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Numerical Solution of the Thermophysical Task of Grinding of the Laminated System Consisting of Polymer-Composite Materials

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Abstract

The generated model of the temperature pattern in the lamellate system takes into account the system structure, thermo-physical parameters of the each layer material, and process factors: acting heat source duration and intensity, heat dissipation to the process liquid allowing calculation of maximum cutting modes permitted per a heating temperature for each layer of the system.

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1. Introduction

Polymeric composite materials (PCM) are more and more extensively used in all branches of industry. Most often PCM are used as covers, frequently multilayered, forming lamellate polymeric composite system.

Grinding is the most effective method of lamellate systems finishing processing. Presence of different materials in a lamellate system forms additional limitations of their creating and processing technology. Well known phenomena when grinding steels and alloys – surface layers original structure rupture under their heating over some threshold temperature, known as burn, can be found during PCM processing. In this case overheating leads to material destruction. As threshold temperatures of lamellate system layers materials can differ (approximately 800°C – for steels and alloys and 100…180°C for majority of industry produced PCM) defect, caused by structural transformations, can appear in any layer of the system. Temperature pattern forming in all its layers analysis is necessary for defects-free processing of lamellate system. This calls for working out a model of the temperature...
pattern in the lamellate system that would take into account thermo-physical parameters of the each system layer material, duration and intensity of acting heat source, as well, as heat dissipation into process liquid.

2. Problem Description

Grinding process is heat-stressed and is accompanied by emitting large quantities of heat in the processing area. A rule of thumb states that grinding lamellate systems often leads to a variety of specific defects of heat nature [1–6]. So, in order to create defect-free technology of processing lamellate systems, it is necessary to study principles of temperature pattern forming in the part during grinding.

Industrial experience demonstrates than it is often necessary to process lamellate system in a form of a rotating part of large diameter and small length, such as bearing journals, shaft journals, etc. Requirements to operating surfaces of such systems are quite high, so final mechanical processing is mainly performed in the form of grinding. As systems of such type are usually short, processing is performed in the form of in-feed grinding for the whole processed length.

For taking into account schematic effect of laminarity on the part temperature pattern, first of all, we'll discuss this scheme.

3. Analytic Model

Taking into account variety of lamellate systems in modern structures, it would be reasonable to analyze general problem of thermal conductivity for n-layered system from different materials.

Conventionally, grinding thermophysics uses theory of fast-moving sources [7, 8]. We'll use this approach in our problem. Then y coordinate on the grinding area moving direction can be transformed into variable \( t \) (time), resulting in uni-dimensional problem. For uni-dimensional problem and system of n layers and different materials we'll have analytic model, presented in Fig. 1.

![Analytic Model of an n-layers System](image)

Fig. 1. Analytic Model of an n-layers System

Here are the parameters of this analytic model:

**Object.** Rod consists of \( n \) layers, each layer length - \( l_i \), thermo-physical parameters of each layer: heat conductivity \( \lambda_i \), density \( \rho_i \), heat capacity \( c_i \).

**Effect on the object.** Heat supply with intensity \( q \) during time interval \([0; \tau]\) - heating grinded layer when passing contact arc \( L_k \). Out of this interval set heat dissipation from the grinded surface with intensity \( \alpha \) (according to Newton's law) into environment during time interval \([\tau; T]\).

This scheme describes feeling of the part point during part turn \( T \): heating in interval \( t \in [0; \tau] \) (passing) of the grinding area and cooling in interval \( t \in [\tau; T] \) (after leaving the grinding area and until end of the turn).
Analysis of the lamellate systems, used in machinery manufacturing all over the world, has demonstrated, that there is a type of lamellate systems with porous top layer [9–11]. This also affects the process thermo-physics. This layer naturally fills with coolant, which will absorb heat.

In order to take into account this phenomena we'll introduce one more condition. By analogy with surface heat loss according to Newton, we'll introduce internal distributed heat loss on the processed layer that should be proportional to temperature – \( vU \), where \( v \) – coefficient of internal distributed heat loss, W/m²·K (see Fig.).

This analytic model reflects main key specifics of lamellate systems and allows specifically taking into account laminarity effect during grinding.

4. Mathematic Statement

Mathematic statement of the same problem of the heat conductivity for the analytic model (see Fig.) is as follows.

