the image plane relation, minimize chromatic aberrations on the Bessel beam and make the illumination point spread function wavelength-independent. In order to generate a multicolour Bessel beam with a wavelength-independent profile over a broad $\lambda$-range, we allow the pupil function to vary. To avoid superposition of different diffraction orders due to dispersion, a good separation between them is necessary. We have designed an iterative Fourier transformation algorithm based on these constraints with the goal to find a suitable SLM phase pattern for a simple multicolour Bessel beam implementation.

## 2. Setup

The setup is shown and described in Fig. 1A. The light from a green (532 nm) and red (635 nm) laser pointer (Coherent,

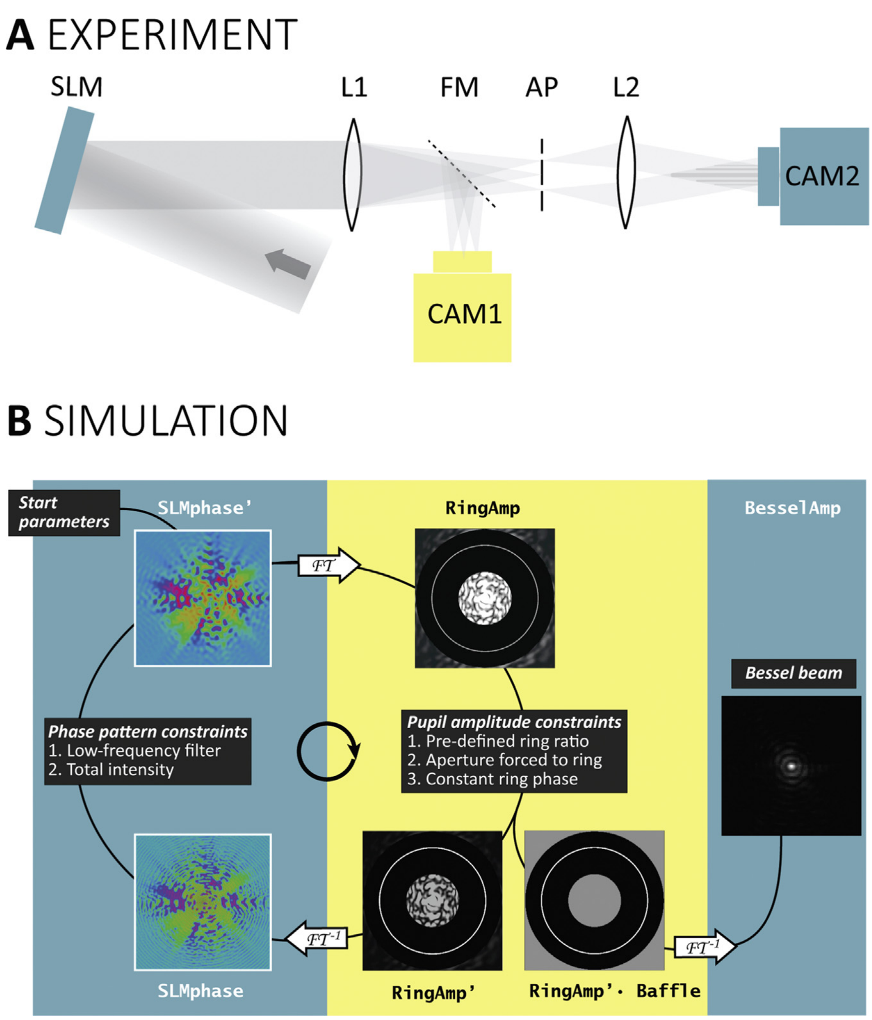


**Fig. 1.** Schematic summary of the setup and algorithm. (A) A collimated beam is reflected by the spatial light modulator (SLM). The light is focused to a ring by lens L1. A flip mirror (FM) allows for observation of this ring on a CCD camera (CAM1). Undesired (zero diffraction order) light is blocked by a ring aperture (AP). Behind lens L2 the resulting interference pattern of the Bessel beam can be observed on camera CAM2. (B) Starting from a given set of parameters, the algorithm jumps back and forth between the planes of the SLM phase pattern (blue) and the ring pupil function (yellow). In both planes certain constraints are applied, the constrained variables are denoted by'. The final pupil function is multiplied with a Boolean `Baffle` mask. At each iteration the error to a desired goal function is evaluated. Function outputs are the final phase pattern (for the SLM) and simulated intensity distribution of the Bessel beam. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

