



# The role of photon polarization modes in the magnetization and instability of the vacuum in a supercritical field

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## ABSTRACT

The response of the QED vacuum in an asymptotically large electromagnetic field is studied. In this regime the vacuum energy is strongly influenced by the vacuum polarization effect. The possible interaction between the virtual electromagnetic radiation and a superstrong magnetic field suggests that a background of virtual photons is a source of magnetization to the whole vacuum. The corresponding contribution to the vacuum magnetization density is determined by considering the individual contribution of each vacuum polarization eigenmode in the Euler–Heisenberg Lagrangian. Additional issues concerning the transverse pressures are analyzed. We also study the case in which the vacuum is occupied by a superstrong electric field. It is discussed that, in addition to the electron–positron pairs, the vacuum could create photons with different propagation modes. The possible relation between the emission of photons and the birefringent character of the vacuum is shown as well.

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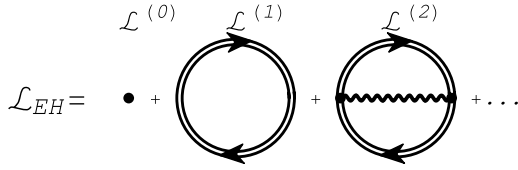
## 1. Introduction

Whilst there is some evidence that very large magnetic fields  $|\mathbf{B}| \gg B_c$ ,  $B_c = m^2/e = 4.42 \times 10^{13} \text{ G}^1$  exist in stellar objects identified as neutron stars [1–3], its origin and evolution remains poorly understood [4]. Some investigations in this area provide theoretical evidence that  $|\mathbf{B}|$  might be generated due to gravitational and rotational effects, whereas other theories estimate that is self-consistent due to the Bose–Einstein condensation of charged and neutral boson gases in a superstrong magnetic field [5–8]. In this framework the nonlinear QED-vacuum possesses the properties of a paramagnetic medium and constitutes a source of magnetization, induced by the external magnetic field. Its properties are primarily determined by the vacuum energy of virtual electron–positron pairs. Because of this, a negative pressure transversal to the external field is generated [9] in similarity with the Casimir effect between metallic plates [10]. Moreover, the vacuum occupied by the external field turns out to be an “exotic” scenario in which processes like photon splitting [11,12] and photon capture [13–15] could take place. These two phenomena depend on the photon dispersion relation which differs from the light cone, due to vacuum polarization effects [16–19]. As a result, the issue of light propagation in empty space, in the presence of an external magnetic field, is similar to the dispersion of light in an anisotropic “medium”.

The phenomenological aspects associated with this problem have been studied for a long time. In the meanwhile, other features of nonlinear electrodynamics in a superstrong magnetic field have been studied such as the dimensional reduction of the Coulomb potential [20–23] and the possible existence of a photon anomalous magnetic moment [24]. However, due to the vacuum polarization effect, virtual photons can carry a magnetization as well. As a consequence, they might be a source of magnetism to the whole vacuum. Motivated by this idea, we address the question in which way the virtual electromagnetic radiation contributes to the vacuum magnetization and therefore to increase the external field strength. The magnetic properties of the vacuum have been studied in [9,25–27] for weak ( $|\mathbf{B}| \ll B_c$ ) and moderate fields ( $|\mathbf{B}| \sim B_c$ ) in one-loop approximation of the Euler–Heisenberg Lagrangian [28] which involves the contribution from virtual electron–positron pairs. The contribution of virtual photons, created and annihilated spontaneously in the vacuum and interacting with  $\mathbf{B}$  by means of  $\Pi_{\mu\nu}$ , is contained within the two-loop term of the Euler–Heisenberg Lagrangian (see Fig. 1). The latter was computed many years ago by Ritus [29,30] and has been recalculated by several authors as well [31–34]. In all these works, however, it is really cumbersome to discern the individual contributions given by each virtual photon propagation mode to the Euler–Heisenberg Lagrangian which should allow to determine the magnetism and pressure associated with each form of virtual mode. In this Letter we analyze these contributions separately for very large magnetic fields ( $|\mathbf{B}| \gg B_c$ ) since these allow to establish relations between the birefringence of the vacuum [19,35,36] and the global properties of it.

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<sup>1</sup> Hereafter  $m$  and  $e$  are the electron mass and charge, respectively.



**Fig. 1.** Two-loop expansion of the Euler–Heisenberg Lagrangian. The double lines represent the electron–positron Green’s functions, whereas the wavy line refers to the photon. Here  $\mathcal{L}^{(0)}$  is the free Maxwell Lagrangian,  $\mathcal{L}^{(1)}$  represents the one loop which gives the contribution of the virtual free electron–positron pairs created and annihilated spontaneously in vacuum and interacting with the external field. The radiative corrections (involved in  $\mathcal{L}^{(2)}$ ) emerge from two loop due to exchange of the virtual photons.