Heat conductivity equation for this model converts into following:

in the first layer:

\[
c_i \rho_i \frac{\partial U_1}{\partial t} = \lambda_i \frac{\partial^2 U_1}{\partial x^2} - \nu U_1; \quad x \in [l_0; l_1]; \quad l_0 = 0
\]

(1)

in inner layers:

\[
c_i \rho_i \frac{\partial U_i}{\partial t} = \lambda_i \frac{\partial^2 U_i}{\partial x^2}; \quad x \in \left[ \sum_{j=0}^{i-1} l_j; \sum_{j=0}^i l_j \right]; \quad i = 2, 3, ..., n.
\]

(2)

Initial conditions: temperature in all layers is taken the same and equal to:

\[
U(x; 0) = 0.
\]

(3)

Edge conditions: left butt is characterized by edge condition of the second and third type, taking into account effect of the heat source and heat dissipation into process media.

\[
\lambda \frac{\partial U_1}{\partial x} \begin{cases} 
- q & t \in \left[ (k-1)T; (k-1)T + \tau \right]; \\
\alpha U_i & t \in \left[ (k-1)T + \tau; kT \right]; \\
\end{cases} \quad k = 1, 2, ..., m.
\]

(4)

Layers interface is characterized by the edge condition of the fourth type, taking into account conditions of the full heat contact:

\[
U_i \left( \sum_{j=0}^{i} l_j; t \right) = U_{i+1} \left( \sum_{j=0}^{i} l_j; t \right); \quad i = 1, 2, ..., n;
\]

(5)

\[
\lambda \frac{\partial U_i}{\partial x} \left( \sum_{j=0}^i l_j; t \right) = \lambda_{i+1} \frac{\partial U_{i+1}}{\partial x} \left( \sum_{j=0}^i l_j; t \right); \quad i = 1, 2, ..., n.
\]

(6)

5. Numerical and Algorithmic Implementation of the Mathematic Model

Thermo-physical constants \( C_i, \rho_i, \lambda_i, \nu_i \) (of heat capacity, heat conductivity, and heat loss) are considered piecewise constants inside of each interval \( (x_i, x_{i+1}) \).
Interval \([0, l]\) is divided into \(n\) subintervals. Dividing points (mesh nodes):

\[
0 = x_0 < x_1 < x_2 < ... < x_n = l.
\]  

(7)

Parameters breaking points are located in nodes. We'll assume that any node point can be a breaking point. Each subinterval \([x_{i-1}, x_i]\) will be assigned a number \(i\).

Parameters values at this subinterval will be shown as \(h_i = x_i - x_{i-1}\), \(C_i\rho_i\), \(\lambda_i\), \(\nu_i\). \(U_i = U(x_i, t)\) — temperature value at this moment of time in node \(x_i\).

Let’s analyze internal subinterval \([x_i - \frac{h_i}{2}, x_i + \frac{h_i}{2}]\).

Taking integral of (1, 2) at this interval leads to

\[
C_i\rho_i \int_{x_i - \frac{h_i}{2}}^{x_i + \frac{h_i}{2}} U_i dx + C_{i+1}\rho_{i+1} \int_{x_i + \frac{h_i}{2}}^{x_{i+1}} U_i dx = \lambda_i U_{xi} \int_{x_i - \frac{h_i}{2}}^{x_i + \frac{h_i}{2}} \left( - \frac{\nu_i}{\lambda_i} \right) dx - \nu_i \int_{x_i}^{x_i + \frac{h_i}{2}} U_i dx + \nu_{i+1} \int_{x_i + \frac{h_i}{2}}^{x_{i+1}} U_i dx.
\]  

(8)

Let’s stipulate natural requirements for passing breaking points \(x_i\) at each moment of time \(t\) on desired solution:

- temperature \(U\) is continuous:
  \[
  U_i = U(x_i, 0) = U(x_i) = U(x_i + 0); \]  

  (9)

- heat flow is continuous:
  \[
  \lambda_i U_{x_i} U_{x_i} = \lambda_{i+1} U_{x_i+0}; \]  

  (10)

- speed \(U_i\) is continuous:
  \[
  U_i(x_i, 0) = U_i(x_i + 0). \]  

(11)

Taking into account Eq. (1, 2) condition (11) becomes equal to following:

\[
\frac{\lambda_i}{C_i\rho_i} U_{x_i} \Big|_{x_i} - \frac{\nu_i}{C_i\rho_i} U_{x_i} \Big|_{x_i} = \frac{\lambda_{i+1}}{C_{i+1}\rho_{i+1}} U_{x_i} \Big|_{x_i} - \frac{\nu_{i+1}}{C_{i+1}\rho_{i+1}} U(x_i). \]  

(12)

At each internal subinterval we'll approximate temperature by second order polynomial:

\[
U = b_0 + b_1 x + b_2 x^2,
\]  

(13)

which coefficients differ to the right and to the left of the point \(x_i\).

