Abstract

The identification of material parameter in geotechnical engineering is a typical complicated nonlinear function optimization problem. The optimum inverse method based on ABAQUS is discussed in the paper. A new evaluation function is established which unites multi objective function to single objective function according to the actually measures and corresponding calculated results. The inversion model combining the Nelder-Mead algorithm and finite element method is proposed, in which the finite element program is embedded as a module in the Nelder-Mead algorithm. This approach is applied to the parameter identification of strongly weathered granite as an example. The results show that the calculated results are similar with the measured ones, which suggests that the reasonability of finite element identification for material parameters.

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Keywords: Geotechnical Engineering; parameter identification; optimum; finite element

1. Introduction

The complexity of rock mass makes parameter identification difficult in geotechnical engineering [1]. In spite of the availability of several experimental means for determining material parameters of rock, if the constitutive model is too complex, it will be very difficult to carry out the work. These difficulties provoke uncertainties in the input parameters for a numerical simulation for constitutive model of rock [2]. In order to overcome these difficulties, a large number of parameter identification methods with the synonym back analysis have been proposed over the past twenty years [3-4].

The first inverse algorithm based on the finite element method was proposed by Kavanagh and Clough,
after which Gioda, Sakurai and Iding et al. made further improvements. Parameter identification is a typical complicated nonlinear function optimization problem. To solve this problem, the choice of global optimization algorithm and good objective function is a very important work. In order to offset the disadvantages of low searching efficiency using traditional methods, a methodology combining Nelder-Mead algorithm and finite element method together with ABAQUS is proposed based on Matlab. Then, a new exact penalty function is constructed between the monitored data and numerical results.

<table>
<thead>
<tr>
<th>Nomenclature</th>
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<tbody>
<tr>
<td>$X$</td>
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<tr>
<td>$m$</td>
</tr>
<tr>
<td>$\lambda$</td>
</tr>
<tr>
<td>$\varphi$</td>
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<tr>
<td>$\sigma_y, \varepsilon_y$</td>
</tr>
<tr>
<td>$h_i$</td>
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<tr>
<td>$g_j$</td>
</tr>
<tr>
<td>$\mu$</td>
</tr>
<tr>
<td>$F$</td>
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<tr>
<td>$q, p$</td>
</tr>
<tr>
<td>$\beta$</td>
</tr>
<tr>
<td>$d_0$</td>
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<tr>
<td>$\alpha, \tau_0$</td>
</tr>
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</table>

2. Parameter identification model

As for parameter identification, the usable data are stress and strain. For a given axial strain under certain confining pressure, the discrepancy between experimental data and the corresponding numerical results are applied as the objective function by least square method. The combined back analysis model can be defined as follows [5]:

$$\varphi(X) = (1-\lambda)\varphi_s + \lambda\varphi_e \rightarrow \min$$  \hspace{1cm} (1)

where, $X$ is the vector of the unknown parameters. $\lambda$ is the relative weigh ratio, which can reflect the different accuracy of fitting for stress or strain. $\varphi_s$ and $\varphi_e$ are the objective function by stress data and strain data, which can be defined as:

$$\begin{align*}
\varphi_s &= \sum_{k=1}^{n} \left( \sigma_{y}^{e_k} - \sigma_{y}^{m_k} \right)^2 \\
\varphi_e &= \sum_{k=1}^{n} \left( \varepsilon_{\sigma}^{e_k} - \varepsilon_{\sigma}^{m_k} \right)^2
\end{align*}$$  \hspace{1cm} (2)
where, $\sigma _{\varepsilon }$ is the stress of experiment corresponding to certain axial strain, $\sigma _{\mu }$ is the stress of numerical calculation, and $n$ is the number of measurements. $\varepsilon _{\sigma }$ is the strain of experiment corresponding to certain axial strain, $\varepsilon _{\mu }$ is the strain of numerical calculation.

3. Numerical implementation by FEM

Considering the complexity of inversion problems, some of constraints should be applied on the model to get stable and reasonable solution. In this situation, the finite element methods approximation in function form is given by

$$\min \varphi (X), \ X \in R^m$$

$$h_i (X) = 0, \ i = 1, 2, \ldots, l$$

$$g_j (X) \leq 0, \ j = 1, 2, \ldots, r$$

where $h_i$ is the $i$th equality constraint; $g_j$ is the $j$th inequality constraint; $m$ is the number of inversion parameters.