Besides the strongly magnetized vacuum, there is another interesting external field configuration which deserves to be analyzed: a supercritical electric field  $|\mathbf{E}| \gg E_c$  with  $E_c = m^2/e = 1.3 \times 10^{16}$  V/cm. In this asymptotic region the Euler–Heisenberg Lagrangian acquires an imaginary term which characterizes the instability of the vacuum. This phenomenon is closely related to the production of observable particles from the own vacuum. Certainly, the creation of electron–positron pairs—the so-called Schwinger mechanism—turns out to be the most remarkable effect predicted through this procedure [28,37,38]. However, the imaginary part of this effective Lagrangian is just a measure of the vacuum decay and does neither give the actual rate of production of particles nor the accessible decay channels [39]. Thereby not only the creation of electron–positron pairs is a plausible effect but also the emission of observable photons [29,30]. The latter phenomenon was analyzed by Gitman, Fradkin and Shvartsman [50–52]. Their results showed that the total probability of photon emission from the vacuum, accompanied by the creation of an arbitrary number of electron–positron pairs, is connected to the decay probability of the vacuum and thus to the imaginary part arising from the two-loop term of the Euler–Heisenberg Lagrangian. In this case the corresponding decomposition in terms of the vacuum polarization modes is particularly illuminating because it reveals that only two of them contribute to the vacuum instability. It seems, therefore, that the vacuum could create photons with different propagation modes, an effect closely related to its own birefringence.

## 2. Preliminary remarks

In a magnetized vacuum the spatial symmetry is explicitly broken by the external field  $\mathbf{B}$ . In this context, there is a vectorial basis  $b_\mu^{(i)}$  [20,40,41] which characterizes the vacuum symmetry properties and fulfills both the orthogonality condition:  $b_\sigma^{(i)} b^\sigma{}^{(j)} = \delta^{ij} (b^{(i)})^2$  and the completeness relation:  $\delta^\mu{}_\nu - \frac{k^\mu k_\nu}{k^2} = \sum_{i=1}^3 b^{\mu(i)} b_\nu^{(i)} / (b^{(i)})^2$ . Explicitly, the basis vectors read  $b_\mu^{(1)} = k^2 \mathcal{F}_{\mu\lambda}^2 k^\lambda - k_\mu (k \mathcal{F}^2 k)$ ,  $b_\mu^{(2)} = \tilde{\mathcal{F}}_{\mu\lambda} k^\lambda$ ,  $b_\mu^{(3)} = \mathcal{F}_{\mu\lambda} k^\lambda$  and  $b_\mu^{(4)} = k_\mu$ . These expressions involve the external field tensor  $\mathcal{F}_{\mu\nu}$  and its dual  $\tilde{\mathcal{F}}^{\mu\nu} = 1/2 \epsilon^{\mu\nu\rho\sigma} \mathcal{F}_{\rho\sigma}$ . In this basis, the vacuum polarization tensor is diagonal i.e.

$$\Pi_{\mu\nu} = \sum_{i=0}^4 \chi_i \frac{b_\mu^{(i)} b_\nu^{(i)}}{(b^{(i)})^2} \quad (1)$$

whereas the dressed photon Green function can be expressed as

$$\mathcal{D}_{\mu\nu} = \sum_{i=1}^3 \frac{1}{k^2 - \chi_i} \frac{b_\mu^{(i)} b_\nu^{(i)}}{(b^{(i)})^2} + \frac{\zeta}{k^2} \frac{k_\mu k_\nu}{k^2}. \quad (2)$$

Here the  $\chi_i$  represent the  $\Pi_{\mu\nu}$ -eigenvalues and  $\zeta$  is the gauge parameter. This diagonal decomposition of  $\Pi_{\mu\nu}$  defines the en-

ergy spectrum of the electromagnetic field which differs from the isotropic vacuum ( $\mathbf{B} = 0$ ).