These coefficients should be determined based on conditions Eq. (9-13). So, on the subinterval \([x_i - \frac{h_i}{2}, x_i + \frac{h_i}{2}]\) we'll assume:
\[ U = \begin{cases} (b_h x_i + b_h b_x b_2) \frac{(x - x_i)}{2 \lambda_i} + b_h \frac{x - x_i}{\lambda_i} + b_0, & \text{if } x < x_i \\ (b_h x_i + b_h b_2 x) \frac{(x - x_i)}{2 \lambda_{i+1}} + b_h \frac{x - x_i}{\lambda_{i+1}} + b_0, & \text{if } x \geq x_i \end{cases} \tag{14} \]

where \( b_0, b_1, b_2 \) are constant for the whole subinterval.

In this case conditions Eq. (14) are fulfilled as including Eq. (13) into these conditions leads to \( b_0 = b_0, b_1 = b_1, b_2 = b_2 \) accordingly.

Denoting \( U_{i-1} = U(x_{i-1}), U_i = U(x_i), U_{i+1} = U(x_{i+1}) \) for determining coefficients \( b_0, b_1, b_2 \), we generate following system of equations:

\[
\begin{aligned}
&b_h x_i + b_h b_2 x = U_{i-1} \\
b_2 = U_i \\
&b_h x_i + b_h b_2 x + b = U_{i+1},
\end{aligned}
\tag{15}
\]

Because of the quadratic polynomial properties we have formulas:

\[
\begin{aligned}
&U_i (x_i - \frac{h_i}{2}) = \frac{U_i - U_{i-1}}{h_i} ; \\
&U_i (x_i - \frac{h_i}{2}) = \frac{U_{i+1} - U_i}{h_{i+1}}.
\end{aligned}
\tag{16}
\]

Introducing designations:

\[
s_i = C_{i} \rho_{i} h_{i}, \quad l_i = \nu_{i} h_{i}, \quad k_i = \frac{\lambda_{i}}{h_{i}} \text{ and}
\]

\[
s_{i+1} = C_{i+1} \rho_{i+1} h_{i+1}, \quad l_{i+1} = \nu_{i+1} h_{i+1}, \quad k_{i+1} = \frac{\lambda_{i+1}}{h_{i+1}},
\]

and solving the system Eq. (18) we receive following for the polynomial coefficients:

\[
\begin{aligned}
&b_0 = \frac{2}{s_i + s_{i+1}} \left( k_i U_{i-1} - \left( \frac{l_i + l_{i+1}}{2} + k_i + k_{i+1} \right) U_i + k_{i+1} U_{i+1} \right), \\
&b_1 = \frac{2}{s_i + s_{i+1}} \left( k_i s_i U_{i-1} - \left( \frac{l_i s_i + l_{i+1} s_{i+1}}{2} + k_i s_i - k_{i+1} s_{i+1} \right) U_i + k_{i+1} s_{i+1} U_{i+1} \right), \\
&b_2 = U_i.
\end{aligned}
\tag{18}
\]

Let's denote time period as \( \Delta t \), temperature at a time as \( U_i \), and temperature at previous period as \( U_{i-1} \). Then for internal points we'll receive subtended numeric scheme:
\[(B_{0i} + B_{0i}' \Delta t - k_i \Delta t)U_i + (B_{1i} + B_{1i}' \Delta t - k_i \Delta t + k_{i+1} \Delta t)U_{i+1} + (B_{2i} + B_{2i}' \Delta t - k_{i+1} \Delta t)U_{i+1} = B_{0i}^i U_{i-1} + B_{1i}^i U_i + B_{2i}^i U_{i+1}, \tag{19}\]

where \(B_{0i}', B_{1i}', B_{2i}', B_{0i}^i, B_{1i}^i, B_{2i}^i\) are determined by Eq. (20–25):

