A unified formula for determination of wellhead pressure and bottom-hole pressure

Mingze Liu, Bing Bai*, Xiaochun Li

State Key Laboratory of Geomechanics and Geotechnical Engineering, Institute of Rock and Soil Mechanics, Chinese Academy of Sciences, Xiaohongshan, Wuhan430071, China

Abstract

The number of gas injection or exploitation projects is on fast increase with growing energy needs and environmental problems. Of these projects, the calculation and the control of wellhead pressure or bottom-hole pressure in wellbores are usually key steps. However, methods used to predict wellhead and bottom-hole pressure are multifarious, making the program writing and practical application inconvenient. In this study, a unified formula derived from the differential equations of one-dimensional steady pipe flow and applicable in both injection wells and production wells was presented for calculation of both wellhead pressure and bottom-hole pressure. The recalculation with a traditional numerical method of the same equations corroborated well the accuracy of the unified formula, and the positive-reverse checking calculation verified its reliability. Basic data from Shenhua CCS project were used for studying the influence of injected gas temperature, geothermal gradient and flow rate on wellbore pressure by applying the unified formula. The results can provide a technical guidance for practical operation.

Keywords: unified formula; wellhead pressure; bottom-hole pressure; CCS

1. Introduction

In recent years, due to the sharply growing environmental problems, gas injection projects appear such as CO₂ storage and sour gas re-injections [1]. For these projects, drilling injection well is the primary means to achieve the goal. When gas is injected into the formation, high bottom-hole pressure may lead to the rupture of both the sidewall and the formation, causing contaminant leakage and diffusion, while low bottom-hole pressure can result in injection failure. A reasonable bottom-hole pressure is therefore the

* Corresponding author. Tel.: +86-027-87197932; fax: +86-027-87198967.
E-mail address: bai_bing2@126.com.
guarantee for the security and the stability of these projects. The bottom-hole pressure cannot be directly controlled in actual operation, but it can be changed by regulating the wellhead pressure. This requires the inverse calculation of injection pressure from a reasonable bottom-hole pressure, usually determined by the formation fracturing pressure and pore pressure, during design phase. Up to now, many attempts have been made for the determination of wellhead pressure. J.J. Carroll and D.W. Lui (1997) [2] presented a model to fetch an appropriate wellhead pressure in sour gas injection well. The model was derived from the basic equations of flow, and the simple Euler Method was used for calculation. J.J. Carroll and James R. Maddocks (1999) [3] later applied the Runge-Kutta-Fehlber method to solve the above model. Bai Bing et al. (2012) [4] put forward a fast explicit finite difference method to back calculating the wellhead pressure from bottom-hole pressure, and assessed the influence degree of acceleration term and friction term on the calculating process.

On the other hand, the number of gas recovery engineering projects is on fast increasing with the urgent energy needs, including not only traditional EOR and natural gas exploitations, but also the enhancement recovery of coalbed methane and shale gas. The major approach to the exploitation of these resources is building production well, in which the bottom-hole pressure prediction is of great significance for productivity forecast and production system dynamic analysis. One of the earliest works on predicting bottom-hole pressure in flowing wells was presented by Rzasa and Katz (1945) [5]. They assumed that the temperature and bias coefficient of wellbore gas were constant, and developed a model based on the energy balance differential equation to predict bottom-hole pressure in static liquid column well. Much of the classic work in this area was developed by Sukkar and Cornell (1955) [6]. They assumed fluid temperature constant as an average value, and wellbore pressure to be a pseudoreduced pressure. Under this simplification, an integral formula deduced from the energy balance differential equation was presented to calculate bottom-hole pressure. However, a corresponding numerical integral values table was needed to solve the formula. Cullender and Smith (1956) [7] improved Sukkar’s method by considering the temperature, the pressure and the bias coefficient as variables along the wellbore, and using single-step trapezoidal method for calculation. Sukkar’s integral formula can also be integrated analytically by iterative method, with assuming temperature, compression factor, viscosity as well as friction factor constant along the wellbore (Aziz, 1967) [8]. This method is so-called single-step method. In contrast, Beggs (1984) [9] applied namely multistep method to get the pressure solution from energy balance differential equation.