The optimization problem is given by Eqs. (3), and the optimal solution set is given by:

$$X = \{X \in R^m | h_i (X) = 0, g_j (X) \leq 0 \}, \ (i = 1, 2, \ldots, l; \ j = 1, 2, \ldots, r)$$

Assuming that:

$$\Phi (y) = \max^2 (0, y)$$

First derivative of $\Phi (y)$ is continuous for $y$, and if $y \geq 0$, then $\Phi ' (y) = 2y$, else if $y < 0$, then $\Phi ' (y) = 0$.

Assuming that:

$$S(X) = \sum [h_i (X)]^2 + \sum \Phi (g_j (X))$$

To the optimization problem of Eqs. (3), the exact penalty function can be expressed as

$$F(X, \mu) = \Phi (\varphi (X) - \mu) + S(X)$$

where, $\mu$ is the minimum estimated value for $\varphi (X)$.

A new penalty function for unconstrained optimization problems is given by:

$$\min F(X, \mu), \ X \in R^m$$

Based on the theory of Neld-Mead and FEM software ABAQUS, the back analysis program GeoInverse.m combines exact Penalty Functions and Nelder-Mead algorithm on Matlab platform.

4. Numerical examples

In order to study the material characteristics of the fully or strongly weathered granite, a series of laboratory tests have been done to study the mechanical behavior. Based on criterion of Drucker-Prager, the developed strain-hardening elasto-plastic model for strongly weathered granite can be written as:

$$f = q - p \tan \beta - d(\sigma_{\mu}) = 0$$

where, $q$ and $p$ are Mises stress and pressure stress, respectively. $\beta$ is internal friction angle. $d(\sigma_{\mu})$ is cohesion, which can be defined as:
\[ d(\tau_p) = d_0 \cdot \alpha \left( A_t e^{-\tau_{\sigma_0}} + B_0 \right) \] (10)

Where, \( d_0 \) is the initial value of cohesion. \( A_t = 1/(e^{\frac{1}{m} - 1}) \), \( B_0 = 1/(e^{\frac{1}{m} - 1}) \), \( \tau_{\sigma_0} = \tau_{\sigma_0} / \max(\tau_{\sigma}) \). \( \alpha \) and \( t_0 \) are the material constants with \( m \geq 1 \) and \( 0 < t_0 < 1 \).

The finite element model for back analysis is shown in Fig. 1. The unknown parameters for inversion model include elastic modulus, Poisson’ ratio, the initial cohesion, internal friction angle, the material constants \( a \) and \( t_0 \).

Allowable tolerance error for calculation is \( \text{eps}=5\% \), and the scope and initial value of unknown parameters are all shown in Table 1. The back analysis satisfies the convergence criteria when the iterations of optimization algorithm are 209 and computation time of finite element are 406.

![Fig. 1 Finite element model (left)](image1.png)
![Fig. 2 Comparison between the stress-strain curve experimentally and the one numerically for confining pressure 1 MPa(right)](image2.png)

Table 1. The initial value and scope of unknown parameters

<table>
<thead>
<tr>
<th>parameters</th>
<th>( E ) (MPa)</th>
<th>( \nu )</th>
<th>( d_0 ) (MPa)</th>
<th>( \beta )</th>
<th>( \alpha )</th>
<th>( t_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>scope</td>
<td>300-900</td>
<td>0.1-0.5</td>
<td>0.3-0.9</td>
<td>30-50</td>
<td>1.0-7.0</td>
<td>0.1-0.7</td>
</tr>
<tr>
<td>initial value</td>
<td>500</td>
<td>0.25</td>
<td>0.6</td>
<td>40</td>
<td>2.0</td>
<td>0.3</td>
</tr>
<tr>
<td>Back analysis</td>
<td>390.680</td>
<td>0.282</td>
<td>0.502</td>
<td>47.424</td>
<td>3.433</td>
<td>0.178</td>
</tr>
</tbody>
</table>

The comparison between numerical results and experimental results are given in Fig. 2. It shows that the new developed model can fit the experimental results very well in the frame of strain-hardening elasto-plastic model. Also, Table 1 shows the value of unknown parameters in the constitutive model of strongly weathered granite.

5. Conclusion

A general procedure is established for the determination of material parameters by stress and strain data from standard rock mechanics tests. The differences between numerical results and experimental data are applied as the objective function of the least square method. A methodology by combining the Nelder-Mead algorithm and finite element method is proposed. Through the identification of mechanical parameter for the strongly weathered granite as an example, the proposed inverse program is verified; and the results show that this method is a very good inverse analysis method and its efficiency is very good.
So the proposed method is a feasible method for back analysis of stress, seepage and displacement in geotechnical engineering.

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References


