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Majorization problem for certain class of p-valently analytic function defined by generalized fractional differintegral operator

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ABSTRACT

In this paper we investigate a majorization problem for a subclass of p-valently analytic function involving a generalized fractional differintegral operator. Some useful consequences of the main result are mentioned and relevance with some of the earlier results are also pointed out.

1. Introduction

Let functions f and g are analytic in the open unit disk

$$\mathbb{U} = \{z : z \in \mathbb{C},\ |z| < 1\}. \tag{1.1}$$

We say that f is majorized by g in $\mathbb{U}$ and write

$$f(z) \ll g(z) \quad (z \in \mathbb{U}), \tag{1.2}$$

if there exist a function $\varphi(z)$, analytic in $\mathbb{U}$ such that

$$|\varphi(z)| \le 1 \quad \text{and} \quad f(z) = \varphi(z)g(z) \quad (z \in \mathbb{U}). \tag{1.3}$$

Note that majorization is closely related to quasi-subordination [1].

Further, we say that the function f is subordinated to g, and write $f(z) \prec g(z)$, $z \in \mathbb{U}$, if there exists a function w analytic in $\mathbb{U}$, with

$$|w(z)| < 1 \quad \text{and} \quad w(0) = 0 \quad (z \in \mathbb{U}), \tag{1.4}$$

such that

$$f(z) = g(w(z)) \quad (z \in \mathbb{U}). \tag{1.5}$$

In particular, if $f(z)$ is univalent in $\mathbb{U}$, we have the following equivalence:

$$f(z) \prec g(z) \quad (z \in \mathbb{U}) \Longleftrightarrow f(0) = g(0) \quad \text{and} \quad f(\mathbb{U}) \subset g(\mathbb{U}).$$

We recall here the following generalized fractional integral and generalized fractional derivative operators due to Srivastava et al. [2] (see also [3]).

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Definition 1. For real numbers $\lambda > 0$, μ and η, Saigo hypergeometrc fractional integral operator $I_{0,z}^{\lambda,\mu,\eta}$ is defined by

$$I_{0,z}^{\lambda,\mu,\eta} f(z) = \frac{z^{-\lambda-\mu}}{\Gamma(\lambda)} \int_0^z (z-t)^{\lambda-1} {}_2F_1\left(\lambda+\mu, -\eta; \lambda; 1-\frac{t}{z}\right) f(t)\, dt, \tag{1.6}$$

where the function $f(z)$ is analytic in a simply-connected region of the complex z-plane containing the origin, with the order

$$f(z) = O(|z|^{\varepsilon}) \quad (z \to 0; \varepsilon > \max\{0, \mu-\eta\} - 1),$$

and the multiplicity of $(z-t)^{\lambda-1}$ is removed by requiring $\log(z-t)$ to be real when $(z-t) > 0$.

Definition 2. Under the hypotheses of Definition 1, Saigo hypergeometric fractional derivative operator $J_{0,z}^{\lambda,\mu,\eta}$ is defined by

$$J_{0,z}^{\lambda,\mu,\eta} f(z) = \begin{cases} \dfrac{1}{\Gamma(1-\lambda)} \dfrac{d}{dz} \left\{ z^{\lambda-\mu} \displaystyle\int_0^z (z-t)^{-\lambda} {}_2F_1\left(\mu-\lambda, 1-\eta; 1-\lambda; 1-\frac{t}{z}\right) f(t)\, dt \right\} & (0 \le \lambda < 1); \\ \dfrac{d^n}{dz^n} J_{0,z}^{\lambda-n,\mu,\eta} f(z) \quad (n \le \lambda < n+1; n \in \mathbb{N}), & \end{cases} \tag{1.7}$$

where the multiplicity of $(z-t)^{-\lambda}$ is removed as in Definition 1.

It may be remarked that

$$I_{0,z}^{\lambda,-\lambda,\eta} f(z) = D_z^{-\lambda} f(z) \quad (\lambda > 0)$$

and

$$J_{0,z}^{\lambda,\lambda,\eta} f(z) = D_z^{\lambda} f(z) \quad (0 \le \lambda < 1),$$

where $D_z^{-\lambda}$ denotes fractional integral operator and D_z^{λ} denotes fractional derivative operator considered by Owa [4].