Germany) is collimated , spatially filtered and expanded such that the beam diameter fits the shorter side of the SLM display (X10468 by Hamamatsu, Japan). The parallel light is reflected from the SLM under a small angle and focussed through lens *L1* ($f$=50 mm, achromatic doublet AC254–050-A-ML, Thorlabs Inc, USA). A flip mirror *FM* allows us to direct the light to a CCD webcam *CAM1* (Philips SPC900NC, Netherlands) to evaluate the intensity distribution in the ring plane. In the back focal plane of lens *L2* ($f$=200 mm, achromatic doublet AC254-200-A-ML, Thorlabs Inc, USA), all undiffracted light is blocked by an opaque disk, such that only the ring illumination passes, which is then focussed onto a second CCD webcam *CAM2* mounted onto a motorized precision translation stage (Standa 8MT167–100, Lithuania) to evaluate the beam profile along the optical axis. Simulations as well as the iterative algorithm for phase pattern optimization were written and performed in Matlab 7.7 (The MathWorks, USA) Image Processing was facilitated by the DIPimage 2.3 & DIPlib toolboxes (Quantitative Imaging Group, TU Delft, Netherlands).

## 3. Algorithm & results

Our first naïve approach was to simply maximize the integrated power in the ring illumination. A circular symmetric grating of triangular phase ripples produces a first diffraction order ring distribution in the Fourier plane to the SLM (see Fig. 2A). However, this ring was observed to be effectively a double ring (Fig. 2B and C). A triangular grating can be considered a superposition of a positive and a negative axicon and Botcherby et al. [11] showed that such a superposition produces not only the desired ring illumination but also additional rings, depending on the relative phase shift between the two axicons. The generated Bessel beam exhibits a pronounced central dip along the optical axis which is undesirable for practical purposes. When working with a monochromatic source, very precise alignment of an annular mask in the ring plane can be used to block one of the rings, however for a broadband source this solution is not applicable.

Consequently we turned to an iterative Fourier Transformation algorithm (IFTA) to seek an SLM phase pattern suitable for a broad light spectrum. The term IFTA describes a family of algorithms which jump back and forth between two spaces connected by a Fourier transformation to iteratively find a desired goal function. In each space, constraints can be applied to the output function and at each iteration the error to the goal function is evaluated. These algorithms are commonly applied for phase retrieval problems in the design of diffractive optical elements (DOE). Originally published by Gerchberg & Saxton in the 1970s [19,20], this group of algorithms has seen many improvements and adaptations since [21].

The goal function of our IFTA is an annular illumination of uniform phase on the ring with an area of zero intensity around it (see Fig. 3A). In the physical setup this corresponds to a wide ring aperture in a baffle. Due to dispersion, the radius of the ring scales with the wavelength of the light source and will change during wavelength tuning (Fig. 3C). This is exemplified by a red and green ring illumination and corresponding Bessel beam in Fig. 5A and B. The aperture on the baffle must thus be much wider than the ring itself. Furthermore, it greatly facilitates alignment to have a dark zone around the ring. Higher diffraction order light can be efficiently blocked even if the aperture is slightly misaligned (Fig. 3B). In the baffle zone (light grey areas in Fig. 3), the algorithm has freedom to allow light intensity (as this corresponds to light that will not pass the annular aperture and contribute to the generation of the Bessel beam).

Fig. 1B summarizes the basic steps of the algorithm. As start values, a Gaussian illumination distribution, a constant phase distribution and desired parameters of the goal function (such as the ratio of intensity in the ring as compared to total intensity in Fourier plane, and the desired size of the ring are defined. At each iteration, the complex amplitude `RingAmp` in the ring plane is calculated and amplitude constraints are applied:

1. The intensity in the ring (as percentage of the total intensity) is set equal to the pre-defined ratio. We achieved good results for 0.1.
2. The amplitude outside the ring but within the aperture area is forced to 0.
3. The phase along the ring is forced to 0.

The variable `RingAmp` which now carries these constraints is denoted here as `RingAmp`'. It represents the ideal result. The error of `RingAmp` compared to `RingAmp`' as the sum of absolute squares of deviations in each pixel is calculated and will be minimized through the iterative process. The complex amplitude `RingAmp`' is backpropagated to the SLM plane and `SLMphase` is retrieved by the Matlab `phase` function which returns phase angles, in radians, of a complex image. Constraints are applied to the phase distribution:

1. A low-frequency filter prevents the algorithm from optimizing for patterns with high-frequency content to reduce higher-order diffraction.