Owing to the transversality property ( $k^\mu \Pi_{\mu\nu} = 0$ ), the eigenvalue corresponding to the fourth eigenvector vanishes identically ( $\chi^{(4)} = 0$ ). Furthermore, not all the remaining eigenmodes are physical. In general, this depend on the direction of wave propagation. To show this we consider  $b_\mu^{(i)}(k)$  as the electromagnetic four vector describing a photon. The corresponding electric and magnetic fields of each mode are  $\mathbf{e}^{(i)} = i(\mathbf{k} b_0^{(i)} - \omega^{(i)} b^{(i)})$ ,  $\mathbf{b}^{(i)} = -i\mathbf{k} \times \mathbf{b}^{(i)}$ . It follows that the mode  $i = 3$  is a wave polarized in the transverse plane to  $\mathbf{k}$  whose electric  $\mathbf{e}^{(3)} \sim \mathbf{k}_\perp \times \mathbf{n}_\parallel$  and magnetic  $\mathbf{b}^{(3)} \sim \mathbf{n}_\parallel k_\perp^2 - \mathbf{k}_\perp k_\parallel$  fields are orthogonal to each other. Here the vectors  $\mathbf{k}_\perp$  and  $\mathbf{k}_\parallel$  are the components of  $\mathbf{k}$  across and along  $\mathbf{B}$  with  $\mathbf{n}_\parallel = \mathbf{B}/|\mathbf{B}|$ . For a pure longitudinal propagation to the external field  $k_\perp = 0$ , the mode  $b_\mu^{(2)}$  is a longitudinal and nonphysical electric wave  $\mathbf{e}^{(2)} \sim \mathbf{n}_\parallel$ . On the other hand,  $b_\mu^{(1)}$  is transverse since the associated electric field is  $\mathbf{e}^{(1)} \sim \mathbf{k}_\perp$  whereas the magnetic one  $\mathbf{b}^{(1)} \sim \mathbf{k}_\perp \times \mathbf{k}_\parallel$ . As a consequence, both  $b_\mu^{(1)}$  and  $b_\mu^{(3)}$  represent physical waves which may be combined to form a circularly polarized transversal wave. In this case both modes propagate along  $\mathbf{B}$  with a dispersion law independents of the magnetic field strength [17, 19,41].

Now, if the photon propagation involves a nonvanishing transversal momentum component  $k_\perp \neq 0$ , we are allowed to perform the analysis in a Lorentz frame that does not change the value  $k_\perp$ , but gives  $k_\parallel = 0$  and does not introduce an external electric field. In this Lorentz frame, the first eigenmode  $b_\mu^{(1)}$  becomes purely electric longitudinal and a nonphysical mode whereas  $b_\mu^{(2)}$  is transverse. Hence, for a photon whose three-momentum is directed at any nonzero angle with the external magnetic field, the two orthogonal polarization states  $b_\mu^{(2)}$  and  $b_\mu^{(3)}$  propagate. In this framework the analytical structures of the corresponding eigenvalues  $\chi_{2,3}$  are different. As a matter of fact, the vacuum behaves like a birefringent medium with refraction indices [19,41]

$$\eta_2 = \frac{|\mathbf{k}|}{\omega_2} = \left(1 + \frac{\chi_2}{\omega_2^2}\right)^{1/2} \quad \text{and} \quad \eta_3 = \frac{|\mathbf{k}|}{\omega_3} = \left(1 + \frac{\chi_3}{\omega_3^2}\right)^{1/2}. \quad (3)$$

Here  $\omega_{2,3}$  are the corresponding solution of the dispersion equations  $k^2 = \chi_{2,3}$  arising from the poles of  $\mathcal{D}_{\mu\nu}$ .

Considering these aspects, we analyze the Euler–Heisenberg Lagrangian

$$\mathcal{L}_{\text{EH}} = \mathcal{L}_{\text{R}}^{(0)} + \mathcal{L}_{\text{R}}^{(1)} + \dots \quad (4)$$

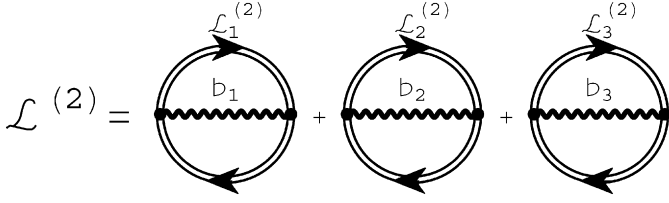
where  $\mathcal{L}_{\text{R}}^{(0)} = -\frac{1}{2} B^2$  is the free renormalized Maxwell Lagrangian, whereas  $\mathcal{L}_{\text{R}}^{(1)}$  denotes the one-loop regularized contribution of virtual electron–positron pairs created and annihilated spontaneously in vacuum and interacting with  $\mathbf{B}$  [28]. In asymptotically large magnetic fields it reads

$$\mathcal{L}_{\text{R}}^{(1)}(b) \approx \frac{m^4 b^2}{24\pi^2} \left\{ \ln\left(\frac{b}{\gamma\pi}\right) + \frac{6}{\pi^2} \zeta'(2) \right\}. \quad (5)$$

Here  $b = |\mathbf{B}|/B_c$ ,  $\ln(\gamma) = 0.577\dots$  denotes the Euler constant whereas  $6\pi^{-2}\zeta'(2) = -0.5699610\dots$  and  $\zeta(x)$  is the Riemann zeta-function.