Let $\mathcal{A}_p$ denote the class of functions of the form

$$f(z) = z^p + \sum_{n=1}^{\infty} a_{n+p} z^{n+p} \quad (p \in \mathbb{N} = \{1, 2, 3, \ldots\}), \tag{1.8}$$

which are analytic and p-valent in the open unit disk $\mathbb{U}$. Recently Goyal and Prajapat [5], introduced generalized fractional differintegral operator $\mathcal{S}_{0,z}^{\lambda,\mu,\eta} : \mathcal{A}_p \to \mathcal{A}_p$, by

$$\mathcal{S}_{0,z}^{\lambda,\mu,\eta} f(z) = \begin{cases} \dfrac{\Gamma(1+p-\mu)\Gamma(1+p+\eta-\lambda)}{\Gamma(1+p)\Gamma(1+p+\eta-\mu)} z^{\mu} J_{0,z}^{\lambda,\mu,\eta} f(z) & (0 \le \lambda < \eta+p+1, z \in \mathbb{U}); \\ \dfrac{\Gamma(1+p-\mu)\Gamma(1+p+\eta-\lambda)}{\Gamma(1+p)\Gamma(1+p+\eta-\mu)} z^{\mu} I_{0,z}^{-\lambda,\mu,\eta} f(z) & (-\infty < \lambda < 0, z \in \mathbb{U}). \end{cases} \tag{1.9}$$

It is easily seen from (1.9) that for a function f of the form (1.8), we have

$$\begin{aligned} \mathcal{S}_{0,z}^{\lambda,\mu,\eta} f(z) &= z^p + \sum_{n=1}^{\infty} \frac{(1+p)_n (1+p+\eta-\mu)_n}{(1+p-\mu)_n (1+p+\eta-\lambda)_n} a_{p+n} z^{p+n} \\ &= z^p {}_3F_2(1, 1+p, 1+p+\eta-\mu; 1+p-\mu, 1+p+\eta-\lambda; z) * f(z) \\ &\quad (z \in \mathbb{U};\ p \in \mathbb{N};\ \mu, \eta \in \mathbb{R};\ \mu < p+1; -\infty < \lambda < \eta+p+1) \end{aligned} \tag{1.10}$$

where $*$ denotes usual Hadamard product of analytic functions and ${}_pF_q$ is well known generalized hypergeometric function.

The operator $\mathcal{S}_{0,z}^{\lambda,\mu,\eta}$ satisfies the following three-term recurrence relation:

$$z(\mathcal{S}_{0,z}^{\lambda,\mu,\eta} f(z))^{(j+1)} = (p+\eta-\lambda)(\mathcal{S}_{0,z}^{\lambda+1,\mu,\eta} f(z))^{(j)} - (\eta+j-\lambda)(\mathcal{S}_{0,z}^{\lambda,\mu,\eta} f(z))^{(j)}. \tag{1.11}$$

Note that

$$\mathcal{S}_{0,z}^{0,0,0} f(z) = f(z), \qquad \mathcal{S}_{0,z}^{1,1,1} f(z) = \mathcal{S}_{0,z}^{1,0,0} f(z) = \frac{zf'(z)}{p}$$

and

$$\mathcal{S}_{0,z}^{2,1,1} f(z) = \frac{zf'(z) + z^2 f''(z)}{p^2}.$$

We also note that

$$\mathcal{S}_{0,z}^{\lambda,\lambda,\eta} f(z) = \mathcal{S}_{0,z}^{\lambda,\mu,0} f(z) = \Omega_z^{\lambda,p} f(z),$$

where $\Omega_z^{\lambda,p}$ is an extended fractional differintegral operator studied very recently by [6] (see also [7]). On the other hand, if we set

$$\lambda = -\alpha, \qquad \mu = 0 \quad \text{and} \quad \eta = \beta - 1,$$

in (1.10) and using

$$I_{0,z}^{\alpha,0,\beta-1} f(z) = \frac{1}{z^\beta \Gamma(\alpha)} \int_0^z t^{\beta-1} \left(1 - \frac{t}{z}\right)^{\alpha-1} f(t)dt,$$