Therefore, the calculations of the wellhead and the bottom-hole pressure are of vital importance in gas injection and exploitation projects, respectively. However, the flow direction and the coordinate system setting of injection well problem differ from those in production well, which leads to the discrepancy in their governing equations. Moreover, these equations can be integrated and solved in various ways. As a result, the methods to predict wellhead and bottom-hole pressure are multifarious, and that brings inconvenience to the calculation programs written and practical application. Besides, most of the past methods needed iteration calculation, making the solution process long and slow, and ignored the acceleration term of fluid balance equation of momentum. This study aims to present a unified formula for calculation of both wellhead pressure and bottom-hole pressure, and applicable in both injection wells and production wells, from the pressure equation that does not ignore any term, in which finite difference method will be applied instead of iteration calculation. Moreover, Basic data from Shenhua CCS project will be used for studying the influence of injected gas temperature, geothermal gradient and flow rate on wellbore pressure.

2. Unified Formula

2.1. Assumptions

The mathematical model of pipe flow discussed should follow some basic assumptions as follows: (1) A single phase Newtonian fluid flows in pipe; (2) The flow in pipe is one dimensional steady, and all
physical parameters are homochromatic at any cross section; (3) There is a heat transfer between wellbore and surrounding wellbore formation, and the formation temperature varies linearly along vertical; (4) The wellbore is vertical, and its diameter variation is ignored.

2.2. Basic equations and calculation method

A coordinate system is set along wellbore whose positive direction is the same as that of the fluid injection. Based on the basic principle of flow dynamics, the equation of continuity for one dimensional steady flow is:

$$\frac{d(\rho v)}{dx} = 0$$  \hspace{1cm} (1)

Where, \( \rho \) is the density of gas, \((\text{kg/m}^3)\); \( v \) is the velocity, \((\text{m/s})\); \( x \) is wellbore coordinates, \((\text{m})\).

The fluid flows along the forward direction of axis in injection well, and that flow direction is opposite in production well. So, the equation of motion in injection well is written as:

$$\frac{dP}{dx} = \rho g - \rho v \frac{dv}{dx} - \lambda v^2 \frac{P}{2D}$$  \hspace{1cm} (2)

For production well, we have:

$$\frac{dP}{dx} = \rho g - \rho v \frac{dv}{dx} + \lambda v^2 \frac{P}{2D}$$  \hspace{1cm} (3)

Where, \( P \) is the pressure of fluid, \((\text{Pa})\); \( g \) is acceleration of gravity, \((\text{m/s}^2)\); \( \lambda \) is friction coefficient; \( D \) is the interior diameter of injection tube, \((\text{m})\). On the right hand side of equation (2) or (3), the first term reflects gas gravity on the pressure gradient, so called gravity term for short. Similarly, the second and the third terms are called acceleration term and friction term, respectively. The equation of state of fluid is:

$$PM = ZRT$$  \hspace{1cm} (4)

Where, \( Z \) is the deviation factor or compression factor; \( R \) is universal gas constant, \((\text{J/mol·K})\); \( T \) is thermodynamic temperature, \((\text{K})\); \( M \) is gas molar mass, \((\text{kg/mol})\). From equation (1), we get:

$$v = \frac{C}{\rho}$$  \hspace{1cm} (5)

Where, \( C \) is constant, independent of wellbore coordinates. Its physical meaning is the mass flow rate of the wellbore. Combining equations (4) and (5), we can obtain:

$$v = \frac{CZRT}{PM}$$  \hspace{1cm} (6)

We deduce the explicit different formula of node pressure for calculating bottom-hole pressure of production well in advance as an instance. Combining equations (3), (4) and (6), we can get:

$$\frac{dP}{dx} = g \frac{PM}{RTZ} - \frac{C^2RT}{M} \frac{d}{dx} \left( \frac{Z}{P} \right) + \frac{\lambda C^2RT}{2D} \frac{Z}{PM}$$  \hspace{1cm} (7)