The contribution of a virtual photon interacting with external field by means of the vacuum polarization tensor is expressed as [29,31,34]

$$\mathcal{L}^{(2)} = \frac{i}{2} \int \frac{d^4 k}{(2\pi)^4} \Pi_{\mu\nu}(k) \mathcal{D}_0^{\mu\nu}(k), \quad (6)$$



**Fig. 2.** Diagrammatic decomposition of  $\mathcal{L}^{(2)}$  in terms of the vacuum polarization eigenmodes.

where  $\mathcal{D}_0^{\mu\nu}(k)$  denotes the bare photon Green function which is obtained by neglecting  $\kappa_i$  in the denominator of  $\mathcal{D}_{\mu\nu}$ . Using this expression we find

$$\mathcal{L}^{(2)} = \frac{i}{2} \sum_{i=1}^3 \int \bar{d}k \frac{\kappa_i}{k^2}, \quad (7)$$

where  $\bar{d}k \equiv d^4k/(2\pi)^4$ . To obtain the above expression we have inserted the diagonal decomposition of  $\Pi_{\mu\nu}$  into Eq. (6) and used the orthogonality condition. Certainly, the quantity  $\kappa_i k^{-2}$  represents the interaction energy density of a virtual photon propagation mode with  $\mathbf{B}$ . Because the analytical properties of the  $\kappa_i$  differ from each other the contribution of each mode will be different. Therefore, the original two-loop graph can be decomposed into three diagrams (see Fig. 2) whose properties are connected to the birefringence property of the vacuum. Therefore, each individual term  $\mathcal{L}_i^{(2)} = \frac{i}{2} \int \bar{d}k \kappa_i/k^2$  will be studied separately. As we shall see very shortly, the decomposition of  $\mathcal{L}^{(2)}$  is also valid in an external electric field in which case some evidence on the possible decay of the vacuum into observable photons emerges.

Obviously, the renormalized two-loop term decomposes as well:  $\mathcal{L}_R^{(2)} = \sum_{i=1}^3 \mathcal{L}_{iR}^{(2)}$  and the leading behavior of  $\mathcal{L}_{iR}^{(2)}$  in an asymptotically large magnetic field is given by (for details see Ref. [24])

$$\begin{aligned} \mathcal{L}_{1R}^{(2)} &\approx -\frac{\alpha m^4 b^2}{16\pi^3} \mathcal{N}_1, \\ \mathcal{L}_{2R}^{(2)} &\approx \frac{\alpha m^4 b^2}{32\pi^3} \left[ \mathcal{N}_2 \ln\left(\frac{b}{\gamma\pi}\right) - \frac{1}{3} \ln^2\left(\frac{b}{\gamma\pi}\right) + \mathcal{A} \right], \\ \mathcal{L}_{3R}^{(2)} &\approx \frac{\alpha m^4 b^2}{32\pi^3} \left[ \mathcal{N}_3 \ln\left(\frac{b}{\gamma\pi}\right) + \frac{1}{3} \ln^2\left(\frac{b}{\gamma\pi}\right) + \mathcal{B} \right]. \end{aligned} \quad (8)$$

These expressions are accurate up to terms that decrease with  $b$  like  $\sim b^{-1} \ln(b)$  and faster. Here the numerical constants are

$$\begin{aligned} \mathcal{N}_1 &= 1.25, \quad \mathcal{N}_2 = 0.71, \quad \mathcal{N}_3 = 0.29, \\ \mathcal{A} &= 4.21 \quad \text{and} \quad \mathcal{B} = 0.69. \end{aligned} \quad (9)$$

Moreover, the asymptotic behavior of the full two-loop term is

$$\mathcal{L}_R^{(2)} \approx \frac{\alpha m^4 b^2}{32\pi^3} \left[ \ln\left(\frac{b}{\gamma\pi}\right) + 2.4 \right], \quad (10)$$

which coincides with the results reported in Refs. [29,31].

### 3. Properties of a highly magnetized vacuum

In the presence of an external magnetic field, the zero point vacuum energy  $\mathcal{E}_{\text{vac}}$  is modified by the interaction between  $\mathbf{B}$  and the virtual QED-particles. The latter is determined by the effective potential coming from the quantum-corrections to the Maxwell Lagrangian which is also contained within the finite temperature formalism. According to Eq. (4) it is expressed as  $\mathcal{E}_{\text{vac}} = -\mathcal{L}_R^{(1)} - \sum_{i=1}^3 \mathcal{L}_{iR}^{(2)} + \dots$ . Consequently the vacuum acquires