we obtain following p-valent generalization of multiplier transformation operator [8,9]:

$$\mathcal{Q}_{\beta,p}^{\alpha} f(z) = \binom{p+\alpha+\beta-1}{p+\beta-1} \frac{\alpha}{z^\beta} \int_0^z t^{\beta-1} \left(1 - \frac{t}{z}\right)^{\alpha-1} f(t)dt$$
$$= z^p + \sum_{n=1}^{\infty} \frac{\Gamma(p+\beta+n)\Gamma(p+\alpha+\beta)}{\Gamma(p+\alpha+\beta+n)\Gamma(p+\beta)} a_{n+p} z^{n+p} \qquad (\beta > -p; \alpha+\beta > -p). \tag{1.12}$$

On the other hand, if we set

$$\lambda = -1, \qquad \mu = 0 \quad \text{and} \quad \eta = \beta - 1,$$

in (1.10), we obtain the *generalized Bernardi–Libera integral operator* $\mathcal{F}_{\beta,p} : \mathcal{A}_p \to \mathcal{A}_p (\beta > -p)$ defined by

$$\mathcal{S}_{0,z}^{-1,0,\beta-1} f(z) = \mathcal{F}_{\beta,p} f(z) = \frac{p+\beta}{z^\beta} \int_0^z t^{\beta-1} f(t)dt$$
$$= z^p + \sum_{n=1}^{\infty} \frac{p+\beta}{p+\beta+n} a_{p+n} z^{p+n} \quad (\beta > -p). \tag{1.13}$$

For the choice $p = 1$, where $\beta \in \mathbb{N}$, the operator defined by (1.13) reduces to the well-known *Bernardi integral operator* [10].

Using the generalized fractional differintegral operator $\mathcal{S}_{0,z}^{\lambda,\mu,\eta}$, we now introduce the following subclass of $\mathcal{A}_p$:

Definition 3. A function $f(z) \in \mathcal{A}_p$ is said to be in the class $\mathcal{M}_{\lambda,\mu,\eta}^{p,j}(A, B; \gamma)$, if and only if

$$\frac{z(\mathcal{S}_{0,z}^{\lambda,\mu,\eta} f(z))^{(j+1)}}{(\mathcal{S}_{0,z}^{\lambda,\mu,\eta} f(z))^{(j)}} - p + j \prec \frac{\gamma(A-B)z}{1+Bz}$$
$$(z \in \mathbb{U}; -1 \le B < A \le 1; j \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}; \gamma \in \mathbb{C} - \{0\}; \mu, \eta \in \mathbb{R}; \mu < p+1; -\infty < \lambda < \eta + p + 1). \tag{1.14}$$

For simplicity, we put

$$\mathcal{N}_{\lambda,\mu,\eta}^{p,j}(\gamma) = \mathcal{M}_{\lambda,\mu,\eta}^{p,j}(1, -1; \gamma),$$

where $\mathcal{N}_{\lambda,\mu,\eta}^{p,j}(\gamma)$ denote the class of functions $f \in \mathcal{A}_p$ satisfying the following inequality:

$$\Re \left\{ \frac{1}{\gamma} \left(\frac{z(\mathcal{S}_{0,z}^{\lambda,\mu,\eta} f(z))^{(j+1)}}{(\mathcal{S}_{0,z}^{\lambda,\mu,\eta} f(z))^{(j)}} - p + j \right) \right\} > -1. \tag{1.15}$$

Clearly, we have the following relationships:

(i) $\mathcal{N}_{\lambda,\mu,0}^{p,j}(\gamma) \equiv S_p^{\lambda;j}(\gamma)$;
(ii) $\mathcal{N}_{0,0,0}^{p,j}(\gamma) \equiv \mathcal{S}_{p,j}(\gamma)$;
(iii) $\mathcal{N}_{0,0,0}^{1,0}(\gamma) \equiv \mathcal{S}(\gamma)$;
(iv) $\mathcal{N}_{1,0,0}^{1,0}(\gamma) \equiv \mathcal{K}(\gamma)$;
(v) $\mathcal{N}_{0,0,0}^{1,0}(1-\alpha) \equiv \mathcal{S}^*(\alpha), 0 \le \alpha < 1$.