Equation (7) is a differential equation about pressure \( P \) and deviation factor \( Z \). However, there are several ways to acquire \( Z \) such as empirical formulas, table or chart-looking up methods and there is not a widely accepted formula about \( Z \). This makes it difficult to discuss the analytical solution of equation (7), but the approximate solution of that equation will be studied still. For this aim, we mesh the one dimensional definitive range of wellbore into line segments-elements and their linkers-nodes, and number the nodes increasingly from the head to bottom of well. Obviously equation (6) is true strictly in each element. The thermo-physical parameters (viscosity, density, temperature, friction factor and deviation factor etc.) in each element are assumed to be constant provided its size is not so large that cannot be accepted. We call this process physical approximation. Thus an ordinary differential equation about pressure \( P \) is obtained as:

$$\frac{dP}{dx} = g \frac{PM}{RTZ} - \frac{C^2RT}{M} \frac{d}{dx} \left( \frac{1}{P} \right) + \frac{\lambda C^2RT}{2D} \frac{Z}{PM}$$  \hspace{1cm} (8)
i.e.
\[
\left(1 - \frac{C^2RT}{MP^2}Z\right) \frac{dP}{dx} = g \frac{PM}{RTZ} + \frac{\lambda C^2RT}{2D} \frac{Z}{PM}.
\]
(9)

Backward finite difference method is applied to solve equation (9). The above segment can be divided into micro-segments, in that way we can get the difference format of (9) as:
\[
\left(1 - \frac{C^2RT}{MP_k^2}Z\right) \frac{P_{i} - P_{i-1}}{\Delta x} = g \frac{PM}{RTZ} + \frac{\lambda C^2RT}{2D} \frac{Z}{P_i M}.
\]
(10)

Rearranging (10) yields:
\[
P_{i+1} = P_i + \left(\Delta x g \frac{PM}{RTZ} + \frac{\Delta t \lambda C^2RT}{2D} \frac{Z}{P_i M} \right) \left(1 - \frac{C^2RT}{MP_i^2}Z\right)
\]
(11)

Equation (11) is an explicit solution of node pressure in each line element. Looping all the line segments from wellhead, the pressure profile and the wellhead pressure can be obtained. In particular, there is no need to divide micro-segments when the firstly meshed segments are small enough, so equation (11) can be used for calculating the bottom-hole pressure directly by meshing the wellbore only once in that case.

Gas temperature will be calculated using Ramy’s formula [10], \(Z\) will be calculated based on Peng-Robinson’s equation and viscosity will be calculated with Guo’s method [11].

2.3. Unified formula

The explicit different formula of node pressure for calculating bottom-hole pressure of production well has been deduced in section 2.2. Similarly, totally four formulas under corresponding conditions can be obtained.

- **The explicit different formula of node pressure for calculating bottom-hole pressure in injection well.**
\[
P_{i+1} = P_i + \left(\Delta x g \frac{PM}{RTZ} + \frac{\Delta t \lambda C^2RT}{2D} \frac{Z}{P_i M} \right) \left(1 - \frac{C^2RT}{MP_i^2}Z\right).
\]
(12)

- **The explicit different formula of node pressure for calculating wellhead pressure in injection well.**
\[
P_{i+1} = P_i + \left(-\Delta x g \frac{PM}{RTZ} + \frac{\Delta t \lambda C^2RT}{2D} \frac{Z}{P_i M} \right) \left(1 - \frac{C^2RT}{MP_i^2}Z\right).
\]
(13)

- **The explicit different formula of node pressure for calculating bottom-hole pressure in production well.**
\[
P_{i+1} = P_i + \left(\Delta x g \frac{PM}{RT_i Z'} + \frac{\Delta t \lambda C^2RT'}{2D} \frac{Z'}{P_i M} \right) \left(1 - \frac{C^2RT'}{MP_i^2}Z'\right).
\]
(14)

- **The explicit different formula of node pressure for calculating wellhead pressure in production well.**
\[
P_{i+1} = P_i - \left(\Delta x g \frac{PM}{RT_i Z'} + \frac{\Delta t \lambda C^2RT'}{2D} \frac{Z'}{P_i M} \right) \left(1 - \frac{C^2RT'}{MP_i^2}Z'\right).
\]
(15)

Where, \(T\) is the fluid temperature in injection well, (K); \(T'\) is the fluid temperature in production well, (K); \(Z\) is the compression factor of fluid in injection well; \(Z'\) is the compression factor of fluid in production well. Considering the similar forms of the above four formulas, sign variable \(m\) and \(n\) are introduced to integrate them to a unified formula as:
\[
P = P_i + (-1)^m \left(\Delta x g \frac{PM}{RB(T,T',Z,Z')} \left(1 - \frac{C^2RB(T,T',Z,Z')}{2DMP_i^2}\right)\right)
\]
(16)