a nontrivial magnetization  $\mathcal{M}_{\text{vac}} = -\partial \mathcal{E}_{\text{vac}}/\partial |\mathbf{B}|$  induced by the external magnetic field. In what follows we will write  $\mathcal{M}_{\text{vac}} = \mathcal{M}_{\text{vac}}^{(1)} + \mathcal{M}_{\text{vac}}^{(2)} + \dots$  in correspondence with the loop-term  $\mathcal{L}_R^{(i)}$ . In this sense, the one-loop contribution at very large magnetic field  $b \gg 1$  can be computed by means of Eq. (5) and gives

$$\begin{aligned} \mathcal{M}^{(1)} &= \frac{\partial \mathcal{L}_R^{(1)}}{\partial |\mathbf{B}|} \\ &\approx \frac{m^4 b}{24\pi^2 B_c} \left[ 2 \ln\left(\frac{b}{\gamma\pi}\right) + 1 + \frac{12\zeta'(2)}{\pi^2} \right]. \end{aligned} \quad (11)$$

Incidentally, the above asymptotic behavior is also manifest in the corresponding magnetization derived from the QCD vacuum in a magnetic field  $|\mathbf{B}| \gg \Lambda_{\text{QCD}}^2/e$ . For more details we refer the reader to Ref. [42].

The two-loop correction is given by  $\mathcal{M}^{(2)} = \sum_{i=1}^3 \mathcal{M}_i^{(2)}$  where  $\mathcal{M}_i^{(2)} = \partial \mathcal{L}_{iR}^{(2)}/\partial |\mathbf{B}|$  is the contribution corresponding to a photon propagation mode. Making use of Eqs. (8) we find

$$\mathcal{M}_1^{(2)} \approx -\frac{\alpha m^4 b}{8\pi^3 B_c} \mathcal{N}_1, \quad (12)$$

$$\begin{aligned} \mathcal{M}_2^{(2)} &\approx -\frac{\alpha m^4 b}{32\pi^3 B_c} \left[ \frac{2}{3} \ln^2\left(\frac{b}{\gamma\pi}\right) \right. \\ &\quad \left. + \frac{8\zeta'(2)}{\pi^2} \ln\left(\frac{b}{\gamma\pi}\right) - 9.13 \right], \end{aligned} \quad (13)$$

$$\begin{aligned} \mathcal{M}_3^{(2)} &\approx \frac{\alpha m^4 b}{32\pi^3 B_c} \left[ \frac{2}{3} \ln^2\left(\frac{b}{\gamma\pi}\right) \right. \\ &\quad \left. + \left( 2 + \frac{8\zeta'(2)}{\pi^2} \right) \ln\left(\frac{b}{\gamma\pi}\right) + 1.67 \right]. \end{aligned} \quad (14)$$

According to these results, in a superstrong magnetic field approximation,  $\mathcal{M}_1^{(2)} < 0$  and  $\mathcal{M}_2^{(2)} < 0$  behave diamagnetically whereas  $\mathcal{M}_3^{(2)} > 0$  is purely paramagnetic. Moreover, while  $\mathcal{M}_1^{(2)}$  depends linearly on  $b$ , the contributions of the second and third propagation mode depend logarithmically on the external field. We find, in particular, that for magnetic fields  $B \sim 10^{18}$  G, the magnetization generated by the first and second polarization mode reaches values the order  $\sim -10^{12}$  erg/(cm<sup>3</sup> G) and  $\sim -10^{13}$  erg/(cm<sup>3</sup> G), respectively. In the same context  $\mathcal{M}^{(3)} \sim +10^{13}$  erg/(cm<sup>3</sup> G). Note that the leading behavior of the complete two-loop contribution is

$$\mathcal{M}^{(2)} \approx \frac{\alpha m^4 b}{16\pi^3 B_c} \left[ \ln\left(\frac{b}{\gamma\pi}\right) + 2.9 \right] > 0, \quad (15)$$

which shows a dominance of the third mode. Indeed, for  $B \sim 10^{18}$  G, one finds  $\mathcal{M}^{(2)} \sim +10^{12}$  erg/(cm<sup>3</sup> G).

As it was expected  $\mathcal{M}^{(1)}/\mathcal{M}^{(2)} \sim \alpha^{-1}$ . This ratio is also manifested between the corresponding magnetic susceptibilities ( $\mathcal{X}^{(i)} = \partial \mathcal{M}^{(i)}/\partial |\mathbf{B}|$ ). Note that

$$\mathcal{X}^{(1)} \approx \frac{m^4}{24\pi^2 B_c^2} \left[ 2 \ln\left(\frac{b}{\gamma\pi}\right) + 1.86 \right] > 0, \quad (16)$$

$$\mathcal{X}^{(2)} \approx \frac{\alpha m^4}{16\pi^3 B_c^2} \left[ \ln\left(\frac{b}{\gamma\pi}\right) + 3.9 \right] > 0. \quad (17)$$

For magnetic fields  $b \sim 10^5$  corresponding to  $|\mathbf{B}| \sim 10^{18}$  G, the magnetic susceptibility reaches values of the order of  $\mathcal{X}^{(1)} \sim 10^{-4}$  erg/(cm<sup>3</sup> G<sup>2</sup>) which exceeds the values of many laboratory materials, for example aluminum ( $\mathcal{X}_{\text{Al}} = 2.2 \times 10^{-5}$  erg/(cm<sup>3</sup> G<sup>2</sup>)).