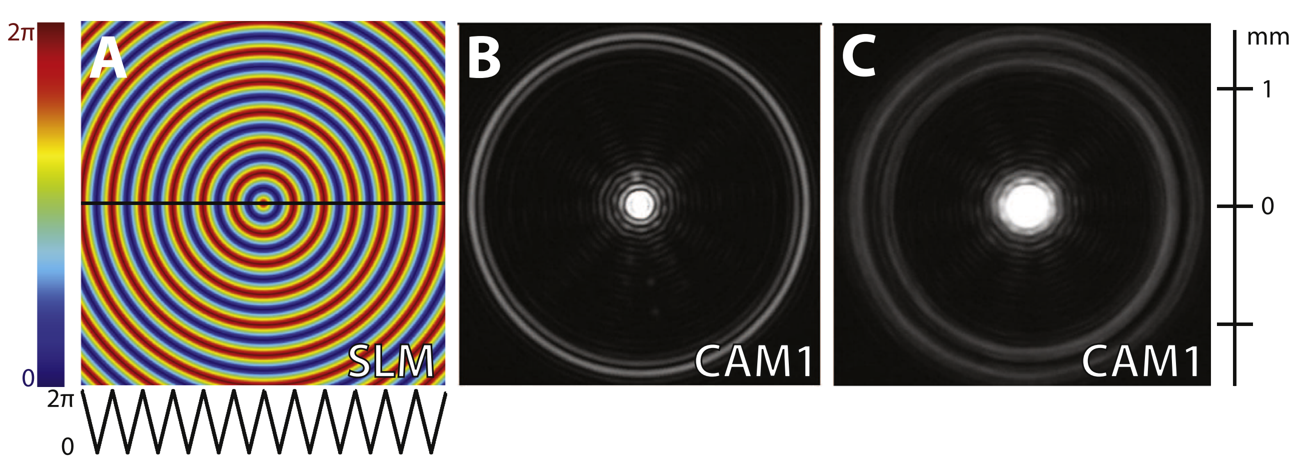


**Fig. 2.** The double ring. (A) For monochromatic light and a circular symmetric grating of triangular phase on the SLM, we observed such double rings in the front focal plane of L1. A precisely aligned annular mask in the ring plane could be used to block one of the rings, which is however not applicable when working with a broadband source. (B) In focus. (C) Slight defocus. The images were intentionally overexposed to highlight the rings.

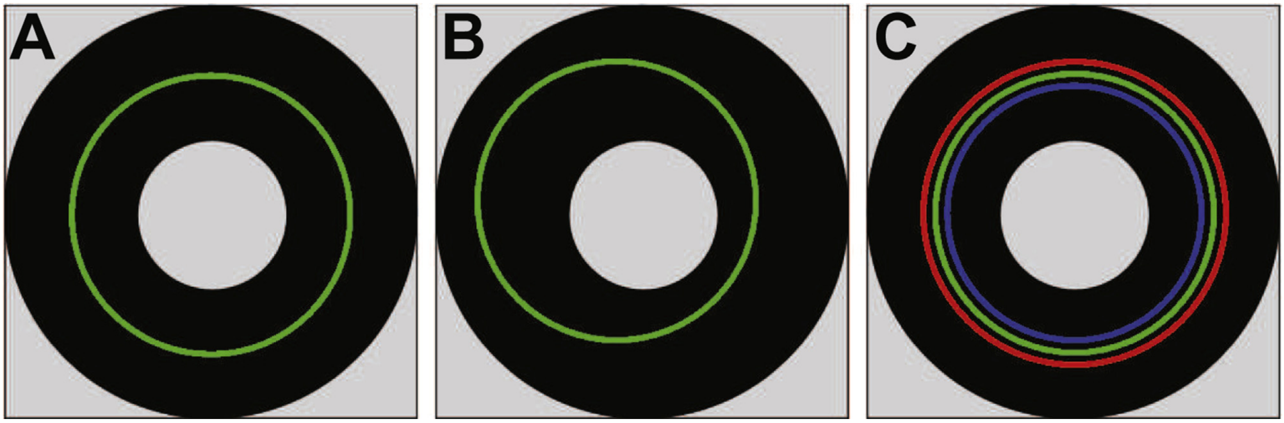


**Fig. 3.** (A) The goal function of our IFTA is an annular illumination (green) of uniform phase on the ring with a zone of zero intensity (black) around it. Both undiffracted and higher order diffraction light is blocked by a circular baffle in the experimental setup (grey). (B) The wide ring aperture around the ring illumination allows for small misalignments. (C) Dispersion effects in the ring plane occur for polychromatic light. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