Where,
\[
B(T,T',Z,Z') = \left[\frac{T + T'}{2} + \frac{(-1)^m T - T'}{2}\right] \left[\frac{Z + Z'}{2} + \frac{(-1)^m Z - Z'}{2}\right]
\]
(17)

Where \(m\) and \(n\) control the calculation direction and flow direction, respectively. When \(m=0,\)
calculation starts from wellhead to bottom-hole, and \( m=1 \) means the opposite situation; When \( n=0 \), gas flows from bottom-hole to wellhead, i.e. exploitation operation, and \( n=1 \) means the injection operation.

3. Reliability Demonstration

3.1. Comparison with numerical method

In the derivation process of unified formula, equation (8) is an ordinary differential equation about pressure \( P \) solved by finite difference method. However, this equation is conventionally solved by numerical method such as Runge-Kutta method. So, in order to validate the new method reliable, it is resolved by using numerical method. Furthermore, several bottom-hole pressures are calculated from given wellbore pressures by using the two different methods, the results are compared as Table 1 shows.

<table>
<thead>
<tr>
<th>NO.</th>
<th>Mass flow(Kg/s)</th>
<th>Wellhead pressure(MPa)</th>
<th>Bottom-hole depth(m)</th>
<th>Calculation step(m)</th>
<th>FDE(MPa)</th>
<th>NM(MPa)</th>
<th>Relative deviation(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.1133</td>
<td>9.5</td>
<td>1500</td>
<td>1</td>
<td>18.4897</td>
<td>18.4922</td>
<td>0.014</td>
</tr>
<tr>
<td>2</td>
<td>9.0572</td>
<td>9.5</td>
<td>1500</td>
<td>1</td>
<td>21.5942</td>
<td>21.5959</td>
<td>0.008</td>
</tr>
<tr>
<td>3</td>
<td>9.0572</td>
<td>15.3</td>
<td>1500</td>
<td>1</td>
<td>29.0253</td>
<td>29.0274</td>
<td>0.007</td>
</tr>
<tr>
<td>4</td>
<td>9.0572</td>
<td>15.3</td>
<td>2700</td>
<td>1</td>
<td>37.9569</td>
<td>37.9592</td>
<td>0.006</td>
</tr>
<tr>
<td>5</td>
<td>9.0572</td>
<td>15.3</td>
<td>2700</td>
<td>5</td>
<td>37.9517</td>
<td>37.9631</td>
<td>0.030</td>
</tr>
</tbody>
</table>

Table 1 shows that for different flow rates, wellbore pressure, depth and calculation step, the relative deviation of each outcome is so small that can be neglected. This implies that the finite difference method used in unified formula is reliable.

3.2. Positive-reverse checking calculation

Applying unified formula to calculate wellhead pressure from given bottom-hole pressure and using the calculated bottom-hole pressure as a datum to inverse calculate the wellhead pressure should obtain the same pressure profile. Otherwise, the unified formula is self-contradictory. Results of the above two calculation are compared as Table 2 shows.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>0</th>
<th>400</th>
<th>800</th>
<th>1200</th>
<th>1600</th>
<th>2000</th>
<th>2400</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse calculation(MPa)</td>
<td>9.2999</td>
<td>12.6225</td>
<td>16.0298</td>
<td>19.4236</td>
<td>22.7689</td>
<td>26.0553</td>
<td>29.2828</td>
</tr>
<tr>
<td>Relative deviation(%)</td>
<td>0.0011</td>
<td>0.0032</td>
<td>0.0031</td>
<td>0.0021</td>
<td>0.0013</td>
<td>0.0007</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

In Table 2, the relative deviation of each outcome is too small to consider, this implies that the unified formula is valid.

4. Influential Factors

In this section, we apply the unified formula to analyze the influential factors of pressure prediction. The wellbore and formation parameters used are based on Shenhua CCS project, which is the first large-scale CO₂ saline aquifer storage project of whole process in China and planned to inject about 0.1Mt CO₂ each year.

