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Asymptotically flat charged rotating dilaton black holes in higher dimensions

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ABSTRACT

We find a class of asymptotically flat slowly rotating charged black hole solutions of Einstein–Maxwell-dilaton theory with arbitrary dilaton coupling constant in higher dimensions. Our solution is the correct one generalizing the four-dimensional case of Horne and Horowitz [J.H. Horne, G.T. Horowitz, Phys. Rev. D 46 (1992) 1340]. In the absence of a dilaton field, our solution reduces to the higher-dimensional slowly rotating Kerr–Newman black hole solution. The angular momentum and the gyromagnetic ratio of these rotating dilaton black holes are computed. It is shown that the dilaton field modifies the gyromagnetic ratio of the black holes.

1. Introduction

General Relativity in higher dimensions has been the subject of increasing attention in recent years. There are several motivations for studying higher-dimensional Einstein's theory, and in particular its black hole solutions. First of all, string theory contains gravity and requires more than four dimensions. In fact, the first successful statistical counting of black hole entropy in string theory was performed for a five-dimensional black hole [1]. This example provides the best laboratory for the microscopic string theory of black holes. Besides, the production of higher-dimensional black holes in future colliders becomes a conceivable possibility in scenarios involving large extra dimensions and TeV-scale gravity. Furthermore as mathematical objects, black hole spacetimes are among the most important Lorentzian Ricci-flat manifolds in any dimension. While the non-rotating black hole solution to the higher-dimensional Einstein–Maxwell gravity was found several decades ago [2], the counterpart of the Kerr–Newman solution in higher dimensions, that is the charged generalization of the Myers–Perry solution [3] in higher-dimensional Einstein–Maxwell theory, still remains to be found analytically. Indeed, the case of charged rotating black holes in higher dimensions has been discussed in the framework of supergravity theories and string theory [4–6]. Recently, charged rotating black hole solutions in higher dimensions with a single rotation parameter in the limit of slow rotation has been constructed in [7] (see also [8,9]).

On the other hand, a scalar field called the dilaton appears in the low energy limit of string theory. The presence of the dilaton field has important consequences on the causal structure and the thermodynamic properties of black holes. Thus much interest has been focused on the study of the dilaton black holes in recent years. While exact static dilaton black hole solutions of Einstein–Maxwell-dilaton (EMd) gravity have been constructed by many authors (see e.g. [10–13]), exact rotating dilaton black hole solutions have been obtained only for some limited values of the dilaton coupling constant [14–16]. For general dilaton coupling constant, the properties of charged rotating dilaton black holes only with infinitesimally small charge [17] or small angular momentum in four [18–20] and five dimensions have been investigated [21]. Recently, one of us has constructed a class of charged slowly rotating dilaton black hole solutions in arbitrary dimensions [22]. However, in contrast to the four-dimensional Horne and Horowitz solution, these solutions [22] have unusual asymptotics. They are neither asymptotically flat nor (A)dS. Besides, they are ill-defined for the string case where $\alpha = 1$. The purpose of the present Letter is to generalize the four-dimensional Horne and Horowitz solution with sensible asymptotics, to arbitrary dimensions. These asymptotically flat solutions describe an electrically charged, slowly rotating dilaton black hole with an arbitrary value of the dilaton coupling constant in various dimensions. It is worth noting that in this Letter, we restrict ourself to the rotation in one plane, so our black hole has only one angular momentum parameter. We also investigate the effects of the dilaton field on the angular momentum and the gyromagnetic ratio of these rotating dilaton black holes.

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2. Field equations and solutions