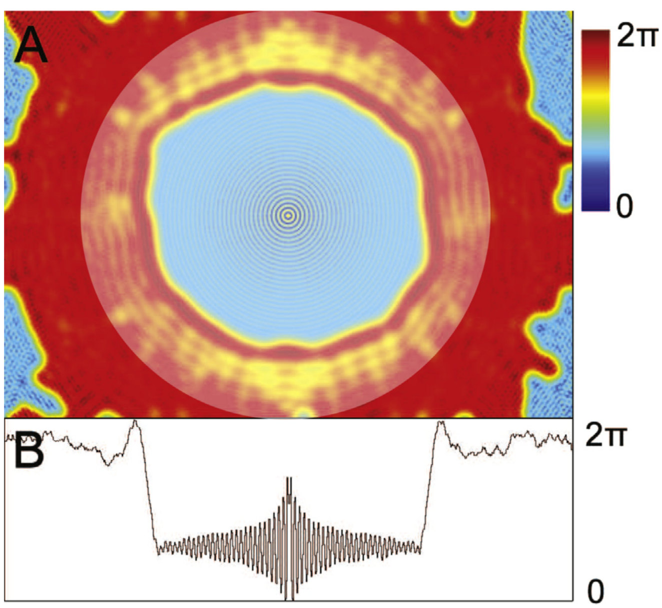


**Fig. 4.** Final phase pattern. (A) After execution of the IFTA we displayed this 800 × 600 periodic pattern on the SLM to modulate the incoming plane wave. The size of the beam incident on the SLM is indicated by the superimposed semi-transparent white circle. (B) Cross-section through the center of *A*.

2. Intensity values at the SLM plane are forced to a predefined illumination intensity.

Then, a new Fourier Transformation is performed and `RingAmp` is calculated and constrained. The process is iterated typically 5000 times, each iteration leading to a smaller error criterium. Finally, a Boolean `Baffle` mask is applied. This corresponds to the annular aperture in the physical setup. (`Baffle` is 0 in the grey areas in Fig. 1B.)

An inverse scale relation exists between the radius of the ring and the radii of the Bessel beam maxima, since they are connected by a Fourier transformation. Here, the final ring diameter ($\rho$ = 4 mm) was adjusted to match the camera sensor size and translation stage range. The illumination incident upon the SLM was simulated as a Gaussian distribution with $\sigma = 268$ pxl. The final phase pattern is shown in Fig. 4 Fig. 4A and B. The final error was $7 \cdot 10^{-9}$.

Addressing the SLM with the resulting optimized `SLMphase`' pattern from the IFTA simulation, yielded the interference patterns presented in Fig. 5 in the BFP of *L1* (ring, Fig. 5A) and behind *L2* (Bessel beam, Fig. 5B–D). Note that the resulting rings are not anymore double rings. The dark line along $k_y$ is a shadow of our zero block holder. The light concentration in the ring (as percentage of the light intensity incident on the SLM) was measured to be 8.9% which is in good agreement with our algorithm input parameter of 10%. Higher light concentrations can be chosen as input parameters if the light source is not strong enough. To demonstrate the effect of a broader wavelength spectrum, as green and red laser were used. While the rings in the BFP show dispersion (Fig. 5A), the resulting Bessel beams overlap well. The thickness changes along the circumference of the red ring are due to a slight misalignment in the optical setup. A cross-section perpendicular to the optical axis and a corresponding radial intensity profile are shown in Fig. 5B and C. The extended focus of the central maximum can be seen in Fig. 5D.

## 4. Discussion

The purpose of this study was to place the spatial light modulator in the nominal image plane in order to achieve a wavelength-independent radial beam profile.

We have shown that a phase-only spatial light modulator together with an iterative Fourier Transformation algorithm can be used to create a multicolour Bessel beam. As a goal pupil function we designed an annular light distribution with a sufficiently wide dark area around it, which is crucial to avoid a superposition of Bessel functions of various scales in multicolour applications. The use of an axicon and a ring plane would not be suitable to achieve this. The 2D Fourier transformation of such a ring pupil function yields a Bessel beam.

We used the SLM to alter the phase distribution of an incoming plane wave with a Gaussian intensity profile. The phase pattern displayed on the SLM was determined by the custom-written IFTA. It has been shown that the resulting phase pattern did indeed yield the desired ring illumination and extended focus Bessel beam.