The class $S_p^{\lambda;j}(\gamma)$ was studied recently by Goyal and Goswami [11], and the class $\mathcal{S}_{p,j}(\gamma)$; was studied by Altintas and Srivastava [12]. Further, the class $\mathcal{S}(\gamma)$, is the class of starlike function of complex order γ in $\mathbb{U}$ and the class $\mathcal{K}(\gamma)$, is the class of convex functions of complex order γ in $\mathbb{U}$, which were considered by Nasr and Aouf [13] and Waitrowski [14], respectively. Moreover $\mathcal{S}^*(\alpha)$ denotes the class of starlike functions of order α in $\mathbb{U}$.

A majorization problem for the class $\mathcal{S}(\gamma)$, has been investigated by Altintas et al. [15] and a majorization problem for the class $\mathcal{S}_{p,j}(\gamma)$, has recently been investigated by Altintas et al. [12]. Very recently, Goswami and Wang [16], Goyal and Goswami [11] and Goyal et al. [17] generalized these results for different function classes. In this present paper we investigate a majorization problem for the class $\mathcal{M}_{\lambda,\mu,\eta}^{p,j}(A, B; \gamma)$, and give some special cases of our main result.

2. Majorization problem for the class $\mathcal{M}_{\lambda,\mu,\eta}^{p,j}(A, B; \gamma)$

We begin by proving

Theorem 1. *Let the function* $f(z) \in \mathcal{A}_p$ *and suppose that* $g(z) \in \mathcal{M}_{\lambda,\mu,\eta}^{p,j}(A, B; \gamma)$. *If*

$$(\mathcal{S}_{0,z}^{\lambda,\mu,\eta} f(z))^{(j)} \ll (\mathcal{S}_{0,z}^{\lambda,\mu,\eta} g(z))^{(j)} \quad (z \in \mathbb{U}),$$

then

$$\left|(\mathcal{S}_{0,z}^{\lambda+1,\mu,\eta} f(z))^{(j)}\right| \le \left|(\mathcal{S}_{0,z}^{\lambda+1,\mu,\eta} g(z))^{(j)}\right| \quad \text{for } |z| \le r_0, \tag{2.1}$$

where $r_0 = r_0(A, B, j, \gamma, \lambda, \mu, \eta)$ *is the smallest positive root of the equation*

$$\begin{aligned} &r^3|\gamma(A-B)+\xi B| - (\xi + 2|B|)r^2 - (|\gamma(A-B)+\xi B| + 2)r + \xi = 0 \\ &\quad (\xi = p + \eta - \lambda)(-1 \le B < A \le 1; j \in \mathbb{N}_0;\ \mu, \eta \in \mathbb{R}; \\ &\quad \mu < p+1;\ -\infty < \lambda < \eta + p + 1;\ p \in \mathbb{N}; \gamma \in \mathbb{C} - \{0\}). \end{aligned} \tag{2.2}$$

Proof. Since $g \in \mathcal{M}_{\lambda,\mu,\eta}^{p,j}(A, B; \gamma)$, we find from (1.13) that

$$\frac{z(\mathcal{S}_{0,z}^{\lambda,\mu,\eta} g(z))^{(j+1)}}{(\mathcal{S}_{0,z}^{\lambda,\mu,\eta} g(z))^{(j)}} - p + j = \frac{\gamma(A-B)w(z)}{1 + Bw(z)}, \tag{2.3}$$

where $w(z) = c_1 z + c_2 z^2 + \cdots$, $w \in \mathcal{P}$, $\mathcal{P}$ denotes the well known class of bounded analytic functions in $\mathbb{U}$ (see [18]), which satisfies the conditions