We consider the n-dimensional ($n \geqslant 4$) theory in which gravity is coupled to dilaton and Maxwell field with an action

$$S = \frac{1}{16\pi}\int_{\mathcal{M}} d^n x\sqrt{-g}\left(R - \frac{4}{n-2}\partial_\mu \Phi \partial^\mu \Phi - e^{-\frac{4\alpha\Phi}{n-2}} F_{\mu\nu}F^{\mu\nu}\right) - \frac{1}{8\pi}\int_{\partial\mathcal{M}} d^{n-1}x\sqrt{-h}\Theta(h), \tag{1}$$

where R is the scalar curvature, Φ is the dilaton field, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor, and A_μ is the electromagnetic potential. α is an arbitrary constant governing the strength of the coupling between the dilaton and the Maxwell field. The last term in Eq. (1) is the Gibbons–Hawking boundary term which is chosen such that the variational principle is well-defined. The manifold $\mathcal{M}$ has metric $g_{\mu\nu}$ and covariant derivative ∇_μ. Θ is the trace of the extrinsic curvature Θ^{ab} of any boundary $\partial\mathcal{M}$ of the manifold $\mathcal{M}$, with induced metric h_{ab}. The equations of motion can be obtained by varying the action (1) with respect to the gravitational field $g_{\mu\nu}$, the dilaton field Φ and the gauge field A_μ which yields the following field equations

$$R_{\mu\nu} = \frac{4}{n-2}\partial_\mu\Phi\partial_\nu\Phi + 2e^{\frac{-4\alpha\Phi}{n-2}}\left(F_{\mu\eta}F_\nu^{\eta} - \frac{1}{2(n-2)}g_{\mu\nu}F_{\lambda\eta}F^{\lambda\eta}\right), \tag{2}$$

$$\nabla^2\Phi = -\frac{\alpha}{2}e^{\frac{-4\alpha\Phi}{n-2}}F_{\lambda\eta}F^{\lambda\eta}, \tag{3}$$

$$\partial_\mu\left(\sqrt{-g}e^{\frac{-4\alpha\Phi}{n-2}}F^{\mu\nu}\right) = 0. \tag{4}$$

We would like to find n-dimensional rotating solutions of the above field equations. For small rotation, we can solve Eqs. (2)–(4) to first order in the angular momentum parameter a. Inspection of the n-dimensional Kerr solutions shows that the only term in the metric that changes to the first order of the angular momentum parameter a is $g_{t\phi}$. Similarly, the dilaton field does not change to $O(a)$ and A_ϕ is the only component of the vector potential that changes. Therefore, for infinitesimal angular momentum we assume the metric being of the following form

$$ds^2 = -U(r)\,dt^2 + \frac{dr^2}{W(r)} - 2af(r)\sin^2\theta\,dt\,d\phi + r^2R^2(r)\left(d\theta^2 + \sin^2\theta\,d\phi^2 + \cos^2\theta\,d\Omega_{n-4}^2\right), \tag{5}$$

where $d\Omega_{n-4}^2$ denotes the metric of a unit $(n-4)$-sphere. The functions $U(r)$, $W(r)$, $R(r)$ and $f(r)$ should be determined. In the particular case $a=0$, this metric reduces to the static and spherically symmetric cases. For small a, we can expect to have solutions with $U(r)$ and $W(r)$ still functions of r alone. The t component of the Maxwell equations can be integrated immediately to give

$$F_{tr} = \sqrt{\frac{U(r)}{W(r)}}\frac{Q\,e^{\frac{4\alpha\Phi}{n-2}}}{(rR)^{n-2}}, \tag{6}$$

where Q, an integration constant, is the electric charge of the black hole. In general, in the presence of rotation, there is also a vector potential in the form

$$A_\phi = -aQ\,C(r)\sin^2\theta. \tag{7}$$

Asymptotically flat static ($a=0$) black hole solutions of the above field equations was found in [23]. Here we are looking for the asymptotically flat solutions in the case $a \neq 0$. Our strategy for obtaining the solution is the perturbative method suggested in [18]. Inserting the metric (5), the Maxwell fields (6) and (7) into the field equations (2)–(4), one can show that the static part of the metric leads to the following solutions [23]

$$U(r) = \left[1 - \left(\frac{r_+}{r}\right)^{n-3}\right]\left[1 - \left(\frac{r_-}{r}\right)^{n-3}\right]^{1-\gamma(n-3)}, \tag{8}$$

$$W(r) = \left[1 - \left(\frac{r_+}{r}\right)^{n-3}\right]\left[1 - \left(\frac{r_-}{r}\right)^{n-3}\right]^{1-\gamma}, \tag{9}$$

$$R(r) = \left[1 - \left(\frac{r_-}{r}\right)^{n-3}\right]^{\gamma/2}, \tag{10}$$