4.1. Flow initial temperature and geothermal gradient

The flow initial temperature and the geothermal gradient concern injection operation and site selection of well, respectively. However, the flow initial temperature of production well equal to the formation temperature and cannot be controlled, thus it is insignificant to study and ignored here. To study different initial temperature and geothermal gradient that influence pressure profile of injection and production well, we use different injection temperature and geothermal gradient parameters. Moreover, flow rate has a great impact on that influence, thus we set a small and a large flow rate both.
In Fig 1(a), the pressure profiles of injection well with different injection temperatures and small flow rate are shown. In Fig 1(b), the pressure profiles of injection well with different injection temperatures and large flow rate are shown. It is clear that: (1) The injection temperature has an explicit impact on pressure profile only under condition of large flow rate. This is because wellbore temperature is dominated by injection temperature as there is not enough time to transfer heat between fluid and surrounding formation under condition of large flow rate; (2) At the same depth, the pressure decrease with an increase in injection temperature. This is because the increase in injection temperature results in a decrease in density, causing a decrease in gravity term of pressure.

In Fig 2(a) and 2(b), the pressure profiles of injection well in different geothermal gradients, with flow rate of small and large is shown, respectively. It implies that: (1) The pressure profiles are influenced by geothermal gradient obviously only when the flow rate is small. This is due to the sufficient heat transfer between wellbore fluid and surrounding formation when flow rate is small, which makes the formation temperature the main factor of wellbore temperature; (2) At the same depth, the pressure decrease with an increase in geothermal gradient. This is because the increase in geothermal gradient results in an increase in wellbore temperature and a decrease in density, causing a decrease in gravity term of pressure.
In Figure 3(a) and 3(b), the pressure profiles of production well in different geothermal gradients with flow rate of small and large is shown, respectively. It is clear that: (1) Under small flow rate condition, the pressure decrease with an increase in geothermal gradient, the reason is the same as that in injection well; (2) Under larger flow rate condition, the pressure increase with an increase in geothermal gradient at the same depth in the depth range from wellhead to approximately 1400m, conversely, it turns to decrease with an increase in geothermal gradient in the depth range from 1400m to bottom-hole.

4.2. Flow rate

To study the different flow rates that influence wellhead pressure inverse calculation, we apply the unified formula to calculate several wellhead pressures from given bottom-hole pressures under different flow rates range from 0 kg/s to 17kg/s, and plot the change of calculated wellhead pressure with the increase of flow rate as Fig 4(a) shows. It is clear that: (1) Wellbore pressure variation tendency is nonlinear; (2) Wellbore pressure decrease firstly and increase afterward with the increase of flow rate in injection well, that tendency is opposite in production well.

Similarly, Fig 4(b) plots the variation tendency of bottom-hole pressure prediction with flow rate increasing. Bottom-hole pressure prediction is the inverse process to wellbore pressure calculation as section 3.2 mentioned, thus its variation tendency is also opposite to the former.
5. Conclusion

The fundamental differential equations of one dimensional pipe flow was studied in depth, based on which the equation of motion of gas was transformed to an ordinary differential equation about pressure considering the flow rate at any position of pipe is constant. Then, finite difference method was applied to solve the ordinary differential equation and four different formulas for node pressure were obtained according to corresponding calculation condition. Further, two sign variables were introduced to integrate them to a unified formula. A comparison with the traditional numerical method of solving ordinary differential equation was conducted, which validated the reliability of this new unified formula. And a positive-reverse checking calculation proves its correctness in relevant respect.

Basic data from Shenhua CCS project was used for influential factors studies. The wellbore pressure profiles of both injection and production wells under different flow initial temperatures and geothermal gradients were plotted. In order to analyze the influence of flow rate on wellhead and bottom-hole pressure calculation, the calculated pressures were plotted with increasing flow rate in both injection and production wells. The results can provide technical guidance for operation and site selection.

The superiority of the unified formula is that it applies to calculate both wellhead pressure and bottom-hole pressure of both injection and production well, which greatly facilitates program writing and practical application. Besides, its derivation does not neglect acceleration term in equation of motion like previous methods, so it can apply to occasions where acceleration term is important. Moreover, the finite difference method used in derivation has specialties of little calculation and simplicity compared to traditional numeral methods. At the same time, the unified formula can be generalized to multiphase flow, horizontal and deviated wells.

Acknowledgements

This work was supported by project 201211063 of the Ministry of Land and Resources of PRC.

References