$$w(0) = 0 \quad \text{and} \quad |w(z)| \le |z| \quad (z \in \mathbb{U}).$$

On using (1.11) in (2.3), we get

$$|(\mathcal{S}_{0,z}^{\lambda,\mu,\eta} g(z))^{(j)}| \le \frac{\xi(1 + |B|\,|z|)}{\xi - |\gamma(A-B) + \xi B|\,|z|} |(\mathcal{S}_{0,z}^{\lambda+1,\mu,\eta} g(z))^{(j)}|, \tag{2.4}$$

where $\xi = p + \eta - \lambda$. Next, since $(\mathcal{S}_{0,z}^{\lambda,\mu,\eta} f(z))^{(j)}$ is majorized by $(\mathcal{S}_{0,z}^{\lambda,\mu,\eta} g(z))^{(j)}$ in the unit disk $\mathbb{U}$, from (1.3), we have

$$(\mathcal{S}_{0,z}^{\lambda,\mu,\eta} f(z))^{(j)} = \varphi(z)(\mathcal{S}_{0,z}^{\lambda,\mu,\eta} g(z))^{(j)}, \tag{2.5}$$

where $|\phi(z)| \le 1$. Differentiating (2.5) w. r. to z and multiplying by z, we get

$$z(\mathcal{S}_{0,z}^{\lambda,\mu,\eta} f(z))^{(j+1)} = z\varphi'(z)(\mathcal{S}_{0,z}^{\lambda,\mu,\eta} g(z))^{(j)} + \varphi(z) z (\mathcal{S}_{0,z}^{\lambda,\mu,\eta} g(z))^{(j+1)},$$

which on using (1.11) once again, gives

$$(\mathcal{S}_{0,z}^{\lambda+1,\mu,\eta} f(z))^{(j)} = \frac{z\varphi'(z)}{\xi}(\mathcal{S}_{0,z}^{\lambda,\mu,\eta} g(z))^{(j)} + \varphi(z)(\mathcal{S}_{0,z}^{\lambda+1,\mu,\eta} g(z))^{(j)}. \tag{2.6}$$

Noting that $\varphi \in \mathcal{P}$ satisfying the inequality (see, e. g., [19])

$$|\varphi'(z)| \le \frac{1 - |\varphi(z)|^2}{1 - |z|^2}, \quad (z \in \mathbb{U}), \tag{2.7}$$

and making use of (2.5) and (2.7) in (2.6), we have

$$|(\mathcal{S}_{0,z}^{\lambda+1,\mu,\eta} f(z))^{(j)}| \le \left(|\varphi(z)| + \frac{1 - |\varphi(z)|^2}{1 - |z|^2} \frac{(1 + |B|\,|z|)|z|}{\xi - |\gamma(A-B) + \xi B|\,|z|}\right) |(\mathcal{S}_{0,z}^{\lambda+1,\mu,\eta} g(z))^{(j)}|,$$

which upon setting

$$|z| = r \quad \text{and} \quad |\varphi(z)| = \rho \quad (0 \le \rho \le 1),$$

leads us to the inequality

$$|(\mathcal{S}_{0,z}^{\lambda+1,\mu,\eta} f(z))^{(j)}| \le \frac{\vartheta(\rho)}{(1 - r^2)(\xi - |\gamma(A-B) + \xi B| r)} |(\mathcal{S}_{0,z}^{\lambda+1,\mu,\eta} g(z))^{(j)}|,$$

where

$$\vartheta(\rho) = -r(1+|B|r)\rho^2 + (1-r^2)(\xi - |\gamma(A-B)+\xi B|r)\rho + r(1+|B|r) \tag{2.8}$$

takes its maximum value at $\rho = 1$, with $r_0 = r_0(A, B, j, \gamma, \lambda, \mu, \eta)$, where r_0 is the smallest positive root of Eq. (2.2). Furthermore, if $0 \le \sigma \le r_0$, then the function $\Psi(\rho)$ defined by

$$\Psi(\rho) = -\sigma(1+|B|\sigma)\rho^2 + (1-\sigma^2)(\xi - |\gamma(A-B)+\xi B|\sigma)\rho + \sigma(1+|B|\sigma) \tag{2.9}$$

is an increasing function on the interval $0 \le \rho \le 1$, so that

$$\Psi(\rho) \le \Psi(1) = (1-\sigma^2)(\xi - |\gamma(A-B)+\xi B|\sigma) \quad (0 \le \rho \le 1; 0 \le \sigma \le r_0). \tag{2.10}$$