Some additional comments are in order. First of all, even though the previous decomposition of  $\mathcal{M}^{(2)}$  is not really observable, it

turns out to be a transparent framework which illustrates, in a phenomenological way, the magnetic property generated by each photon propagation mode and thus, a connection with the birefringent property of the vacuum. The decomposition of  $\mathcal{M}^{(2)}$  in terms of the photon modes allows, in addition, to establish similarities and differences with the magnetization carried by observable photons.<sup>2</sup> Indeed, similar to the latter the virtual radiation carries a magnetization which depends on the polarization vector. However, while an observable second mode has a paramagnetic response [45], our result points out that the corresponding virtual polarization generates a purely diamagnetic magnetization. Besides, we have seen that the first propagation mode contributes to the magnetization of the vacuum. This is not expected for an observable mode-1 photon since its dispersion law is independent on the external field strength [17,19,41] and therefore does not carry a magnetization.

Because of the anisotropy generated by  $\mathbf{B}$  a magnetized vacuum exerts two different pressures [5–8]. One of them is positive ( $\mathcal{P}_{\parallel} = -\mathcal{E}_{\text{vac}}$ ) and along  $\mathbf{B}$ , whereas the remaining is transverse to the external field direction ( $\mathcal{P}_{\perp} = -\mathcal{E}_{\text{vac}} - \mathcal{M}|\mathbf{B}|$ ). For  $b \sim 1$  the latter acquires negative values. At very large magnetic fields ( $b \gg 1$ ) the one-loop approximation of  $\mathcal{P}_{\perp}$  can be computed by making use of Eq. (5) and Eq. (11). In fact

$$\mathcal{P}_{\perp}^{(1)} \approx -\frac{m^4 b^2}{24\pi^2} \left[ \ln\left(\frac{b}{\gamma\pi}\right) + 1 + \frac{6\zeta'(2)}{\pi^2} \right] < 0. \quad (18)$$

Therefore, at asymptotically large values of the external field, the interaction between  $\mathbf{B}$  and the virtual electron–positron pairs generates a negative pressure which would tend to shrink inserted matter in the plane transverse to  $\mathbf{B}$ .

Again, the two-loop contribution can be written as the sum of the corresponding terms due to the vacuum polarization modes  $\mathcal{P}_{\perp}^{(2)} = \sum_{i=1}^3 \mathcal{P}_{\perp i}^{(2)}$ . According to Eqs. (8) and Eq. (14) they read:

$$\mathcal{P}_{\perp 1}^{(2)} \approx \frac{\alpha m^4 b^2}{16\pi^3} \mathcal{N}_1 > 0, \quad (19)$$

$$\mathcal{P}_{\perp 2}^{(2)} \approx \frac{\alpha m^4 b^2}{32\pi^3} \left[ \frac{1}{3} \ln^2\left(\frac{b}{\gamma\pi}\right) + \left( \mathcal{N}_2 + \frac{8\zeta'(2)}{\pi^2} \right) \times \ln\left(\frac{b}{\gamma\pi}\right) - 4.92 \right] > 0, \quad (20)$$

$$\mathcal{P}_{\perp 3}^{(2)} \approx -\frac{\alpha m^4 b^2}{32\pi^3} \left[ \frac{1}{3} \ln^2\left(\frac{b}{\gamma\pi}\right) + \left( 1.71 + \frac{8\zeta'(2)}{\pi^2} \right) \times \ln\left(\frac{b}{\gamma\pi}\right) + 0.98 \right] < 0, \quad (21)$$

with the complete two-loop term given by

$$\mathcal{P}_{\perp}^{(2)} \approx -\frac{\alpha m^4 b^2}{32\pi^3} \left[ \ln\left(\frac{b}{\gamma\pi}\right) + 3.4 \right] < 0. \quad (22)$$

For  $b \sim 10^5$ , corresponding to magnetic fields  $B \sim 10^{18}$  G, the transverse pressure generated by the first and second polarization mode are positive and reaches values of the order  $\sim 10^{30}$  dyn/cm<sup>2</sup> and  $\sim 10^{31}$  dyn/cm<sup>2</sup>, respectively. In contrast, the contribution given by the third mode is negative with  $\mathcal{P}_{\perp 3}^{(2)} \sim -10^{31}$  dyn/cm<sup>2</sup>. The combined result is negative and achieves values of the order  $\mathcal{P}_{\perp}^{(2)} \sim -10^{31}$  dyn/cm<sup>2</sup>. This fact strengthens the analogy of the