$$\Phi(r) = \frac{n-2}{4}\sqrt{\gamma(2+3\gamma-n\gamma)}\ln\left[1 - \left(\frac{r_-}{r}\right)^{n-3}\right], \tag{11}$$

while the rotating part of the metric admits a solution

$$f(r) = (n-3)\left(\frac{r_+}{r}\right)^{n-3}\left[1 - \left(\frac{r_-}{r}\right)^{n-3}\right]^{\frac{n-3-\alpha^2}{n-3+\alpha^2}} + \frac{(\alpha^2-n+1)(n-3)^2}{\alpha^2+n-3}r_-^{n-3}r^2\left[1 - \left(\frac{r_-}{r}\right)^{n-3}\right]^{\gamma} \times \int\left[1 - \left(\frac{r_-}{r}\right)^{n-3}\right]^{\gamma(2-n)}\frac{dr}{r^n}, \tag{12}$$

$$C(r) = \frac{1}{r^{n-3}}. \tag{13}$$

We can also perform the integration and express the solution in terms of hypergeometric function

$$f(r) = (n-3)\left(\frac{r_+}{r}\right)^{n-3}\left[1 - \left(\frac{r_-}{r}\right)^{n-3}\right]^{\frac{n-3-\alpha^2}{n-3+\alpha^2}} + \frac{(\alpha^2-n+1)(n-3)^2}{(1-n)(\alpha^2+n-3)}\left(\frac{r_-}{r}\right)^{n-3}\left[1 - \left(\frac{r_-}{r}\right)^{n-3}\right]^{\gamma} \times {}_2F_1\left(\left[(n-2)\gamma, \frac{n-1}{n-3}\right], \left[\frac{2n-4}{n-3}\right], \left(\frac{r_-}{r}\right)^{n-3}\right). \tag{14}$$

Here r_+ and r_- are the event horizon and Cauchy horizon of the black hole, respectively. The constant γ is

$$\gamma = \frac{2\alpha^2}{(n-3)(n-3+\alpha^2)}. \tag{15}$$

The charge Q is related to r_+ and r_- by

$$Q^2 = \frac{(n-2)(n-3)^2}{2(n-3+\alpha^2)}r_+^{n-3}r_-^{n-3}, \tag{16}$$

and the physical mass of the black hole is obtained as follows [24]

$$M = \frac{\Omega_{n-2}}{16\pi}\left[(n-2)r_+^{n-3} + \frac{n-2-p(n-4)}{p+1}r_-^{n-3}\right], \tag{17}$$

where Ω_{n-2} denotes the area of the unit $(n-2)$-sphere and the constant p is

$$p = \frac{(2-n)\gamma}{(n-2)\gamma-2}. \tag{18}$$

The metric corresponding to (8)–(14) is asymptotically flat. In the special case $n=4$, the static part of our solution reduces to

$$U(r) = W(r) = \left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right)^{\frac{1-\alpha^2}{1+\alpha^2}}, \tag{19}$$

$$R(r) = \left(1 - \frac{r_-}{r}\right)^{\frac{\alpha^2}{1+\alpha^2}}, \tag{20}$$

$$\Phi(r) = \frac{\alpha}{(1+\alpha^2)}\ln\left(1 - \frac{r_-}{r}\right), \tag{21}$$

while the rotating part reduces to

$$f(r)=\frac{r^2(1+\alpha^2)^2(1-\frac{r_-}{r})^{\frac{2\alpha^2}{1+\alpha^2}}}{(1-\alpha^2)(1-3\alpha^2)r_-^2} - \left(1-\frac{r_-}{r}\right)^{\frac{1-\alpha^2}{1+\alpha^2}}\left(1+\frac{(1+\alpha^2)^2r^2}{(1-\alpha^2)(1-3\alpha^2)r_-^2} + \frac{(1+\alpha^2)r}{(1-\alpha^2)r_-}-\frac{r_+}{r}\right), \tag{22}$$

which is the four-dimensional charged slowly rotating dilaton black hole solution of Horne and Horowitz [18]. One may also note that in the absence of a non-trivial dilaton ($\alpha=0=\gamma$), our solutions reduce to

$$U(r)=W(r) =\left[1-\left(\frac{r_+}{r}\right)^{n-3}\right]\left[1-\left(\frac{r_-}{r}\right)^{n-3}\right], \tag{23}$$

$$f(r)=(n-3)\left[\frac{r_-^{n-3}+r_+^{n-3}}{r^{n-3}}-\left(\frac{r_+r_-}{r^2}\right)^{n-3}\right], \tag{24}$$

which describe n-dimensional Kerr–Newman black hole in the limit of slow rotation [7].