Hence upon setting $\rho = 1$, in (2.9), we conclude that (2.1) of Theorem 1 holds true for

$$|z| \le r_0 = r_0(A, B, j, \gamma, \lambda, \mu, \eta),$$

where $r_0(A, B, j, \gamma, \lambda, \mu, \eta)$ is the smallest positive root of Eq. (2.2). This complete the proof of Theorem 1. □

Remark 1. Setting $\eta = 0$ in Theorem 1, we get a result due to Goswami and Wang [16, Theorem 1; pp. 100].

Putting $\lambda = -\alpha$, $\mu = 0$ and $\eta = \beta - 1$ in Theorem 1, we get

Corollary 1. *Let the function $f(z) \in \mathcal{A}_p$ and suppose that $g(z)$ satisfy*

$$\frac{z(\mathcal{Q}^{\alpha}_{\beta,p}g(z))^{(j+1)}}{(\mathcal{Q}^{\alpha}_{\beta,p}g(z))^{(j)}} \prec \frac{p-j+[\gamma A+(p-j-\gamma)B]z}{1+Bz}.$$

If

$$(\mathcal{Q}^{\alpha}_{\beta,p}f(z))^{(j)} \ll (\mathcal{Q}^{\alpha}_{\beta,p}g(z))^{(j)} \qquad (z \in \mathbb{U}),$$

then

$$\left|(\mathcal{Q}^{\alpha+1}_{\beta,p}f(z))^{(j)}\right| \le \left|(\mathcal{Q}^{\alpha+1}_{\beta,p}g(z))^{(j)}\right| \quad \text{for } |z| \le r_1,$$

where $r_1 = r_1(j, \gamma, \alpha, \beta, p)$ is the smallest positive root of the equation

$$r^3|\gamma(A-B)+\chi B| - (\chi + 2|B|)r^2 - (|\gamma(A-B)+\chi B|+2)r + \chi = 0.$$
$$(\chi = p+\beta+\alpha-1;\ \beta > -p;\ \alpha+\beta > -p;\ p, j \in \mathbb{N};\ \gamma \in \mathbb{C}-\{0\}).$$

Setting $A = 1$ and $B = -1$ in Theorem 1, we have

Corollary 2. *Let the function $f(z) \in \mathcal{A}_p$ and suppose that $g(z) \in \mathcal{N}^{p,j}_{\lambda,\mu,\eta}(\gamma)$. If*

$$(\mathcal{S}^{\lambda,\mu,\eta}_{0,z}f(z))^{(j)} \ll (\mathcal{S}^{\lambda,\mu,\eta}_{0,z}g(z))^{(j)} \quad (z \in \mathbb{U}),$$

then

$$\left|(\mathcal{S}^{\lambda+1,\mu,\eta}_{0,z}f(z))^{(j)}\right| \le \left|(\mathcal{S}^{\lambda+1,\mu,\eta}_{0,z}g(z))^{(j)}\right| \quad \text{for } |z| \le r_1,$$

where

$$r_2 = r_2(j, \gamma, \lambda, \mu, \eta) = \frac{\tau - \sqrt{\tau - 4\xi|2\gamma-\xi|}}{2|2\gamma-\xi|}$$
$$(\tau = 2+\xi+|2\gamma-\xi|;\ \xi = p+\eta-\lambda;\ \lambda < \eta+p+1;\ \mu < p+1; \mu, \eta \in \mathbb{R}; p, j \in \mathbb{N};\ \gamma \in \mathbb{C}-\{0\}).$$

Remark 2. (i) Setting $\eta = 0$ in Corollary 2, we get a result due to Goyal and Goswami [11, Theorem 2.1; pp. 1856].
(ii) Putting $j = \lambda = \mu = \eta = 0$ and $p = 1$ in Corollary 2, we get the result obtained by [15].
(iii) For $j = \lambda = \mu = \eta = 0$ and $\gamma = p = 1$ Corollary 2, gives the result obtained by MacGregor [18].

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