<sup>2</sup> The magnetic response of an observable photon was studied in [45] in two different regimes of the vacuum polarization tensor. On the one hand for low energies in weak fields ( $|\mathbf{B}| \ll B_c$ ) and on the other hand (originally studied in Ref. [44]) near the first pair creation threshold and for a moderate fields ( $|\mathbf{B}| \sim B_c$ ).

considered problem with the Casimir effect in which the pressure transversal to the parallel plates is also negative and dominated by the virtual electromagnetic radiation [43].

#### 4. The vacuum instability in a supercritical electric field

Ultra-high electric fields  $|\mathbf{E}| \gg E_c = m^2/e = 1.3 \times 10^{16}$  V/cm have been predicted to exist at the surface of strange stars [46–49]. In this electric field regime, the asymptotic behavior of  $\mathcal{L}_{\text{ir}}^{(2)}$  is obtained from Eqs. (8) by means of the duality transformation  $b \rightarrow -i\epsilon$  with  $\epsilon = |\mathbf{E}|/E_c$ . As a consequence we can write

$$\mathcal{L}_{\text{EH}} = \Re[\mathcal{L}_{\text{EH}}] + \text{Im}[\mathcal{L}_{\text{EH}}] \quad (23)$$

where  $\Re[\mathcal{L}_{\text{EH}}]$  contributes to the dispersive effects. Because of the imaginary part the vacuum becomes unstable and creation of particles could take place. The probability associated with the vacuum decay is  $\mathcal{P} = 1 - |\langle 0_{\text{out}} | 0_{\text{in}} \rangle|^2$  with  $\langle 0_{\text{out}} | 0_{\text{in}} \rangle = e^{i\text{VT}\mathcal{L}_{\text{EH}}}$ , where VT is the volume element in 3 + 1 dimensions. With this in mind and by considering Eq. (23) one has

$$\mathcal{P} = 1 - e^{-2\text{Im}[\mathcal{L}_{\text{EH}}]\text{VT}}. \quad (24)$$

As we have already mentioned in the introduction, the emission of observable photons from the vacuum is also a plausible effect. Indeed, the probability  $\mathfrak{P}$  of photon emission from the vacuum, accompanied by the creation of an arbitrary number of electron–positron pairs, can be determined by using the unitarity condition for the dispersion matrix  $\mathcal{S} = 1 + i\mathcal{T}$  which leads to a relation of the optical-theorem type

$$\sum_{\text{out}} |\langle \text{out} | \mathcal{T} | \text{in} \rangle|^2 = 2 \text{Im}(\langle \text{in} | \mathcal{T} | \text{in} \rangle), \quad (25)$$

where  $\mathcal{T}$  is the sum of all Feynman graphs. However, the electron causal Green function for the matrix elements  $\langle \text{in} | \dots | \text{in} \rangle$  is quite different from the standard propagator in the Schwinger proper-time representation of the out–in matrix elements. Let us consider the relation above with the in-state having no photons i.e.  $|\text{in}\rangle = |0_{\text{in}}\rangle$ . In addition, we will confine ourselves in the right- and left-hand sides of Eq. (25) to the second order radiational interaction. In this approximation, one can take the operator  $\mathcal{T}$  on the left-hand side to first order:  $\mathcal{T}^{(1)} = -\int d^4x j^\mu(x) a_\mu(x)$ . Here  $j^\mu = \frac{e}{2} [\bar{\psi}(x) \gamma^\mu \psi(x)]$  is the current and  $a_\mu(x)$  denotes the radiation field. Therefore, only one-photon states contribute to the sum over the out-states  $\langle \text{out} | = \langle 0_{\text{in}} | c_{k_i} b_{q_1} \dots b_{q_n} d_{p_1} \dots d_{p_n}$ . Whilst  $c_{k_i}$  is understood as the annihilation operator of a mode- $i$  photon,  $b$  and  $d$  are interpreted as the annihilation operators of electrons and positrons, respectively. On the right-hand side of Eq. (25)  $\mathcal{T}$  has to be taken in second order  $\mathcal{T}^{(2)} = -\frac{i}{2} \int d^4x d^4x' T[j^\mu(x) j^\nu(x') \times a_\mu(x) a_\nu(x')]$ , where  $T$  represents the time ordering operator. Considering the normal mode expansions of the dynamical fields and by using the Wick theorem one obtains

$$\mathfrak{P} = 2 \text{Im}(\langle \text{in} | \mathcal{T}^{(2)} | \text{in} \rangle) = 2 \text{Im} \mathcal{L}_{\text{in}}^{(2)} \quad (26)$$

where  $\mathcal{L}_{\text{in}}^{(2)}$  is given by Eq. (6) with the causal Feynman propagators replaced by the corresponding electron Green function appearing in the mean values  $\langle \text{in} | \dots | \text{in} \rangle$  of Eq. (25).