Next, we calculate the angular momentum and the gyromagnetic ratio of these rotating dilaton black holes which appear in the limit of slow rotation parameter. The angular momentum of the dilaton black hole can be calculated through the use of the quasilocal formalism of the Brown and York [25]. According to the quasilocal formalism, the quantities can be constructed from the information that exists on the boundary of a gravitating system alone. Such quasilocal quantities will represent information about the spacetime contained within the system boundary, just like the Gauss's law. In our case the finite stress-energy tensor can be written as

$$T^{ab}=\frac{1}{8\pi}\left(\Theta^{ab}-\Theta h^{ab}\right), \tag{25}$$

which is obtained by variation of the action (1) with respect to the boundary metric h_{ab}. To compute the angular momentum of the spacetime, one should choose a spacelike surface $\mathcal{B}$ in $\partial\mathcal{M}$ with metric σ_{ij}, and write the boundary metric in ADM form

$$\gamma_{ab}\,dx^a\,dx^a=-N^2\,dt^2+\sigma_{ij}\left(d\varphi^i+V^i\,dt\right)\left(d\varphi^j+V^j\,dt\right),$$

where the coordinates φ^i are the angular variables parameterizing the hypersurface of constant r around the origin, and N and V^i are the lapse and shift functions respectively. When there is a Killing vector field ξ on the boundary, then the quasilocal conserved quantities associated with the stress tensors of Eq. (25) can be written as

$$Q(\xi)=\int_{\mathcal{B}} d^{n-2}\varphi\,\sqrt{\sigma}T_{ab}n^a\xi^b, \tag{26}$$

where σ is the determinant of the metric σ_{ij}, ξ and n^a are, respectively, the Killing vector field and the unit normal vector on the boundary $\mathcal{B}$. For boundaries with rotational ($\varsigma=\partial/\partial\varphi$) Killing vector field, we can write the corresponding quasilocal angular momentum as follows

$$J=\int_{\mathcal{B}} d^{n-2}\varphi\,\sqrt{\sigma}T_{ab}n^a\varsigma^b, \tag{27}$$

provided the surface $\mathcal{B}$ contains the orbits of ς. Finally, the angular momentum of the black holes can be calculated by using Eq. (27). We find

$$J=\frac{a\Omega_{n-2}}{8\pi}\left(r_+^{n-3}+\frac{(n-3)(n-1-\alpha^2)r_-^{n-3}}{(n-3+\alpha^2)(n-1)}\right). \tag{28}$$

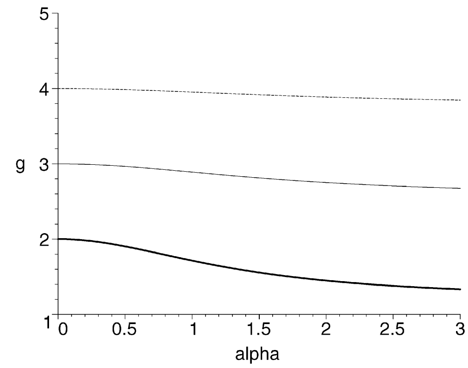

Fig. 1. The behaviour of the gyromagnetic ratio g versus α in various dimensions for $r_-=1$, $r_+=2$. $n=4$ (bold line), $n=5$ (continuous line), and $n=6$ (dashed line).