\[B_{0i}^i = \frac{s_i k_{i+1}(s_i + 3s_{i+1}) - s_{i+1} k_i 2s_{i+1}}{24k_{i+1}(s_i + s_{i+1})}; \tag{20}\]

\[B_{1i}^i = \frac{s_i}{24} \left( \frac{s_i l_{i+1} - s_{i+1} l_i + 2(s_i k_{i+1} - s_{i+1} k_i)}{(s_i + s_{i+1}) k_i} + 11 \right) + \frac{s_i + 1}{24} \left( \frac{s_i l_{i+1} - s_{i+1} l_i + 2(s_i k_{i+1} - s_{i+1} k_i)}{(s_i + s_{i+1}) k_i} + 11 \right); \tag{21}\]

\[B_{2i}^i = \frac{s_i k_i (s_i + 3s_{i+1}) - s_k k_{i+1} 2s_{i+1}}{24k_i (s_i + s_{i+1})}; \tag{22}\]

\[B_{0i} = \frac{l_i k_{i+1}(s_i + 3s_{i+1}) - l_{i+1} k_i 2s_{i+1}}{24k_{i+1}(s_i + s_{i+1})}; \tag{23}\]

\[B_{1i} = \frac{l_i}{24} \left( \frac{s_i l_{i+1} - s_{i+1} l_i + 2(s_i k_{i+1} - s_{i+1} k_i)}{(s_i + s_{i+1}) k_i} + 11 \right) + \frac{l_i + 1}{24} \left( \frac{s_i l_{i+1} - s_{i+1} l_i + 2(s_i k_{i+1} - s_{i+1} k_i)}{(s_i + s_{i+1}) k_i} + 11 \right); \tag{24}\]

\[B_{2i}^i = \frac{l_i k_i (s_i + 3s_{i+1}) - l_{i+1} k_{i+1} 2s_{i+1}}{24k_i (s_i + s_{i+1})}. \tag{25}\]

We'll also approximate left edge of the subinterval \(0; \frac{1}{2} h_0\) by second order polynomial Eq. (13). Then conditions at the subinterval end will convert into:

\[
\begin{bmatrix}
    b_2 = U_0 \\
    b_0 h_0^2 + b_1 h_0 + b_2 = U \\
    \lambda U'_0(0, t) = i \alpha U(0, t) - (1 - i) q(t)
\end{bmatrix} \tag{26}
\]

This results into refinement of the numeric scheme at the left edge:

\[(B_0^0 + B_0^0 \Delta t + (k_0 + i \alpha_0) \Delta t)U_0 + (B_{2i}^0 + B_{2i}^0 \Delta t - k_{i+1} \Delta t)U_{i+1} = B_0^0 U_{i-1}^0 + B_{2i}^0 U_{i+1}^0 + (1 - i)Q \Delta t. \tag{27}\]

Right edge of the subinterval \(0; \frac{1}{2} h_0\): same approximation of temperature by second order polynomial Eq. (17). Then condition at the subinterval end will convert into:
\[
\begin{align*}
\begin{cases}
 b_2 & = U_n \\
b_2 h_2^2 + b_3 h_2 + b_3 & = U_{n+1} \\
\lambda \ U_x (0, t) & = i\alpha U (0, t) - (1 - i) q (t)
\end{cases}
\end{align*}
\]  
(28)

Denoting \( B_{0i}^n = \frac{s_n}{24} \), \( B_{li}^n = \frac{l_n}{24} \); \( B_{1i}^n = s_n \left( \frac{11}{24} + n_{nn} \right) \), \( B_{1i}^n = l_n \left( \frac{11}{24} + n_{nn} \right) \);

after integrating we'll have refinement of the numeric scheme at the right end:

\[
(B_{0i}^n + B_{li}^n \Delta t - k_n \Delta t)U_{n+1} + (B_{1i}^n + B_{1i}^n \Delta t + (k_n + i\alpha_n) \Delta t)U_n = B_{0i}^n \overline{U_{n+1}} + B_{1i}^n \overline{U_n}.
\]  
(29)

So, complex of equations:

- for internal subintervals – Eq. (19);
- refinement at the left end – Eq. (27);
- refinement at the right end – Eq. (29)

gives subtended numeric scheme for solving stated combined boundary problem.

6. Conclusion

In such a way, temperature pattern in the lamellate system under grinding was worked out. For the first time such model takes into account not only heat dissipation into process fluid and temperatures distribution over the system layers, but also possible porosity of the system external layer.

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