In principle we could have also searched for a purely radial solution, that is a phase modulation of the type $\varphi(r)$ by the algorithm instead of $\varphi(x, y)$. However, we modulate the phases of the incoming light by means of a phase-only SLM which has a discrete number of pixels arranged in a square grid pattern without radial symmetry. This phase image $\varphi(x, y)$ is optimized during each iteration. It is not immediately clear, whether allowing the

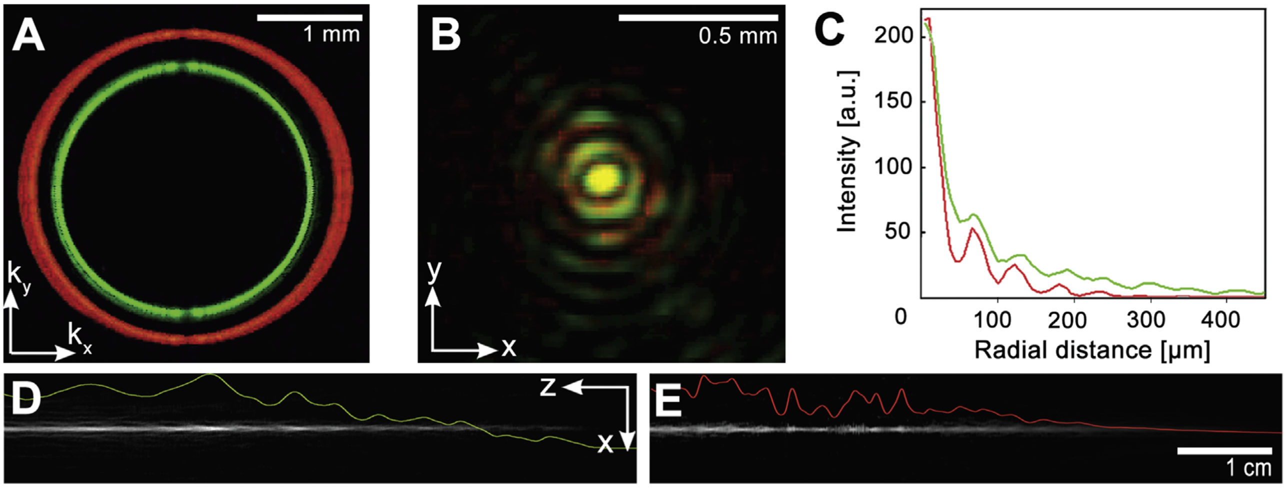


**Fig. 5.** Experimental results. (A) Dispersion effects occur in the intermediate ring plane upon illumination with a red & green laser. Zero order diffraction is blocked by a baffle. (B) Overlap of achromatic Bessel beams shown for red and green illumination. (C) Radial profiles of red and green beams shown in B. (D and E) Extended focus performance for the green (D) and red (E) laser with axial intensity [a.u.] plot along the beam. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

algorithm to break the symmetry might allow more freedom.

Because the SLM and the Bessel beam are in conjugate planes, our approach is applicable to broadband applications. We designed constraints such that a sufficiently sized area of zero intensity around the ring illumination exists. This allows for a dispersion in the intermediate ring plane as well as little misalignment. A wide ring aperture in the mechanical baffle is sufficient to spatially filter undesired zero- and higher-order diffraction light.

In contrast to previous work on achromatic Bessel beams by Leach et al. [18], no chromatic compensation techniques (such as a grating or prism) are required in our setup due to the image plane-to-image plane relation of the SLM and the Bessel beam center.

One major drawback of Bessel beams is that the central lobe contains only a small part of the total intensity, while the remaining intensity is in the ring system around the central lobe from where it is continuously transported to the beam center [4]. Depending on the application, this can deteriorate image contrast. The intensity of Bessel beams generated by the method presented here will depend mostly on the diffraction efficiency of the SLM and of course the intensity of the laser source. Most light will be lost in the zero order diffraction and some in higher-order diffraction. The remaining light intensity in the ring that produces the Bessel beam is one of the input parameters of our algorithm, as detailed in the algorithm description. We achieved a measured efficiency of 8.9% and this will be sufficient for many practical applications, but higher ring intensity values could be chosen in the IFTA if required.

Our solution for engineering a Bessel beam promises to be particularly useful for multicolour and broadband applications that require an extended field of view. For example, Bessel illumination in Swept Source Optical Coherence Tomography can help to achieve an axially extended PSF without compromising the lateral resolution and circumvent the classic dilemma between lateral resolution and depth of field. This has been demonstrated previously with an axicon lens [22,23].

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