For $a=0$, the angular momentum vanishes, and therefore a is the rotational parameter of the dilaton black hole. In the case $n=4$, the angular momentum reduces to

$$J=\frac{a}{2}\left(r_++\frac{3-\alpha^2}{3(1+\alpha^2)}r_-\right), \tag{29}$$

which restores the angular momentum of the four-dimensional Horne and Horowitz solution [18], while in the absence of dilaton field ($\alpha=0$), the angular momentum reduces to

$$J=\frac{a\Omega_{n-2}}{8\pi}\left(r_+^{n-3}+r^{n-3}\right), \tag{30}$$

which is the angular momentum of the n-dimensional Kerr–Newman black hole. Next, we calculate the gyromagnetic ratio of this rotating dilaton black holes. The magnetic dipole moment for this asymptotically flat slowly rotating dilaton black hole can be defined as

$$\mu=Qa. \tag{31}$$

The gyromagnetic ratio is defined as a constant of proportionality in the equation for the magnetic dipole moment

$$\mu=g\frac{QJ}{2M}. \tag{32}$$

Substituting M and J from Eqs. (17) and (28), the gyromagnetic ratio g can be obtained as

$$g=\frac{(n-1)(n-2)[(n-3+\alpha^2)r_+^{n-3}+(n-3-\alpha^2)r_-^{n-3}]}{(n-1)(n-3+\alpha^2)r_+^{n-3}+(n-3)(n-1-\alpha^2)r_-^{n-3}}. \tag{33}$$

It was argued in [18] that the dilaton field modifies the gyromagnetic ratio of the asymptotically flat four-dimensional black holes. Our general result here in n-dimensions confirms their arguments. We have shown the behaviour of the gyromagnetic ratio g of the dilatonic black holes versus α in Fig. 1. From this figure we find out that the gyromagnetic ratio decreases with increasing α in any dimension. In the absence of a non-trivial dilaton ($\alpha=0=\gamma$), the gyromagnetic ratio reduces to

$$g=n-2, \tag{34}$$

which is the gyromagnetic ratio of the n-dimensional Kerr–Newman black hole (see e.g. [7]). When $n=4$, Eq. (33) reduces to

$$g=2-\frac{4\alpha^2r_-}{(3-\alpha^2)r_-+3(1+\alpha^2)r_+}, \tag{35}$$

which is the gyromagnetic ratio of the four-dimensional Horne and Horowitz dilaton black hole.

3. Summary and conclusion

To sum up, we found a class of asymptotically flat slowly rotating charged dilaton black hole solutions in higher dimensions. Our strategy for obtaining this solution was the perturbative method suggested by Horne and Horowitz [18] and solving the equations of motion up to the linear order of the angular momentum parameter. We stared from the asymptotically flat non-rotating charged dilaton black hole solutions in n-dimensions [23]. Then, we considered the effect of adding a small amount of rotation parameter a to the solution. We discarded any terms involving a^2 or higher powers in a. Inspection of the Kerr–Newman solutions shows that the only term in the metric which changes to $O(a)$ is $g_{t\phi}$. Similarly, the dilaton field does not change to $O(a)$. The vector potential is chosen to have a non-radial component $A_\phi = -aQ\,C(r)\sin^2\theta$ to represent the magnetic field due to the rotation of the black hole. As expected, our solution $f(r)$ reduces to the Horne and Horowitz solution for $n=4$, while in the absence of dilaton field ($\alpha = 0 = \gamma$), it reduces to the n-dimensional Kerr–Newman modification thereof for small rotation parameter [7]. We calculated the angular momentum J and the gyromagnetic ratio g which appear up to the linear order of the angular momentum parameter a. Interestingly enough, we found that the dilaton field modifies the value of the gyromagnetic ratio g through the coupling parameter α which measures the strength of the dilaton-electromagnetic coupling. This is in agreement with the arguments in [18].

In this Letter we only considered the higher-dimensional generalization of the Horowitz and Horne solution with a single rotation parameter. In general, in more than three spatial dimensions, black holes can rotate in different orthogonal planes, so the general solution has several angular momentum parameters. Indeed, an n-dimensional black hole can have $N = [(n-1)/2]$ independent rotation parameters, associated with N orthogonal planes of rotation where $[x]$ denotes the integer part of x. The generalization of the present Letter to the case with more than one rotation parameter and arbitrary dilaton coupling constant is now under investigation and will be addressed elsewhere.

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