Now, the decay probability  $\mathcal{P}$  is connected to the total probability of photon emission from the vacuum, accompanied by the creation of an arbitrary number of electron–positron pairs  $\mathcal{P} = \mathfrak{P} + \dots$ <sup>3</sup> Obviously, the corresponding expansion up to second order in  $\alpha$  involves the imaginary part coming from the two-loop

<sup>3</sup> For details, see the cited references of Fradkin, Gitman, Shvartsman.



contribution of the Euler–Heisenberg Lagrangian. The latter can be written as the sum of the corresponding terms due to the individual vacuum polarization modes  $\text{Im}[\mathcal{L}_R^{(2)}] = \sum_{i=1}^3 \text{Im}[\mathcal{L}_{iR}^{(2)}]$  with

$$\begin{aligned} \text{Im}[\mathcal{L}_{1R}^{(2)}] &\approx 0, \\ \text{Im}[\mathcal{L}_{2R}^{(2)}] &\approx \frac{\alpha m^4 \epsilon^2}{32\pi^2} \left\{ \frac{1}{2} \mathcal{N}_2 - \frac{1}{3} \ln\left(\frac{\epsilon}{\gamma\pi}\right) \right\} < 0, \\ \text{Im}[\mathcal{L}_{3R}^{(2)}] &\approx \frac{\alpha m^4 \epsilon^2}{32\pi^2} \left\{ \frac{1}{2} \mathcal{N}_3 + \frac{1}{3} \ln\left(\frac{\epsilon}{\gamma\pi}\right) \right\} > 0. \end{aligned} \quad (27)$$

Note that, because of the dominance of the third propagation mode, the complete imaginary part arising from the two-loop term is positive

$$\text{Im}[\mathcal{L}_R^{(2)}] \approx \frac{3\alpha}{4\pi} \text{Im}[\mathcal{L}_R^{(1)}] > 0 \quad (28)$$

where  $\text{Im}[\mathcal{L}_R^{(1)}] \approx \frac{m^4 \epsilon^2}{48\pi}$  is the imaginary part coming from the one-loop contribution at very large electric field  $\epsilon \gg 1$ .

There is some interesting aspects in Eq. (27) which deserves some comments. First of all, it reveals that the first propagation mode does not contribute to the imaginary part of  $\mathcal{L}_{EH}$ . This suppression provides an evidence that the vacuum does not generate electromagnetic waves propagating along the external field  $\mathbf{E}$ . On the other hand, only the second and third mode contribute to the vacuum instability. This is a signal that the vacuum could create the corresponding propagation modes. However, the birefringence character of the vacuum imposes that there is not a single observable mode-2 photon without the existence of a corresponding mode-3 photon. Therefore, in addition to the electron–positron pairs, the vacuum could create photons with different propagation modes. Note, however, that the described photon emission has been predicted within the framework of equilibrium quantum field theory, even though it is a far-from-equilibrium, time-dependent phenomenon. Only further studies can tell us how far this process can be stretched because a realistic treatment of this issue requires a time evolution analysis of the photon number distribution functions, similar to that developed by Hebenstreit et al. for electron–positron pairs within a quantum kinetic approach [53].

## 5. Summary and outlook

In summary, we have examined the magnetization of the QED vacuum in the presence of a constant magnetic field in the strong field regime,  $|\mathbf{B}| \gg B_c$ . We have seen that the virtual electromagnetic radiation is a source of magnetization to the whole vacuum. In a superstrong magnetic field approximation, the two-loop contribution of the magnetization density corresponding to the second and third propagation mode depends nonlinearly on the external magnetic field and their behavior is diamagnetic and paramagnetic, respectively. On the other hand, the contribution coming from the first mode is diamagnetic and depends linear on  $B$ . We have seen that for very large magnetic field the contribution of the third mode strongly dominates the analyzed quantities. In this regime the latter tends to shrink inserted matter by exerting a negative transverse pressures to the external field. On the contrary those contributions coming from the first and second virtual mode are positive and tend to expand the matter.

In the last section of this Letter we showed that only two photon propagation modes contribute to the instability of the vacuum in an strong electric field. This instability is associated with the emission of photon whose propagation modes differ each other. A plausible connection between this mechanism and the birefringent character of the vacuum occupied by a supercritical electric

field was established and the suppression of vacuum decay into pair of modes propagating along an external electric field was analyzed as well.

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